

# ISyE 6669 Project - Team 7 - Fall 2020

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## Introduction

*You are in charge of developing an optimization solution for a large online retailer. The retailer sells  $K$  types of products. These products are stored in  $N$  warehouses spread around the country. Each warehouse has limited stock of products available. These are given in `Warehouses.csv`. We want to satisfy  $M$  customer orders. Each order may consist of varying quantities of different products. For now, you can assume that products are infinitely divisible.*

*For instance, all of the following are valid orders:*

- *Order A requires 3 units of Product 1.*
- *Order B requires 5 units of Product 2 and 5 units of Product 3*
- *Order C requires 1.5 unit of Product 1*

*Additionally, an order can be fulfilled from multiple warehouses. For example, if an order requires 2 units of Product 1 and 3 units of Product 2, you could send:*

- *1 unit of Product 1 from Warehouse 1.*
- *1 unit of Product 1 and 1.5 units of Product 2 from Warehouse 2*
- *1.5 units of Product 2 from Warehouse 3*

*The orders are given in `Orders.csv`.*

*Sending products from the warehouse to the customer has a cost. The cost is proportional to both the distance between the warehouse and the customer and the total weight of the items sent. The costs of sending one pound from a warehouse to satisfy a order are given in `DeliveryCost.csv`. The per-unit weight of the products is given in `ProductWeight.csv`.*

*Your goal is to assign customer orders to warehouses so that you satisfy all the orders while minimizing fulfillment costs.*

## Part A (50 pts)

### Question 1 (20 pts)

Formulate a Linear Programming model to solve this problem. Your submission must be typed. Clearly define all variables, parameters, and constraints.

To begin we list products, warehouses, and orders according to the following notation:

$$k = 1, \dots, K \text{ Products,} \quad (1)$$

$$j = 1, \dots, N \text{ Warehouses, and} \quad (2)$$

$$i = 1, \dots, M \text{ Orders} \quad (3)$$

We then define,

$$d_{ik} \in \mathbb{R}, \text{ the demand for product } k \text{ in order } i, \quad (4)$$

$$s_{jk} \in \mathbb{R}, \text{ stock of product } k \text{ in warehouse } j, \quad (5)$$

$$c_{ij} \in \mathbb{R}, \text{ cost of sending 1 lbs. of product from warehouse } j \text{ to customer } i, \quad (6)$$

$$w_k \in \mathbb{R}, (k = 1, \dots, K) \text{ denotes the weight of product } k \text{ in lbs.} \quad (7)$$

Our decision variables are formulated as,

$$x_{ijk} \in \mathbb{R}, \text{ number of units shipped of product } k \text{ from warehouse } j \text{ to customer } i, \text{ and} \quad (8)$$

$$\delta_{ik}^* = \text{the number of units of product } k \text{ not fulfilled in order } i \quad (9)$$

and our objective is then to minimize the cost of shipping units to customers:

$$\min z = \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K x_{ijk} c_{ij} w_k + M \sum_{i=1}^M \sum_{k=1}^K \delta_{ik}^* \quad (10)$$

subject to,

$$\sum_{j=1}^N x_{ijk} + \delta_{ik}^* = d_{ik}, \forall i \in M, k \in K, \quad (11)$$

$$\sum_{i=1}^M x_{ijk} \leq s_{jk}, \forall j \in N, k \in K \quad (12)$$

$$x_{ijk} \geq 0 \forall i \in M, j \in N, k \in K \quad (13)$$

$$\delta_{ik}^* \geq 0 \forall i \in M, k \in K \quad (14)$$

$$M > 0, \text{ arbitrary large positive number (i.e. 'Big M')}. \quad (15)$$

where (11) ensures that all demand must be satisfied or accounted for, (12) ensures that stock levels are not exceeded, and (13) and (14) ensure that non-negativity is enforced.

## Question 2 (20 pts)

*Implement your model in Xpress or Gurobi/Python. Your program must read the data from the given files. Hard-coded data will be (severely) penalized. In your submission, this script should be named ModelA.mos or ModelA.py.*

*Clearly explain your data structures and how different parts of your code correspond to different parts of your formulation. Specifically, explain which parts of the code generate the variables, objective function, and constraints.*

*Your program's output should clearly show the optimal solution in an easy-to-read way.*

## Question 3 (5 pts)

*Solve your model. What is the objective function value of your solution? What does it mean in words?*

*What is the optimal solution? Make sure to specify which orders are satisfied from which warehouse and what quantities of different items have been sent. In your write up, summarize your solution in a human readable format, e.g. a table.*

## Question 4 (5 pts)

*Notice that all orders in the data require whole quantities of products. In practice, the products are not infinitely divisible. Suppose we were to impose this restriction (you do not have to implement it or change the formulation yet). Would the optimal solution change? If your answer is yes, would the optimal objective value be higher, lower, or stay the same? Why? If your answer is no, why not?*

## Part B (20 pts)

### Question 5 (5 pts)

*Now assume that products are not infinitely divisible and can only be sold in whole quantities. Furthermore, sending a package from a warehouse to a customer incurs a fixed cost in addition to the weight-based cost. Fixed costs are given in FixedCosts.csv. How would you change your formulation to reflect the new assumptions? You can either write out the new formulation or specify what you had to add and which parts of the old formulation had to be changed and how. Is your new formulation an LP or IP?*

### Question 6 (10 pts)

*Implement the changes to your model in Question 5 in Xpress or Gurobi/Python. In your submission, this script should be named ModelB.mos or ModelB.py. As in Question 2, make sure you explain any new lines of code you have added.*

### Question 7 (5 pts)

*Solve your model. What is the objective value of your solution? What does it mean in words? How does it compare to the solution of Question 3?*

*What is the optimal solution? Which orders are satisfied from which warehouse? What quantities of different items have been sent? In your write up, summarize your solution in a human readable format, e.g. a table.*

## Part C (30 pts)

### Question 8 (15 pts)

*Often businesses have to take into account considerations beyond just the cost. In our case, due to trade regulations, our warehouses and customer orders have been assigned to one of four different regions. These assignments are given in WarehouseRegions.csv and OrderRegions.csv. Orders from one region should be fulfilled by warehouses from the same region until the supplies are depleted. After that, they can be fulfilled from any region.*

*For instance, suppose Region 1 has only one warehouse, say Warehouse 1. This warehouse holds 5 units of Product 1. Also suppose there is only one order coming from Region 1, say Order 1. Order 1 requires 7 units of Product 1. With the new regional constraints, we have to send 5 units of Product 1 from Warehouse 1 to Order 1. We can satisfy the remaining demand of 2 units from any warehouse. Notice that this constraint takes precedence over any cost considerations, i.e. we have to send 5 units from Warehouse 1 even if it is cheaper to satisfy the order from another warehouse.*

*Write the model for the updated problem. Explain any additional parameters, variables, and constraints you had to introduce. Your new formulation should include the changes you have made in Question 5. You don't have to explain these again.*

### Question 9 (10 pts)

*Implement your new model in Xpress or Gurobi/Python. In your submission, this script should be named ModelC.mos or ModelC.py. As in Question 2, make sure you explain any new lines of code you have added.*

### Question 10 (5 pts)

*Solve your model. What is the objective value of your solution? What does it mean in words? How does it compare to the solution of Question 8? What is the optimal solution? Which orders are satisfied from which warehouse? What quantities of different items have been sent? In your write up, summarize your solution in a human readable format, e.g. a table.*