



Computer Engineering Department

# Exploration in RL

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Courtesy: Most of slides are adopted from CS 285, UC Berkeley.

# What's the problem?

this is easy (mostly)



Why?

↙ exploration ↘ issue  
this is impossible



# Montezuma's revenge

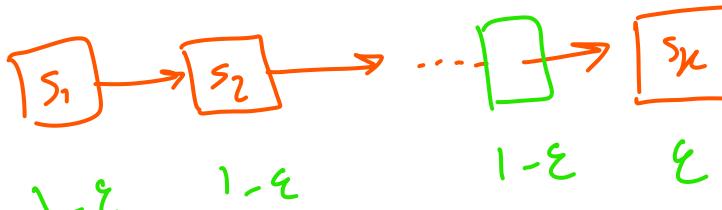


- Getting key = reward
- Opening door = reward
- Getting killed by skull = nothing (is it good? bad?)
- Finishing the game only weakly correlates with rewarding events
- We know what to do because we understand what these sprites mean!

فیجی، reward، حذرل، حزیز، عزیز

# Why exploration can be difficult?

- **Temporally extended** tasks like Montezuma's revenge become increasingly difficult based on
  - How extended the task is
  - How little you know about the rules
- Let's assume a **complex task**
  - Consisting of **multiple sub-task**, each is a **prerequisite** for the next sub-task.
  - Each should be solved **in a sequence** to get a high reward
  - Epsilon greedy does not obviously help:
  - Suppose you mastered up to the  $k^{\text{th}}$  sub-task.
  - You have to **exploit up to the  $k^{\text{th}}$  task** and then **explore onwards**.
  - Now the chance to only explore in the sub-task ( $k+1$ ) is  $(1 - \varepsilon)^{O(k)} \varepsilon^{O(1)}$ .
  - For  $\varepsilon = 0.1$ , this is  $\sim 6\%$ . For  $\varepsilon = 0.5$ , this is  $\sim 2\%$ .



$$\xrightarrow[k \uparrow]{TE} O$$

...  
J.  
1,

# Exploration and exploitation

- Two potential definitions of exploration problem:
  - How can an agent **discover high-reward strategies** that require a temporally **extended sequence of complex behaviors** that, **individually, are not rewarding?**
  - How can an agent decide whether to **attempt new behaviors** (to discover ones with higher reward) or **continue to do the best thing it knows so far?**

# Optimal Exploration?

- Bayesian model of the environment. (POMDP with belief state)
- Optimize the expected reward under **all uncertainties**.
- Requires knowledge of **state dynamic distribution class**, the **prior**, and **maintaining the belief state**.
- Here we seek **simpler** solutions which could be extended to more **complex** scenarios.
- Compare the regret in such models against the Bayes' optimal approach.

*is low?* ↗

$$\text{Reg}(T) = TE[r(a^*)] - \sum_{t=1}^T r(a_t)$$

expected reward of best action  
(the best we can hope for in expectation) ↗

actual reward of action  
actually taken ↗

# Bandits

assume  $r(a_i) \sim p_{\theta_i}(r_i)$

e.g.,  $p(r_i = 1) = \theta_i$  and  $p(r_i = 0) = 1 - \theta_i$

$\theta_i \sim p(\theta)$ , but otherwise unknown

this defines a POMDP with  $s = [\theta_1, \dots, \theta_n]$

belief state is  $\hat{p}(\theta_1, \dots, \theta_n)$

- solving the POMDP yields the optimal exploration strategy
- but that's overkill: belief state is huge!
- we can do very well with much simpler strategies

how do we measure goodness of exploration algorithm?

regret: difference from optimal policy at time step  $T$ :

$$\text{Reg}(T) = TE[r(a^*)] - \sum_{t=1}^T r(a_t)$$

expected reward of best action  
(the best we can hope for in expectation)

actual reward of action  
actually taken

# Optimistic exploration

keep track of average reward  $\hat{\mu}_a$  for each action  $a$

exploitation: pick  $a = \arg \max \hat{\mu}_a$

optimistic estimate:  $a = \arg \max \hat{\mu}_a + \frac{C}{\overline{\sigma_a}}$

some sort of variance estimate

**intuition: try each arm until you are *sure* it's not great**

example (Auer et al. Finite-time analysis of the multiarmed bandit problem):

$$a = \arg \max \hat{\mu}_a + \sqrt{\frac{2 \ln T}{N(a)}}$$

 number of times we  
picked this action

$\text{Reg}(T)$  is  $O(\log T)$ , provably as good as any algorithm

# Probability matching/posterior sampling

assume  $r(a_i) \sim p_{\theta_i}(r_i)$

this defines a POMDP with  $\mathbf{s} = [\theta_1, \dots, \theta_n]$

belief state is  $\hat{p}(\theta_1, \dots, \theta_n)$

this is a *model* of our bandit

idea: sample  $\theta_1, \dots, \theta_n \sim \hat{p}(\theta_1, \dots, \theta_n)$

pretend the model  $\theta_1, \dots, \theta_n$  is correct

take the optimal action

update the model

- This is called posterior sampling or Thompson sampling
- Harder to analyze theoretically
- Can work very well empirically
- See: Chapelle & Li, “An Empirical Evaluation of Thompson Sampling.”

# Information gain

## Bayesian experimental design:

say we want to determine some latent variable  $z$  (e.g.,  $z$  might be the optimal action, or its value)  
which action do we take?

let  $\mathcal{H}(\hat{p}(z))$  be the current entropy of our  $z$  estimate

let  $\mathcal{H}(\hat{p}(z)|y)$  be the entropy of our  $z$  estimate after observation  $y$  (e.g.,  $y$  might be  $r(a)$ )

the lower the entropy, the more precisely we know  $z$

$$\text{IG}(z, y) = E_y[\mathcal{H}(\hat{p}(z)) - \mathcal{H}(\hat{p}(z)|y)]$$

typically depends on action, so we have  $\text{IG}(z, y|a)$

# Information gain example

$$\text{IG}(z, y|a) = E_y[\mathcal{H}(\hat{p}(z)) - \mathcal{H}(\hat{p}(z)|y)|a]$$

how much we learn about  $z$  from action  $a$ , given current beliefs

Example bandit algorithm:

Russo & Van Roy “Learning to Optimize via Information-Directed Sampling”

$y = r_a, z = \theta_a$  (parameters of model  $p(r_a)$ )

$g(a) = \text{IG}(\theta_a, r_a|a)$  – information gain of  $a$

$\Delta(a) = E[r(a^*) - r(a)]$  – expected suboptimality of  $a$

choose  $a$  according to  $\arg \min_a \frac{\Delta(a)^2}{g(a)}$

don't take actions that you're  
sure are suboptimal

don't bother taking actions if  
you won't learn anything

# General themes

UCB:

$$a = \arg \max \hat{\mu}_a + \sqrt{\frac{2 \ln T}{N(a)}}$$

Thompson sampling:

$$\begin{aligned}\theta_1, \dots, \theta_n &\sim \hat{p}(\theta_1, \dots, \theta_n) \\ a &= \arg \max_a E_{\theta_a}[r(a)]\end{aligned}$$

Info gain:

$$\text{IG}(z, y|a)$$

- Most exploration strategies require some kind of uncertainty estimation (even if it's naïve)
- Usually assumes some value to new information
  - Assume unknown = good (optimism)
  - Assume sample = truth
  - Assume information gain = good

# Optimistic exploration in RL

UCB: 
$$a = \arg \max \hat{\mu}_a + \sqrt{\frac{2 \ln T}{N(a)}}$$
  
“exploration bonus”

lots of functions work, so long as they decrease with  $N(a)$

can we use this idea with MDPs?

count-based exploration: use  $N(\mathbf{s}, \mathbf{a})$  or  $N(\mathbf{s})$  to add *exploration bonus*

use  $r^+(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \mathcal{B}(N(\mathbf{s}))$



bonus that decreases with  $N(\mathbf{s})$

use  $r^+(\mathbf{s}, \mathbf{a})$  instead of  $r(\mathbf{s}, \mathbf{a})$  with any model-free algorithm

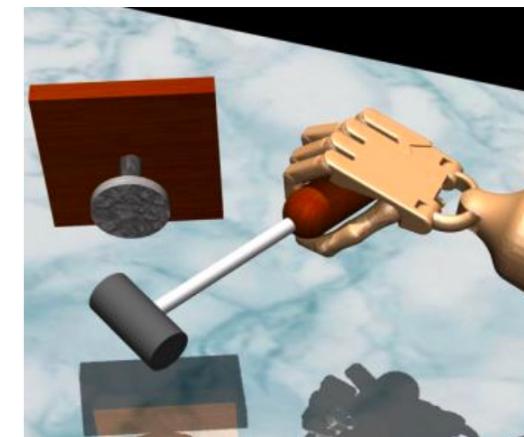
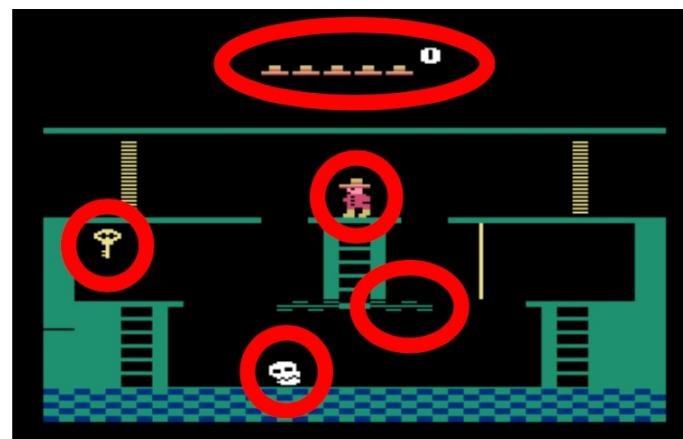
+ simple addition to any RL algorithm

- need to tune bonus weight

# The trouble with counts

use  $r^+(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \mathcal{B}(N(\mathbf{s}))$

But wait... what's a count?



Uh oh... we never see the same thing twice!

But some states are more similar than others

# Fitting generative models

idea: fit a density model  $p_\theta(\mathbf{s})$  (or  $p_\theta(\mathbf{s}, \mathbf{a})$ )

$p_\theta(\mathbf{s})$  might be high even for a new  $\mathbf{s}$

if  $\mathbf{s}$  is similar to previously seen states

can we use  $p_\theta(\mathbf{s})$  to get a “pseudo-count”?

if we have small MDPs

the true probability is:

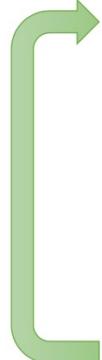
after we see  $\mathbf{s}$ , we have:

$$P(\mathbf{s}) = \frac{N(\mathbf{s})}{n}$$

↑                          ← count  
probability/density      total states visited

$$P'(\mathbf{s}) = \frac{N(\mathbf{s}) + 1}{n + 1}$$

# Exploring with pseudo-counts

- 
- fit model  $p_\theta(\mathbf{s})$  to all states  $\mathcal{D}$  seen so far
  - take a step  $i$  and observe  $\mathbf{s}_i$
  - fit new model  $p_{\theta'}(\mathbf{s})$  to  $\mathcal{D} \cup \mathbf{s}_i$
  - use  $p_\theta(\mathbf{s}_i)$  and  $p_{\theta'}(\mathbf{s}_i)$  to estimate  $\hat{N}(\mathbf{s})$
  - set  $r_i^+ = r_i + \mathcal{B}(\hat{N}(\mathbf{s}))$  ← “pseudo-count”
- how to get  $\hat{N}(\mathbf{s})$ ? use the equations

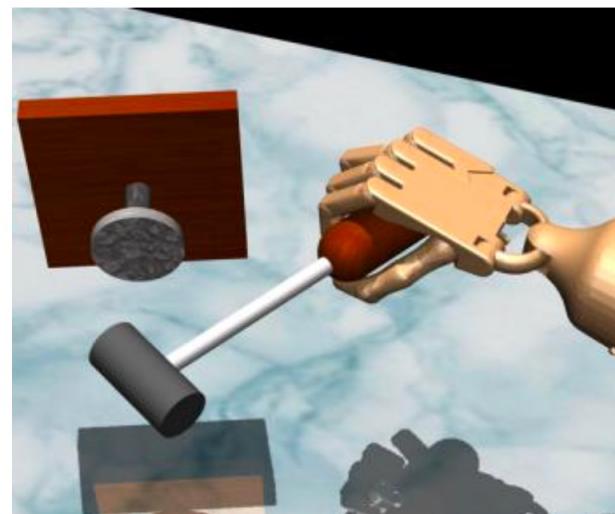
$$p_\theta(\mathbf{s}_i) = \frac{\hat{N}(\mathbf{s}_i)}{\hat{n}}$$

$$p_{\theta'}(\mathbf{s}_i) = \frac{\hat{N}(\mathbf{s}_i) + 1}{\hat{n} + 1}$$

two equations and two unknowns!

$$\hat{N}(\mathbf{s}_i) = \hat{n}p_\theta(\mathbf{s}_i) \quad \hat{n} = \frac{1 - p_{\theta'}(\mathbf{s}_i)}{p_{\theta'}(\mathbf{s}_i) - p_\theta(\mathbf{s}_i)}p_\theta(\mathbf{s}_i)$$

# What kind of generative modeling to use?

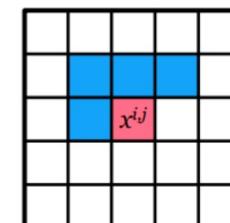


$$p_{\theta}(\mathbf{s})$$

need to be able to output densities, but doesn't necessarily need to produce great samples

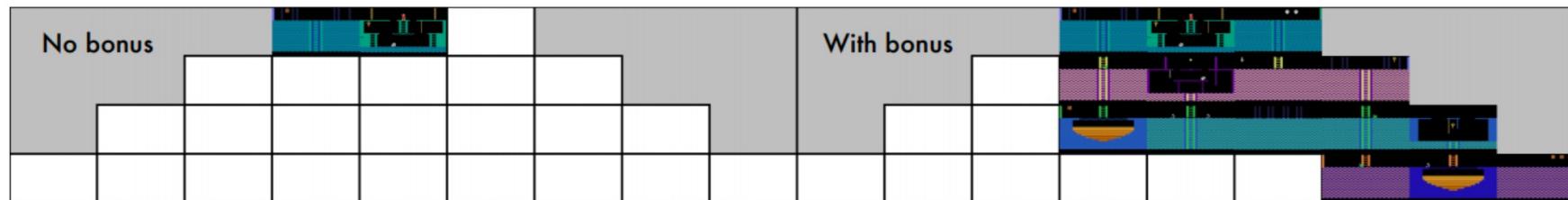
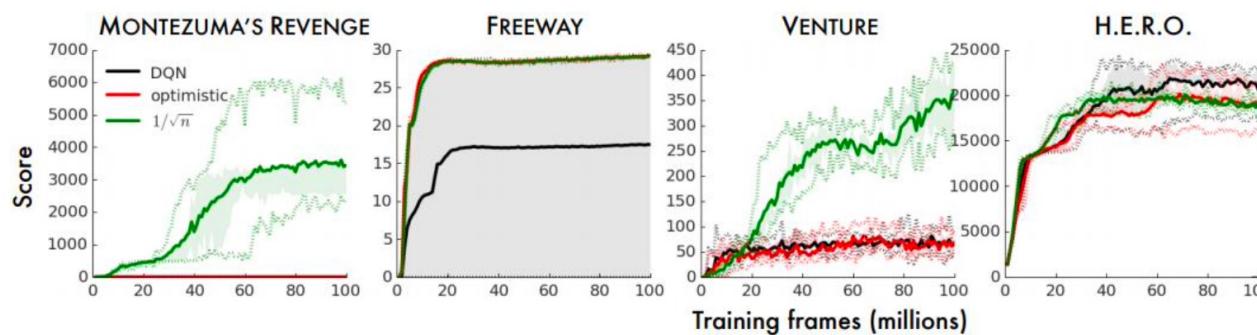
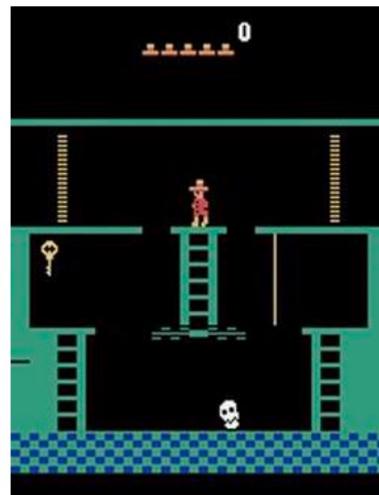
opposite considerations from many popular generative models in the literature (e.g., GANs)

Bellemare et al.: "CTS" model:  
condition each pixel on its top-left neighborhood



Other models: stochastic neural networks, compression length, EX2

# Does it work?



Bellemare et al. "Unifying Count-Based Exploration..."

# Counting with hashes

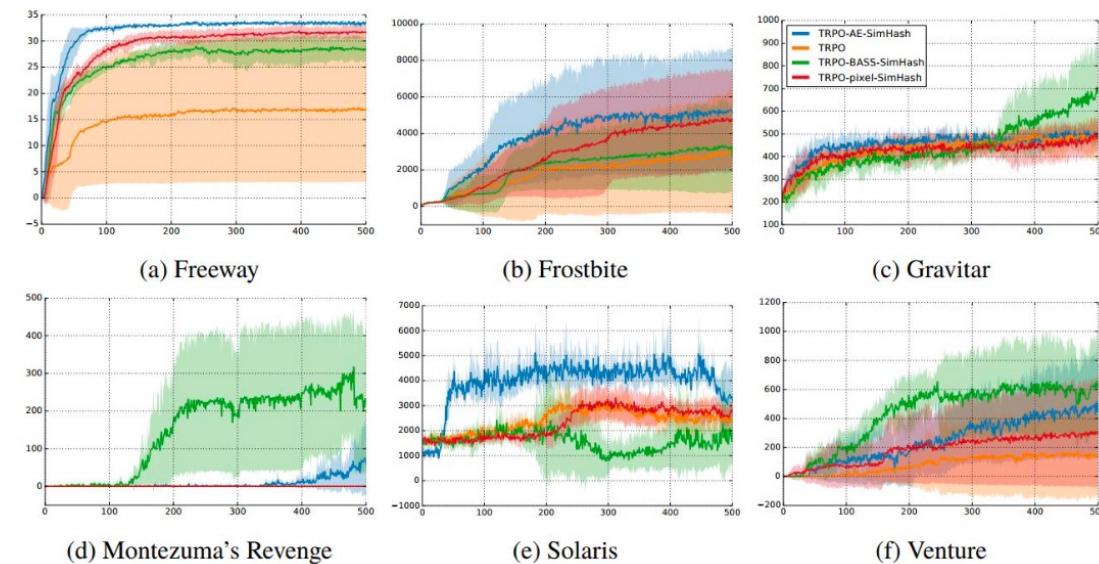
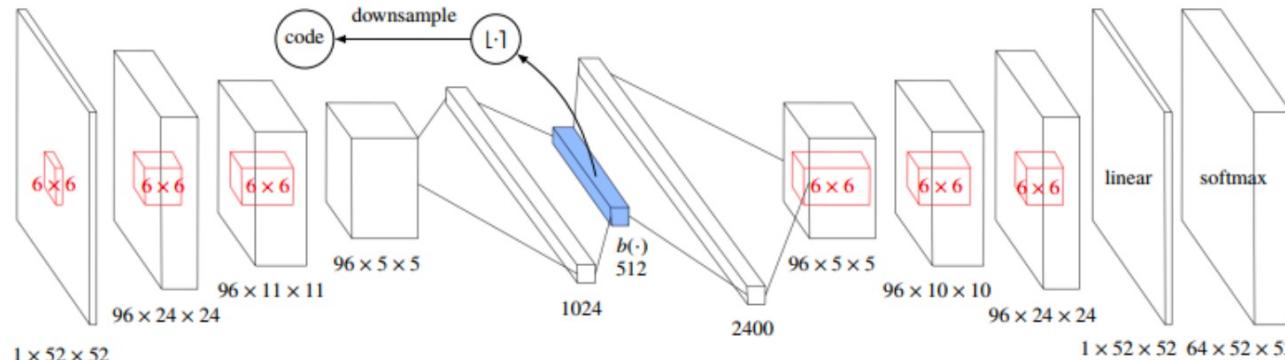
What if we still count states, but in a different space?

idea: compress  $\mathbf{s}$  into a  $k$ -bit code via  $\phi(\mathbf{s})$ , then count  $N(\phi(\mathbf{s}))$

shorter codes = more hash collisions

similar states get the same hash? maybe

improve the odds by *learning* a compression:



# Implicit density modeling with exemplar models

$p_\theta(\mathbf{s})$  need to be able to output densities, but doesn't necessarily need to produce great samples

Can we explicitly compare the new state to past states?

Intuition: the state is **novel** if it is **easy** to distinguish from all previous seen states by a classifier

for each observed state  $\mathbf{s}$ , fit a classifier to classify that state against all past states  $\mathcal{D}$ , use classifier error to obtain density

$$p_\theta(\mathbf{s}) = \frac{1 - D_{\mathbf{s}}(\mathbf{s})}{D_{\mathbf{s}}(\mathbf{s})} \leftarrow \begin{array}{l} \text{probability that classifier assigns that } \mathbf{s} \text{ is "positive"} \\ \text{positives: } \{\mathbf{s}\} \\ \text{negatives: } \mathcal{D} \end{array}$$

# Implicit density modeling with exemplar models

hang on... aren't we just checking if  $\mathbf{s} = \mathbf{s}$ ?

if  $\mathbf{s} \in \mathcal{D}$ , then the optimal  $D_{\mathbf{s}}(\mathbf{s}) \neq 1$

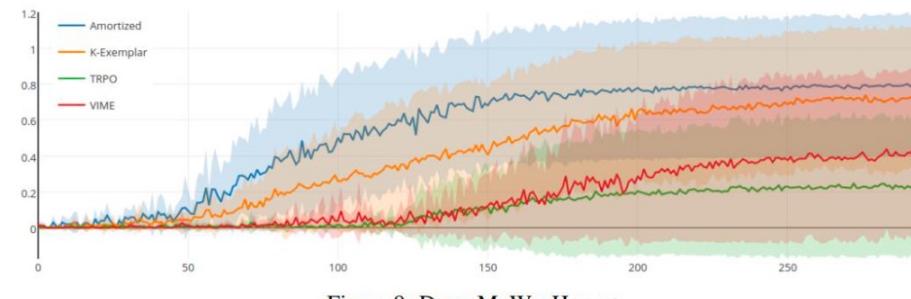
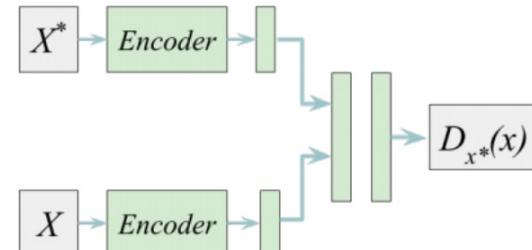
in fact:  $D_{\mathbf{s}}^*(\mathbf{s}) = \frac{1}{1 + p(\mathbf{s})}$


$$p_{\theta}(\mathbf{s}) = \frac{1 - D_{\mathbf{s}}(\mathbf{s})}{D_{\mathbf{s}}(\mathbf{s})}$$

in reality, each state is unique, so we *regularize* the classifier

isn't one classifier per state a bit much?

train one *amortized* model: single network that takes in exemplar as input!



Fu et al. "EX2: Exploration with Exemplar Models..."

# Heuristic estimation of counts via errors

$p_\theta(\mathbf{s})$

need to be able to output densities, but doesn't necessarily need to produce great samples

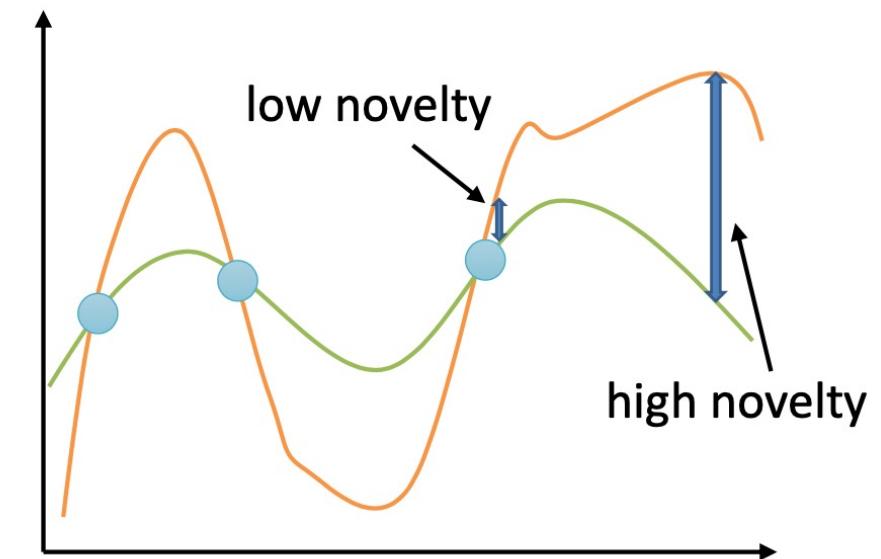
...and doesn't even need to output great densities

...just need to tell if state is **novel** or not!

let's say we have some **target** function  $f^*(\mathbf{s}, \mathbf{a})$

given our buffer  $\mathcal{D} = \{(\mathbf{s}_i, \mathbf{a}_i)\}$ , fit  $\hat{f}_\theta(\mathbf{s}, \mathbf{a})$

use  $\mathcal{E}(\mathbf{s}, \mathbf{a}) = \|\hat{f}_\theta(\mathbf{s}, \mathbf{a}) - f^*(\mathbf{s}, \mathbf{a})\|^2$  as bonus



# Heuristic estimation of counts via errors

what should we use for  $f^*(\mathbf{s}, \mathbf{a})$ ?

one common choice: set  $f^*(\mathbf{s}, \mathbf{a}) = \mathbf{s}'$  – i.e., next state prediction

even simpler:  $f^*(\mathbf{s}, \mathbf{a}) = f_\phi(\mathbf{s}, \mathbf{a})$ , where  $\phi$  is a *random* parameter vector

# Posterior sampling in deep RL

Thompson sampling:

$$\theta_1, \dots, \theta_n \sim \hat{p}(\theta_1, \dots, \theta_n)$$

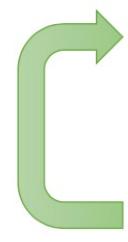
$$a = \arg \max_a E_{\theta_a}[r(a)]$$

bandit setting:  $\hat{p}(\theta_1, \dots, \theta_n)$  is distribution over *rewards*

MDP analog is the *Q*-function!

What do we sample?

How do we represent the distribution?

- 
1. sample Q-function  $Q$  from  $p(Q)$
  2. act according to  $Q$  for one episode
  3. update  $p(Q)$
- since Q-learning is off-policy, we don't care which Q-function was used to collect data

how can we represent a distribution over functions?

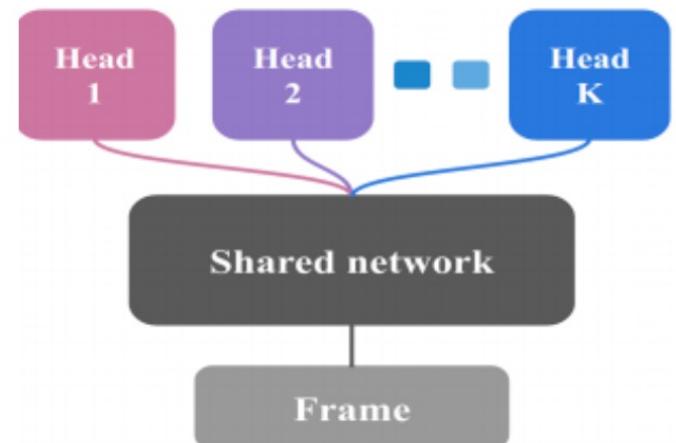
# Bootstrap

given a dataset  $\mathcal{D}$ , resample with replacement  $N$  times to get  $\mathcal{D}_1, \dots, \mathcal{D}_N$

train each model  $f_{\theta_i}$  on  $\mathcal{D}_i$

to sample from  $p(\theta)$ , sample  $i \in [1, \dots, N]$  and use  $f_{\theta_i}$

training  $N$  big neural nets is expensive, can we avoid it?



Osband et al. “Deep Exploration via Bootstrapped DQN”

# Bootstrap

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**Algorithm 1** Bootstrapped DQN

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- 1: **Input:** Value function networks  $Q$  with  $K$  outputs  $\{Q_k\}_{k=1}^K$ . Masking distribution  $M$ .
- 2: Let  $B$  be a replay buffer storing experience for training.
- 3: **for** each episode **do**
- 4:   Obtain initial state from environment  $s_0$
- 5:   Pick a value function to act using  $k \sim \text{Uniform}\{1, \dots, K\}$
- 6:   **for** step  $t = 1, \dots$  until end of episode **do**
- 7:     Pick an action according to  $a_t \in \arg \max_a Q_k(s_t, a)$
- 8:     Receive state  $s_{t+1}$  and reward  $r_t$  from environment, having taking action  $a_t$
- 9:     Sample bootstrap mask  $m_t \sim M$
- 10:    Add  $(s_t, a_t, r_{t+1}, s_{t+1}, m_t)$  to replay buffer  $B$
- 11:   **end for**
- 12: **end for**

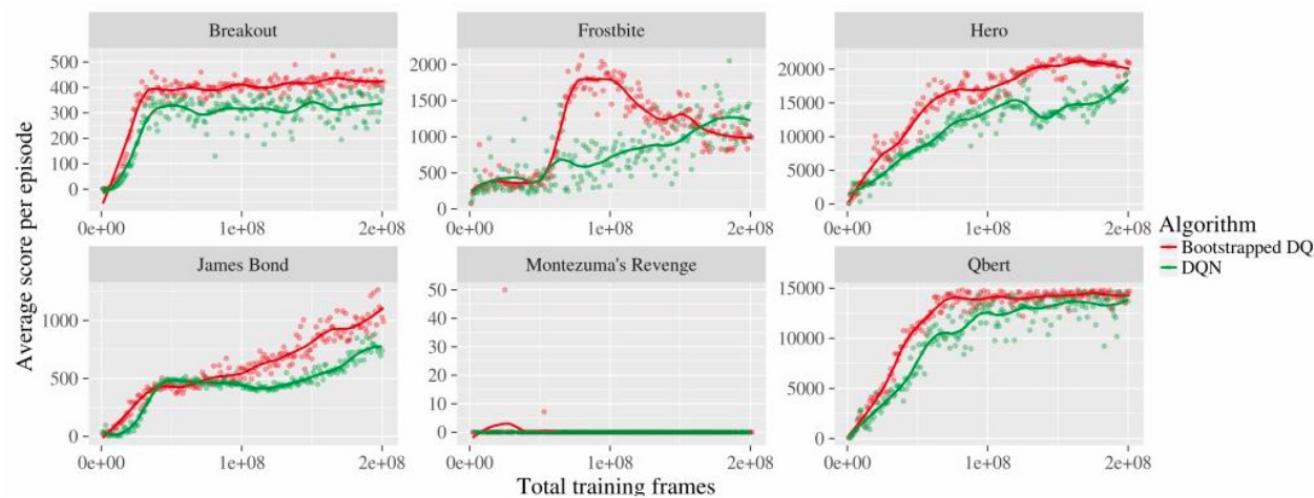
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Osband et al. “Deep Exploration via Bootstrapped DQN”

# Why does this work?

Exploring with random actions (e.g., epsilon-greedy): oscillate back and forth, might not go to a coherent or interesting place

Exploring with random Q-functions: commit to a randomized but internally consistent strategy for an entire episode



+ no change to original reward function  
- very good bonuses often do better

Osband et al. "Deep Exploration via Bootstrapped DQN"

# Reasoning about information gain (approximately)

Info gain:  $\text{IG}(z, y|a)$

information gain about *what*?

information gain about reward  $r(\mathbf{s}, \mathbf{a})$ ?

not very useful if reward is sparse

state density  $p(\mathbf{s})$ ?

a bit strange, but somewhat makes sense!

information gain about dynamics  $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ ?

good proxy for *learning* the MDP, though still heuristic

Generally intractable to use exactly, regardless of what is being estimated!

# Reasoning about information gain (approximately)

A few approximations:

prediction gain:  $\log p_{\theta'}(\mathbf{s}) - \log p_{\theta}(\mathbf{s})$  (Schmidhuber '91, Bellemare '16)

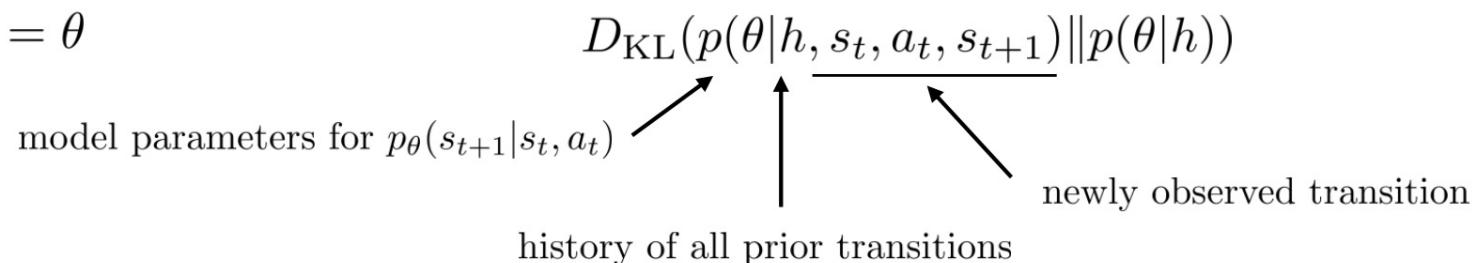
intuition: if density changed a lot, the state was novel

variational inference: (Houthooft et al. "VIME")

IG can be equivalently written as  $D_{\text{KL}}(p(z|y) \| p(z))$

learn about *transitions*  $p_{\theta}(s_{t+1}|s_t, a_t)$ :  $z = \theta$

$y = (s_t, a_t, s_{t+1})$



intuition: a transition is more informative if it causes belief over  $\theta$  to change

idea: use variational inference to estimate  $q(\theta|\phi) \approx p(\theta|h)$

given new transition  $(s, a, s')$ , update  $\phi$  to get  $\phi'$

# Reasoning about information gain (approximately)

VIME implementation:

IG can be equivalently written as  $D_{\text{KL}}(p(\theta|h, s_t, a_t, s_{t+1}) \| p(\theta|h))$

model parameters for  $p_\theta(s_{t+1}|s_t, a_t)$

history of all prior transitions

newly observed transition

$$q(\theta|\phi) \approx p(\theta|h)$$

specifically, optimize variational lower bound  $D_{\text{KL}}(q(\theta|\phi) \| p(h|\theta)p(\theta))$

represent  $q(\theta|\phi)$  as product of independent Gaussian parameter distributions

with mean  $\phi$

(see Blundell et al. “Weight uncertainty in neural networks”)

given new transition  $(s, a, s')$ , update  $\phi$  to get  $\phi'$

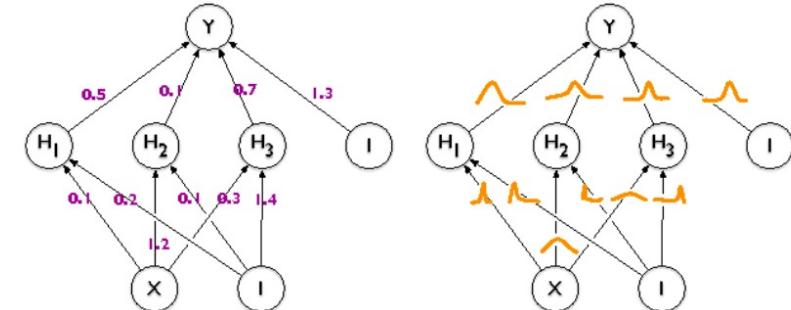
i.e., update the network weight means and variances

use  $D_{\text{KL}}(q(\theta|\phi') \| q(\theta|\phi))$  as approximate bonus

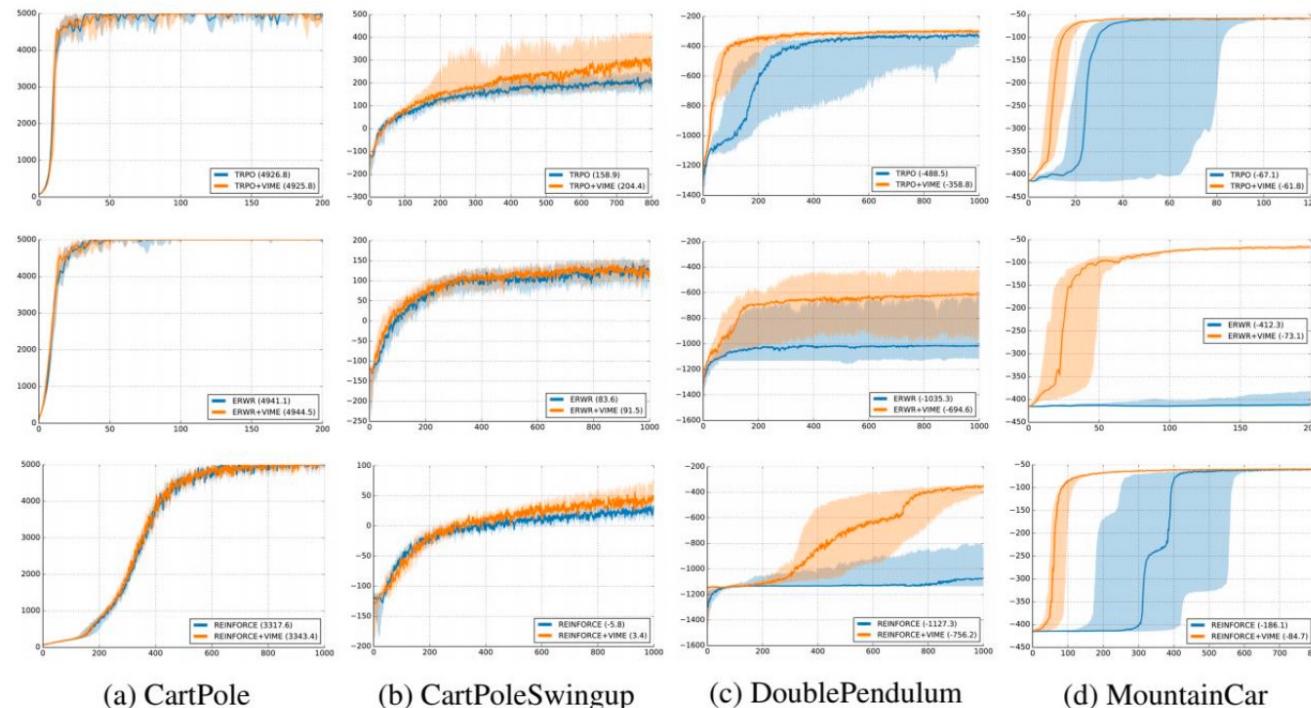
$$p(\theta|\mathcal{D}) = \prod_i p(\theta_i|\mathcal{D})$$

$$p(\theta_i|\mathcal{D}) = \mathcal{N}(\mu_i, \sigma_i)$$

$$\phi$$



# Reasoning about information gain (approximately)



Approximate IG:

- + appealing mathematical formalism
- models are more complex, generally harder to use effectively

Houthooft et al. “VIME”