Math 4317 (Prof. Swiech, S'18): HW #3

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Section 14

A. Let $b \in \mathbb{R}$, show $\lim \frac{b}{n} = 0$.
B. Show that $\lim(\frac{1}{n} - \frac{1}{n+1}) = 0$.
D. Let $X = (x_n)$ be a sequence in $\mathbb{R}^p$ which is convergent to x. Show that $\lim ||x_n|| = ||x||$. (Hint: use the Triangle Inequality.)
G. Let $d \in \mathbb{R}$ satisfy $d > 1$. Use Bernoulli's Inequality to show that the sequence (d_n) is not bounded in $\mathbb{R}$. Hence it is not convergent.$
H. Let $b \in \mathbb{R}$ satisfy $0 < b < 1$; show that $\lim(nb^n) = 0$. (Hint: use the Binomial Theorem as in Example 14.8(e).)
I. Let $X = (x_n)$ be a sequence of strictly positive real numbers such that $\lim(\frac{x_{n+1}}{x_n}) < 1$. Show that for some r with $0 < r < 1$ and some $C > 0$, then we have $0 < x_n < Cr^n$ for all sufficiently large $n \in \mathbb{N}$. Use this to show that $lim(x_n) = 0$
J. Let $X = (x_n)$ be a sequence of strictly positive real numbers such that $\lim(\frac{x_{n+1}}{x_n}) > 1$. Show that X is not a bounded sequence and hence is not convergent.
K. Give and example of a convergent sequence (x_n) of strictly positice real numbers such that $\lim(\frac{x_n+1}{x_n}) = 1$. Give an example of a divergent sequence with this property. L. Apply the results of Exercises 14.I and 14.J to the following sequences. (Here $0 < a < 1, 1 < b, c > 0$) (a) (a^n)
(b) (na^n)
(c) (b^n)
(d) $(\frac{b^n}{n})$
(e) $(\frac{c^n}{n!})$
(f) $(\frac{2^{3n}}{3^{2n}})$

Section 15

C(a-e),E,F,L,N

Section 16

A,B,E,G,J,M(a)(c)(d),N

Section 17

A,B,D,E,L,M

Section 18

A(a-c),D,F,I