Exam 2 for ISYE 6413 with Professor Wu at GT

No description

ISyE6413

Second Midterm Examination April 1st, 2008

(Total : 50 points)

Name :

Problem	1	2	3	4	5	Total
Max Points	8	4	10	20	8	50
Your score						

Problem 1 (8 pts)
Consider the following Latin square design of order 4.

	Column			
Row	1	2	3	4
1	A	C	B	D
2	D	B	C	A
3	C	A	D	B
4	B	D	A	C

	Column			
Row	1	2	3	4
1	235	236	218	268
2	251	241	227	229
3	234	273	274	226
4	195	270	230	225

Recall that the linear model for this Latin square design is

$$y_{ijl} = \eta + \alpha_i + \beta_j + \tau_l + \varepsilon_{ijl}, \ i = 1, \ldots, 4; j = 1, \ldots, 4; l = 1, \ldots, 4,$$

where l = Latin letter in the (i, j) cell of the Latin Square,
$\alpha_i = i$th row effect,
$\beta_j = j$th column effect,
$\tau_l = l$th treatment (i.e., Latin letter) effect,
ε_{ijl} are independent $N(0, \sigma^2)$.
Assume the zero-sum constraints $\sum_{i=1}^{4} \alpha_i = \sum_{j=1}^{4} \beta_j = \sum_{l=1}^{4} \tau_l = 0$.

(a) (2+2+2=6 pts) Calculate $\hat{\alpha}_3$, $\hat{\beta}_1$, $\hat{\tau}_2$ (i.e., estimates of $\alpha_3, \beta_1, \tau_2$). You can use the fact $\bar{y}_{...} = 239.5$.

$\hat{\tau}_2 = \bar{y}_{..2} - \bar{y}_{...}$ = average response corresponding to B - grand mean $= 220 - 239.5 = -19.5$. Similarly, $\hat{\alpha}_3 = \bar{y}_{3..} - \bar{y}_{...} = 251.75 - 239.5 = 12.25$ and $\hat{\beta}_1 = \bar{y}_{.1.} - \bar{y}_{...} = 228.75 - 239.5 = -10.75$.

(b) (2 pts) What is the residual degrees of freedom for this design?

$(k-1)(k-2) = 3 \times 2 = 6$.

Problem 2 (4 pts) To study the strength of plastic, an experimenter prepares 16 batches of plastic with two for each of the eight treatment combinations for T: baking temperature with 2 levels and A: additive percentage with 4 levels. Four batches with different additive percentages are baked for one temperature setting at the same time. Analysis of Split-plot design is used to study the effects of T and A.

(a) (2 pt) Which factor is whole plot and which is subplot?

T is the whole plot, and A is the subplot.

(b) (2 pt) What is the degrees of freedom for the whole plot error? What is the degrees of freedom for the subplot error?

df(whole plot error) = $(I-1)(n-1) = 1$;
df(subplot plot error) = $I(J-1)(n-1) = 6$.

Problem 3 (10 pts)

A taste panel will convene to compare five different brands of ice cream - A, B, C, D, and E. However, in order to assess and compare the tastes properly, *not more than three brands* should be offered to an expert taster.

(a) (2 pt) What experimental design would be the best to use in this situation (just name the design)?

Balanced Incomplete Block Design (BIBD).

(b) (2 pts) Argue that you cannot construct such a design with five tasters.

Here, we have $t = 5$ and $k = 3$. If $b = 5$, then from the identity $bk = rt$, we have $r = 3$. Then, from the second identity, $\lambda = r(k-1)/(t-1) = 3/2$, which is not an integer.

(c) (2 pts) Find out the minimum number of tasters needed to construct such a design.

Since $b = rt/k = 5r/3$, b has to be multiple of 5 and r has to be a multiple of 3 so that b is an integer. Clearly, $r = 3$ (which means $b = 5$) is not a solution. The next choice is $r = 6$, for which $b = 10$, and $\lambda = r(k-1)/(t-1) = 3$ (integer). All the inequalities of BIBD are also satisfied. This means the minimum number of blocks (tasters) needed to construct a BIBD is 10.

Alternative Solution: Since $\lambda = r(k-1)/(t-1) = r/2$, r has to be multiple of 2 to ensure that λ is an integer. Let $r = 2n$, where n is any integer. From the first identity of BIBD, $b = rt/k = (2n \times 5)/3 = 10n/3$. The minimum n which makes b an integer is therefore 3, for which $b = 10$. This means the minimum number of blocks (tasters) needed to construct a BIBD is 10.

(d) (4 pts) Construct the design with the minimum possible number of expert tasters (i.e., show which expert will taste which brands of ice creams).

With $b = 10$, $k = 3$, $t = 5$, $r = 6$, and $\lambda = 3$, a possible design is

Taster	A	B	C	D	E
1	X	X	X		
2	X	X		X	
3	X	X			X
4	X		X	X	
5	X		X		X
6	X			X	X
7		X	X	X	
8		X	X		X
9		X		X	X
10			X	X	X

or

Taster	A	B	C	D	E
1	X	X		X	
2	X	X			X
3	X	X	X		
4	X		X		X
5	X		X	X	
6	X			X	X
7		X	X	X	
8		X		X	X
9		X	X		X
10			X	X	X

Problem 4 (20 pts)

Speedometer cables can be noisy because of shrinkage in the plastic casing material, so an experiment was conducted to find out what caused shrinkage. The engineers identified four factors at two levels: A = wire braid type, B = braiding tension, C = wire diameter, D = line speed. Response y is percentage shrinkage per specimen. A full factorial design with 16 runs was used. Full data, collapsed data and output of analysis are given below. Note that the fit given here is a regression output and the plot is a half-normal plot.

Data

Run	A	B	C	D	y
1	-1	-1	-1	-1	0.2750
2	-1	-1	-1	1	0.1700
3	-1	-1	1	-1	0.0875
4	-1	-1	1	1	0.1750
5	-1	1	-1	-1	0.1750
6	-1	1	-1	1	0.2250
7	-1	1	1	-1	0.1250
8	-1	1	1	1	0.1200
9	1	-1	-1	-1	0.5350
10	1	-1	-1	1	0.4550
11	1	-1	1	-1	0.1950
12	1	-1	1	1	0.1450
13	1	1	-1	-1	0.5750
14	1	1	-1	1	0.4850
15	1	1	1	-1	0.3425
16	1	1	1	1	0.5825

Collapsed data with factors A and C

Run	A	C	Average	Variance	Log Variance
1	-	-	0.211250	0.002423	-6.02278
2	-	+	0.126875	0.001306	-6.64099
3	+	-	0.512500	0.002825	-5.86925
4	+	+	0.316250	0.038535	-3.25618

Regression output

```
Call:
lm(formula = y ~ A + B + C + D + AB + AC + AD + BC + BD + CD
                   +  ABC + ABD + BCD)

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.2917188  0.0261181  11.169  0.00792 **
A            0.1226563  0.0261181   4.696  0.04247 *
B            0.0370312  0.0261181   1.418  0.29199
C           -0.0701563  0.0261181  -2.686  0.11515
D            0.0029687  0.0261181   0.114  0.91988
AB           0.0448438  0.0261181   1.717  0.22812
AC          -0.0279688  0.0261181  -1.071  0.39633
AD          -0.0004687  0.0261181  -0.018  0.98731
BC           0.0339063  0.0261181   1.298  0.32376
BD           0.0214063  0.0261181   0.820  0.49858
CD           0.0310938  0.0261181   1.191  0.35599
ABC          0.0304688  0.0261181   1.167  0.36367
ABD          0.0135937  0.0261181   0.520  0.65462
BCD          0.0032812  0.0261181   0.126  0.91151
```

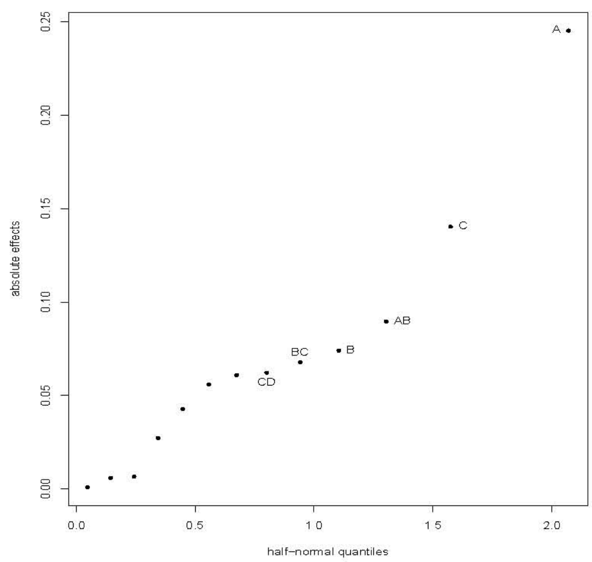

Figure 1 : Half-Normal Plot

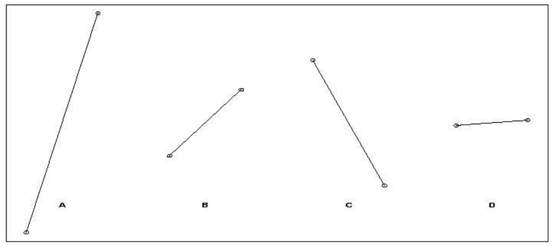

Figure 2 : Main Effects Plot

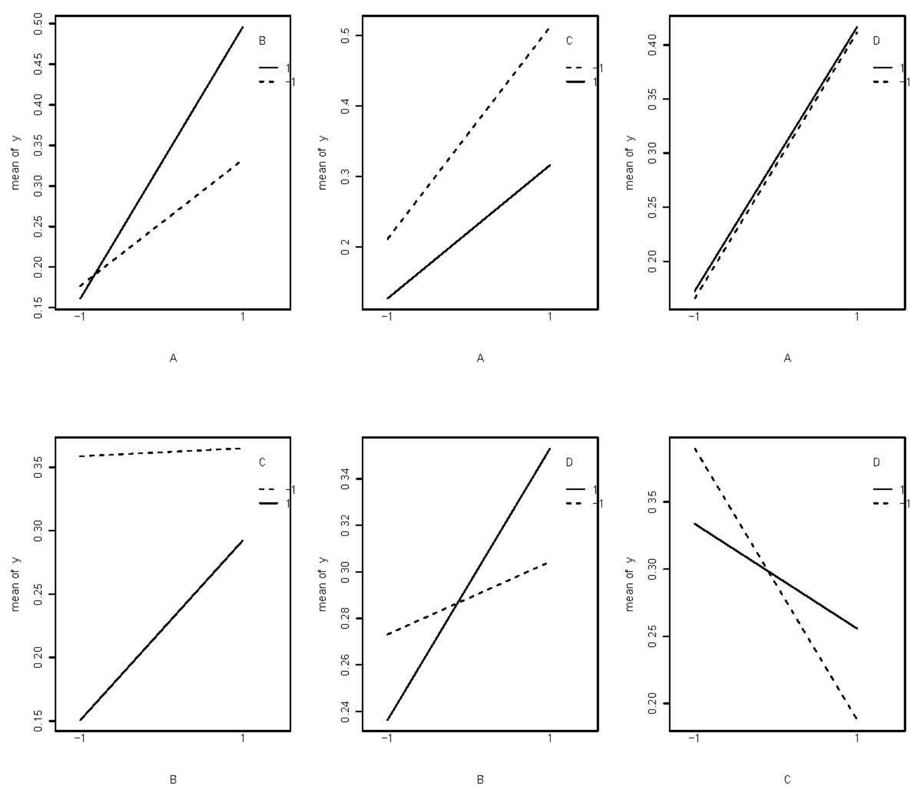

Figure 3 : Interaction Plots

Questions (a)-(d), (h) do not pertain to the collapsed data.

(a) (1 pts) Compute the factorial effect for B.

$2 \times 0.0370312 = 0.0740624$.

(b) (2 pts) Based on the output, which effects seem to be the most significant? (You need to choose at least two.)

A and C.

(c) (2 pts) Based on the effect estimates and plots, determine the optimal setting of the two most important factors. (Hint: what shrinkage values are considered optimal?)

$A = -1$ and $C = +1$.

(d) (2 pts) Is the setting you choose supported by the information in the original data (first table)?

Yes. The two lowest values of the response correspond to A- and C+.

(e) (3+1 pts) The second table gives the data collapsed onto factors A and C. It is a 2^2 design with 4 replicates. Compute the three factorial effects for the log variance (last column of table). Which one is the largest?

ME(A) = 1.76917, ME(C) = 0.99743, INT(A, C)=1.61564. ME(A) is largest.

(f) (2 pts) Identify the optimal factor setting in terms of minimizing the variance.

$A = -1$ and $C = +1$.

(g) (2+1 pts) Based on your findings in (c) and (f), what is your overall recommendation for choosing the factor setting? Any conflict?

$A = -1$ and $C = +1$, choose B and D to accommodate any other economic or engineering criteria. No Conflict.

(h) (2+1+1 pts) Draw the interaction plot of $D \times B$ (D on the horizontal axis) and state whether this plot is synergystic or antagonistic. Comparing it with the interaction plot of $B \times D$ (B on the horizontal axis), is there any contradiction? Explain.

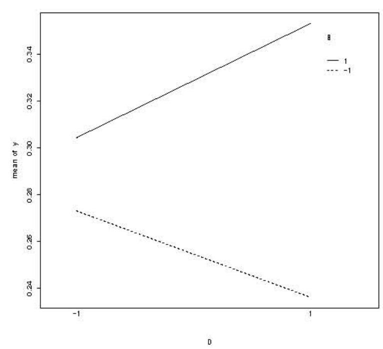

Figure 4 : Interaction Effects Plot for $D \times B$ (D on the horizontal axis).

$D \times B$ is antagonistic, and $B \times D$ is synergystic. No contradiction, but this suggests that underlying response surface is more complicated.

Problem 5 (8 pts) Suppose a 2^7 design is arranged in 8 blocks with the generators: $B_1 = 1234$, $B_2 = 2345$ and $B_3 = 1457$. For any blocking scheme b, denote by $g_i(b)$ the number of i-factor interactions that are confounded with block effects.

(a) (2 pts) Find all the interactions that are confounded with the block effects.

$B_1 = 1234$, $B_2 = 2345$, $B_3 = 1457$
$B_1B_2 = 15, B_1B_3 = 2357, B_2B_3 = 1237, B_1B_2B_3 = 47.$

(b) (2 pts) Compute the $g_i(b)$ for $i \geq 2$.

$(g_2(b), \ldots, g_7(b)) = (2, 0, 5, 0, 0, 0).$

(c) (4 pts) An alternative blocking scheme is $B_1 = 1234$, $B_2 = 1256$, and $B_3 = 1357$. Compare the two schemes and show the clear advantage of the second scheme. State the criterion you use for comparing the two schemes.

For the alternative blocking scheme b_2, $B_1 = 1234$, $B_2 = 1256$, $B_3 = 1357$
$B_1B_2 = 3456, B_1B_3 = 2457, B_2B_3 = 2367, B_1B_2B_3 = 1467.$
And $(g_2(b_2), \ldots, g_7(b_2)) = (0, 0, 7, 0, 0, 0)$
Based on the **Minimum Aberration criterion**, the second blocking scheme has clear advantage. In the second scheme all the 2-factor and 3-factor interactions are estimable, whereas in the first scheme the 2-factor interactions 15 and 47 are confounded with block effects.