quant_case_study

October 20, 2022

```
[]: import os
     import numpy as np
     import pandas as pd
     from typing import Dict, List
     from datetime import datetime, timezone
     import matplotlib.pyplot as plt
     import seaborn as sns
     %matplotlib inline
     import warnings
     warnings.filterwarnings('ignore')
     path = "./mkt_data/"
     dir_list = os.listdir(path)
     all_data = []
     for rd in dir_list:
         # print('reading ', rd, '...')
         raw_data = pd.read_csv(path + '/' + rd)
         raw_data['ticker'] = str(rd)[:-4]
         all_data.append(raw_data)
     # clean data column name
     def cln_df(df: pd.DataFrame):
         cols: List[str] = df.columns
         for col in cols:
             if ' ' in col:
                 col_name = col.strip()
                 df[col_name] = df[col]
                 del df[col]
         return df
     all_data = [cln_df(data) for data in all_data]
```

/Users/jeffreychen/opt/anaconda3/lib/python3.8/sitepackages/pandas/core/computation/expressions.py:21: UserWarning: Pandas requires version '2.7.3' or newer of 'numexpr' (version '2.7.1' currently installed). from pandas.core.computation.check import NUMEXPR_INSTALLED

convert timestamp to datetime object, easier for resample and groupping for analysis

```
[]: def datetime_from_millis(millisec):
    return pd.Timestamp.fromtimestamp(millisec/1000) #.tz_localize('UTC').
    →tz_convert('US/Eastern').strftime('%H:%M:%S.%f')[:-3] #depends on pd version
    def convert_timestamp(df: pd.DataFrame):
        df['time'] = [datetime_from_millis(x) for x in df['timestamp']]
        return df
    all_data = [convert_timestamp(data) for data in all_data]
```

creating target columns for later usage

```
[]: def get_target_cols(df:pd.DataFrame):
    df['non_cum_vol'] = df["volume"].diff().fillna(df['volume'].iloc[0])
    df['vol %'] = df['non_cum_vol']/(df['volume'].iloc[-1])
    df['spread'] = df['ask_price'] - df['bid_price']
    df['mid_price'] = (df['ask_price'] + df['bid_price'])/2
    df['time'] = pd.to_datetime(df['time'])
    return df
all_data = [get_target_cols(data) for data in all_data]
```

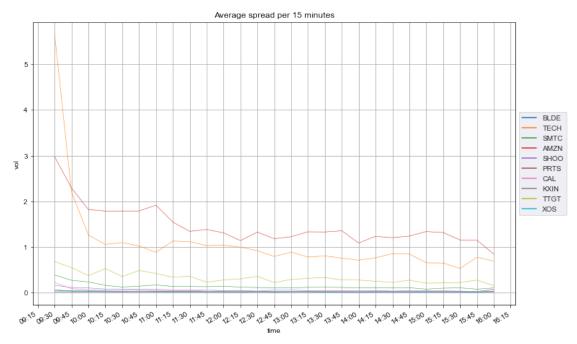
- 1. Create a visualization to show the following in 15-minute intervals for each stock in the included data
 - a. Average spread (Ask Bid),
 - b. Average total bin volume as a percent of the full day's volume
 - c. Volume-weighted average price (VWAP)

First, let create 15 minutes intervals data for plotting

a. Average spread (Ask - Bid) Plot

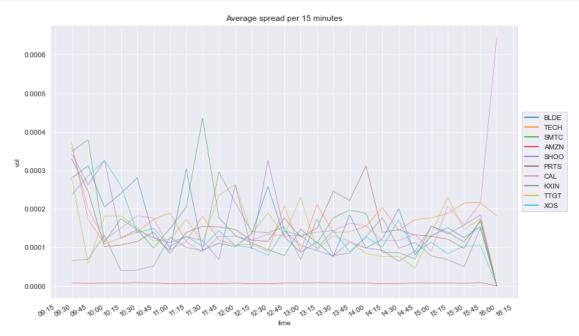
```
[]: import seaborn as sns
import matplotlib.pyplot as plt
import matplotlib.ticker as mticker
from matplotlib.pyplot import subplot, subplots
from matplotlib import dates as mdates
from pandas.plotting import register_matplotlib_converters
register_matplotlib_converters()
```

```
def plot_15min_spread(data_15min: List[pd.DataFrame]):
   meta_data = pd.concat(data_15min).reset_index()
    # meta_data['time'] = str(meta_data['time'].dt.time)
   fig,ax = subplots(figsize=(12, 8))
   sns.set_style("darkgrid")
   sns.lineplot(y = 'spread',
                x = 'time',
                hue='ticker',
                markers=True,
                lw=0.5.
                data=meta_data)
   ax.set(title='Average spread per 15 minutes', ylabel="vol")
   xformatter = mdates.DateFormatter('%H:%M')
   xlocator = mdates.MinuteLocator(byminute=[0,15, 30, 45], interval = 1)
→##Set ticks to show 00 and 30 minutes only
    ## Set xtick labels to appear every 30 minutes
   ax.xaxis.set_major_locator(xlocator)
   ## Format xtick labels as HH:MM
   ax.xaxis.set_major_formatter(xformatter)
   ax.legend(loc='center left', bbox_to_anchor=(1, 0.5))
   fig.autofmt_xdate()
   ax.grid(True)
plot_15min_spread(all_15min_data)
```



b. Average total bin volume as a percent of the full day's volume

```
[]: def plot_15min_volume(data_15min: List[pd.DataFrame]):
         meta_data = pd.concat(data_15min)
         fig,ax = subplots(figsize=(12, 8))
         sns.set_style("darkgrid")
         sns.lineplot(y = 'vol %',
                     x = 'time',
                     hue='ticker',
                     markers=True,
                     lw=.5,
                     data=meta_data.reset_index())
         ax.set(title='Average spread per 15 minutes', ylabel="vol")
         xformatter = mdates.DateFormatter('%H:%M')
         xlocator = mdates.MinuteLocator(byminute=[0,15, 30, 45], interval = 1)__
      →##Set ticks to show 00 and 30 minutes only
         ## Set xtick labels to appear every 30 minutes
         ax.xaxis.set_major_locator(xlocator)
         ## Format xtick labels as HH:MM
         ax.xaxis.set_major_formatter(xformatter)
         ax.legend(loc='center left', bbox_to_anchor=(1, 0.5))
         fig.autofmt_xdate()
         ax.grid(True)
     plot_15min_volume(all_15min_data)
```



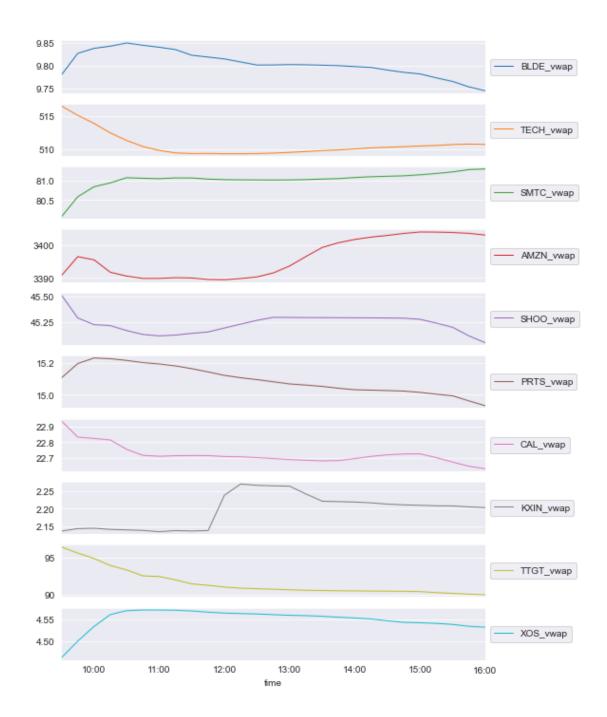
c. Volume-weighted average price (VWAP)

due to vwap value for each stock vary drastically, thus here presented individual graph instead of aggregate one

```
[]: def plot_vwap(pdf):
    dats = pd.DataFrame()
    for data in pdf:
        if 'time' not in dats.columns:
            dats['time'] = data.index
        ticker = data['ticker'].iloc[0]
        dats[ticker + '_vwap'] = data['vwap'].values
    dats = dats.set_index('time')

# print(dats.head())
    cols_plot = list(dats.columns)
    axes = dats[cols_plot].plot(linewidth=1, figsize=(8, 12), subplots=True)
    for ax in axes:
        ax.set_ylabel('')
        ax.legend(loc='center left', bbox_to_anchor=(1, .5))

plot_vwap(all_15min_data)
```



Question 2 - Cross Sectional Regression

First step - Create Response Variable

Creating the required columns for future usage

```
[]: from copy import deepcopy all_data_2 = deepcopy(all_data)
```

Next, calculate the normalized mid price return from the time of trade to the midpoint price at 10 seconds after the time of a trade. Since using built-in resample on 10s will result in aggregating the data, here I build an algo to retreive the proper t+10 midprice without losing any data.

```
[]: from collections import deque
     from typing import Deque
     # runtime O(n)
     def get_t_p_10_midprc(pdf: pd.DataFrame):
         time, midprcs = pdf['time'], pdf['mid_price']
         qt: Deque = deque([]) # we use queue we keep tracks the next t+10 time,
      → initialized with deque here
         time_10 = [0]*len(time)
         price_10 = [0]*len(midprcs)
         prevt = time[0]
         prevp = midprcs[0]
         for idx, currt in enumerate(time):
                 qt.append(idx) # using que to keep tracks on the idx of time that
      \rightarrowsmaller than t+10
                 # whenever theres a time larger than the que's (t+10), we find the
      \rightarrow 'next' t that larger than t+10
                 while qt and currt > (time[qt[0]] + pd.Timedelta('10s')):
                     time_10[qt[0]] = prevt # thus, when we found time > t+10, we_
      →store the previous time
                     price_10[qt[0]] = prevp # store previous mid price
                     qt.popleft()
                 prevt = currt
                 prevp = midprcs[idx]
             except Exception as e:
                 print('err', e, end=',')
```

```
# we might have left queue due to time already larger than the last time well
 \hookrightarrow have
    while qt:
        time_10[qt[0]] = prevt
        price_10[qt[0]] = prevp
        qt.popleft()
    return time_10, price_10
# after getting the mid price t+10 series, we could calculate the rest and
⇔create our response data
def create_respond_data(pdf:pd.DataFrame):
    timep10, prcp10 = get t p 10 midprc(pdf)
    respond = pdf.copy()
    # respond['timep10'] = timep10 ; unmark to check the t+10s time
    respond['prcp10'] = prcp10
    respond['mid_prc_ret'] = (respond['mid_price']/respond['prcp10']) - 1
    respond['returns'] = respond['mid_prc_ret']/respond['denom']
    respond['time'] = respond['time']
    # pdf.drop(columns=['timep10', 'prcp10', 'mid_prc_ret', 'denom'], inplace=True)
    respond = respond[['time', 'returns']]
    return respond
respond_data = [create_respond_data(df) for df in all_data_2]
```

Second step - Create Predictors

Creating the following predictors for modelling

- Bid Offer Imbalance, calculated as (Nbb_agg_size Nbo_agg_size)/(Nbb_agg_size + Nbo_agg_size)
- Trade sign: 1 if the last trade price is above the average of bid and ask price -1 if the last trade price is below the average of bid and ask price
- Trade Size imbalance over the last 10 seconds: Trade size imbalance for any period can be calculated as: Sum (Trade Sign x Trade Size)

```
predictor['trade_sign'] = np.where(pdf['trade_price'] > pdf['mid_price'],__
 \hookrightarrow 1, 0)
    predictor['trade_sign2'] = np.where(pdf['trade_price'] < pdf['mid_price'],__</pre>
    predictor['trade_sign'] = predictor['trade_sign'] + predictor['trade_sign2']
    ## Trade Size imbalance over last 10 seconds
    predictor['trade_sz_imbal'] = predictor['trade_sign']*pdf['trade_size']
    predictor['trade_sz_imbal_10s'] = predictor.

→set_index('time')['trade_sz_imbal'].rolling('10s').sum().values
    ## get ticker and timestamp for grouping
    predictor['ticker'] = pdf['ticker']
    predictor.drop(columns=['trade_sign2', 'trade_sz_imbal'], axis=1,__
 →inplace=True)
    if not label:
        return predictor[['time', 'bid_offer_imbal', 'trade_sign',_
 return predictor
predictors = [create_predictors(data) for data in all_data_2]
aggregate responses and predictors for modelling
```

```
[]: agg_predictors = pd.concat(predictors).set_index('time')
    # agg_predictors.index = agg_predictors.index.strftime('%H:%M:%S:%f')
    agg_response = pd.concat(respond_data).set_index('time')
    # agg_response.index = agg_response.index.strftime('%H:%M:%S:%f')

## check dimension
    print(agg_predictors.shape)
    print(agg_response.shape)

(195107, 3)
    (195107, 1)

[]: agg_predictors.index.round('10s')
```

bid_offer_imbal trade_sign trade_sz_imbal_10s time 2021-10-27 09:30:00.380 0.833333 -1 -10.0 2021-10-27 09:30:00.386 0.428571 1 200.0 2021-10-27 09:30:00.386 617.0 0.428571 1 2021-10-27 09:30:00.386 0.428571 1 1175.0 2021-10-27 09:30:00.386 0.428571 1200.0

```
bid_offer_imbal trade_sign trade_sz_imbal_10s
time
2021-10-27 09:30:10.374
                                      0.2
                                                    1
                                                                   -652.0
2021-10-27 09:30:10.375
                                      0.2
                                                    1
                                                                   -649.0
2021-10-27 09:30:10.627
                                      0.2
                                                    0
                                                                   -649.0
2021-10-27 09:30:10.669
                                      0.2
                                                   -1
                                                                   -650.0
2021-10-27 09:30:10.751
                                      0.2
                                                    1
                                                                   -649.0
```

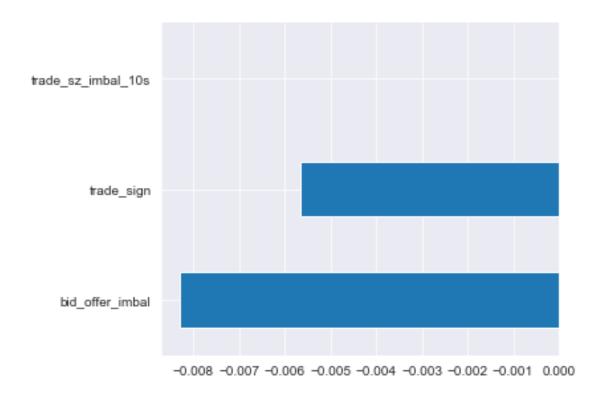
Here we use linear regression model from sklearn as base class. It gives a flexibility to build more statistic objects on it

```
[]: from sklearn import linear model
     from scipy.stats import t as scpt
     import numpy as np
     from sklearn.linear_model import LinearRegression
     class LinearRegression(linear_model.LinearRegression):
         LinearRegression class after sklearn's, but calculate t-statistics
         and p-values for model coefficients (betas).
         Additional attributes available after .fit()
         are 't' and 'p' which are of the shape (y.shape[1], X.shape[1])
         which is (n_features, n_coefs)
         This class sets the intercept to 0 by default, since usually we include it
         in X.
         11 11 11
         def __init__(self, *args, **kwargs):
             if not "fit_intercept" in kwargs:
                 kwargs['fit intercept'] = False
             super(LinearRegression, self)\
                     .__init__(*args, **kwargs)
         def fit(self, X, y, n_jobs=1):
             self = super(LinearRegression, self).fit(X, y, n_jobs)
             mse = np.sum((self.predict(X) - y) ** 2, axis=0) / float(X.shape[0] - X.
      \rightarrowshape[1])
             self.se = np.array([
                 np.sqrt(np.diagonal(mse[i] * np.linalg.inv(np.dot(X.T, X))))
                                                          for i in range(mse.shape[0])
                         ])
             self.t = self.coef_ / self.se
             self.p = 2 * (1 - scpt.cdf(np.abs(self.t), y.shape[0] - X.shape[1]))
             self.residual = self.predict(X) - y
             return self
```

```
[]: # initialize date range for looping
     # start_time_range = pd.date_range("2021-10-27 09:30:00", "2021-10-27 15:59:
     \rightarrow 50", freq="10s").time
     # end_time_range = pd.date_range("2021-10-27 09:30:10", "2021-10-27 16:00:00",
     \rightarrow freq="10s"). time
     predictor_gp = agg_predictors.groupby(pd.Grouper(freq='10s'))
     response_gp = agg_response.groupby(pd.Grouper(freq='10s'))
     betas = []
     stats = \Pi
     residuals = []
     for (n1, X), (n2, y) in zip(predictor_gp, response_gp): # since just linear_
      →model, we loop thru all time interval and run regression
         # if ind/100 == ind//100:
              print("running", ind, end=',') # keep tracks the batch runner
         \# X = aqq \ predictors.loc[start:end] \ \#  use the date range to select time.
      →periods, pd Datetimeindex has good performance
         \# y = agg\_response.loc[start:end]
         ols = LinearRegression()
         reg = ols.fit(X, y)
         betas.append(reg.coef_)
         stats.append((
             reg.t,
             reg.p,
             reg.se,
             ))
         residuals.append(reg.residual)
```

Lets check the coefficient and the statistic we applied

check coefficients



statistic summary

```
[]: print("Summary : ")
     beta_summary = pd.DataFrame({
         'Mean Return':coef_sum.mean().values,
         'Std Dev': coef_sum.std().values,
         'Sharp Ratio':coef_sum.mean()/coef_sum.std()},
     print(beta_summary)
    Summary :
                        Mean Return
                                      Std Dev Sharp Ratio
    bid_offer_imbal
                          -0.008304 0.067198
                                                 -0.123570
    trade_sign
                          -0.005672 0.037572
                                                 -0.150967
    trade_sz_imbal_10s
                          -0.000004 0.000053
                                                 -0.079769
[]: tstats1 = np.mean([t[0].tolist()[0] for t in stats], axis=0)
     pstats = np.mean([t[1].tolist()[0] for t in stats], axis=0)
     stderr = np.mean([t[2].tolist()[0] for t in stats], axis=0)
     print('statistic summary: ')
     stats_sum = pd.DataFrame([tstats1, pstats, stderr],
                            index=['t statistic', 'p value', 'standard error'],
```

```
→columns=['bid_offer_imbal','trade_sign','trade_sz_imbal_10s'])

print(stats_sum)
```

statistic summary:

```
bid_offer_imbal trade_sign trade_sz_imbal_10s
t statistic -1.357984 -1.467850 -0.383953
p value 0.220367 0.258368 0.290972
standard error 0.006566 0.003004 0.000006
```

We can see that all coefficient are negative, and trade_sz_imbal_10s has almost zero explained power over the model. the t-stats are pretty small and p value are quite large, indicate our dependend variable is not significant enough?

```
[]: from linearmodels.panel import PanelOLS, FamaMacBeth from patsy import dmatrices
```

```
[]:
                                         bid_offer_imbal trade_sign \
                                 returns
    ticker time
           2021-10-27 09:30:00 0.011046
                                                  0.833333
    BLDE
                                                                    -1
            2021-10-27 09:30:00 0.009239
                                                  0.047619
                                                                    -1
            2021-10-27 09:30:00 0.009239
                                                  0.047619
                                                                    -1
            2021-10-27 09:30:00 0.009239
                                                  0.047619
                                                                     0
            2021-10-27 09:30:00 0.009239
                                                                     0
                                                  0.047619
    XOS
           2021-10-27 16:00:00 0.000000
                                                 -0.458333
                                                                     0
           2021-10-27 16:00:00 0.000000
                                                 -0.521739
                                                                     0
           2021-10-27 16:00:00 0.000000
                                                 -0.227273
                                                                     1
           2021-10-27 16:00:00 0.000000
                                                 -0.209302
                                                                     1
           2021-10-27 16:00:00 0.000000
                                                 -0.444444
                                                                     1
```

trade_sz_imbal_10s

ticker time
BLDE 2021-10-27 09:30:00 -10.0
2021-10-27 09:30:00 -727.0
2021-10-27 09:30:00 -809.0
2021-10-27 09:30:00 -809.0

```
2021-10-27 09:30:00
                                         -809.0
    XOS
           2021-10-27 16:00:00
                                          546.0
           2021-10-27 16:00:00
                                          546.0
           2021-10-27 16:00:00
                                          646.0
           2021-10-27 16:00:00
                                          651.0
           2021-10-27 16:00:00
                                          926.0
    [195107 rows x 4 columns]
[]: y, X = dmatrices('returns~bid_offer_imbal+trade_sign+trade_sz_imbal_10s',__
     →agg_predictors2,return_type='dataframe')
    res1 = FamaMacBeth(y,X).fit()
    res1
[]:
                               FamaMacBeth Estimation Summary
    ______
    ======
    Dep. Variable:
                                                R-squared:
                                       returns
    0.0059
    Estimator:
                                   FamaMacBeth
                                                R-squared (Between):
    0.6070
    No. Observations:
                                        195107
                                                R-squared (Within):
    0.0027
                              Thu, Oct 20 2022
                                                R-squared (Overall):
    Date:
    0.0059
    Time:
                                      00:51:21
                                                Log-likelihood
    6.963e+04
    Cov. Estimator:
                      Fama-MacBeth Standard Cov
                                                F-statistic:
    384.29
                                                P-value
    Entities:
                                            10
    0.0000
    Avg Obs:
                                     1.951e+04
                                                Distribution:
    F(3,195103)
    Min Obs:
                                        5982.0
    Max Obs:
                                                F-statistic (robust):
                                     1.401e+05
    47.721
                                                P-value
    0.0000
                                                Distribution:
    Time periods:
                                          2341
    F(3,195103)
```

83.343

12.000

757.00

Avg Obs:

Min Obs:

Max Obs:

Parameter Estimates

=======================================		========	========		==========
 Upper CI	Parameter	Std. Err.	T-stat	P-value	Lower CI
Intercept	0.0010	0.0015	0.6940	0.4877	-0.0018
0.0039					
<pre>bid_offer_imbal -0.0066</pre>	-0.0086	0.0011	-8.1915	0.0000	-0.0107
trade_sign	-0.0049	0.0006	-8.1612	0.0000	-0.0061
-0.0037	4 691 - 06	1 605- 06	0.0001	0 0040	7 967- 06
trade_sz_imbal_10s -1.496e-06	-4.681e-06	1.625e-06	-2.8801	0.0040	-7.867e-06
=======================================	-=======	========	========		=========

====== FamaMacBethResults, id: 0x7fd3540db6d0

from our model, we observed that the trade_sign and bid_offer_imbalance both shows significant result. However, both factors indicate negative returns in terms of profitability.

Model Improvement Analysis

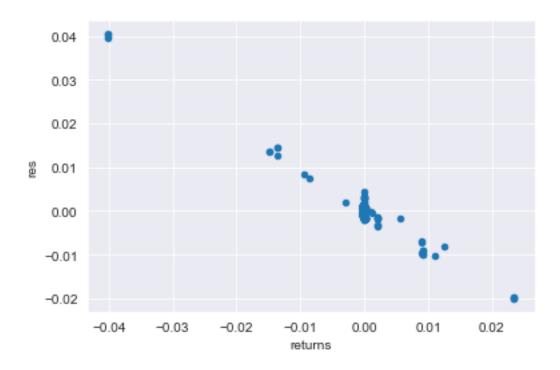
we can investigate on the residual of our model

```
[]: res = []
for ind, (n, g) in enumerate(response_gp):
    r = residuals[ind].copy()
    r = r.rename(columns={'returns':'res'}).reset_index()
    r['returns'] = g['returns'].values
    res.append(r)
len(res)
```

[]: 2341

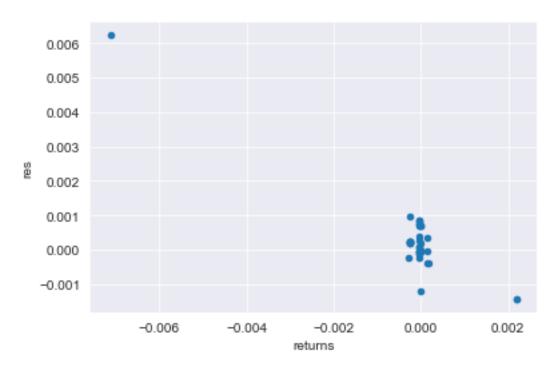
```
[]: res[0].plot.scatter(x='returns', y='res')
```

[]: <AxesSubplot:xlabel='returns', ylabel='res'>



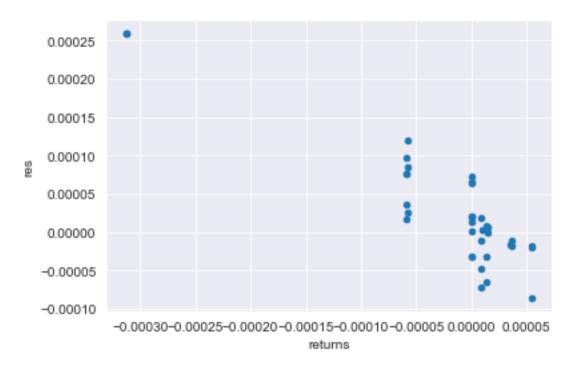
[]: res[1000].reset_index().plot.scatter(x='returns',y='res')

[]: <AxesSubplot:xlabel='returns', ylabel='res'>



```
[]: res[800].reset_index().plot.scatter(x='returns',y='res')
```

[]: <AxesSubplot:xlabel='returns', ylabel='res'>



we randomly pick few point of time to check the residual vs fitted value. We could observed that few condition that are not idea for our model. Theres outlier inline with our data, and also from figure suggests that the error variances are not equal from time to time. To improve our model, we can truncate our model to make our depend variable more standardize and could avoid outlier in our model to avoid overfitting. We can also use do more robust testing on Heteroscedasticity by using tools like HAC estimator and white std error to further update our predictors, either tranform the feature or drop it.

4. Calculate the 15-second volatility (standard deviation of mid price returns) for SHOO in each 30 minute interval. Present the volatility in annualized terms.

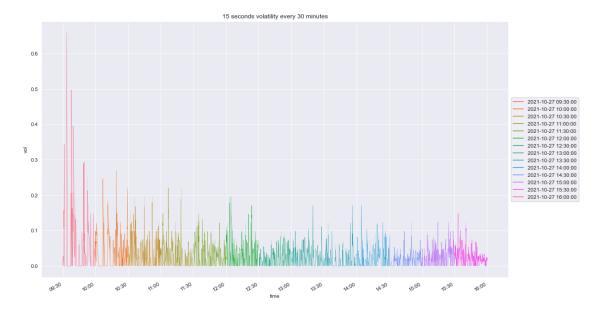
```
[]: shoo_data = pd.read_csv(path+'SHOO.csv')
    shoo_data = cln_df(shoo_data)
    shoo_data = convert_timestamp(shoo_data)
    shoo_data = get_target_cols(shoo_data)
    shoo_data = create_model_variables(shoo_data)
```

```
[]: import math
## calculating mid price return
shoo_data['mid_prc_ret'] = shoo_data['mid_price'].pct_change(1)
```

```
shoo_data = shoo_data[['time','mid_prc_ret']]
     ## calculate the rolling 15 seconds volatility for each 30 minutes interval
     gp = shoo data.set_index('time').groupby(pd.Grouper(freq='30Min'))
     vol15 = []
     intervals = []
     for t, g in gp:
        # print(g)
        series = g.rolling('15s')['mid_prc_ret'].std().fillna(0)
         # print(series)
        interval = len(series)*[t]
        vol15.extend(series)
         intervals.extend(interval)
     # print(vol15)
     shoo_data['15 sec volatility'] = np.array(vol15)*math.sqrt(390*252)
     shoo_data['intervals'] = intervals
     shoo_data.head()
[]:
                          time mid_prc_ret 15 sec volatility
                                                                         intervals
                                                      0.000000 2021-10-27 09:30:00
     0 2021-10-27 09:30:04.396
                                        {\tt NaN}
     1 2021-10-27 09:30:04.607
                                                      0.000000 2021-10-27 09:30:00
                                   0.000000
     2 2021-10-27 09:30:05.124
                                                      0.000000 2021-10-27 09:30:00
                                  0.000000
     3 2021-10-27 09:30:15.390 -0.000109
                                                      0.019760 2021-10-27 09:30:00
     4 2021-10-27 09:30:15.411
                                                      0.027946 2021-10-27 09:30:00
                                 0.000109
[]: from matplotlib import dates as mdates
     from matplotlib.pyplot import subplot, subplots
     # sns.set theme()
     fig,ax = subplots(figsize=(16, 10))
     sns.lineplot(y = '15 sec volatility',
     x = 'time',
     hue='intervals',
     data=shoo_data,
     lw=.5
     )
     ax.set(title='15 seconds volatility every 30 minutes', ylabel="vol")
     xformatter = mdates.DateFormatter('%H:%M')
     xlocator = mdates.MinuteLocator(byminute=[0,30], interval = 1) ##Set ticks to_
     →show 00 and 30 minutes only
     ## Set xtick labels to appear every 30 minutes
     ax.xaxis.set_major_locator(xlocator)
     ## Format xtick labels as HH:MM
     ax.xaxis.set_major_formatter(xformatter)
```

```
ax.legend(loc='center left', bbox_to_anchor=(1, 0.5))
fig.autofmt_xdate()
ax.grid
```

[]: <bound method _AxesBase.grid of <AxesSubplot:title={'center':'15 seconds volatility every 30 minutes'}, xlabel='time', ylabel='vol'>>



We can observe that during the Market On Openm mid price return volatility has the highest variance, and the variance starting to decrease along with the trading hour goes. We can take a step further for volatility analysis. In the variance calculation, which is equal weighting squared deviation, we can further weight based off of volume, or off of sqrt/log voume so could aviod overweight observations that have a lot more volume.