WELCOME

Introduction to Wavelet Transform.

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OUTLINE

- Overview
- Limitations of Fourier Transform
- Historical Development
- Principle of Wavelet Transform
- Examples of Applications
- Conclusion
- References

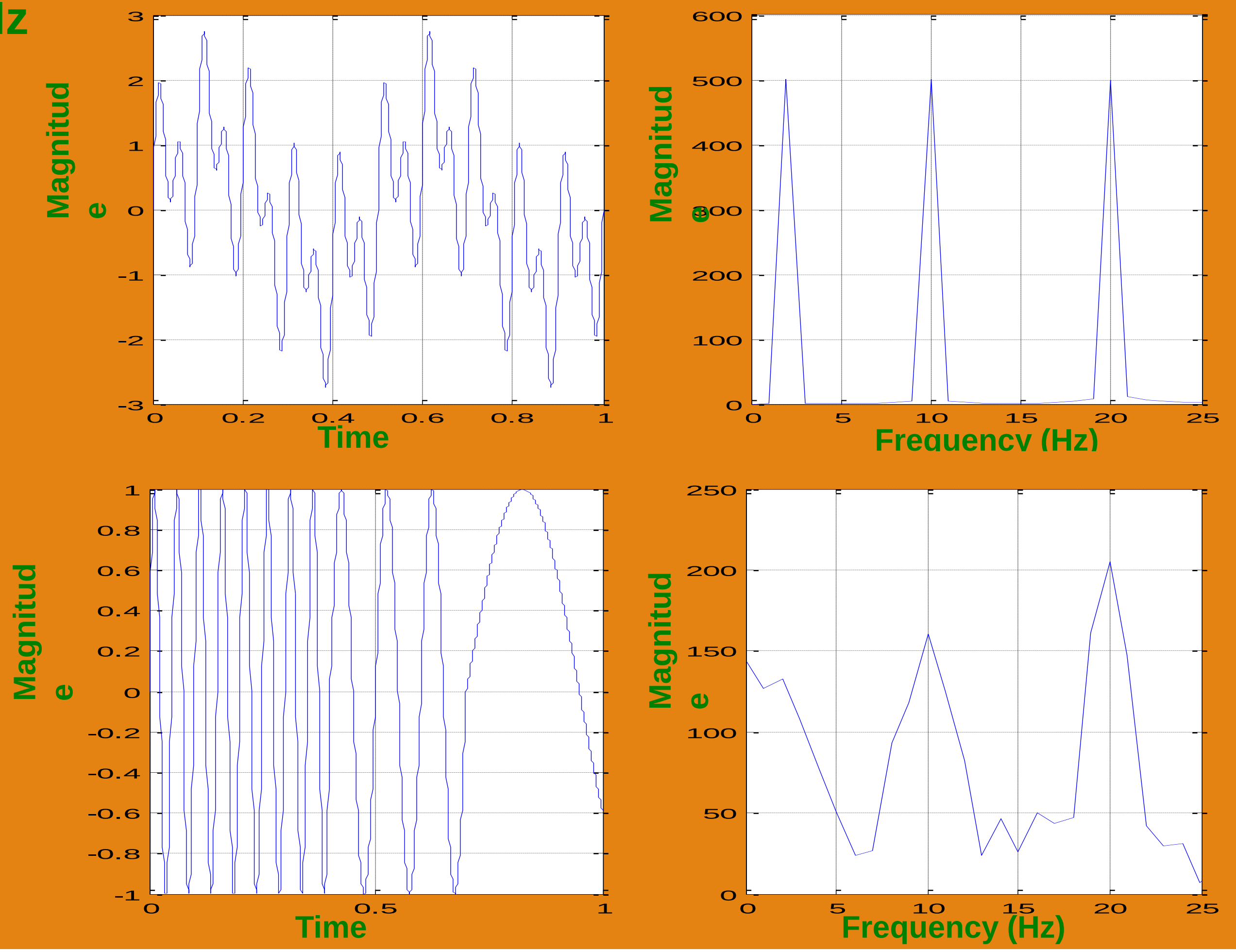
STATIONARITY OF SIGNAL

2 Hz + 10 Hz + 20Hz

Stationary

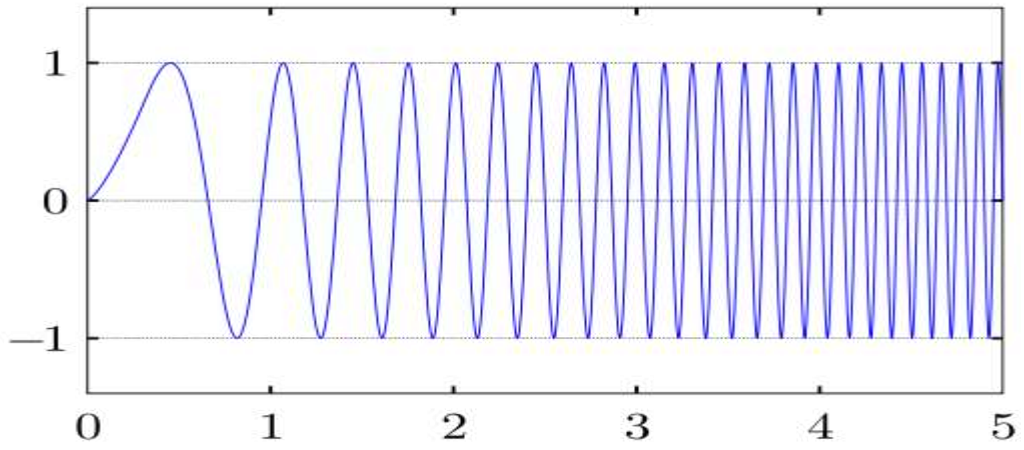
0.0-0.4: 2 Hz +
0.4-0.7: 10 Hz +
0.7-1.0: 20Hz

Non-Stationary

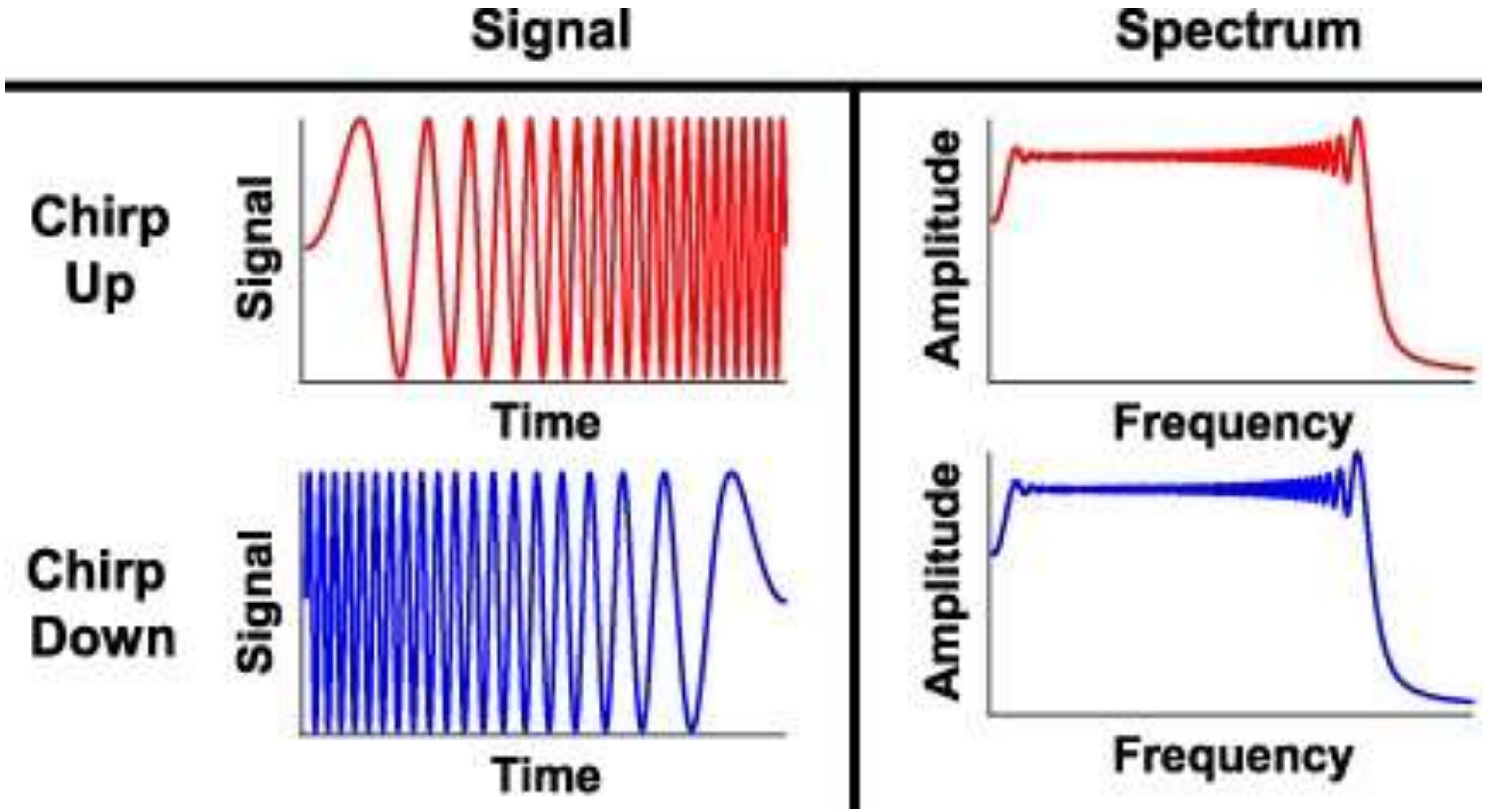


Limitations of Fourier Transform:

- To show the limitations of Fourier Transform, we chose a well-known signal in SONAR and RADAR applications, called the Chirp.
- A Chirp is a signal in which the frequency increases (‘up-chirp’) or decreases (‘down-chirp’).



Fourier Transform of Chirp Signals:



Result:

- Different in time but same frequency representation!!!

- Fourier Transform only gives what frequency components exist in a signal.
- Fourier Transform cannot tell at what time the frequency components occur.
- However, Time-Frequency representation is needed in most cases.

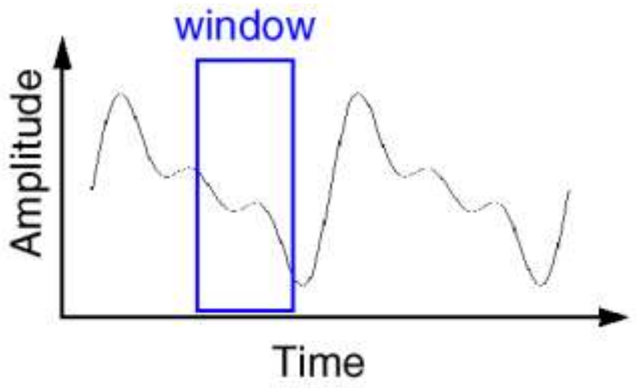
SOLUTION

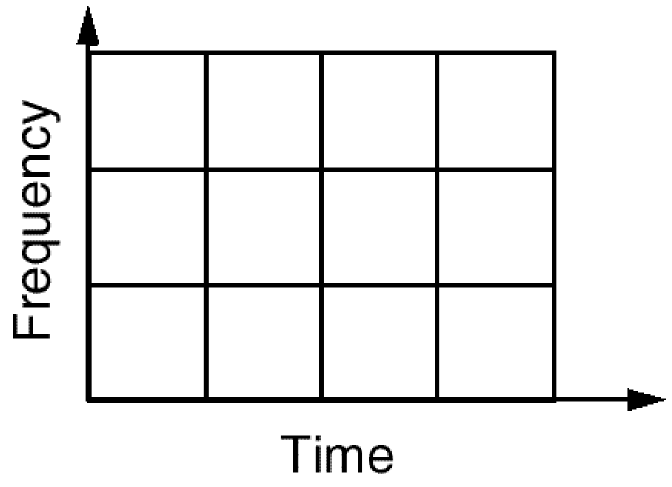
?

SOLUTION 1

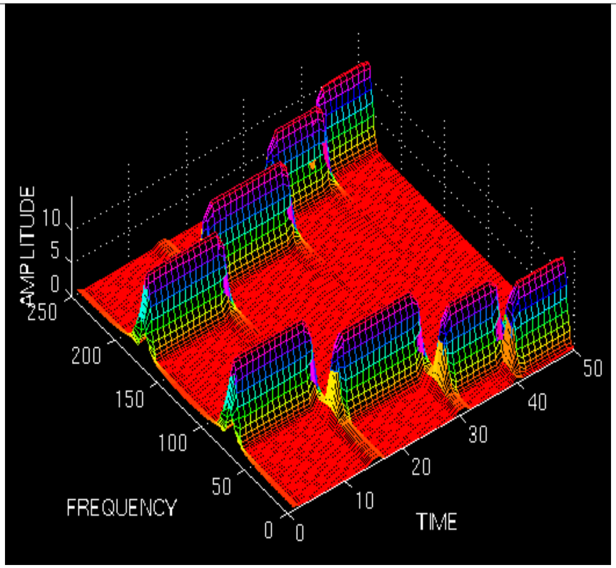
Short Time Fourier Analysis

- In order to analyze small section of a signal, Denis Gabor (1946), developed a technique, based on the FT and using *windowing* : STFT





STFT At Work


AMPLITUDE
10
5
0
250
200
150
100
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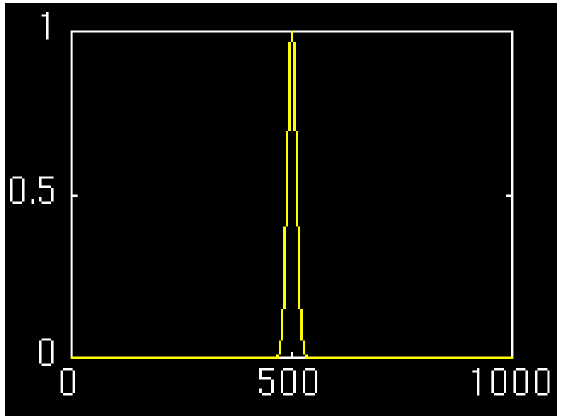
FREQUENCY
0
10
20
30
40
50
TIME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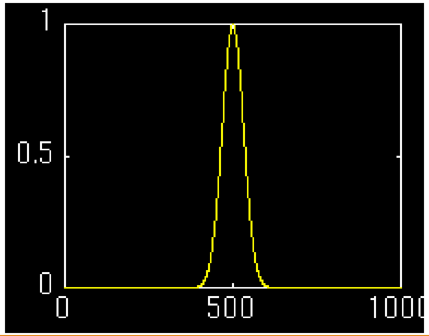
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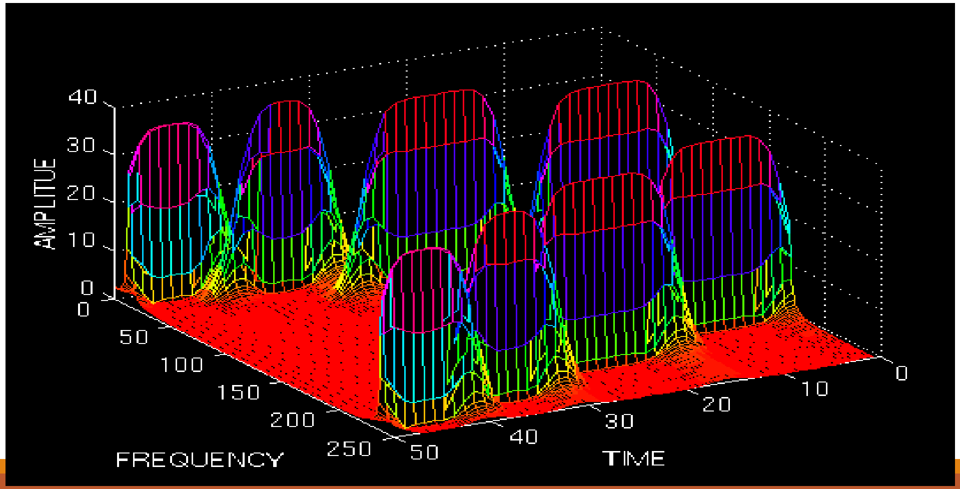
40
30
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AMPLITUE
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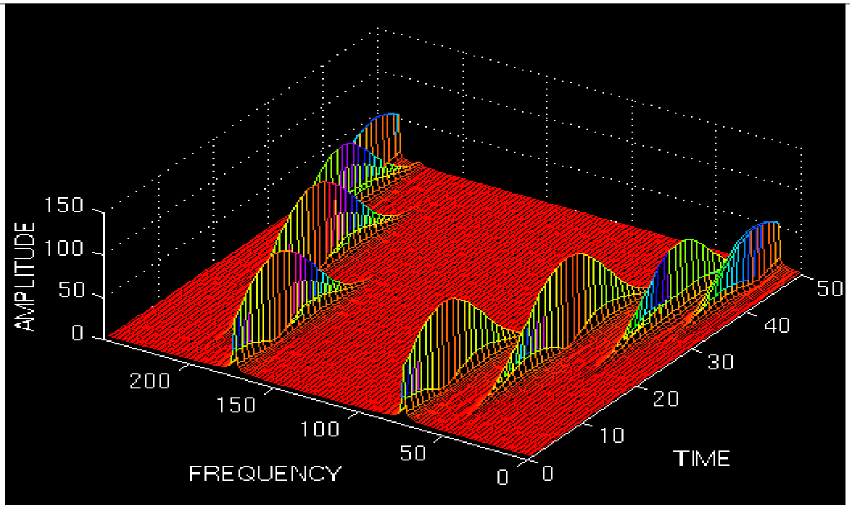
TIME

STFT At Work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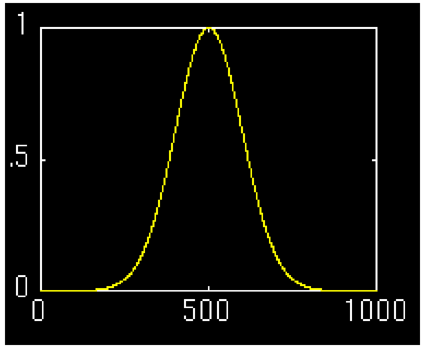
TIME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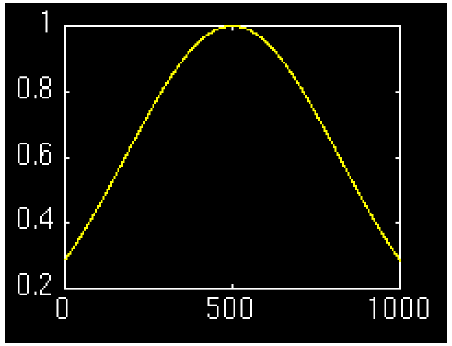
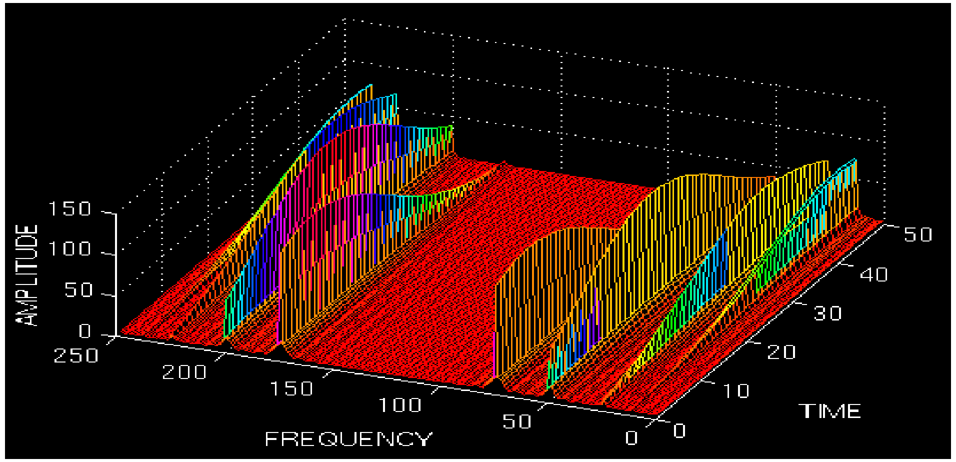

1
0.8
0.6
0.4
0.2
0
500
1000


AMPLITUDE
150
100
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0
250
200
150
100
50
0
FREQUENCY
0
10
20
30
40
50
TIME

What’s wrong with Gabor?

- Many signals require a more flexible approach - so we can vary the window size to determine more accurately <u>either time or frequency.</u>

SOLUTION 2

WAVELET TRANSFORM

Overview of wavelet:

What does Wavelet mean?

Oxford Dictionary: A wavelet is a small wave.

Wikipedia: A wavelet is a mathematical function used to divide a given function or continuous-time signal into different scale components.

A Wavelet Transform is the representation of a function by wavelets.

Historical Development:

1909 : ***Alfred Haar*** – Dissertation “On the Orthogonal Function Systems” for his Doctoral Degree. The first wavelet related theory .

1910 : ***Alfred Haar*** : Development of a set of rectangular basis functions.

1930s : - ***Paul Levy*** investigated “The Brownian Motion”.

- ***Littlewood*** and ***Paley*** worked on localizing the contributing energies of a function.

1946 : ***Dennis Gabor*** : Used Short Time Fourier Transform .

1975 : ***George Zweig*** : The first Continuous Wavelet Transform CWT.

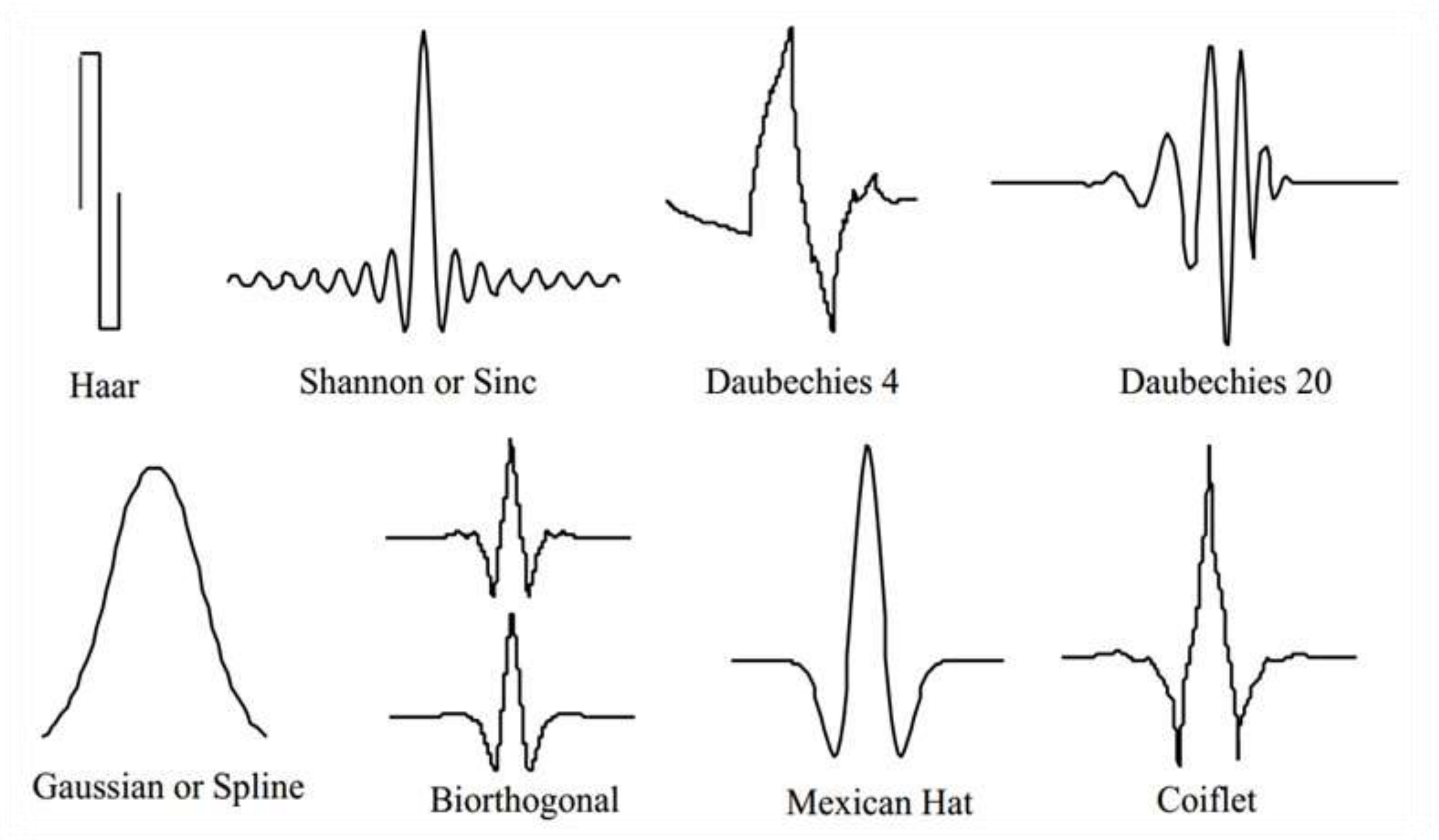
1985 : ***Yves Meyer*** : Construction of orthogonal wavelet basis functions with very good time and frequency localization.

1986 : ***Stephane Mallat*** : Developing the Idea of Multiresolution Analysis “MRA” for Discrete Wavelet Transform “DWT”.

1988 : The Modern Wavelet Theory with ***Daubechies*** and ***Mallat.***

1992 : ***Albert Cohen, Jean fauveaux*** and ***Daubechies*** constructed the compactly supported biorthogonal wavelets.

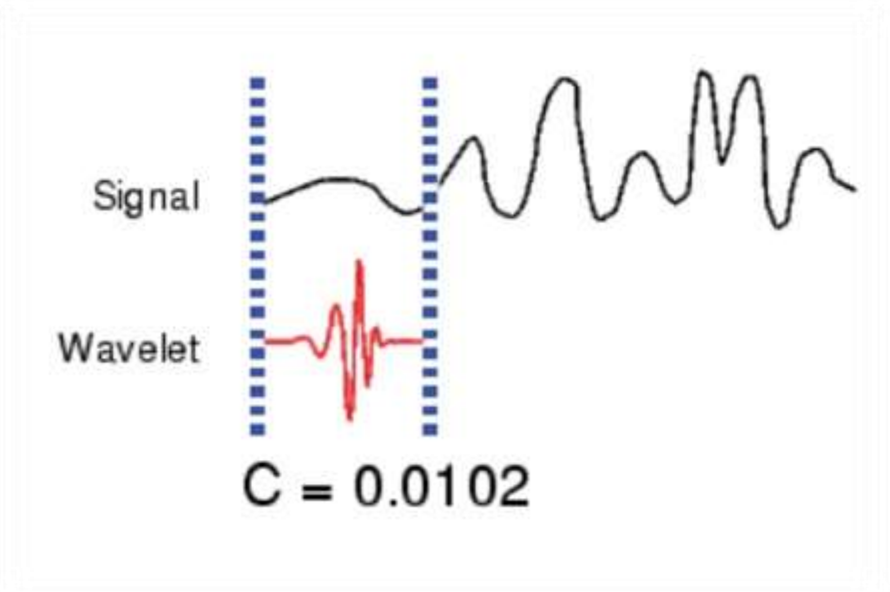
Here are some of the most popular mother wavelets :



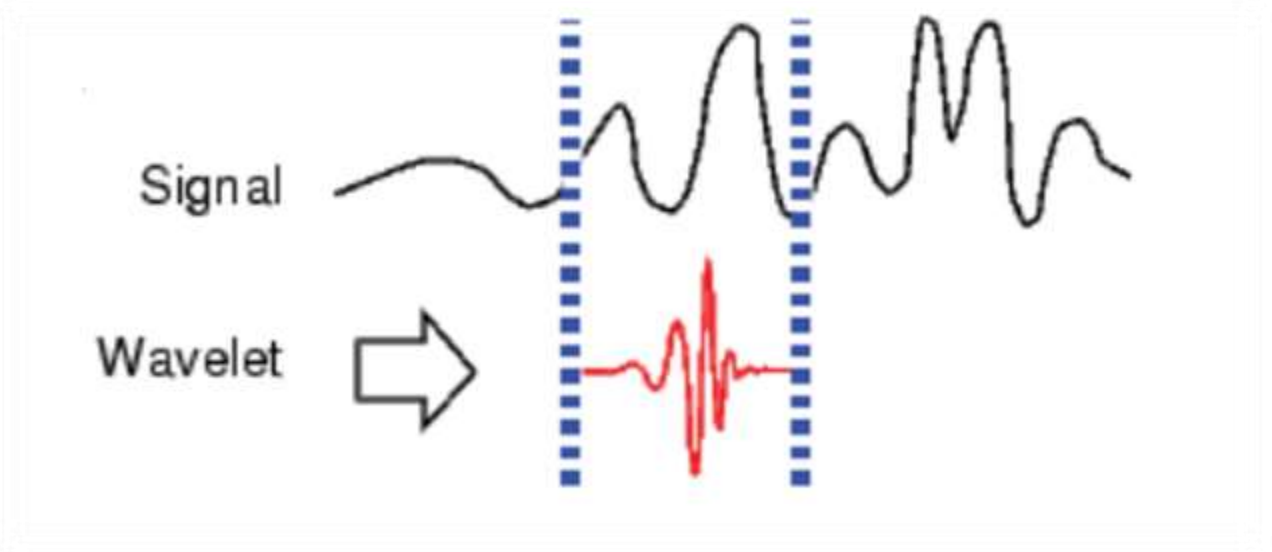
Steps to compute CWT of a given signal :

1. Each Mother Wavelet has its own equation

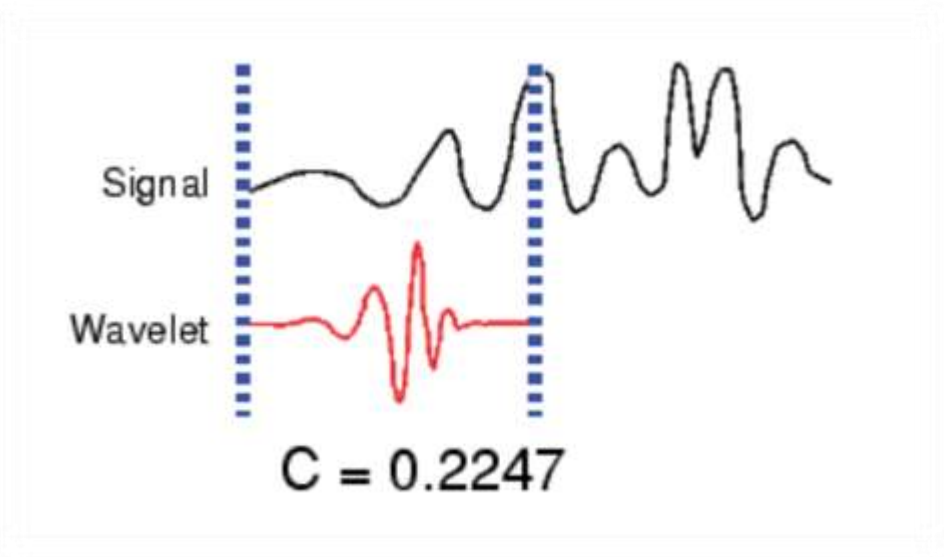
2. Take a wavelet and compare it to section at the start of the original signal, and calculate a correlation coefficient C.



2. Shift the wavelet to the right and repeat step 1 until the whole signal is covered.



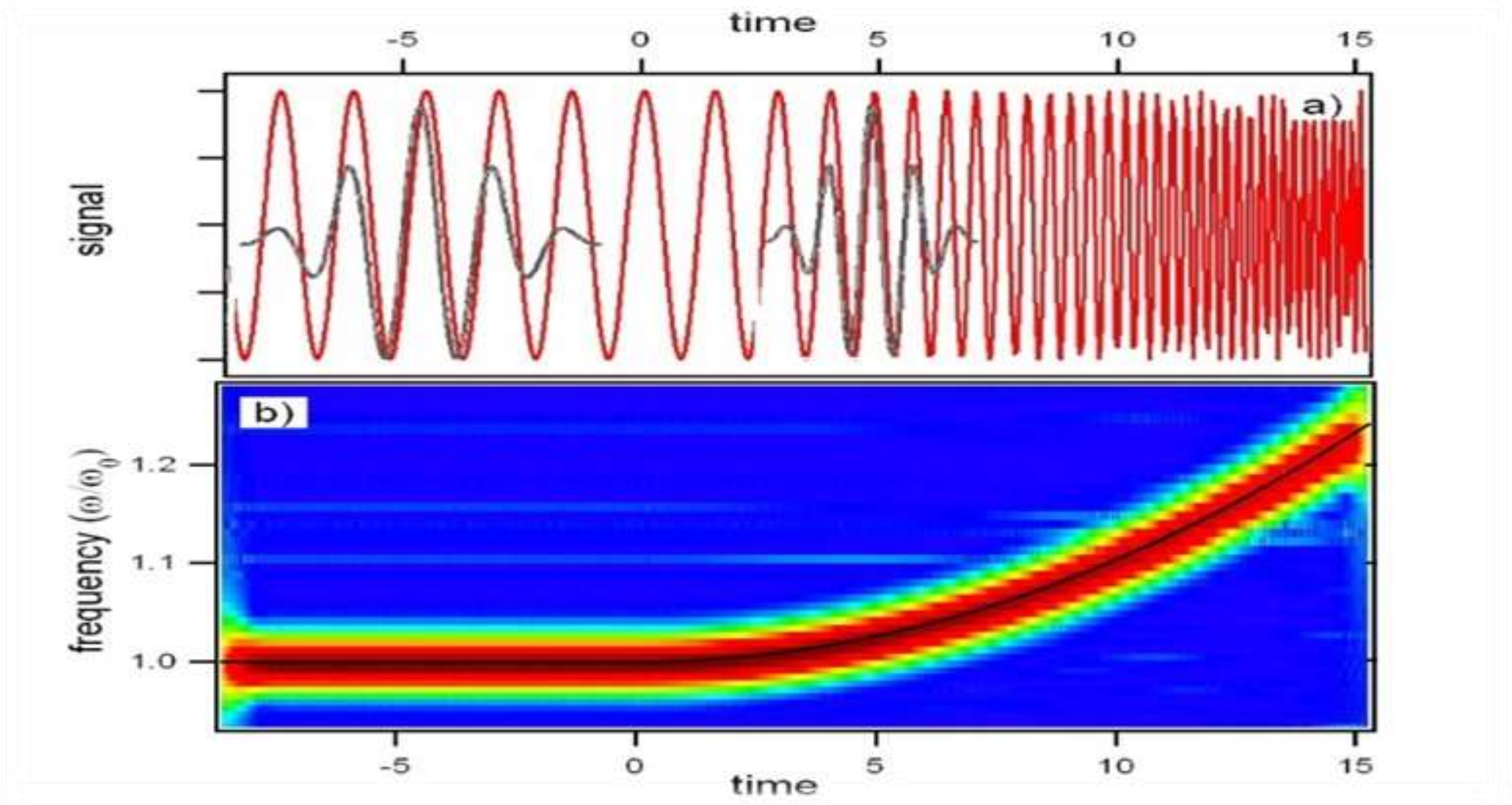
3. Scale (stretch) the wavelet and repeat steps 1 through 2.



4. Repeat steps 1 through 3 for all scales.

Haar transform

Time-frequency representation of « up-chirp » signal using CWT :



Applications:


DEPARTMENT OF JUSTICE
FEDERAL BUREAU OF INVESTIGATION
FIDELITY
BRAVERY
INTEGRITY
F.B.I. TERMINAL
This is a secured and monitored Federal Goverment system. Unauthorized access is strictly prohibited. All activity is fully monitored. Individuals who attempt to gain unauthorized access or attempt any modification of information on this sytem is subject to criminal prosecution. All persons who are hereby notified that use of this sytem constitutes concent to monitoring and auditing.

FBI Fingerprints Compression:

- ***Since 1924, the FBI Collected about 200 Million cards of fingerprints.***
- ***Each fingerprints card turns into about 10 MB, which makes 2,000 TB for the whole collection. Thus, automatic fingerprints identification takes a huge amount of time to identify individuals during criminal investigations.***

- The FBI decided to adopt a wavelet-based image coding algorithm as a national standard for digitized fingerprint records.
- The WSQ (Wavelet/Scalar Quantization) developed and maintained by the FBI, Los Alamos National Lab, and the National Institute for Standards and Technology involves:

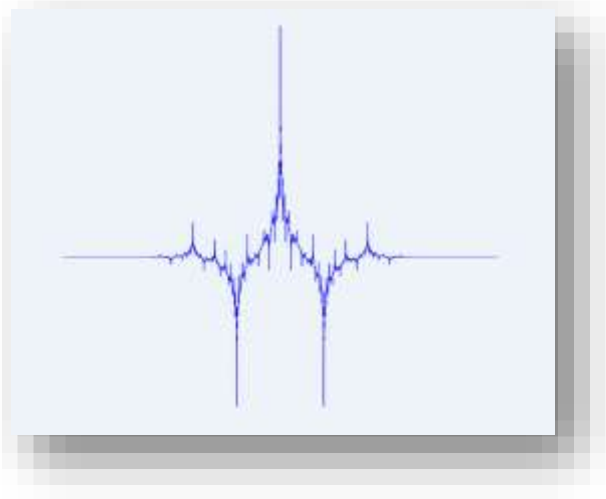
JPEG 2000

Image compression standard and coding system.

Created by Joint Photographic Experts Group committee in 2000.

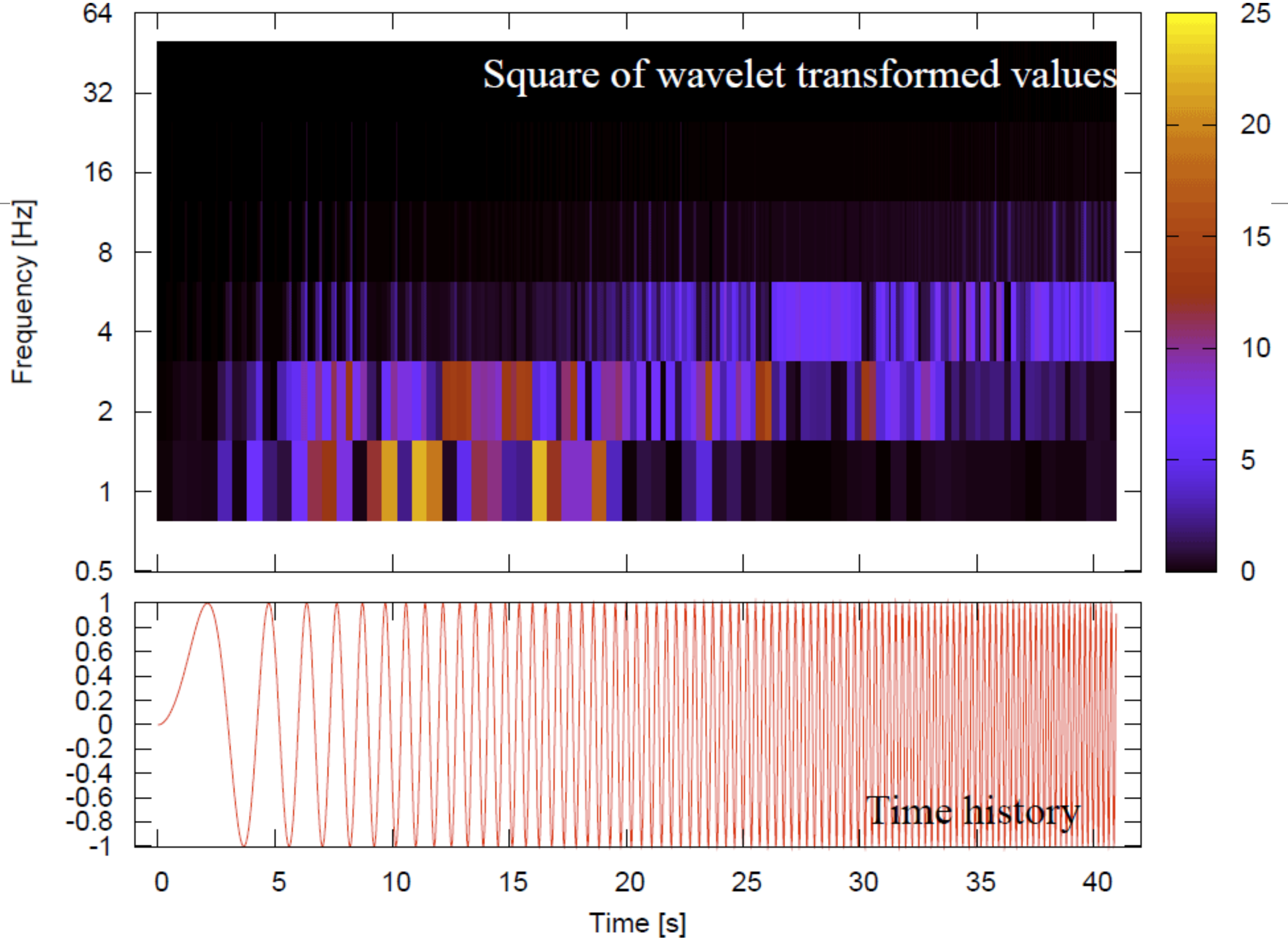
Wavelet based compression method.

1:200 compression ratio



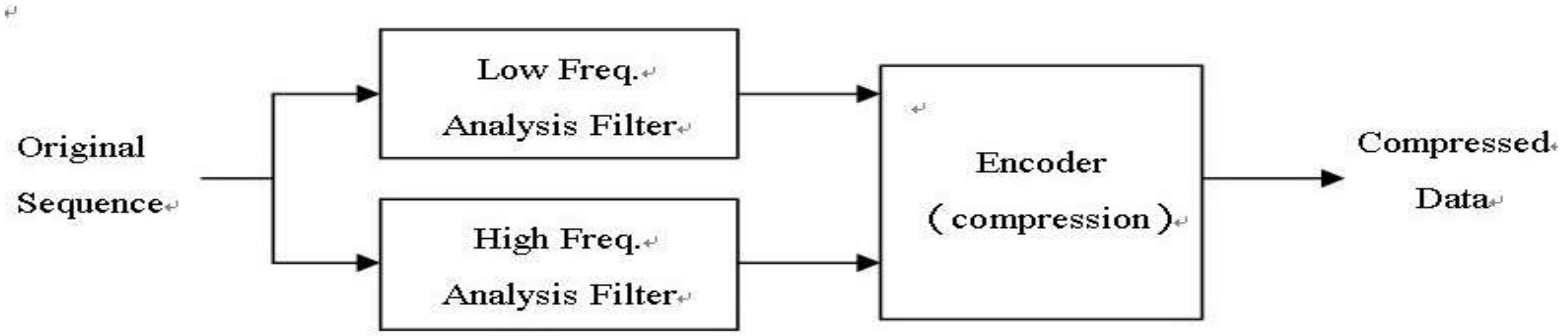
Mother Wavelet used in JPEG2000 compression

Comparison between JPEG and JPEG2000

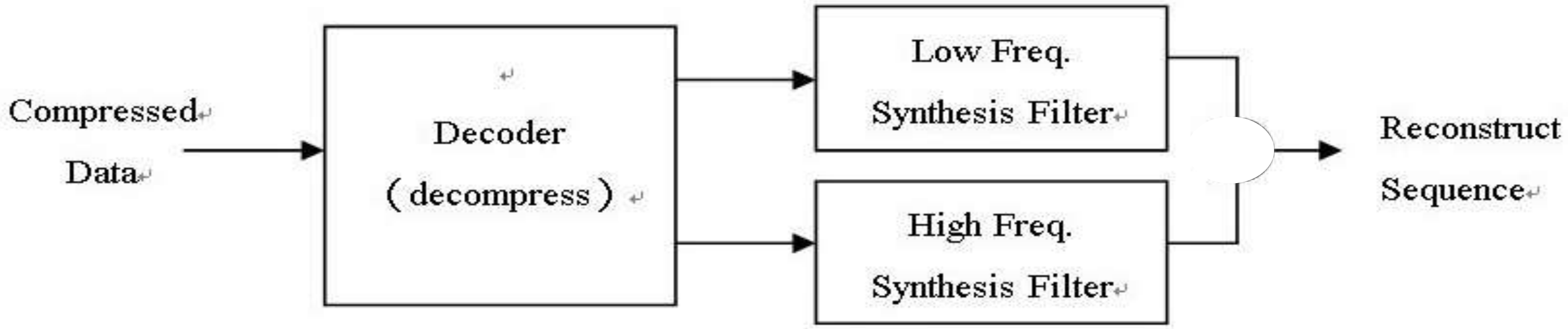


MRA time-frequency Representation of Chirp Signal

WT compression

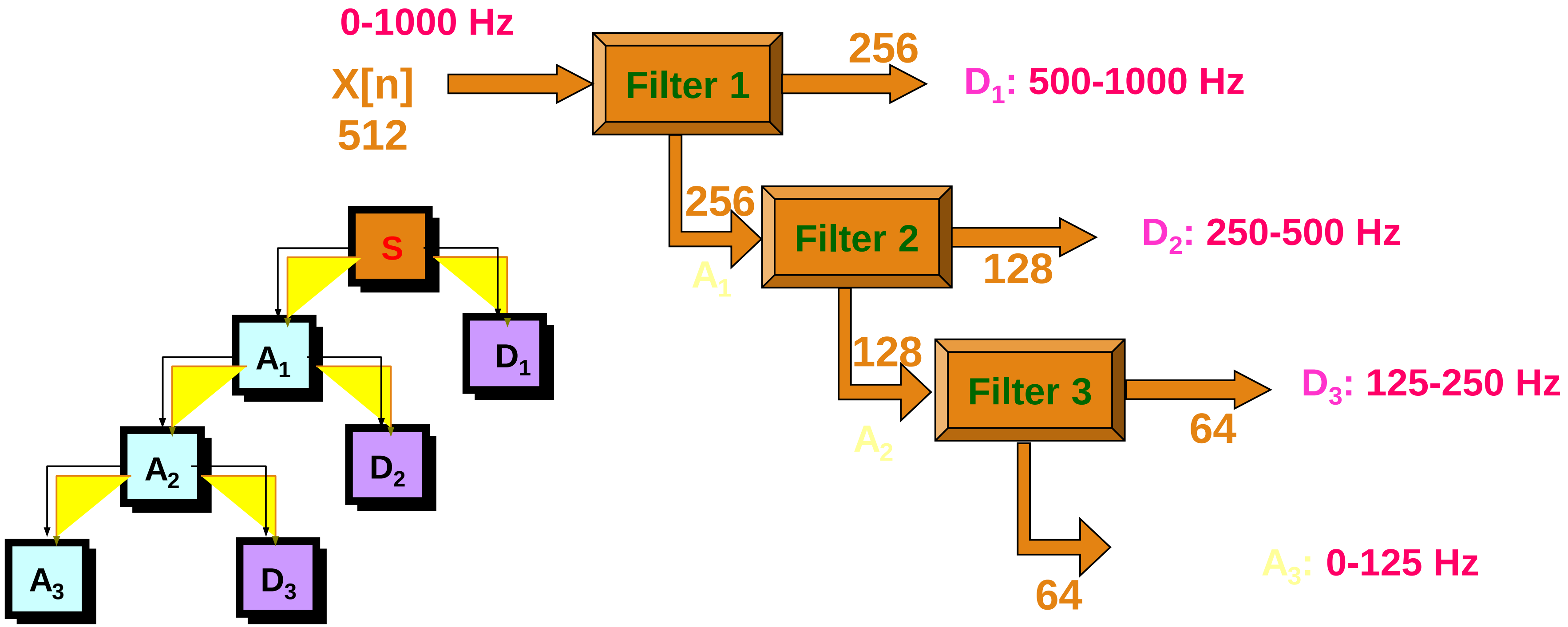


Wavelet Decomposition & Compression



Wavelet Decompression & Synthesis

SUBBABD CODING ALGORITHM



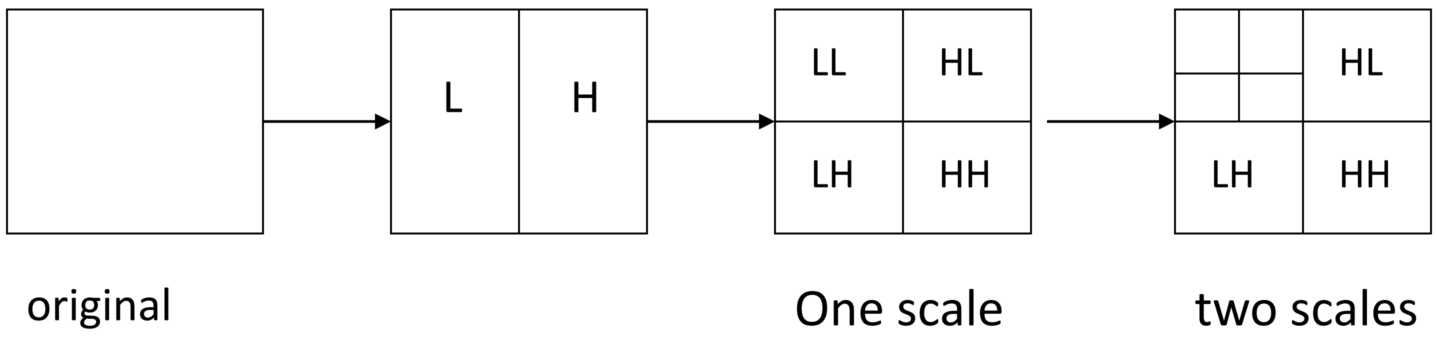
- 2-D Discrete Wavelet Transform
- A 2-D DWT can be done as follows:

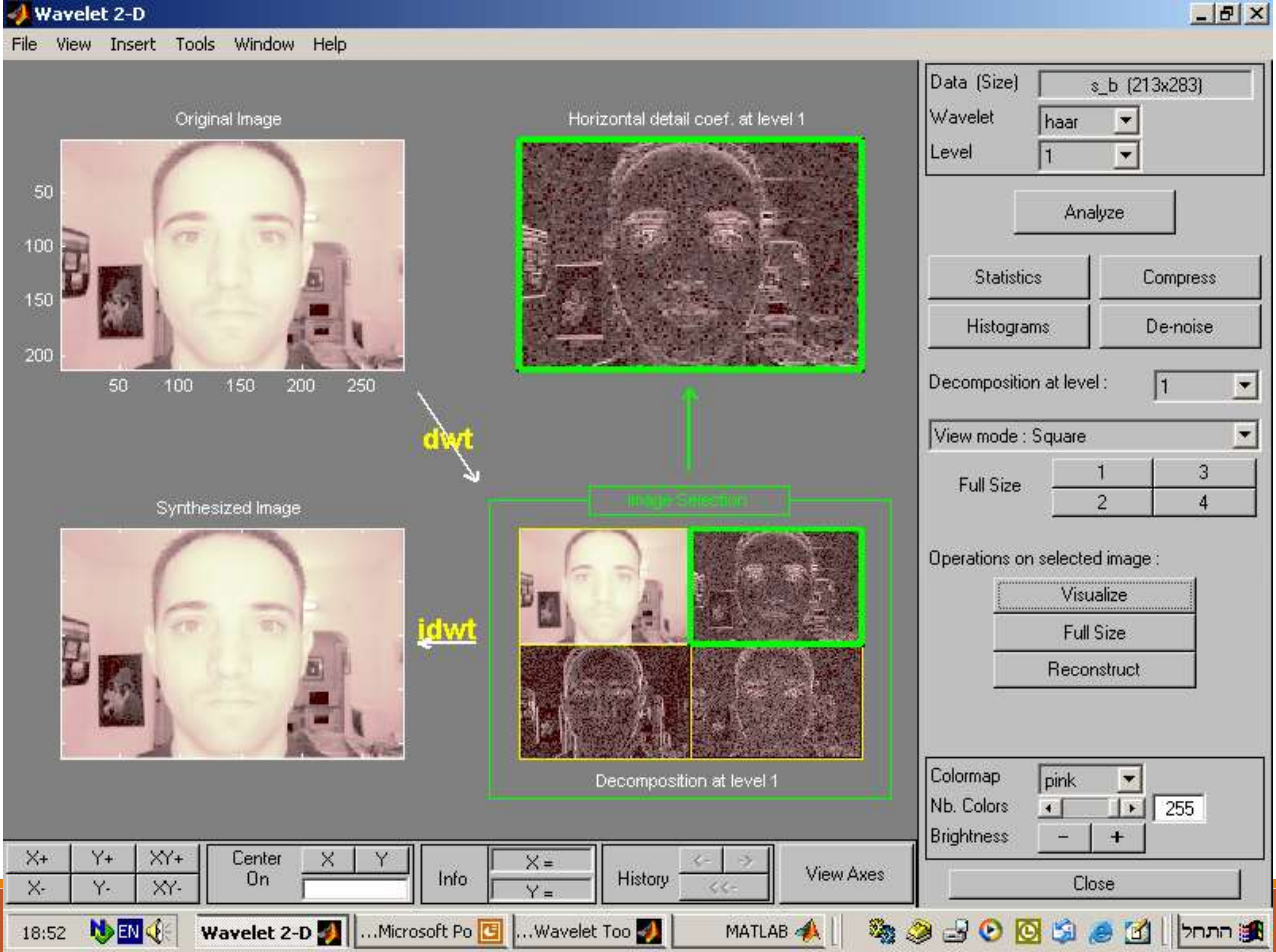
Step 1: Replace each row with its 1-D DWT;

Step 2: Replace each column with its 1-D DWT;

Step 3: repeat steps (1) and (2) on the lowest subband for the next scale

Step 4: repeat steps (3) until as many scales as desired have been completed




Wavelet 2-D
File View Insert Tools Window Help
Original Image
Horizontal detail coef. at level 1
dwt
idwt
Synthesized Image
Image Selection
Decomposition at level 1
Data (Size)
s_b (213x283)
Wavelet
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Statistics
Compress
Histograms
De-noise
Decomposition at level :
View mode : Square
Full Size
Operations on selected image :
Visualize
Full Size
Reconstruct
Colormap
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255
Brightness
Close
Center On
Info
History
View Axes
18:52
Wavelet 2-D
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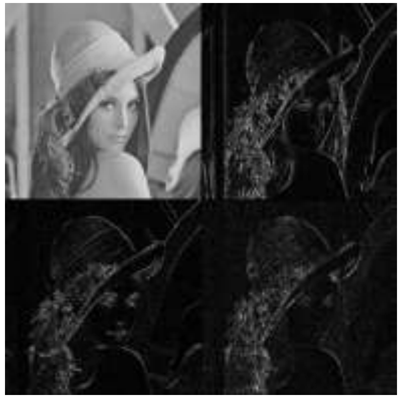
MATLAB

Why is wavelet-based compression effective?


Original


1 level Haar


1 level linear spline

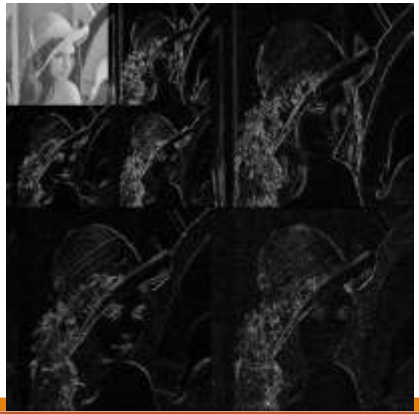

2 level Haar

Image at different scales



Why is wavelet-based compression effective?

- Coefficient entropies

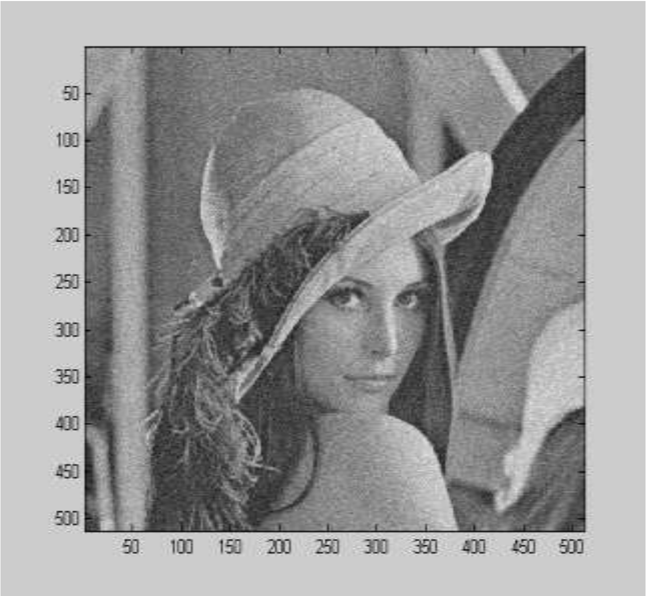
	Entropy
Original image	7.22
1-level Haar wavelet	5.96
1-level linear spline wavelet	5.53
2-level Haar wavelet	5.02
2-level linear spline wavelet	4.57

Introduction to image compression

- For human eyes, the image will still seems to be the same even when the Compression ratio is equal 10
- Human eyes are **less sensitive** to those **high frequency** signals
- Our eyes will **average fine details** within the small area and record only the overall intensity of the area, which is regarded as a **lowpass** filter.

Application: Image Denoising Using Wavelets

Noisy Image:



Denoised Image:

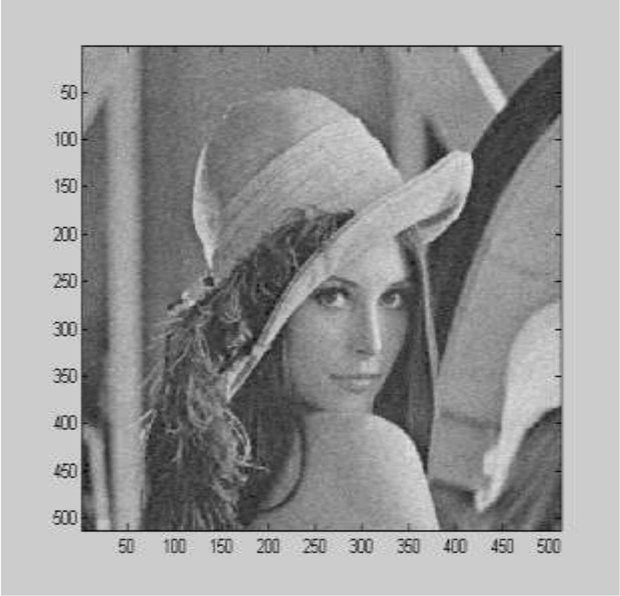


Image Denoising Using Wavelets

Calculate the DWT of the image.

Threshold the wavelet coefficients. The threshold may be universal or subband adaptive.

Compute the IDWT to get the denoised estimate.

Soft thresholding is used in the different thresholding methods. Visually more pleasing images.

Advantages of using Wavelets:

- Provide a way for analysing waveforms in both frequency and duration.
- Representation of functions that have discontinuities and sharp peaks.
- Accurately deconstructing and reconstructing finite, non-periodic and/or non-stationary signals.
- Allow signals to be stored more efficiently than by Fourier transform.

Wavelets can be applied for many different purposes :

Audio compression.

Speech recognition.

Image and video compression

Denoising Signals

Motion Detection and tracking

Conclusion :

- Wavelet is a relatively new theory, it has enjoyed a tremendous attention and success over the last decade, and for a good reason.
- Almost all signals encountred in practice call for a time-frequency analysis, and wavelets provide a very simple and efficient way to perform such an analysis.
- Still, there’s a lot to discover in this new theory, due to the infinite variety of non-stationary signals encountred in real life.

Questions?

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