Homework 1: Direction of Arrival estimation

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Introduction

In this homework assignment, we will estimate the angular position of a target using a Uniform Linear Array. This involves each component transmitting a radar signal and capturing the reflected echo from the intended target. Through the analysis of these received echoes, we can precisely determine the angular location of the target.

To achieve this, we must design the ULA, comprising nine antennas with appropriate spacing between the elements, positioned around a single target situated at an angular position θ. Each antenna is responsible for transmitting a signal and receiving the echo back with specified characteristics.

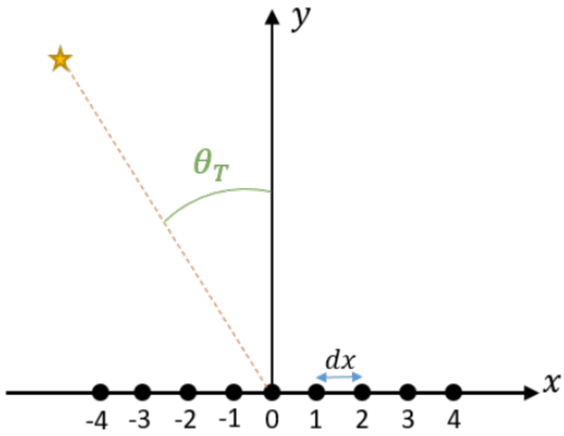

Figure 1: System geometry

1 Direction of Arrival estimation

In this section of the homework, we will design and simulate a ULA able to detect the angular position θ_T of the target in Figure 1 with a resolution at the boresight ($\theta = 0$) of 2 degrees.

1.1. System model

In this section, our aim is to formulate the model for the signal received by the nth antenna. Given that the signal undergoes perfect demodulation at the receiver, our objective is to linearize the phase term of the received signal with respect to variable x around the central element. In accordance with the provided question, the signal transmitted by the nth antenna is:

$$S_{TX} = g\,(t) = e^{j2\pi f_0 t}$$

The received signal by the n-th antenna is:

$$S_{RX} = g\,(t-\tau) = e^{j2\pi f(t-t0)}$$

which t_0 is $\frac{d+n.dx.\sin\theta}{c}$ as shown in the figure below and d is the distance between target and central antenna.

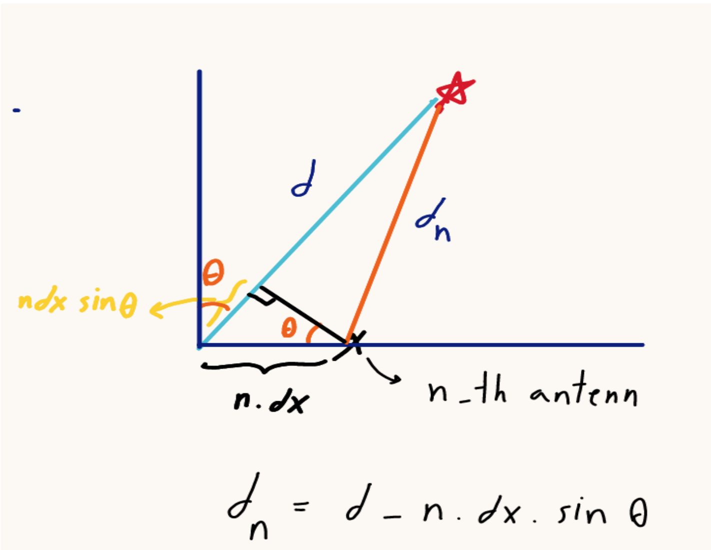

Figure 2: System geometry

Also, for demodulating the signal, we multiply it by $e^{-j2\pi f_0 t}$. Therefore, the received signal will be:

$$S_{RX} = e^{-j2\frac{\pi}{\lambda}d}.e^{-j2\frac{\pi}{\lambda}ndxsin(\theta)}$$

As we can see, the phase difference between antenna is:

$$\Delta\varphi = 2.\frac{\pi}{\lambda}.n.dx.sin(\theta)$$

Therefore, we can calculate the DoA using phase differential.

$$\theta = \sin^{-1}\frac{\Delta\varphi.\lambda}{2.\pi.n.dx}$$

1.2. Array design and DoA estimation

In this section we will design an ULA and propose a method for DoA estimation.

To avoid aliasing when sampling the signal, the spacing between antennas should be less than or equal to half the wavelength of the signal:

$$\lambda = \frac{c}{f} = \frac{3.10^8}{77.10^9} = 3.896 \text{ mm}$$

$$dx \leq \frac{\lambda}{2} = \frac{3.896}{2} = 1.948 \text{ mm}$$

Now we can detect DoA by the following method:

One of the ways to detect the direction of arrival is Phase Comparison Technique which involves measuring the phase differences between signals received by different antennas in an array. By analyzing these differences, we can calculate angles of arrival for incoming signals:

$$\theta = \sin^{-1}\frac{\Delta\varphi.\lambda}{2.\pi.n.dx}$$

To achieve the angular resolution of 2 degrees, the array length L must satisfy the following relationship:

$$L = \frac{\lambda}{\theta} = \frac{3.896\,mm}{0.0349} = 111.6\,mm$$

So, the number of antennas will be:

$$N = \frac{L}{dx} + 1 = 58$$

2 MATLAB implementation

Now that the design is ready, it is time to implement it in MATLAB. After setting up initial parameters and ULA system, we use Phase differential method to determine the DoA and we can see that estimated DoA is equivalent to the true DoA, as shown in the figures below:

```
True Angles: 37.50 degrees
Estimated Angles: 37.50 degrees
```

Figure 3: estimated target

Now we repeat the simulation with two targets located on 37 and 39 degrees and we can see that the result is accurate:

```
True Angles: 37.00 degrees
True Angles: 39.00 degrees
Estimated Angles: 37.00 degrees
Estimated Angles: 39.00 degrees
```

Figure 4: two targets

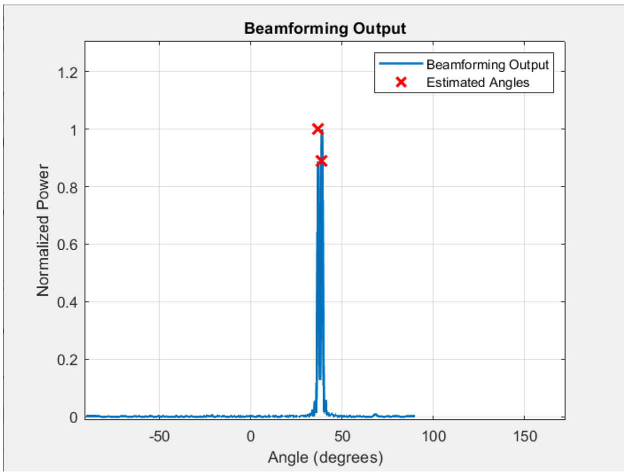

Figure 5: Beamforming output

For targets located on 37 and 38 degrees, the estimation is not correct, and only one target is detected, so the resolution is 2 degrees.

```
True Angles: 37.00 degrees
True Angles: 38.00 degrees
Estimated Angles: 37.60 degrees
```

Figure 6: DoA estimation with two targets

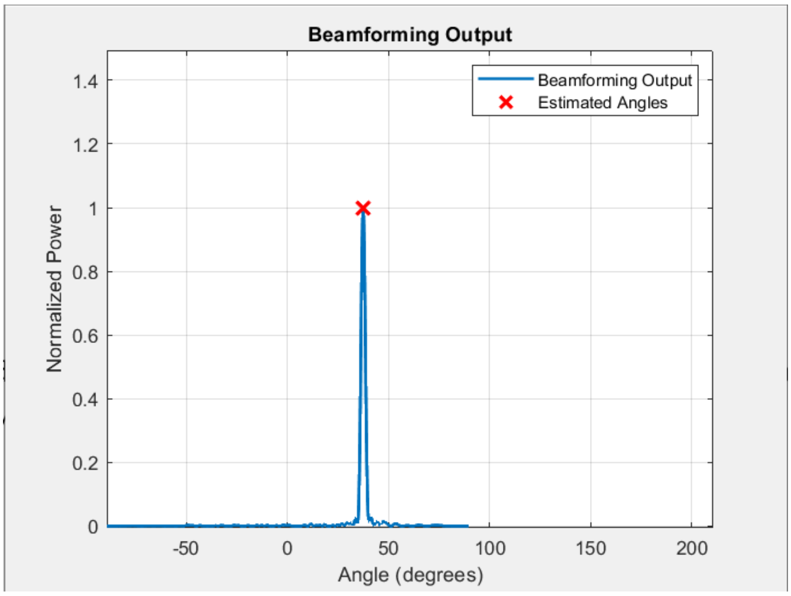

Figure 7: Target detection

Increasing dx beyond $\frac{\lambda}{2}$ introduces grating lobes, causing ambiguity in angle estimation. We can see that the estimation for $dx = \lambda$ is:

```
True Angle: 50.00 degrees
Estimated Angle: -13.53 degrees
```

Figure 8: effect of increasing dx

Which is not accurate.

3 2D position estimation

In the previous section, the position estimation is just angular, meaning that the system is not able to estimate the distance between the array and the target (in radar jargon, this distance is called range). By changing the transmitted signal, it is possible to estimate the 2D position of the target (i.e., both in range and angle).

3.1. System model

The transmitted signal in the time domain is now a cardinal sin function with a bandwidth of 1 GHz. Such a signal is modulated around a central frequency of 77 GHz as the previous one. The model of the transmitted signal by each antenna is then:

$$g(t) = sinc(Bt).\, e^{j2\pi f_0 t}$$

The received signal at the n-th antenna is:

$$r_n(t) = sinc\,[B\,(t-\tau)]e^{j2\pi f_0(t-\tau)}e^{-jknd_x \sin\theta}$$

Which τ is the round-trip time delay:

$$\tau = \frac{2r}{c}$$

And k is the wavenumber:

$$k = \frac{2\pi f_0}{c}$$

The equation of the received signal after demodulation will be:

$$r_n(t) = sinc\,[B\,(t-\tau)]e^{-j2\pi f_0 \tau}e^{-jknd_x \sin\theta}$$

Now we compute time and space resolution of the signal. The time resolution of a signal is inversely proportional to its bandwidth

$$\Delta t = \frac{1}{B} = \frac{1}{1 \times 10^{9}} = 1 \times 10^{-9}\, s$$

The space resolution is related to the time resolution and speed of the light:

$$\Delta r = \frac{c.\Delta t}{2} = \frac{3 \times 10^{8} \times 1 \times 10^{-9}}{2} = 0.15\, m$$

3.2. MATLAB Implementation

Now we implement MATLAB code by defining initial parameters N, λ, d_x, B and antennas position. Then, we generate the transmitted signal and simulate received signal according to the formulas described in the previous section.

For range estimation, we perform cross correlation between demodulated received signal and reference signal. The result for range estimation is shown in the following picture:

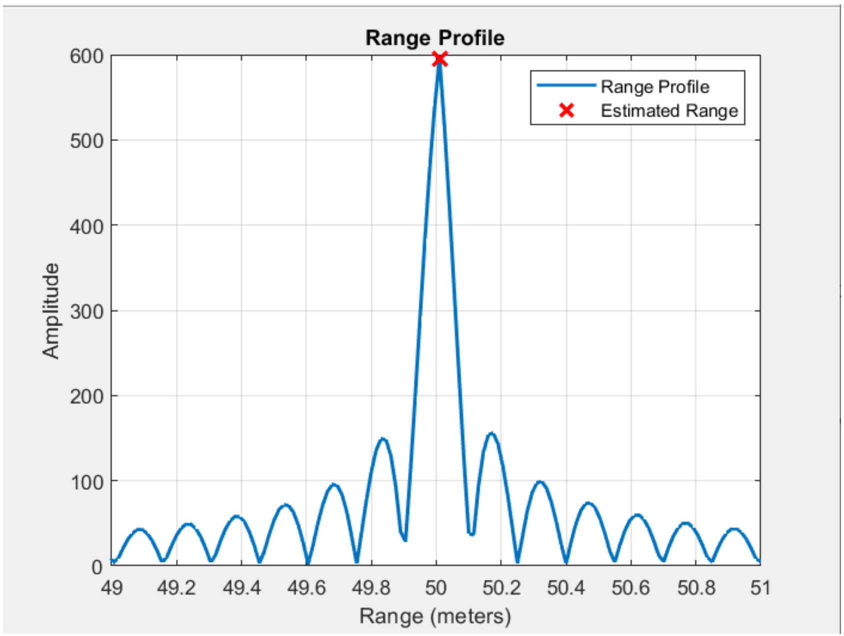

Figure 9: Range estimation

As we can see, the range is estimated accurately, and the estimated range is 50.01 m which is almost the same as true range.

Phase across the antennas at estimated range is shown in the figure below:

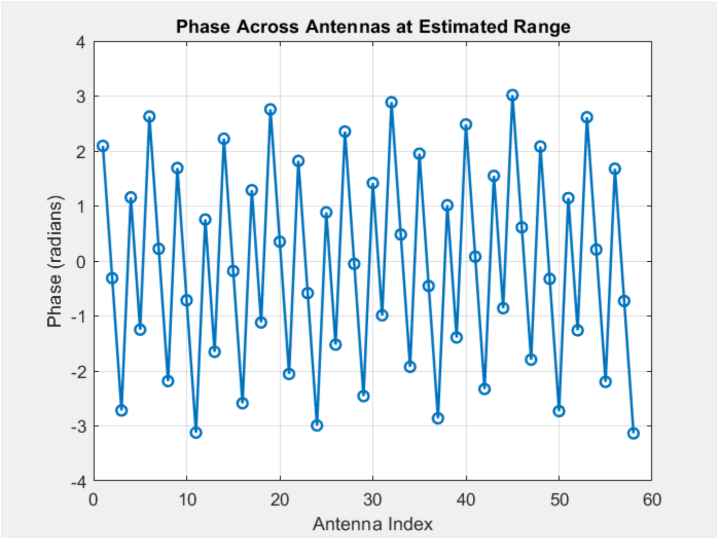

Figure 10: Phase across antennas at estimated range

Then, by computing the phase differential between the adjacent antennas, we estimate the angle of arrival.

delta_phi = angle(signal_at_tau(2: end) . conj(signal_at_tau(1:end-1)))*

Finally, we print the true and estimated angle of arrival and range:

```
True Range: 50.00 meters
Estimated Range: 50.01 meters
True Angle: 50.00 degrees
Estimated Angle: 50.00 degrees
```

Figure 11: results

The result for angle is equal to the true angle of arrival.

4 MIMO array

In real-world systems, arrays are frequently implemented using Multiple-Input Multiple-Output (MIMO) radar technology. In the scenario previously described the N antennas both transmit and receive signals, a configuration known as monostatic. In contrast, a MIMO system consists of NTx transmitting antennas and NRx receiving antennas.

4.1. system design

In this system, the number of virtual antennas is 58, so we set the number of transmit antennas equal to 2 and the number of receive antennas equal to 29.

$$N_{Tx}.N_{Rx} = 2 \times 29 = 58$$

This confirms that with 2 transmitters and 29 receivers, we achieve the desired 58 virtual elements.

Then, we set antennas position according to the following formulas:

$$Tx\_positions \;=\; (0{:}NTx-1)\; *\; NRx\; *\; dx;$$

$$Rx\_positions = (0{:}\, NRx\text{-}1) * dx;$$

4.2. MATLAB implementation

Now, we simulate the system in MATLAB for 2D case. After defining the initial parameters and simulating the transmitted signal, we calculate the received signal at each antenna. To simulate the received signal, we calculate the phase shifts for each transmitter-receiver pair based on their relative positions and the target's angle:

$$\varphi_{ij} = -k(Rx_j + Tx_i)\sin\theta$$

And the received signal is:

$$received\ signal(j,:) = \sum_{i=1}^{NTx} g_{tx}(t-\tau)\,.\,e^{j\varphi_{ij}}$$

Then, we demodulate the received signal by multiplying it by $e^{-j2\pi f_0 t}$.

For range estimation, we cross-correlate the demodulated received signal with a reference sinc pulse to estimate the time delay (τ), thereby determining the range.

$$R = ifft(fft(receied\ signal\ demodulate(j,:))\,.\,conj\left(fft(ref_{signal})\right))$$

Also, for DoA estimation, we compute phase differential at the estimated time delay across all receive antennas:

$$\Delta\varphi = \angle(signal\_at\_tau(j+1)\cdot conj(signal\_at\_tau(j)))$$

$$\theta est = arcsin(\frac{-\Delta\varphi_{avg}\cdot\lambda}{2\pi dx})$$

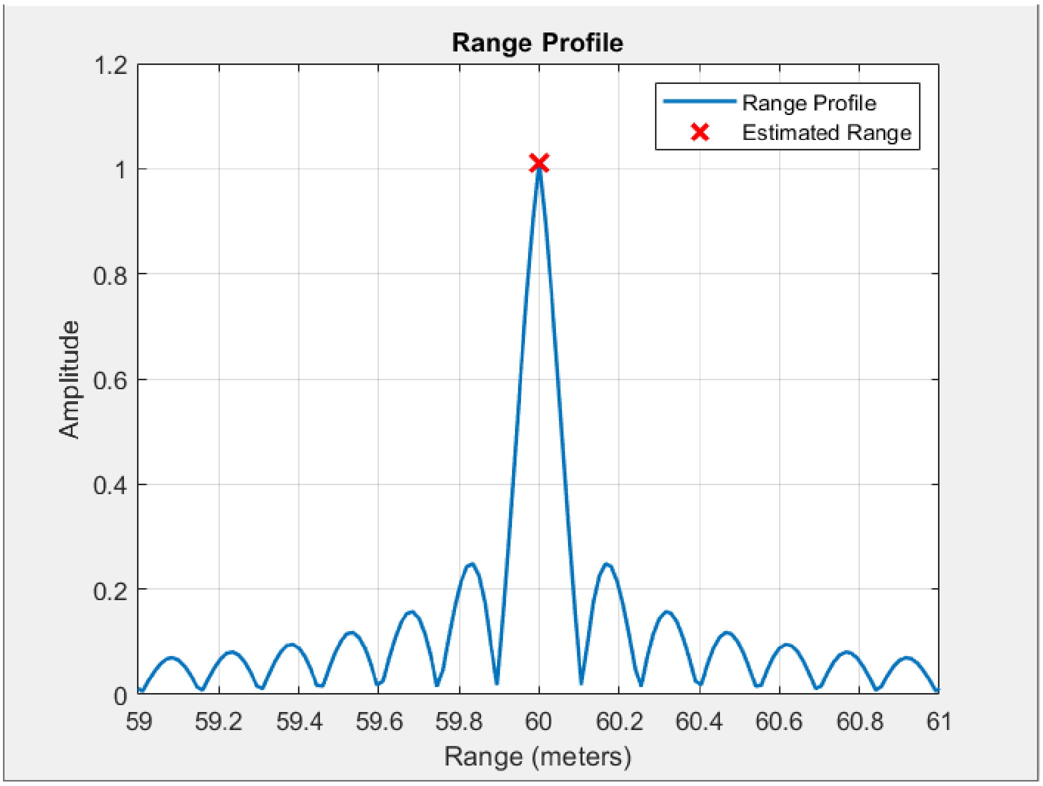

Figure 12: range profile

Finally, we display the results and plot range profile:

```
True Range: 60.00 meters
Estimated Range: 60.00 meters
True Angle: 41.00 degrees
Estimated Angle: 41.00 degrees
```

Figure 13: estimated range and angle

5 Orthogonal waveform

In this section, we will implement a MIMO radar system where each antenna transmits an orthogonal waveform simultaneously. This method allows us to avoid mutual interference and enables us to separate the contributions of each transmitter at the receiver through match filtering.

For this implementation, we will use orthogonal frequency division multiplexing (OFDM) waveforms, where each transmitter uses a unique subcarrier frequency. This method allows for simultaneous transmission while maintaining orthogonality.

5.1. MATLAB implementation

After defining system parameters, we generate orthogonal waveforms such that each transmitter uses a unique subcarrier frequency. Then, we model the transmitted signal's propagation to the target and back to the receivers, including phase shifts due to the target's angle and range, as described in the previous section.

At the receiver, we perform matched filtering using the conjucate of the transmitted signal.

$$R = ifft(fft(rx\_signal) .* fft(ref\_signal))$$

The range is estimated by finding the peak of the matched filter output. Since each transmitter's signal is orthogonal, we can perform range estimation for each transmitter separately.

Moreover, we perform DoA estimation using the virtual array formed by the transmit-receive pairs. The phase progression across the virtual array elements is used to estimate the angle.

$$delta\_phi = angle(V\_signal\_sorted(2\!:\!end) .* conj(V\_signal\_sorted(1\!:\!end-1)))$$

Finally, we display the results:

```
True Range: 100.00 meters
Estimated Range: 184.88 meters
True Angle: 30.00 degrees
Estimated Angle: 35.32 degrees
```

Figure 14: range and angle estimation

However, we can see that range estimation is not accurate enough, but angle of arrival is almost close.