

Homework 3 for math6262

1) Let X denote the outcome of a random experiment with a four sided die. Assume that the die is not symmetric, so that the probabilities for the different sides are:

$$P_\theta(X = 1) = c \cdot \theta, P_\theta(X = 2) = 2c \cdot \theta, P_\theta(X = 3) = 3c \cdot \theta, P_\theta(X = 4) = 4c \cdot \theta$$

where $c = c(\theta)$ is a constant which depends on the parameter $\theta > 0$.

a) Calculate $c(\theta)$.

b) Estimate Fisher-Information.

c) What is maximum-likelihood estimate of θ . Suppose that you have hundred i.i.d. observations x_1, x_2, \dots, x_{100} all following the above model for one unknown θ . (All x_i 's have been generated with the same θ .)

d) Find 95%-confidence interval.

2) Say you are given n observations: x_1, x_2, \dots, x_n which are drawn from an exponential distribution with parameter λ .

a) Calculate the maximum-likelihood estimate of λ .

b) Find Fisher-Information.

c) Find a 95% confidence interval using Fisher Information. What do you say?

3) We have a parameter θ which can take three different values: 1, 2 and 3 corresponding to three different dice. On each dice we have the number 4, 5 and 6. So, these are three sided dices. Now, the probabilities for each dice are as follows:

θ	$P_\theta(X = 4)$	$P_\theta(X = 5)$	$P_\theta(X = 6)$
1	0.2	0.3	0.5
2	0.6	0.2	0.2
3	0.8	0.1	0.1

a) Find maximum likelihood estimate of θ .

b) find only non-biased estimate of θ .

c) Assume that the table gets changed and we have the following table:

θ	$P_\theta(X = 4)$	$P_\theta(X = 5)$	$P_\theta(X = 6)$
1	0.2	0.3	0.5
2	0.6	0.2	0.2
3	0.6	0.2001	0.1999

Calculate the only unbiased estimator. Is that estimator good?

4) The probability to catch disease a is 10%. The probability to get diseases b is 17%. The probability to get at least one of the two a or b is 20%.

a) What is the probability to get sick with both a and b ?

b) What is the probability to get a or b but not both at the same time?

5) In a randomly selected group of 100 people from a population in a certain country we find that

- gene a , is present in 30% of the people in our sample.
- gene b , present in 40%.
- gene a and gene b at the same time, are present in 10% of sampled people.

Let p_a , p_b resp. $p_{a \cap b}$ be the probabilities that gene a , b , resp. a and b at the same time are present in a randomly selected individual.

a) Find a maximum-Likelihood estimate for these three probabilities.

b) (Optional+more difficult) What would be a 95% confidence interval for the parameter vector $(p_1, p_b, p_{a \cap b})$?