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Exam 2 for ISYE 6413 with Professor Wu at GT

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ISyE6413
Second Midterm Examination April 1st, 2008
(Total : 50 points)

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Problem	1	2	3	4	5	Total
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Problem 1 (8 pts)
Consider the following Latin square design of order 4.

Row	Column 1	2	3	4
1	A	C	B	D
2	D	B	C	A
3	C	A	D	B
4	B	D	A	C

Row	Column 1	2	3	4
1	235	236	218	268
2	251	241	227	229
3	234	273	274	226
4	195	270	230	225

Recall that the linear model for this Latin square design is

$$y_{ijl} = \eta + \alpha_i + \beta_j + \tau_l + \varepsilon_{ijl},\ i = 1,\dots,4;\ j = 1,\dots,4; l = 1,\dots,4,$$

where l = Latin letter in the (i, j) cell of the Latin Square,
α_i = ith row effect,
β_j = jth column effect,
τ_l = lth treatment (i.e., Latin letter) effect,
ε_{ijl} are independent $N(0, \sigma^2)$.
Assume the zero-sum constraints $\sum_{i=1}^4 \alpha_i = \sum_{j=1}^4 \beta_j = \sum_{l=1}^4 \tau_l = 0$.

(a) (2+2+2=6 pts) Calculate $\hat{\alpha}_3$, $\hat{\beta}_1$, $\hat{\tau}_2$ (i.e., estimates of $\alpha_3, \beta_1, \tau_2$). You can use the fact $\bar{y}_{...} = 239.5$.

$\hat{\tau}_2 = \bar{y}_{..2} - \bar{y}_{...}$ = average response corresponding to B - grand mean $= 220 - 239.5 = -19.5$. Similarly, $\hat{\alpha}_3 = \bar{y}_{3..} - \bar{y}_{...} = 251.75 - 239.5 = 12.25$ and $\hat{\beta}_1 = \bar{y}_{.1.} - \bar{y}_{...} = 228.75 - 239.5 = -10.75$.

(b) (2 pts) What is the residual degrees of freedom for this design?

$(k-1)(k-2) = 3 \times 2 = 6$.

Problem 2 (4 pts) To study the strength of plastic, an experimenter prepares 16 batches of plastic with two for each of the eight treatment combinations for T: baking temperature with 2 levels and A: additive percentage with 4 levels. Four batches with different additive percentages are baked for one temperature setting at the same time. Analysis of Split-plot design is used to study the effects of T and A.

(a) (2 pt) Which factor is whole plot and which is subplot?

T is the whole plot, and A is the subplot.

(b) (2 pt) What is the degrees of freedom for the whole plot error? What is the degrees of freedom for the subplot error?

df(whole plot error) = $(I-1)(n-1) = 1$;
df(subplot plot error) = $I(J-1)(n-1) = 6$.

Problem 3 (10 pts)
A taste panel will convene to compare five different brands of ice cream - A, B, C, D, and E. However, in order to assess and compare the tastes properly, *not more than three brands* should be offered to an expert taster.

(a) (2 pt) What experimental design would be the best to use in this situation (just name the design)?

Balanced Incomplete Block Design (BIBD).

(b) (2 pts) Argue that you cannot construct such a design with five tasters.

Here, we have $t = 5$ and $k = 3$. If $b = 5$, then from the identity $bk = rt$, we have $r = 3$. Then, from the second identity, $\lambda = r(k-1)/(t-1) = 3/2$, which is not an integer.

(c) (2 pts) Find out the minimum number of tasters needed to construct such a design.

Since $b = rt/k = 5r/3$, b has to be multiple of 5 and r has to be a multiple of 3 so that b is an integer. Clearly, $r = 3$ (which means $b = 5$) is not a solution. The next choice is $r = 6$, for which $b = 10$, and $\lambda = r(k-1)/(t-1) = 3$ (integer). All the inequalities of BIBD are also satisfied. This means the minimum number of blocks (tasters) needed to construct a BIBD is 10.
Alternative Solution: Since $\lambda = r(k-1)/(t-1) = r/2$, r has to be multiple of 2 to ensure that λ is an integer. Let $r = 2n$, where n is any integer. From the first identity of BIBD, $b = rt/k = (2n \times 5)/3 = 10n/3$. The minimum n which makes b an integer is therefore 3, for which $b = 10$. This means the minimum number of blocks (tasters) needed to construct a BIBD is 10.

(d) (4 pts) Construct the design with the minimum possible number of expert tasters (i.e., show which expert will taste which brands of ice creams).

With $b = 10$, $k = 3$, $t = 5$, $r = 6$, and $\lambda = 3$, a possible design is

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