Geometri $\mathbb{R}^2$ (planet)

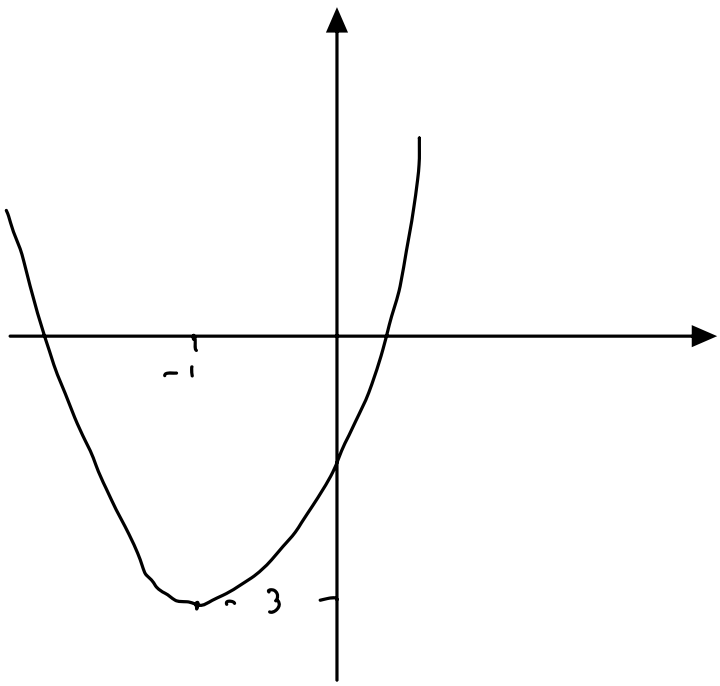
Ex: Rita kurvan

$$y = x^2 + 2x - 2$$

kvadratkomp

$$\Leftrightarrow y = (x+1)^2 - 1 - 2 = (x+1)^2 - 3$$

parabel



- ekvation i x o y motsvarar kurvor i planet.

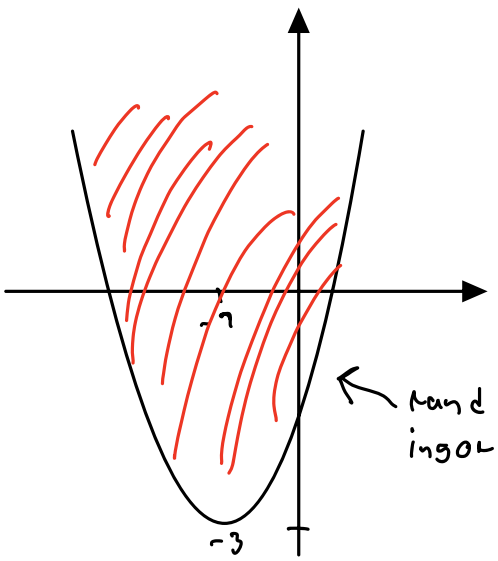
Ex:

Rita de mängden i $\mathbb{R}^2$ som ges av

a) $y \geq x^2 + 2x - 2$ b) $y < x^2 + 2x - 2$

---

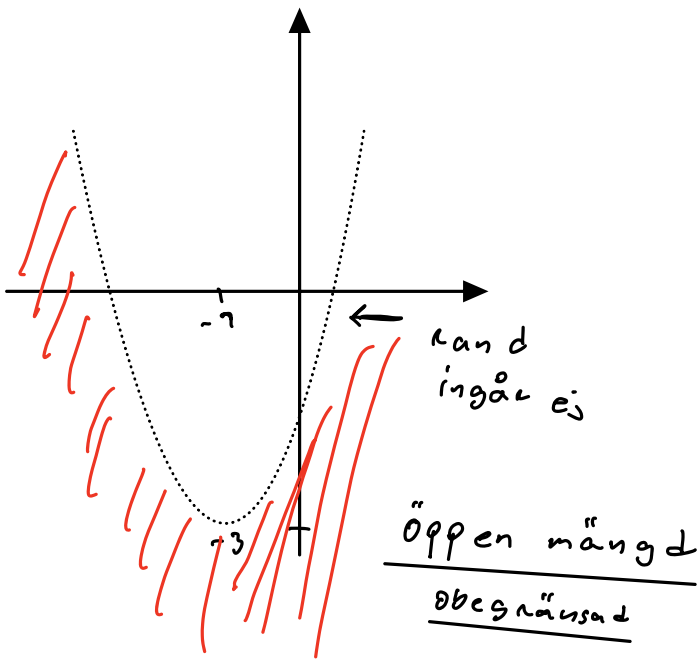
a) $y = (x+1) - 3$



Slutet mängd

Obegränsad

b)



Öppen mängd

Obegränsad

- Olikhet i x, y motsvarar område

Ex:

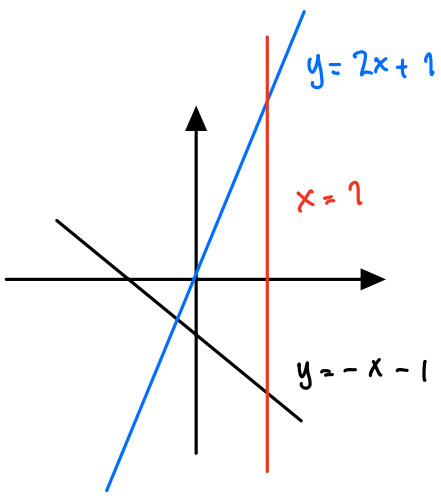
Rita området

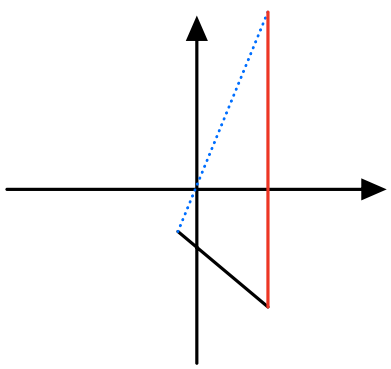
$$M = \{(x,y) : -x - 1 \leq y < 2x + 1,\ x \leq 1\}$$

Tre olikheten

$$-x - 1 \leq y,\quad y < 2x + 1,\quad x \leq 1$$

Rita randen först (dvs likhet):





Var ken öppen eller sluten.

begränsad

Elliptisk skiva

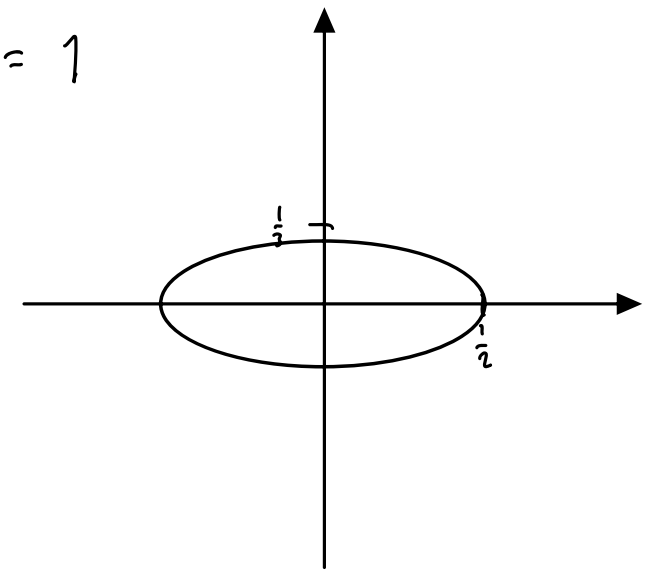
Ex:

Rita kurvan

$$4x^2 + 9y^2 = 1$$

$$4x^2 + 9y^2 = 1 \iff \frac{x^2}{1/4} + \frac{y^2}{1/9} = 1$$

$$\iff \frac{x^2}{(\frac{1}{2})^2} + \frac{y^2}{(\frac{1}{3})^2} = 1$$



ellips med halvaxlar $\frac{1}{2}$ & $\frac{1}{3}$, mp i origo

Alt:

Skär kurvan med koordinat axlarna

x-axeln: har ekv $y = 0$, insättning ger $4x^2 = 1 \iff x = \pm \frac{1}{2}$

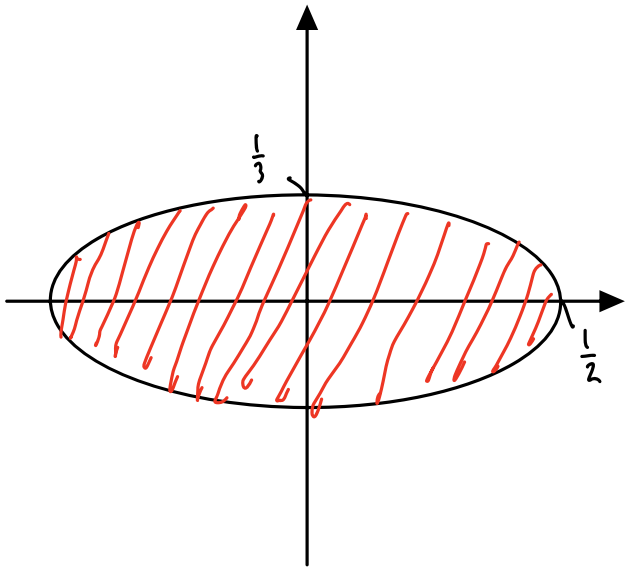
y-axeln: —"— $x = 0$, —"— $9y^2 = 1 \iff y = \pm \frac{1}{3}$

Ex:

Rita mängderna

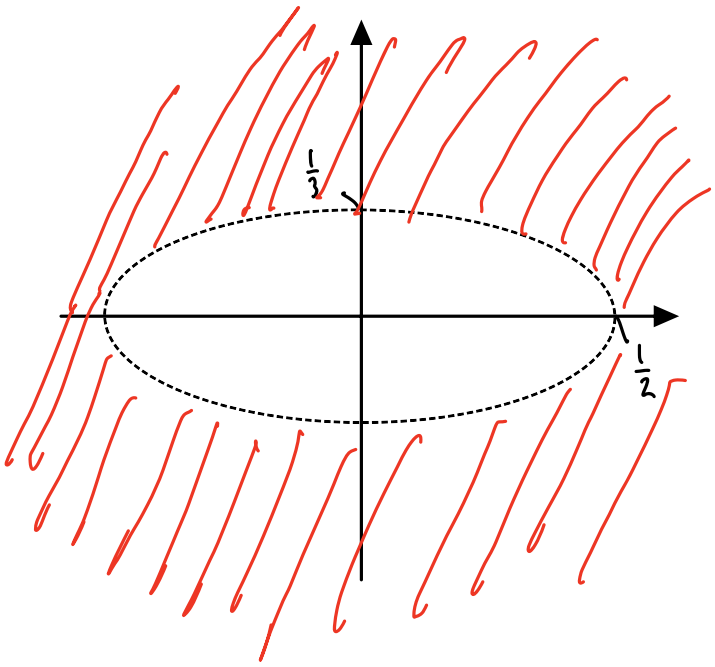
$M_1 = \{(x, y) : 4x^2 + 9y^2 \leq 1\}$

$M_2 = \{(x, y) : 4x^2 + 9y^2 > 1\}$



Kompakt mängd

Sluten och begränsan



öppen och obegränsad.

Absolut Belopp

Ex:

Rita området

$$M = \{(x, y) : |x - y| \leq 1,\ |y| \leq 3\}$$

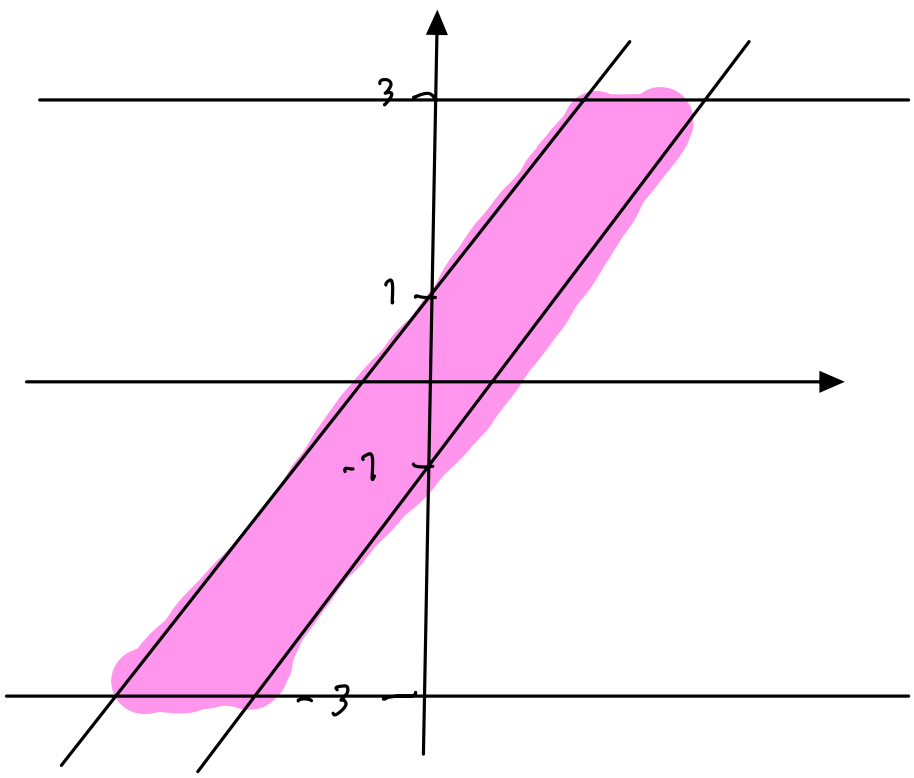
- $|y| \leq 3 \iff -3 \leq y \leq 3 \iff y \leq 3 \ \text{o} \ y \geq -3$

- $|x - y| \leq 1 \iff -1 \leq x - y \geq 1 \iff -1 \leq x - y \ \text{o}$
  $x - y \leq 1$

  $\iff y \leq x + 1 \ \text{o} \ y \geq x - 1$

Fyra olikheter

Rita randen.



Kompakt mängd

Geometri i Rummet, $\mathbb{R}^3$

Paraboloid

Ex: Rita alla punkter (x, y, z) i $\mathbb{R}^3$ som uppfyller

$$z = x^2 + y^2 \quad !$$

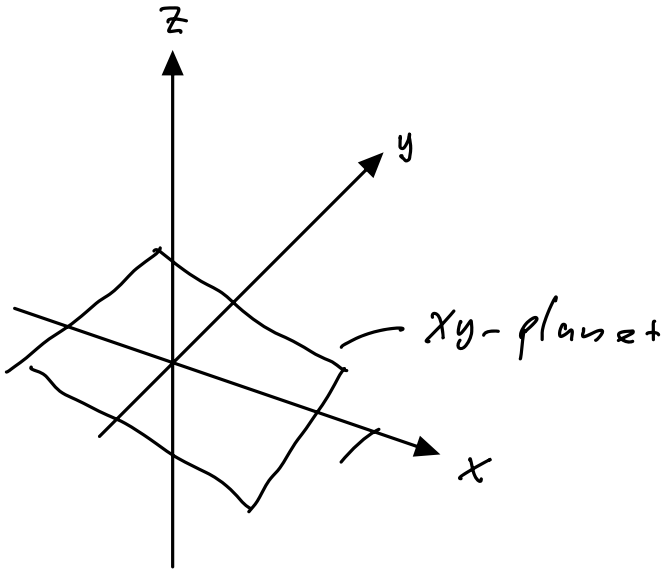
Skär med koordinat planen

xz-planet har ekv. $y = 0$

$\Rightarrow z = x^2$

yz-planet —"— $x = 0$

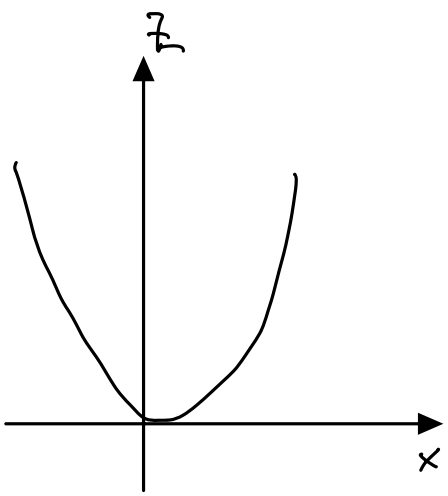
$\Rightarrow z = y^2$

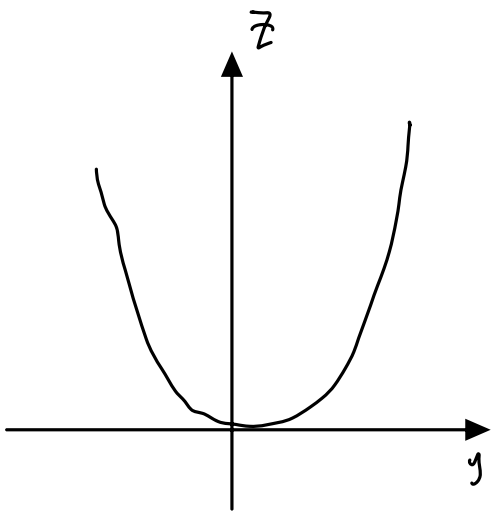


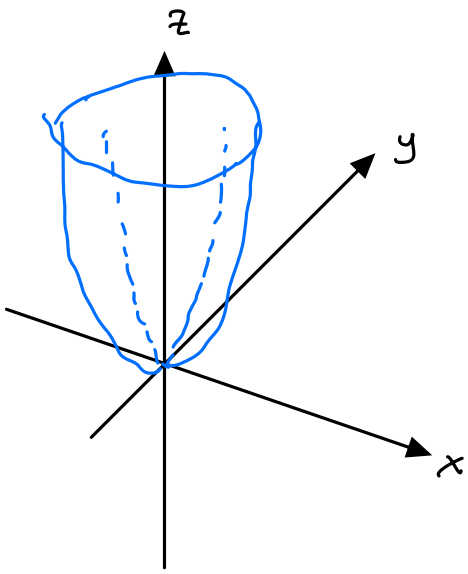
(xy-planet —"— $z = 0$

$\Rightarrow 0 = x^2 + y^2$, dvs, punkten $(0,0,0)$)

$z = x^2 + y^2$







paraboloid.

ekv i $x, y, z \rightarrow$ yta i rummet.

Allmänt:

Om $z = f(r)$ där $r = \sqrt{x^2 + y^2}$ ← avstånd till origo

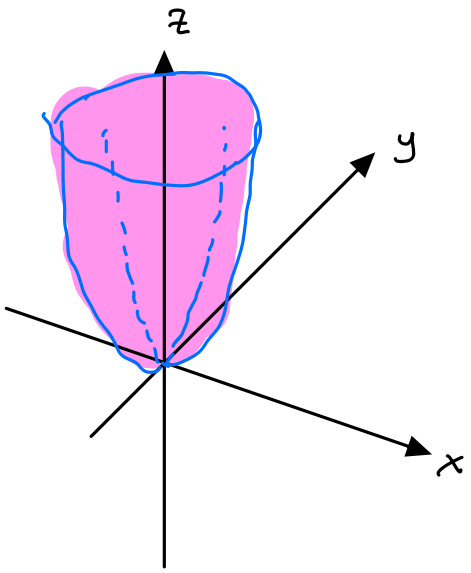
som t.ex ovan $z = x^2 + y^2 = \left(\sqrt{x^2 + y^2}\right)^2 = r^2$

så får vi en rotations symmetrisk yta

kring z - axeln

Ex:

Skissera mängden $M = \{(x, y, z) : z \geq x^2 + y^2\}$



Paraboloiden blir randen

Sluten mängd, obegränsad.

olikhet i $x, y, z \rightarrow$ kropp i rummet.

Kon

Ex:

Rita ytan $z = \sqrt{x^2 + y^2}$

$z = f(r) = r$

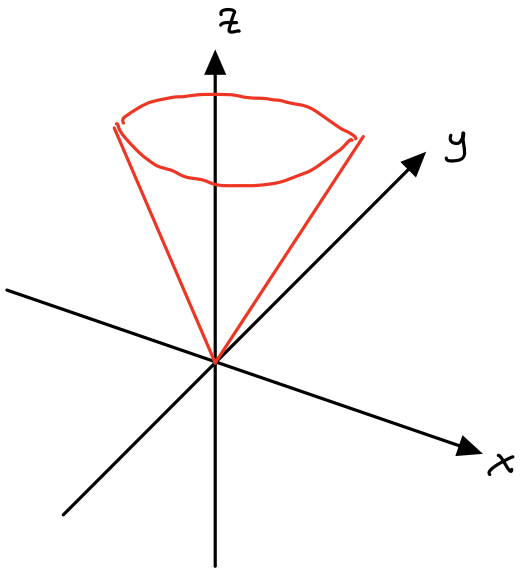
där $r = \sqrt{x^2 + y^2}$

Rotationssymmetrisk kring z-axeln

Skärning med t.ex xz-planet.

ekv $y = 0$

$\Rightarrow z = \sqrt{x^2} = |x|$



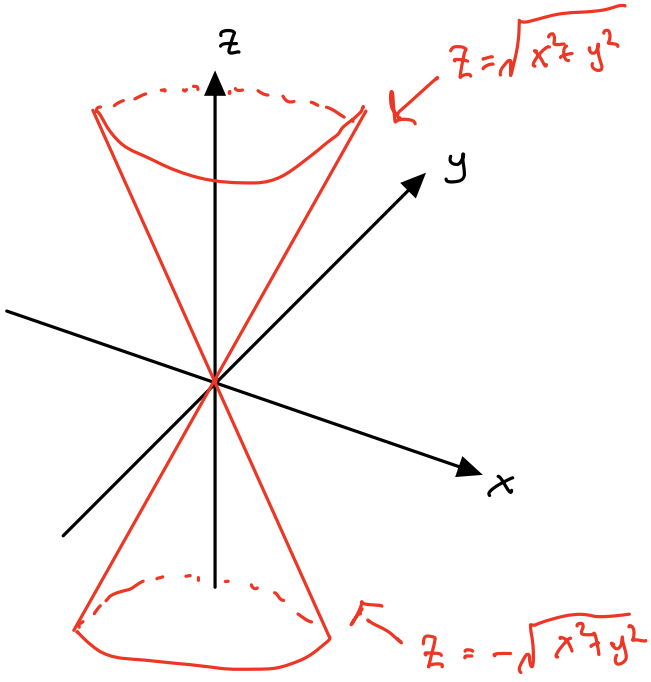
Ex:

Rita ytan $z^2 = x^2 + y^2$

$z^2 = x^2 + y^2 \iff z = \pm\sqrt{x^2 + y^2}$

"dubbelkon"

kägla



Sfär och klot

Ex:

Skissera mängden

$$M = \{(x, y, z) : x^2 + y^2 + z^2 < 4\}$$

Randen

$$x^2 + y^2 + z^2 = 4 = 2^2$$

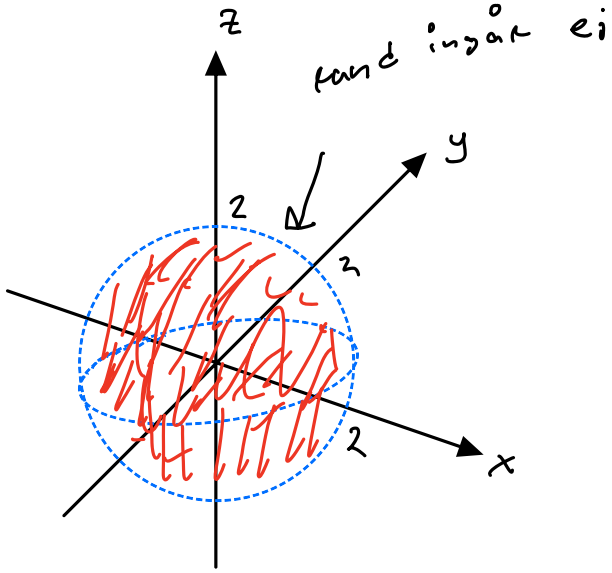
$\sqrt{x^2 + y^2 + z^2}$ = avstånd från (x, y, z) till origo

Sfären med radie 2 och medelpunkt i origo.

$$x^2 + y^2 + z^2 < 4$$

klot

öppen och begränsad mängd



Ellipsoid

Ex

Rita ytan

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} - \frac{x}{2} + \frac{2}{9}z - \frac{23}{36} = 0$$

Kvadrat komplettera

$$\frac{1}{4}(x^2 - 2x) + y^2 + \frac{1}{9}(z^2 + 2z) - \frac{23}{36} = 0$$

$\Leftrightarrow$

$$\frac{1}{4}((x-1)^2 - 1) + y^2 + \frac{1}{9}((z+1)^2 - 1) - \frac{23}{36} = 0$$

$$\Leftrightarrow \frac{(x-1)^2}{4} + y^2 + \frac{(z+1)^2}{9} \underbrace{- \frac{1}{4} - \frac{1}{9} - \frac{23}{36}}_{\frac{-9-4}{36} - \frac{23}{36} = -1} = 0$$

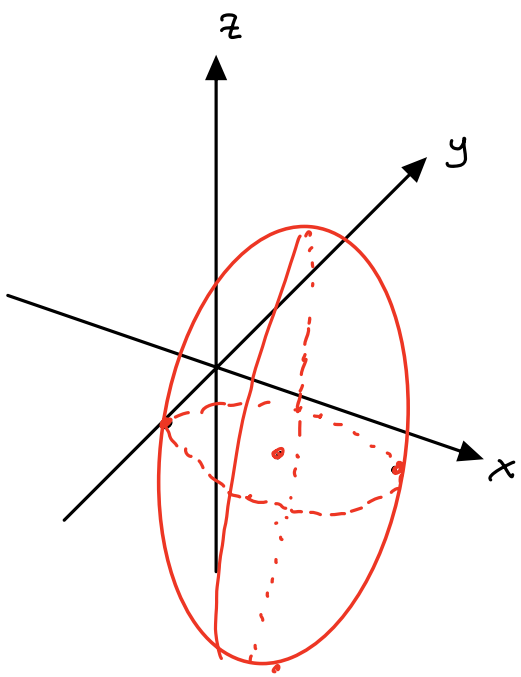
$\Leftrightarrow$

$$\left(\frac{x-1}{2}\right)^2 + y^2 + \left(\frac{z+1}{3}\right)^2 = 1$$

Ellipsoid!

Halvaxlarna 2, 1, 3

medelpunkt $(1, 0, -1)$



Hyperbolisk paraboloid

Ex:

Skissera ytan $z = x^2 - y^2$

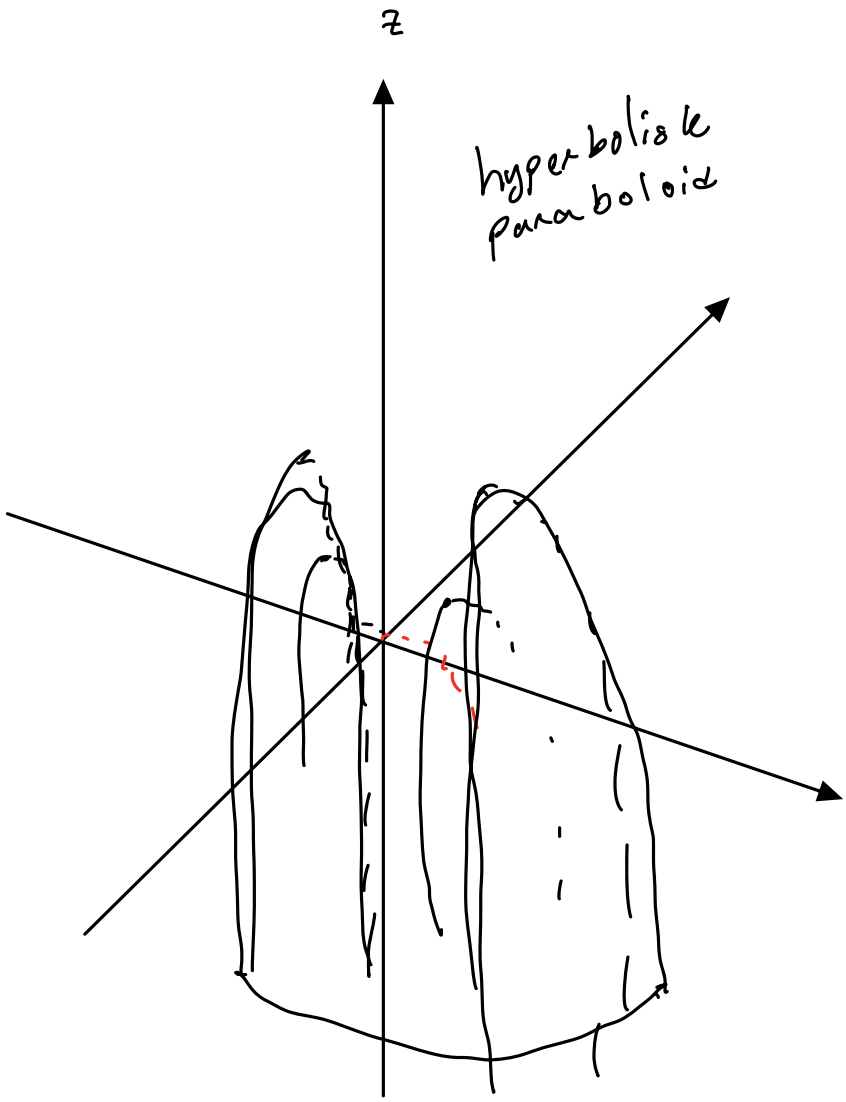
yz-planet: $z = -y^2$

xz-planet: $z = x^2$

$x = 1$: $z = 1 - y^2$

$x = -1$: $z = 1 - y^2$

$x = \pm 2$: $z = 4 - y^2$



Cylinder

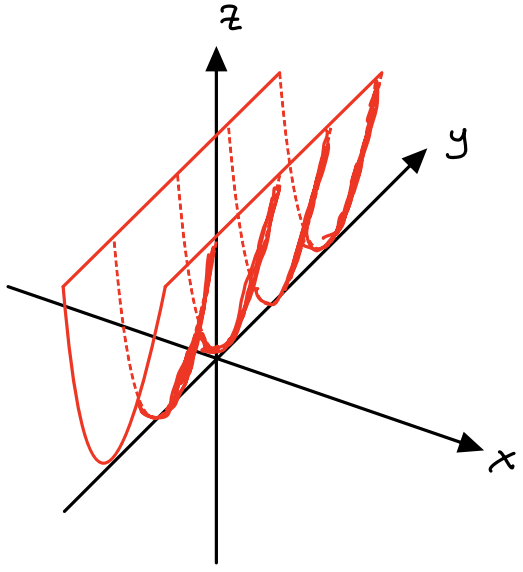
Ex:

Skissera ytan $z = x^2$!

$y = 0$: $z = x^2$

$y = 1$: $z = x^2$

$y = -1$: $z = x^2$



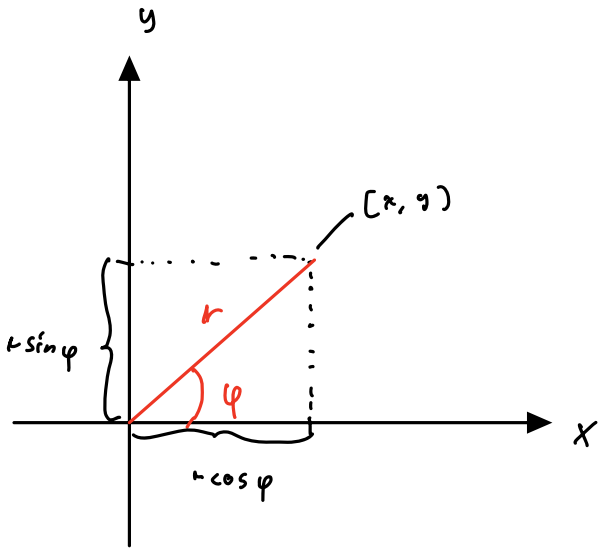
parabolisk cylinder

Polära koordinater

$(x, y) \longleftrightarrow (r, \varphi)$

$\varphi \in [0, 2\pi]$

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$$



Ex:

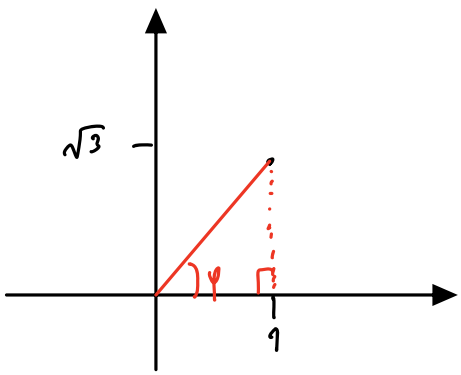
Punkten $(1, \sqrt{3})$

$r = \sqrt{1 + \sqrt{3}} = \sqrt{4} = 2$

$\varphi = \frac{\pi}{3}$

i polära koord.

$(2, \frac{\pi}{3})$

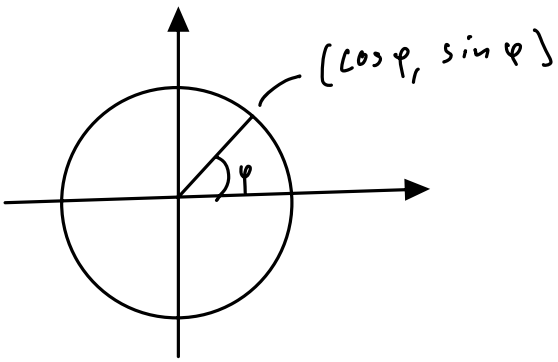


Ex:

Enhetscirkeln $x^2 + y^2 = 1$

kan beskrivas

$$\begin{cases} x = \cos\varphi \\ y = \sin\varphi \end{cases}$$

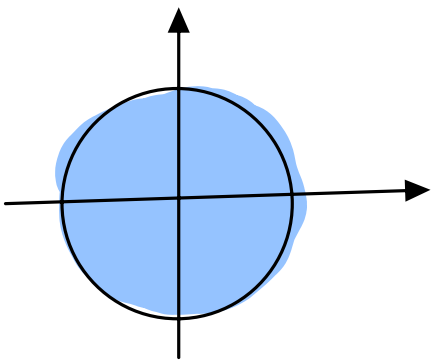


i pol.koord ges av $r = 1,\ 0 \le \varphi \le 2\pi$

Ex:

Enhetscirkelskivan $x^2 + y^2 \le 1$ ges av

$$\begin{cases} x = r\cos\varphi \\ \varphi = y\sin\varphi \end{cases}, \quad 0 \le r \le 1,\ 0 \le \varphi \le 2\pi$$



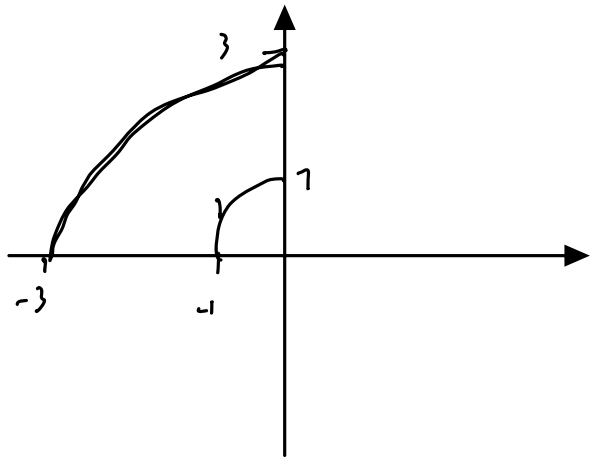
så i pol. koord ges denna av

$$\{(r, \varphi) : 0 \le r \le 1,\ 0 \le \varphi \le 2\pi\}$$

Ex:

Skitsering

$$1 \le x^2 + y^2 \le 9, \quad x \le 0, \quad y \ge 0$$



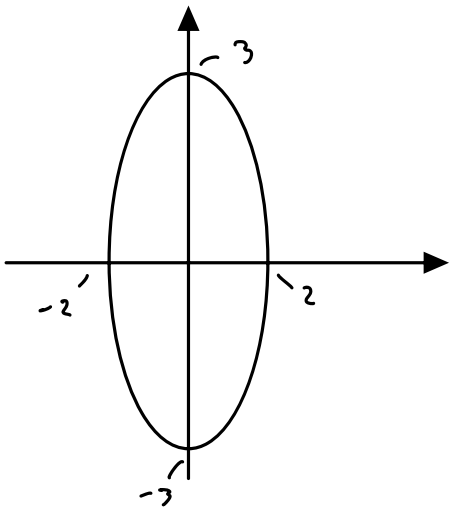
ges i polära koord

$$1 \le r \le 3, \quad \frac{\pi}{2} \le \varphi \le \pi$$

Ellipspolära koordinater

Ex:

Ellipsen $\frac{x^2}{2^2} + \frac{y^2}{3^2} = 1$



$\Leftrightarrow \left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$ går

att beskriva med

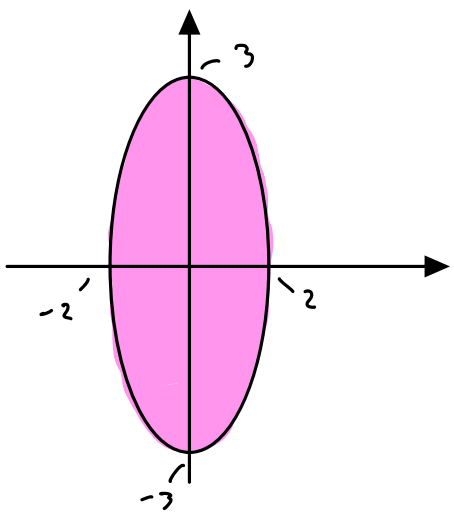
$$\begin{cases} \frac{x}{2} = \cos\varphi \\ \frac{y}{3} = \sin\varphi \end{cases}, \quad 0 \le \varphi \le 2\pi$$

dvs $\begin{cases} x = 2\cos\varphi \\ y = 3\sin\varphi \end{cases}, \quad 0 \le \varphi \le 2\pi, \quad r = 1$

Ex:

Ellipsskivan $\frac{x^2}{2^2} + \frac{y^2}{3^2} \le 1$

$$\begin{cases} x = 2r\cos\varphi \\ y = 3r\sin\varphi \end{cases}, \quad 0 \le r \le 1, \quad 0 \le \varphi \le 2\pi$$



Ex:

Beskriv ellipsskivan

$$4x^2 + 9y^2 + 8x - 36y + 39 \leq 0$$

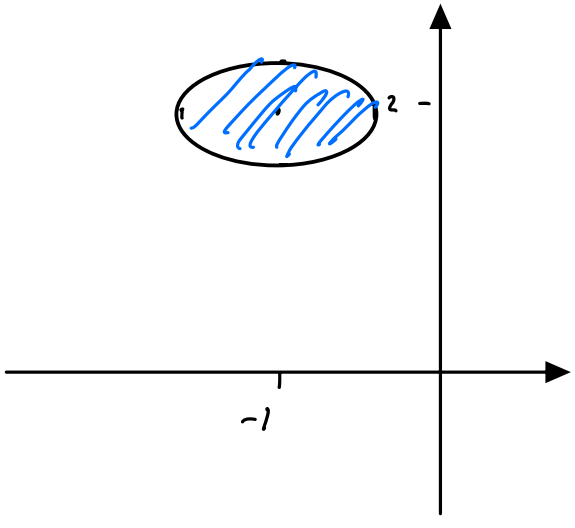
i ellipspolära koord.

$$4(x^2+2x) + 9(y^2-4y) + 39 \leq 0$$

$$4(x+1)^2 + 9(y-2)^2 + 39 - 4 - 36 \leq 0$$

$$4(x+1)^2 + 9(y-2)^2 \leq 1$$

$$\frac{(x+1)^2}{(1/2)^2} + \frac{(y-2)^2}{(1/3)^2} \leq 1$$



$$\begin{cases} \dfrac{x+1}{1/2} = r\cos\varphi \\ \dfrac{y-2}{1/3} = r\sin\varphi \end{cases} \qquad 0 \leq r \leq 1, \quad 0 \leq \varphi \leq 2\pi$$

Svar:

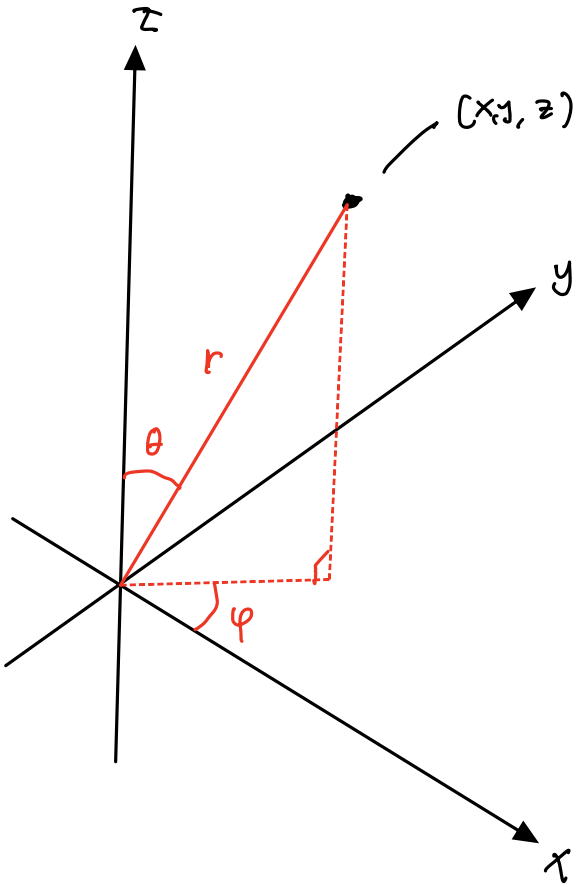
$$\begin{cases} x = -1 + \dfrac{r}{2}\cos\varphi \\ y = 2 + \dfrac{r}{3}\sin\varphi \end{cases} \qquad 0 \leq r \leq 1, \quad 0 \leq \varphi \leq 2\pi$$

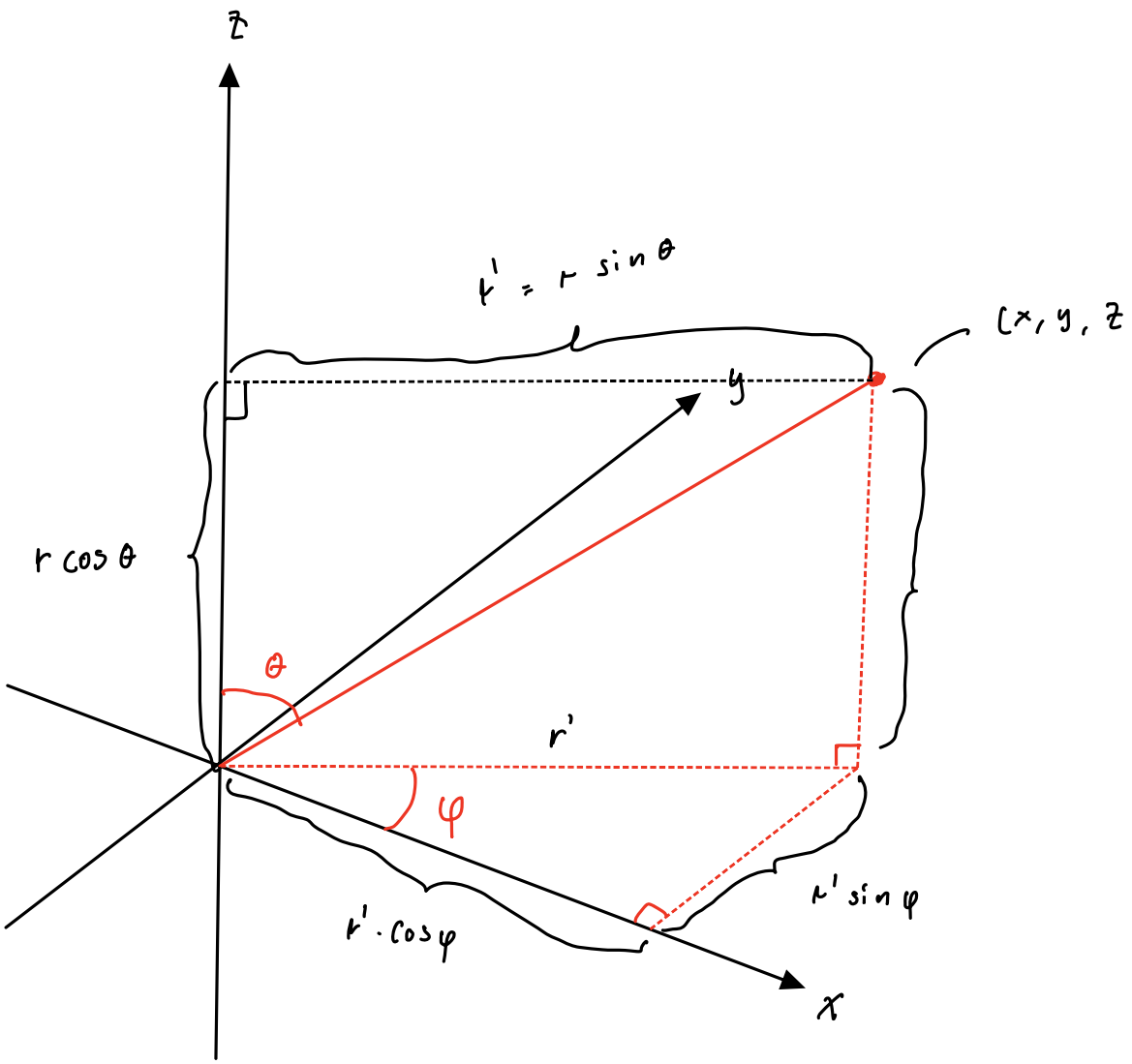
Rymdpolära koordinater

$(x, y, z) \leftrightarrow (r, \theta, \varphi)$

$0 \leq \varphi \leq 2\pi$

$0 \leq \theta \leq \pi$





$$\begin{cases} x = r' \cos\varphi = r \sin\theta \cos\varphi \\ y = r' \sin\varphi = r \sin\theta \sin\varphi \\ z = r \cos\theta \end{cases}$$

$0 \le \theta \le \pi$

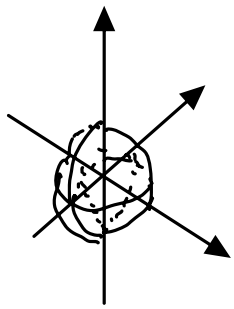
$0 \le \varphi \le 2\pi$

Ex

Sfären med radie 2, m.p i origo

i rymd. pol. koord. av $r = 2$, $0 \le \theta \le \pi$,

$0 \le \varphi \le 2\pi$



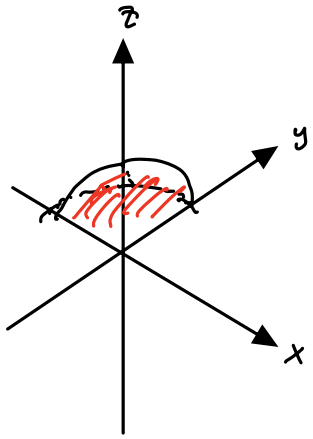
Klotet — " — $0 \le r \le 2$, $0 \le \theta \le \pi$,

$0 \le \varphi \le 2\pi$

Ex:

Rita mängden med rymd. pol. koord

$0 \le r \le 1$, $0 \le \theta \le \frac{\pi}{2}$, $\frac{\pi}{2} \le \varphi \le \pi$



Ellipsoiden

$$\begin{cases} x = \alpha + a \cdot r \cdot \sin\theta \cdot \cos\varphi \\ y = \beta + b \cdot r \cdot \sin\theta \cdot \sin\varphi \\ z = \gamma + c \cdot r \cos\theta \end{cases}$$

a, b, c beskriver halv axlarnas storlek

α, β, γ beskriver medelpunkten

$0 \le r \le 1$, $0 \le \theta \le \pi$, $0 \le \varphi \le 2\pi$