Exam 1 for ISYE 6413 with Professor Wu at GT

No description

ISyE6413
First Midterm – Sample Exam
(Total : 100 points)

Name :

Problem	1	2	3	4	5	Total
Max Points	4	6	6	12	12	40
Your score						

Problem 1 (4 pts) If the plot of residuals against the fitted values exhibits a reverse funnel shape, i.e., the spread of residuals is larger for smaller fitted values and vice versa. What does it suggest you regarding model inadequacy?

Problem 2 (6 pts) Consider a two-way layout with the model $y_{ij} = \eta + \alpha_i + \beta_j + \varepsilon_{ij}$, and $i = 1,2,3$ and $j = 1,2$, where α_i and β_j represent the main effects of two qualitative factors. Suppose it is determined that $\alpha_3 = 0$ and $\beta_2 = 0$ be chosen as the baseline constraints. Find estimates of α_1, α_2, and β_1 in terms of the y_{ij} values.

Problem 3 (6 pts) In the typing experiment in Chapter 1, suppose there are two keyboards (denoted by A and B) to be compared on three manuscripts and three typists (chosen to be representative of the population). A complete comparison would require 18 trials. (A trial means a typist typing one manuscript on a keyboard.) If for resource limitation only 12 trials are allowed, devise a valid and efficient experimental plan for this study.

Problem 4 (12 pts)

The bioactivity of four different drugs A, B, C, D for treating a particular illness was compared in a study and the following ANOVA table was given for the data:

Source	Sum of squares	Degrees of freedom
Between treatments	64.42	3
Within treatments	64.12	26
Total	128. 54	29

(a) (3 pts) Describe a proper design of the experiment to allow valid inferences to be made from the data.

(b) (3 pts) Use an F test to test at 0.05 level the null hypothesis that the four treatments have the same bioactivity. Compute the p value of the observed F statistic. (Hint: interpolate between values in different F tables.)

(c) (3 pts) The treatment averages are as follows: $\bar{y}_A = 66.10$ (7 samples), $\bar{y}_B = 65.75$ (8 samples), $\bar{y}_C = 62.63$ (9 samples), $\bar{y}_D = 63.85$ (6 samples). Use the Tukey method to perform multiple comparisons of the four treatments at the 0.05 level.

(d) (3 pts) It turns out that A and B are brand-name drugs and C and D are generic drugs. To compare brand-name vs. generic drugs, the contrast $\frac{1}{2}(\bar{y}_A + \bar{y}_B) - \frac{1}{2}(\bar{y}_C + \bar{y}_D)$ is computed. Obtain the p-value of the computed contrast and test its significance at the 0.05 level. Comment on the difference between brand-name and generic drugs.

Problem 5 (12 pts) The data considered in this problem are taken from 20 incoming shipments of chemicals in drums arriving at a warehouse. The response Y is the number of minutes required to handle a shipment. There are two predictors:

(i) X_1 : number of drums in the shipment.

(ii) X_2 : total weight of the shipment (hundreds of pounds).

After running a multiple regression code in a standard statistical software, the following results are obtained:

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Predictor      Coef  SE Coef      T
Constant      3.324    3.111   1.07
X1           3.7681   0.6142   6.13
X2           5.0796   0.6655
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(a) (2 pt) Compute the t-value corresponding to X_2.

(b) (3 pts) Test whether the predictor variables X_1 and X_2 significantly affect Y at level 0.01.

(c) (4 pts) Estimate the average value of Y at $X_1 = 5$ and $X_2 = 16.00$. Find the 95% confidence interval for the estimated average. The $(\mathbf{X}'\mathbf{X})^{-1}$ matrix is given below to aid your calculation:

$$\begin{bmatrix} 0.306662 & -0.0329521 & 0.0148670 \\ -0.032952 & 0.0119557 & -0.0119974 \\ 0.014867 & -0.0119974 & 0.0140366 \end{bmatrix}.$$

(d) (3 pts) Obtain the 95% prediction interval for a single future observation Y corresponding to $X_1 = 5$ and $X_2 = 16.00$.