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UAV-SAR Imaging with Time Domain Back Projection

Radar Imaging, Homework 2

Author: **Peyman Javaheri Neyestanak**

Student ID: 10971058
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Introduction

In this homework we use Time Domain Back projection to obtain an image of environment based on data acquired by Unmanned Aerial Vehicles (UAVs). The UAV flies at an altitude of roughly 60 meters, following an almost rectilinear trajectory. While traveling, the radar onboard transmits chirps and receives the echo from the scatterers in the scene. The echoes are range-compressed (i.e., matched filtered) at the receiver. Then, they are coherently processed with TDBP algorithm to obtain an image of the environment.

1 Range resolution assessment

The range resolution of a radar system can be calculated using the following formula:

$$\Delta r = \frac{c}{2B}$$

Where c is the speed of light and B is the bandwidth of transmitted radar pulse which is given in the dataset.mat file. So, we can see that the range resolution of radar system is:

$$\Delta r = 0.375$$

Now we imagine that we don't know the bandwidth. To calculate the resolution, first we transform pulses to frequency domain by taking Fast Fourier Transform of each pulse (column of RCData).

Then, we average the magnitude of the frequency spectra across all pulses. This gives the overall frequency response of the radar system.

After which, we determine the effective bandwidth from the averaged frequency spectrum by identifying the range of frequencies where the power (magnitude squared) is significant.

Finally, we use the formula above to calculate estimated resolution and the result is:

```
True Resolution (m): 0.375
Estimated Bandwidth (Hz): 313626830.5013
Estimated Range Resolution (m): 0.47828
```

Figure 1:Estimated resolution

2 Time Domain Back Projection

TDBP is an algorithm that processes several radar pulses to generate an image of the scene under observation. The starting point is the set of range-compressed echoes $s_{rc}(r, \tau)$ in which r is the range variable, while τ is the so-called slow-time variable related to the movement of the platform. In other words, τ has a sample every Pulse Repetition Interval (PRI, the time span between one pulse and the next one). The TDBP can be easily expressed as:

$$I(x) = \int s_{rc}(r = R(x, \tau), \tau).e^{+j\frac{4\pi}{\lambda}R(x,\tau)} d\tau$$

in which $\mathrm{x} = [\mathrm{x}, \mathrm{y}, \mathrm{z}]^T$ are the coordinates of a pixel in the back projection grid, R (x, τ) is the distance from the pixel in x and the sensor at time τ, λ is the wavelength.

2.1. Implementation

First, we generate the TDBP grid by defining x vector from -200 to 200 and y vector from 80 to 200 with sampling interval of 0.5. Then, we use meshgrid function in MATLAB to generate the coordinate maps of each pixel.

After that, we compute the distances from the first trajectory sample to each pixel in the grid according to the following formula:

$$R = \sqrt{(X - S_x(1))^2 + (Y - S_y(1))^2 + (Z - S_z(1))^2}$$

Now, we calculate range compression data at these ranges by using interp1 function in MATLAB:

$$interp1(r\_ax, RCData(:,1), R)$$

Following which, we rephase the data using the phase term $e^{+j\frac{4\pi}{\lambda}R(x,\tau)}$ and we add the result to the TDBP matrix.

Finally, we plot the image using imagesc function and compare it with optical image:

Figure 2: Optical image

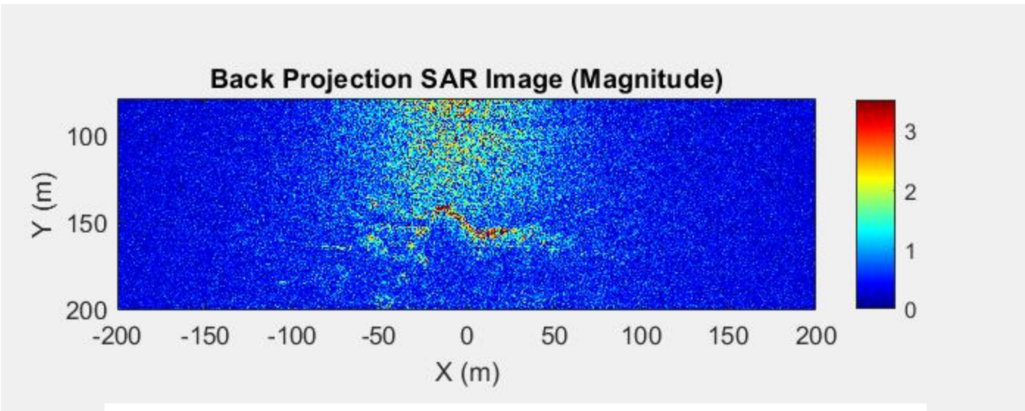

Figure 3: Back projection SAR image

As we can see, the image is noisy so we should remove noise from the picture. In the next section we will discuss it.

3 Despeckling

As we saw in section 2, the image is very noisy. This "noise" is called speckle, and it is due to the interaction of many elementary scatterers inside the resolution cell. This noise can be abated with a procedure called despeckling.

To do this operation, we use a 2D moving average. First, we use the magnitude of SAR image as the input for despeckling. Then, we define a moving average filter of size $N \times N$ as defined below:

$$\text{Average filter} = \text{ones (N, N) / (N*N)}$$

And we design the filter for different values of N like 3 and 5 and we compare the results.

Finally, we use conv2 function in MATLAB to take convolution between image and filter.

The results are shown in the figure below:

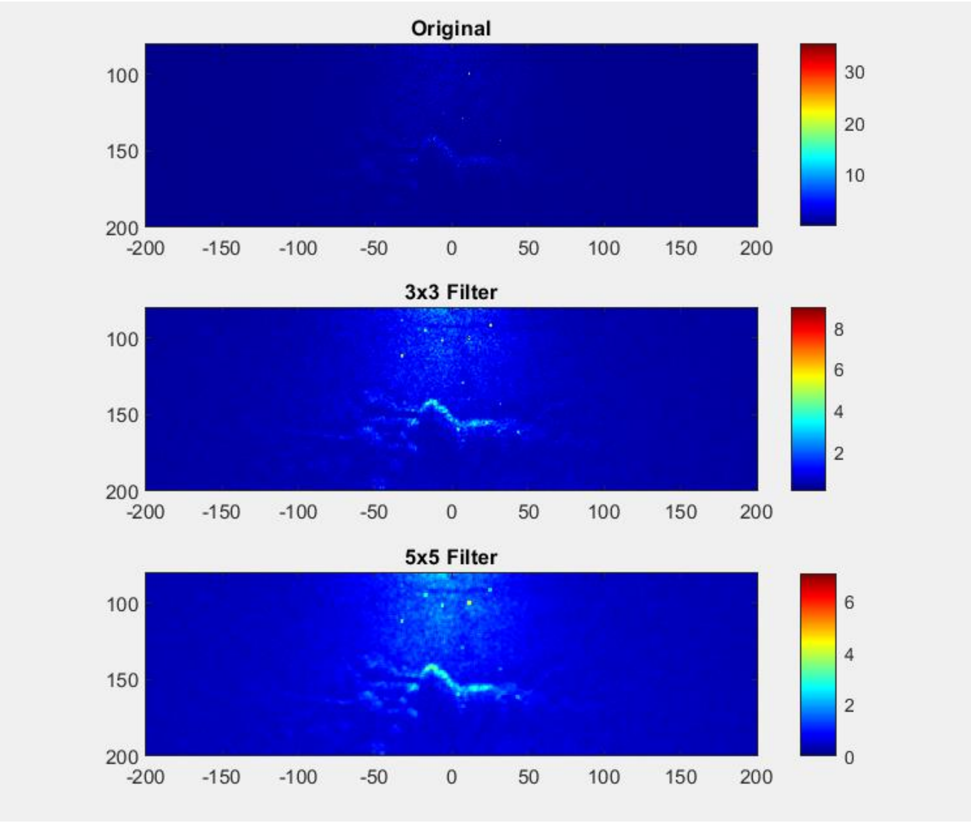

Figure 4:The image after despeckling

As we can see, the noise is abated with despeckling procedure.

4 Resolution assessment using corner reflectors

If we look closely at the image, we can find very bright targets in the near range which are corner reflectors that are placed as calibration targets in the scene.

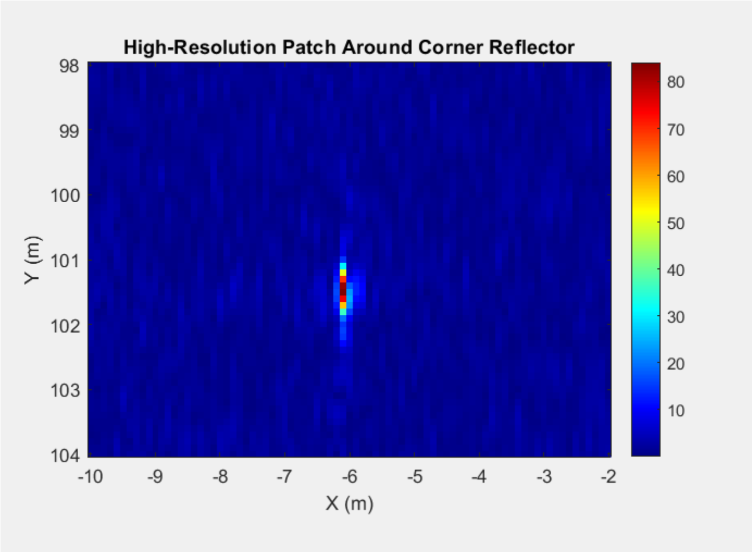

Figure 5: corner reflector

To find resolution, we focus to a small region near $x = -6m\ and\ y = 101m$ and, we define smaller grid spacing of 0.1 to enhance resolution and accurate presentation of the corner reflector. Then, we use back projection algorithm to display the focused SAR image (figure 5).

After which, we extract amplitude profiles by finding the maximum of absolute value of SAR image and we visualize amplitude profiles:

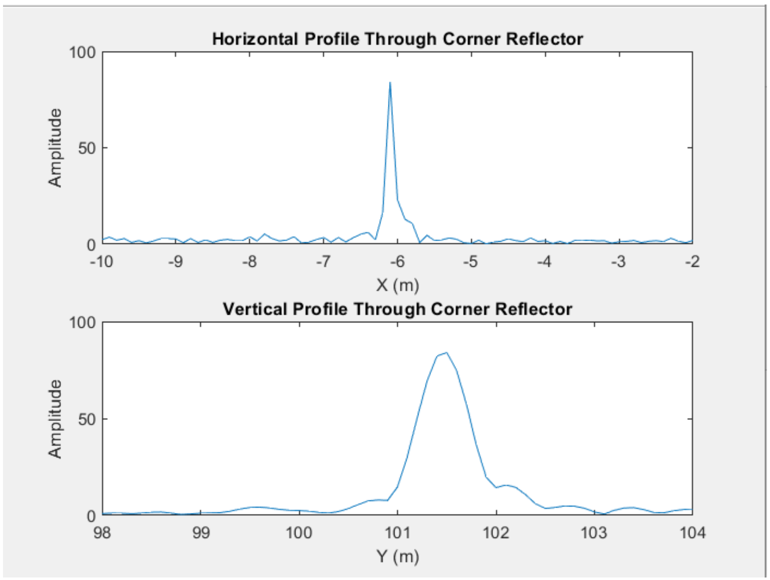

Figure 6: Amplitude profiles

These profiles reveal the reflector's shape and width in the SAR image, allowing resolution measurement.

The resolution can be assessed in the spatial frequency domain. To do this, we compute the 2D Fourier transform of the SAR patch using fft2 function in MATLAB. The FFT shows the image's spatial frequency content, which helps evaluate resolution in the frequency domain.

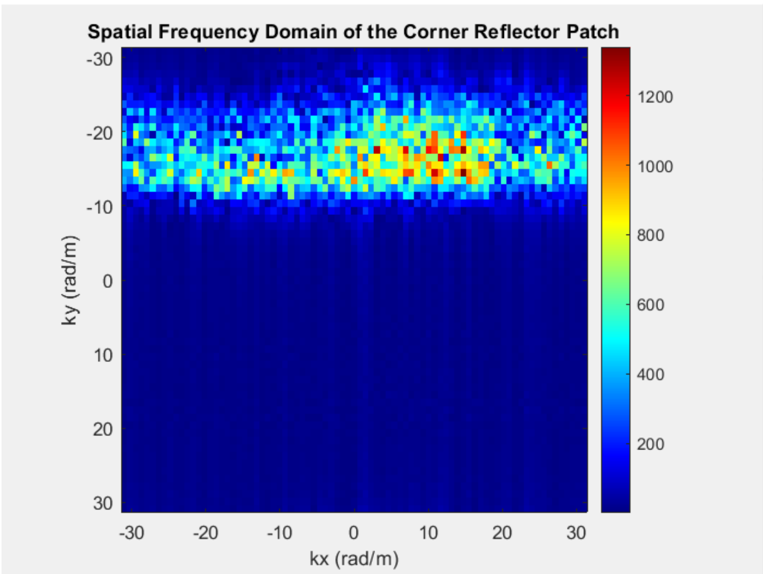

Figure 7: Spatial frequency domain of the corner reflector patch

Bright regions in the frequency domain correspond to strong signal components and the width of these regions is inversely proportional to the image resolution.

To compute the resolution of the SAR image patch, we will determine the Full Width at Half Maximum (FWHM) of the reflector's response in both the horizontal and vertical directions.

First, we identify the peak value in the horizontal and vertical profiles. The resolution is the distance between the points where the signal amplitude falls to half the maximum value. The results are shown below:

```
Horizontal Resolution: 0.131 m
Vertical Resolution: 0.614 m
```

Figure 8: Results

Now, if we define grid spacing equal to 0.01, we will have more accurate picture of corner reflector:

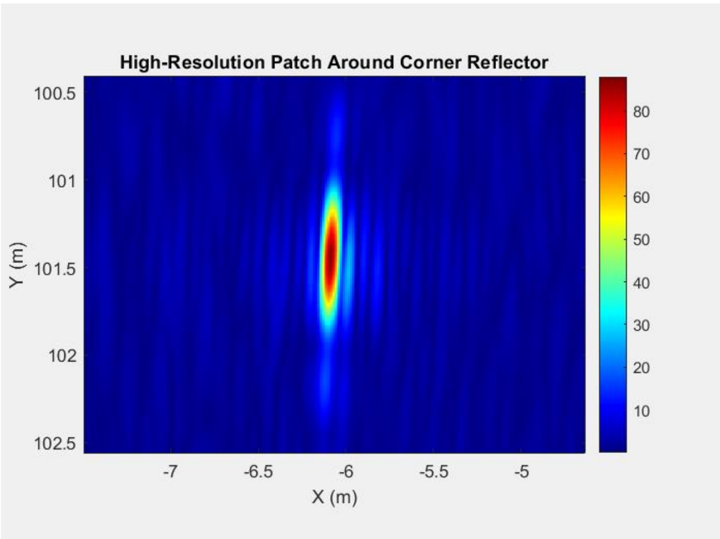

Figure 9: Corner reflector

And the amplitude profiles will be shown in the figure below:

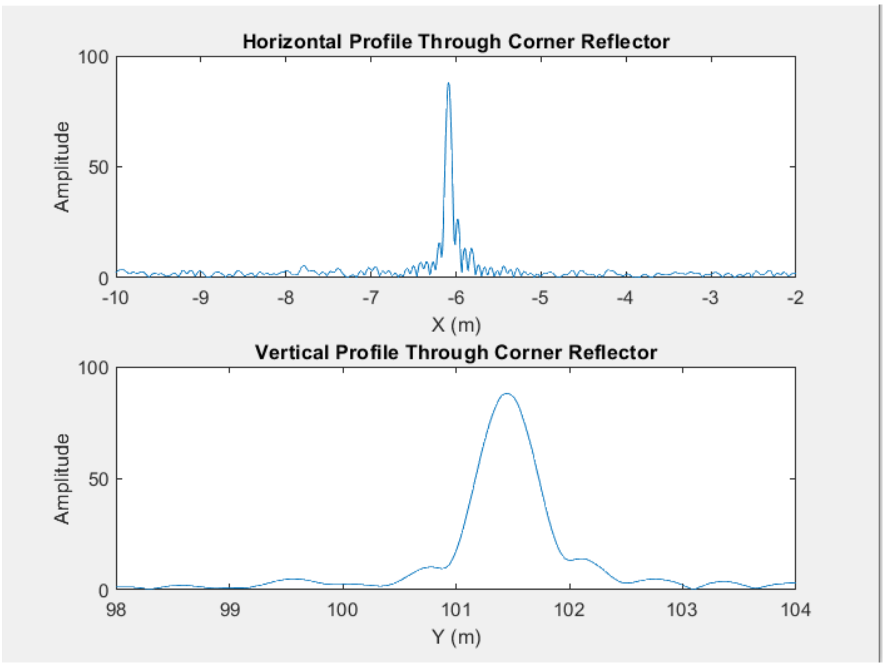

Figure 10: Amplitude profiles

And the results of resolution will be:

```
Horizontal Resolution: 0.093 m
Vertical Resolution: 0.614 m
```

Figure 11: Resolution

5 The effect of trajectory errors

When we perform the back projection, we use the platform coordinates for every slow time. Those coordinates are just estimates of the true trajectory the UAV performs, and they come from the GNSS and the Inertial Measurement Unit (IMU) onboard the platform.

If the platform's trajectory coordinates are inaccurate, this introduces errors in the Time Domain Back Projection (TDBP) process, leading to several effects on the Synthetic Aperture Radar (SAR) image like:

Defocusing of the Image, Geometric Distortions and reduction in Signal-to-Noise Ratio (SNR).

To see the effects of trajectory errors, we add a random white noise with standard deviation $\lambda/10, \lambda/6, \lambda/4, \lambda/2$ to the values of S_x, S_y, S_z and we compare the results in the figure which is shown in the next page:

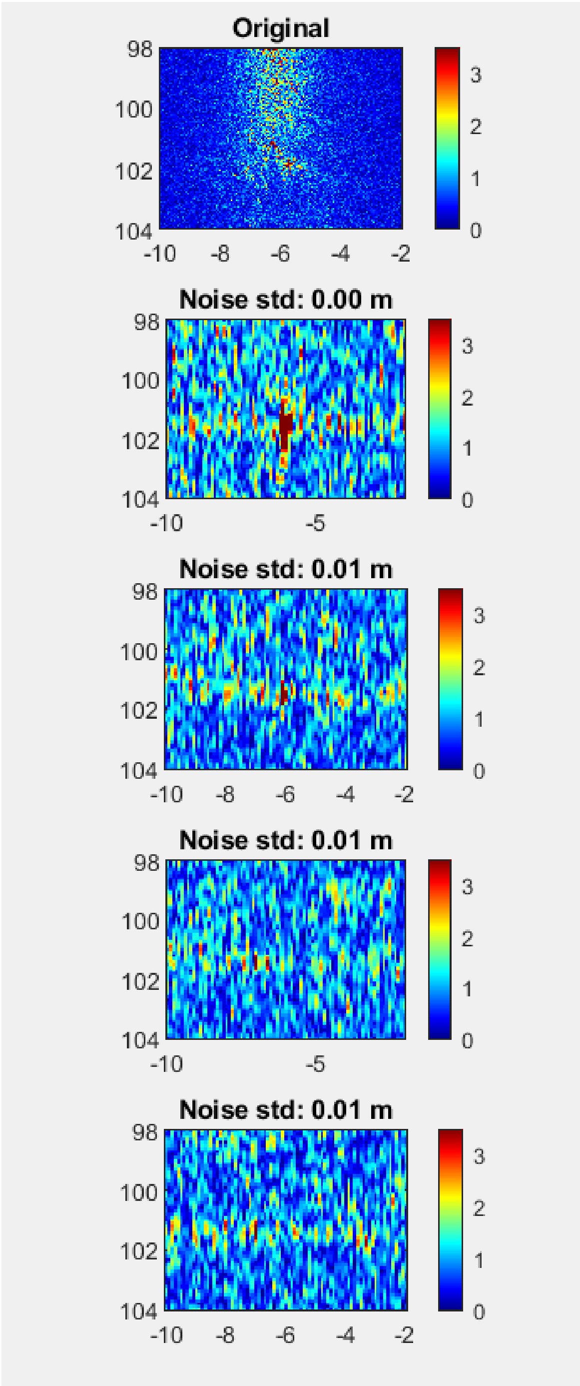

Figure 12: The effect of trajectory error

It is obvious that images are not clear due to inaccuracies.