Math 4317 (Prof. Swiech, S'18): HW #1

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1/25/2018

Section 1

F. Show that the symmetric difference D, defined in the preceding exercise is also given by $D = (A \cup B) \setminus (A \cap B)$

I. If $\{A_1, A_2, ..., A_n\}$ is a collection of sets, and if E is any set, show that:

$$E \cap \cup_{j=1}^{n} A_j = \cup_{j=1}^{n} (E \cap A_j), \quad E \cup \cup_{j=1}^{n} A_j = \cup_{j=1}^{n} (E \cup A_j)$$

J. If $\{A_1, A_2, ..., A_n\}$ is a collection of sets, and if E is any set, show that:

$$E \cap \cap_{j=1}^{n} A_j = \cap_{j=1}^{n} (E \cap A_j), \quad E \cup \cap_{j=1}^{n} A_j = \cap_{j=1}^{n} (E \cup A_j)$$

K. Let E be a set and $\{A_1, A_2, ..., A_n\}$ be a collection of sets. Establish the De Morgan laws:

$$E \setminus \cap_{j=1}^{n} A_j = \cup_{j=1}^{n} (E \setminus A_j), \quad E \setminus \cup_{j=1}^{n} A_j = \cap_{j=1}^{n} (E \setminus A_j)$$

Section 2

C. Consider the subset of $\mathbb{R} \times \mathbb{R}$ defined by $D = \{(x, y) \ : |x| + |y| = 1\}$. Describe this set in words. Is it a function?

E. Prove that if f is an injection from A to B, than $f^{-1} = \{(b, a) : (a, b) \in f\}$ is a function. Then prove it is an injection.

H. Let f, g be functions such that

$$g \circ f(x) = x, \quad for\ all\ x\ in\ D(f)$$

$$f \circ g(y) = y, \quad for\ all\ y\ in\ D(g)$$

Prove that $g = f^{-1}$

J. Let f be the function on $\mathbb{R}$ to $\mathbb{R}$ given by $f(x) = x^2$, and let $E = \{x \in \mathbb{R} - 1 \leq x \leq 0\}$ and $F = \{x \in \mathbb{R} : 0 \leq x \leq 1\}$.Then $E \cap F = \{0\}$ and $f(E \cap F) = \{0\}$ while $f(E) = f(F) = \{y \in \mathbb{R} : 0 \leq y \leq 1\}$. Hence $f(E \cap F)$ is a proper subset of $f(E) \cap f(F)$. Now delete 0 from E and F.