6404 In-class Exercise 10/04/18

Given Data in Table 10.1 and $\hat{P}_i$, $S_{KM}(x_{i:n})$ in Eq. (10.3),

- calculate $\hat{P}_1$, $\hat{P}_3$, $\hat{P}_{95}$, $\hat{P}_{102}$

$n=49$ and

$S_{KM}(x_{(1)})$, $S_{KM}(x_{(3)})$, $\cdots$, $S_{KM}(x_{(\cancel{95}\,47)})$, $S_{KM}(x_{(48)})$.

Solution:

① "complete sample"

$$\hat{P}_1 = \frac{1}{49-1+1} \cancel{\boxed{}}^{\,1} = \frac{1}{49}$$

for both obs. at $x_{(1)} = x_{(2)} = 1$
since there are two cases of "1" obs.
$\hat{P}_1 = \hat{P}_2 = 1/49$ or $\hat{P}("1") = 2/49$

②

$$\hat{P}_3 = \frac{1}{49-3+1} \prod_{j=1}^{3-1=2} \boxed{}$$

$$= \frac{1}{47}\left[1-\frac{1}{49-1+1}\right]\left[1-\frac{1}{49-2+1}\right]$$

$$= \frac{1}{\cancel{47}} \times \underbrace{\frac{\cancel{48}}{49}}_{j=1} \times \underbrace{\frac{\cancel{47}}{\cancel{48}}}_{j=2} = \boxed{\frac{1}{49}}$$

works out.

$= \hat{P}_4$ (ties)

③ "complete sample"

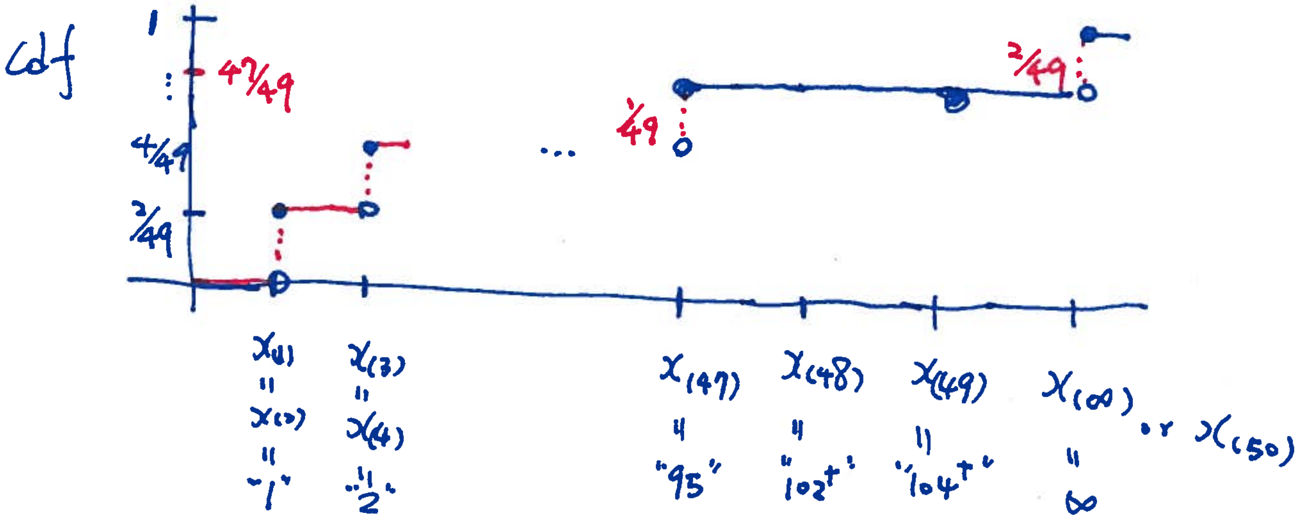
$$\hat{P}_{95} = \hat{P}_{47} = \frac{1}{49-47+1} \prod_{j=1}^{47-1=46} \boxed{} \qquad \underbrace{\left[1-\frac{1}{49-45+1}\right]}_{j=45}$$

$$= \frac{1}{3}\left[1-\frac{1}{n-1+1}\right]\left[1-\frac{1}{n-2+1}\right]\left[1-\frac{1}{n-3+1}\right]\cdots\left[1-\frac{1}{49-46+1}\right]$$

$$= \frac{1}{\cancel{3}}\underbrace{\left[\frac{\cancel{48}}{49}\right]}_{j=1}\underbrace{\left[\frac{\cancel{47}}{\cancel{48}}\right]}_{j=2}\underbrace{\left[\frac{46}{\cancel{47}}\right]}_{j=3}\cdots\left[\frac{\cancel{4}}{\cancel{5}}\right]\underbrace{\left[\frac{\cancel{3}}{\cancel{4}}\right]}_{j=46}$$

$$= \boxed{\frac{1}{49}}.$$

④ $\hat{P}_{\text{"102"}} = \hat{P}_{48} = \frac{0}{49-48+1} = 0 = \hat{P}_{49}$



Survival function : $S_{KM}(t)$

① $i=1$ $\quad S_{KM}(x_{(1)}) = \left(1 - \frac{1}{49-1+1}\right)^{1}_{j=i=1} = \frac{48}{49}$ $\quad \cancel{S_{KM}(x_{(2)})}$

$i=2$ $\quad$ since $x_{(1)} = x_{(2)} = \text{"1"}$, $\quad S_{KM}(\text{"1"}) = \frac{47}{49}$

i.e. drop $2/49$.

② $i=3$ $\quad S_{KM}(\text{"2"}) = \frac{45}{49}$

$i=4$

$$\underset{i=3}{=} \underbrace{\left[1 - \frac{1}{49-1+1}\right]}_{j=1} \underbrace{\left[1 - \frac{1}{49-2+1}\right]}_{j=2} \underbrace{\left[1 - \frac{1}{49-3+1}\right]}_{j=3}$$

$$= \left(\frac{\cancel{48}}{49}\right)\left(\frac{\cancel{47}}{\cancel{48}}\right)\left(\frac{46}{\cancel{47}}\right) = \frac{46}{49} \text{ at } \boxed{i=3}$$

Since $x_{(3)} = x_{(4)} = \text{"2"}$, $\quad S_{KM}(\text{"2"}) = \frac{45}{49}$.

3 $i = 47$

$$S_{KM}(\text{"}95\text{"}) = S_{KM}(x_{(47)})$$

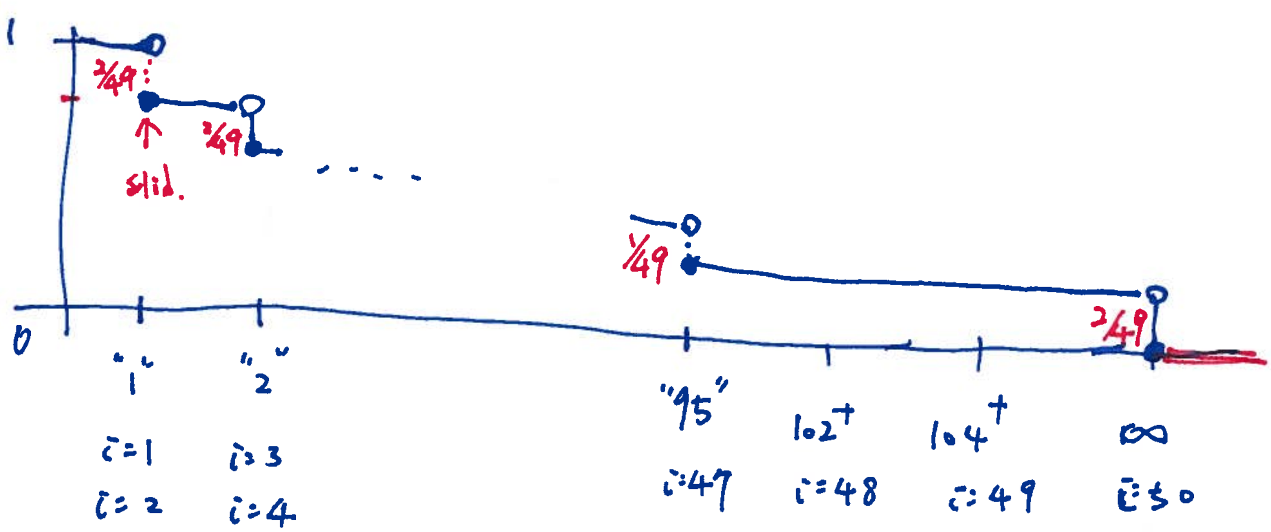
$$= \prod_{j=1}^{i=47} \boxed{\vdots}$$

$$= \left(\frac{\cancel{48}}{49}\right)\left(\frac{\cancel{47}}{\cancel{48}}\right)\left(\frac{\cancel{46}}{\cancel{47}}\right) \cdots \left(1 - \frac{1}{49-46+1}\right)\left(1 - \frac{1}{49-47+1}\right)$$

$j=1 \quad j=2 \quad j=3 \quad \ldots \quad j=46 \quad j=47$

$$\left(\frac{\cancel{3}}{4}\right)\left(\frac{2}{\cancel{3}}\right)$$

$$= \frac{2}{49} \quad \text{(yes there are two more obs. left in the data set)}$$



p. 187 Eq. (10.4)

$$F_{KM}(t) = 1 - \prod_{x_j \le t} \left(1 - \frac{d_j}{m_j}\right)$$

where d_j = # failures/deaths at x_j,
m_j = # obs. has survived up to x_j^-.

① $t = x_{(1)} = x_{(2)} = $ "1":

$$F_{KM}(\text{"1"}) = 1 - \left(1 - \frac{2}{49}\right) = 1 - \frac{47}{49} = \boxed{\frac{2}{49}}$$

for "1" ≤ "1" $x_{(1)}^- = $ "1⁻".

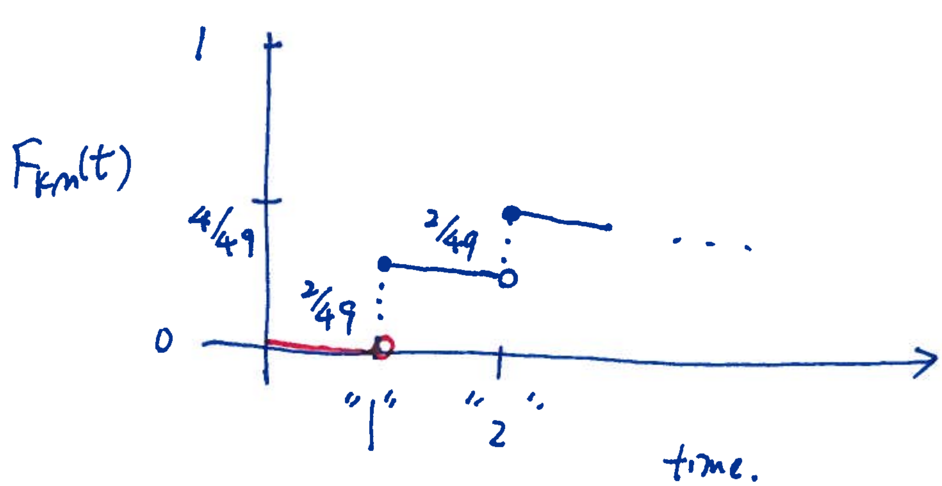
② $t = x_{(3)} = x_{(4)} = $ "2" :

$$F_{KM}(\text{"2"}) = 1 - \left[1 - \frac{2}{49}\right]\left[1 - \frac{2}{47}\right] = 1 - \left(\frac{\cancel{47}}{49}\right)\left(\frac{45}{\cancel{47}}\right)$$

for "1" ≤ "2"   for "2" ≤ "2"
$j = 1, 2$ $j = 3, 4$
$x_j^- = x_{(1)}^- = $ "1⁻"

$$= 1 - \frac{45}{49}$$

$$= \boxed{\frac{4}{49}}$$



③

p.5

$F_{KM}(\text{"}95\text{"}) = 1 - \left[1-\frac{2}{49}\right]\left[1-\frac{2}{47}\right] \cdots \left[1-\frac{1}{4}\right]$

$x_{(47)}$

"1" ≤ "95"

$x_1^- = 1^-$

"83" ≤ "95"

83

$x_{46}^- = 83^-$

$\left[1-\frac{1}{3}\right]$

"95" ≤ "95"

$x_{47}^- = 95^-$

$= 1 - \left(\frac{\cancel{47}}{49}\right)\left(\frac{45}{\cancel{47}}\right) \cdots \left(\frac{3}{4}\right)\left(\frac{2}{\cancel{3}}\right)$

$= 1 - \frac{2}{49} = \boxed{\frac{47}{49}}$

No death

④ $F_{KM}(\text{"}102\text{"}) = 1 - \left(\frac{\cancel{47}}{49}\right)\left(\frac{45}{\cancel{47}}\right) \cdots \left(\frac{\cancel{3}}{4}\right)\left(\frac{2}{\cancel{3}}\right)\left(1-\frac{0}{2}\right)$

$x_{(48)}$

"102" ≤ "102"

$x_{48}^- = 102^-$

$= 1 - \left(\frac{1}{49}\right) \cdots \left(\frac{2}{1}\right)(1)$

$= 1 - \frac{2}{49} = \boxed{\frac{47}{49}}$

$\parallel$

$F_{KM}(\text{"}104\text{"})$

Not adding any point mass

⑤ $F_{KM}(\text{"}\infty\text{"}) = 1-(\cdots)\left(\cancel{1-\frac{2}{2}}\right)^{\to 0} = 1-0 = \boxed{1}$ (until ~~$x_{(48)}$~~ $x_{(50)} = \text{"}\infty\text{"}$)