



Computer Engineering Department

Reinforcement Learning: Model Based RL

Mohammad Hossein Rohban, Ph.D.

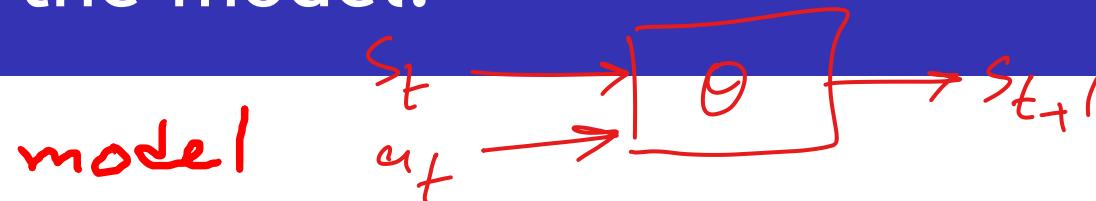
Spring 2024

Courtesy: Most of slides are adopted from CS 285 Berkeley.

Today's Lecture

1. Basics of model-based RL. ~~learn a model, use model for control~~
 - Why does naïve approach not work?
 - The effect of distributional shift in model-based RL
2. ~~Uncertainty~~ in model-based RL
3. Model-based Policy Learning
 - Goals:
 - Understand how to build model-based RL algorithms
 - Understand the important considerations for model-based RL
 - Understand the tradeoffs between different model class choices

Why learn the model?



If we knew $f(s_t, a_t) = s_{t+1}$, we could use the tools from last week.
(or $p(s_{t+1}|s_t, a_t)$ in the stochastic case)

So let's learn $f(s_t, a_t)$ from data, and *then* plan through it!

$$\left\{ \begin{array}{l} \text{max}_{\theta} \sum_{t=1}^T r(s_t, a_t) \\ \text{s.t. } s_t = f(s_{t-1}, a_{t-1}) \end{array} \right.$$

(θ learned), s_t from π

or π MCT from r ,

but we can't do this yet,

model-based reinforcement learning version 0.5:

1. run base policy $\pi_0(a_t|s_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(s, a, s')_i\}$
2. learn dynamics model $f(s, a)$ to minimize $\sum_i \|f(s_i, a_i) - s'_i\|^2$
3. plan through $f(s, a)$ to choose actions

CEM, MTS

ground truth

Does it work?

Yes!

قراءه خیزیک تغیرات سنت در ۸۰ (دوین)

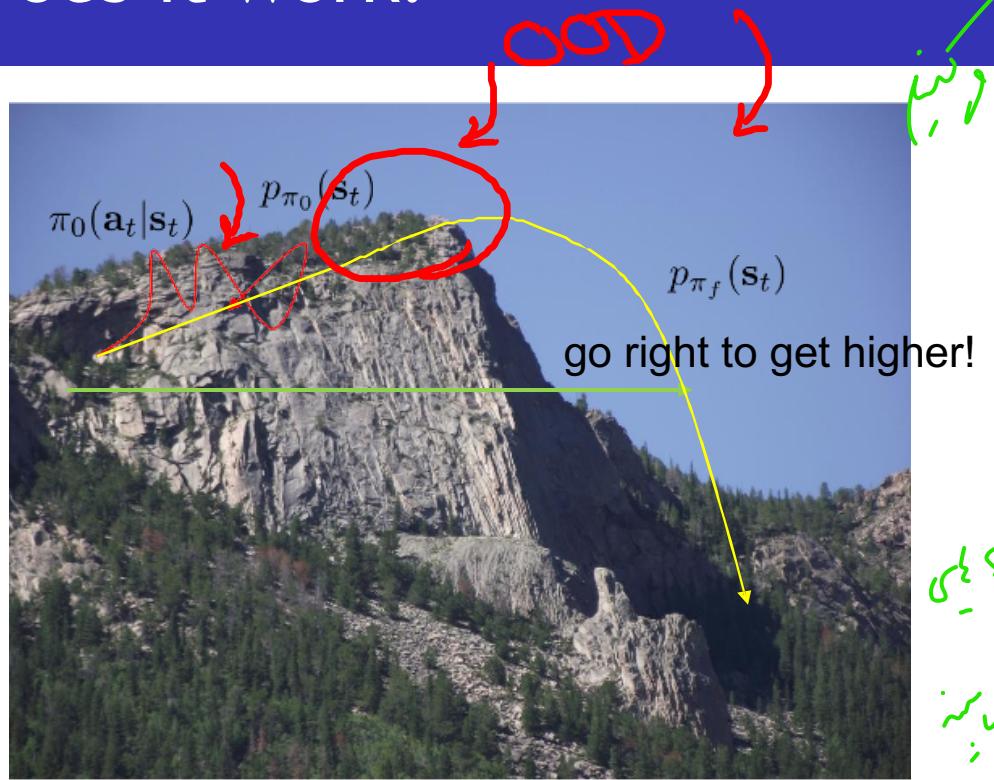
- Essentially how system identification works in classical robotics
 - Some care should be taken to design a good base policy
 - Particularly effective if we can hand-engineer a dynamics representation using our knowledge of physics, and fit just a few parameters

وَيُنْهَا إِلَى الْمَسْكَنِ الْأَعْلَى، فَإِنَّمَا يَرَى مِنْ أَنْفُسِهِمْ أَنَّهُمْ
كَافِرٌ وَلَا يُؤْمِنُونَ



$\hat{y}_i \in \{\text{learner}\} \cup \{\text{Out of Distribution}\}$

Does it work?



- Distribution mismatch problem becomes exacerbated as we use more expressive model classes

$$r(s) \leftarrow P(s) | s, a)$$

No!

1. run base policy $\pi_0(\mathbf{a}_t | \mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
 2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
 3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions

Planning with

$$p_{\pi_f}(\mathbf{s}_t) \neq p_{\pi_0}(\mathbf{s}_t)$$

Can we do better?

can we make $p_{\pi_0}(\mathbf{s}_t) = p_{\pi_f}(\mathbf{s}_t)$?

where have we seen that before? need to collect data from $p_{\pi_f}(\mathbf{s}_t)$

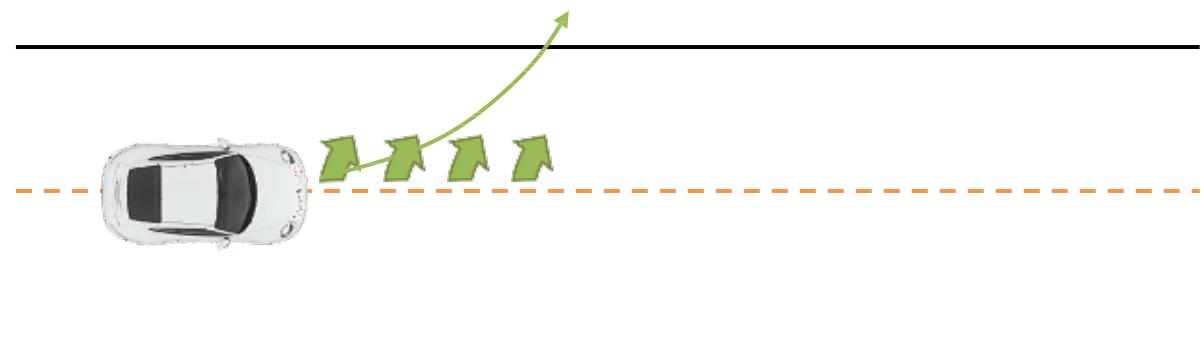
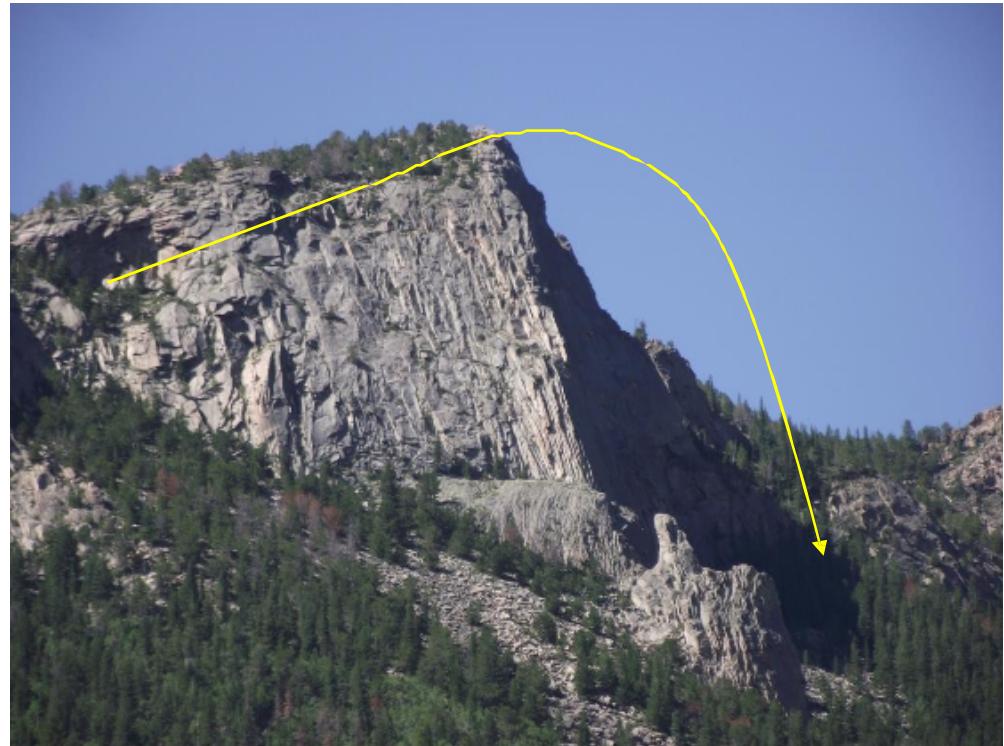
$$\begin{matrix} \textcircled{D} \\ \uparrow \\ \pi_0 \rightarrow \pi_f \rightarrow \pi_f' \end{matrix}$$

model-based reinforcement learning version 1.0:

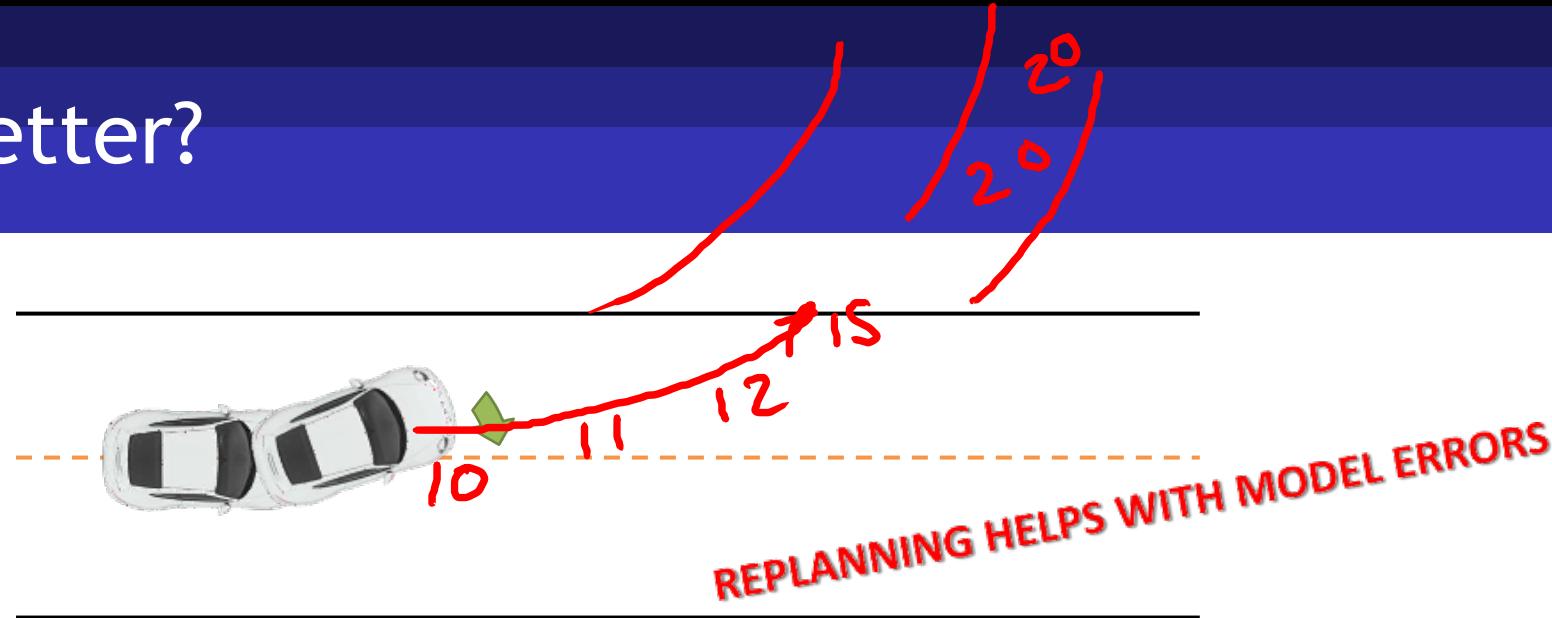
1. run base policy $\pi_0(\mathbf{a}_t | \mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions π_f
4. execute those actions and add the resulting data $\{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_j\}$ to \mathcal{D}

step in env.

What if we make a mistake?



Can we do better?



model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t | \mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$

2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$

3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions

an opt. happens here (CEM)

- MPC*
4. execute the first planned action, observe resulting state \mathbf{s}' (MPC)
 5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset \mathcal{D}

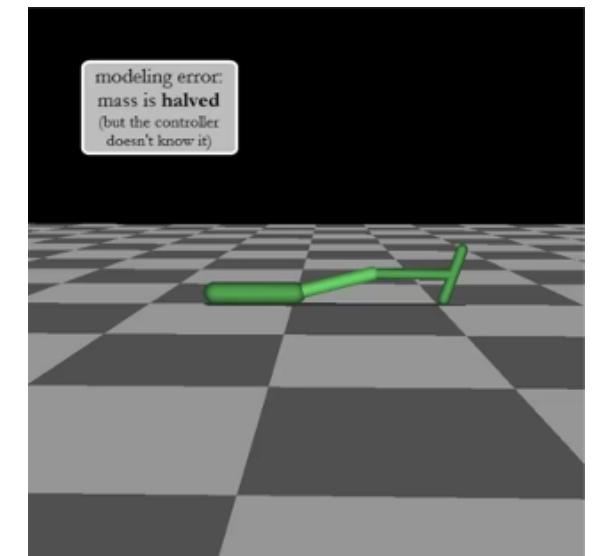
How to replan?

model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute the first planned action, observe resulting state \mathbf{s}' (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset \mathcal{D}



- The more you replan, the less perfect each individual plan needs to be
- Can use shorter horizons
- Even random sampling can often work well here!

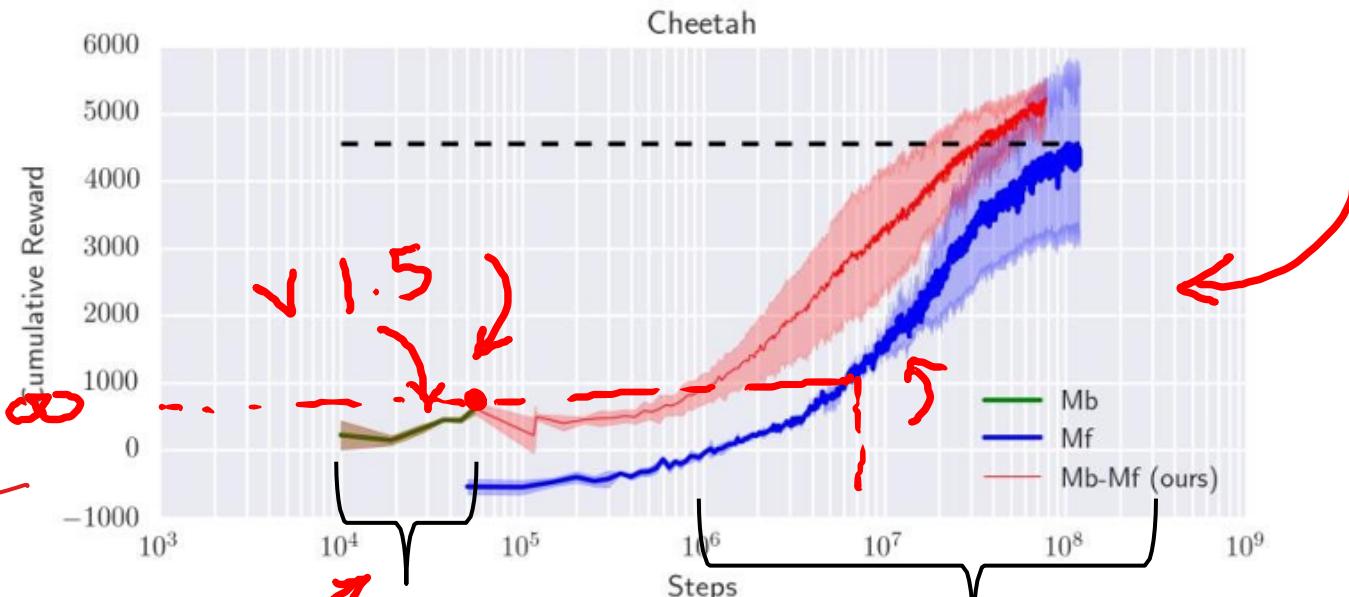


Uncertainty in Model-Based RL

A performance gap in model-based RL

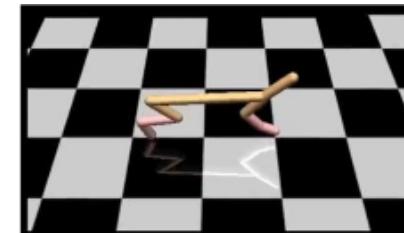
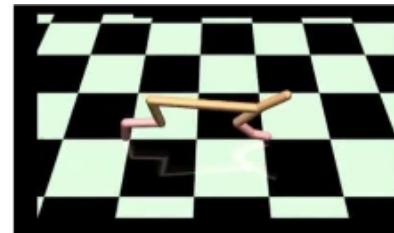


↑
so
Lijstje Mb zijn λ_p ←

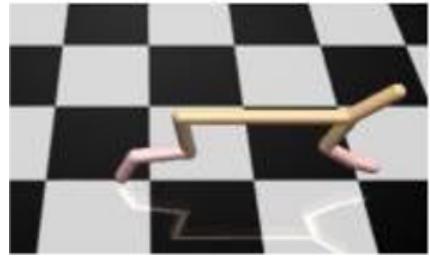


**pure model-based
(about 10 minutes real
time)**

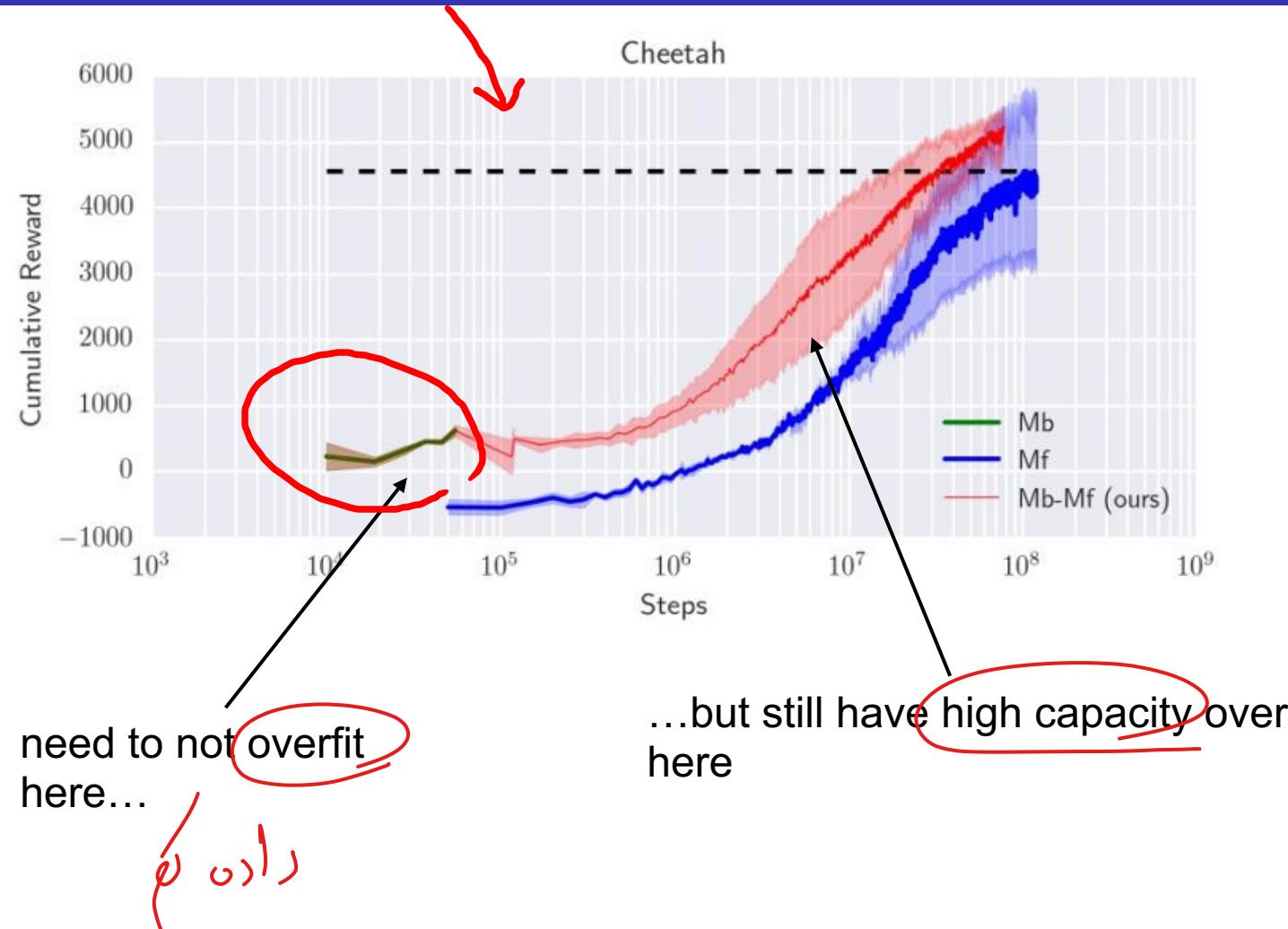
model-free training
(about 10 days)



Why the performance gap?



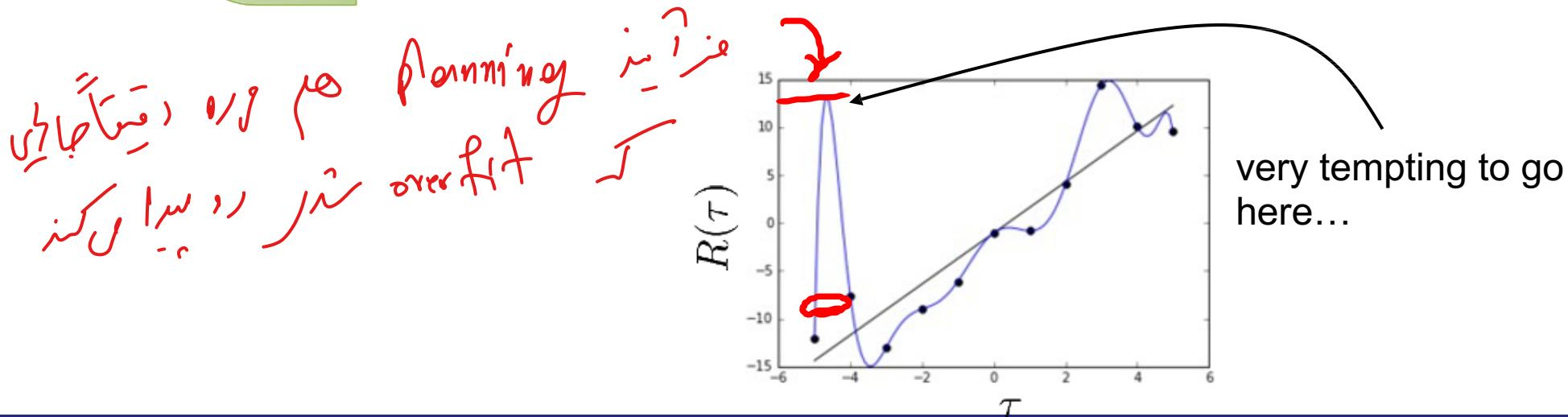
Overfit



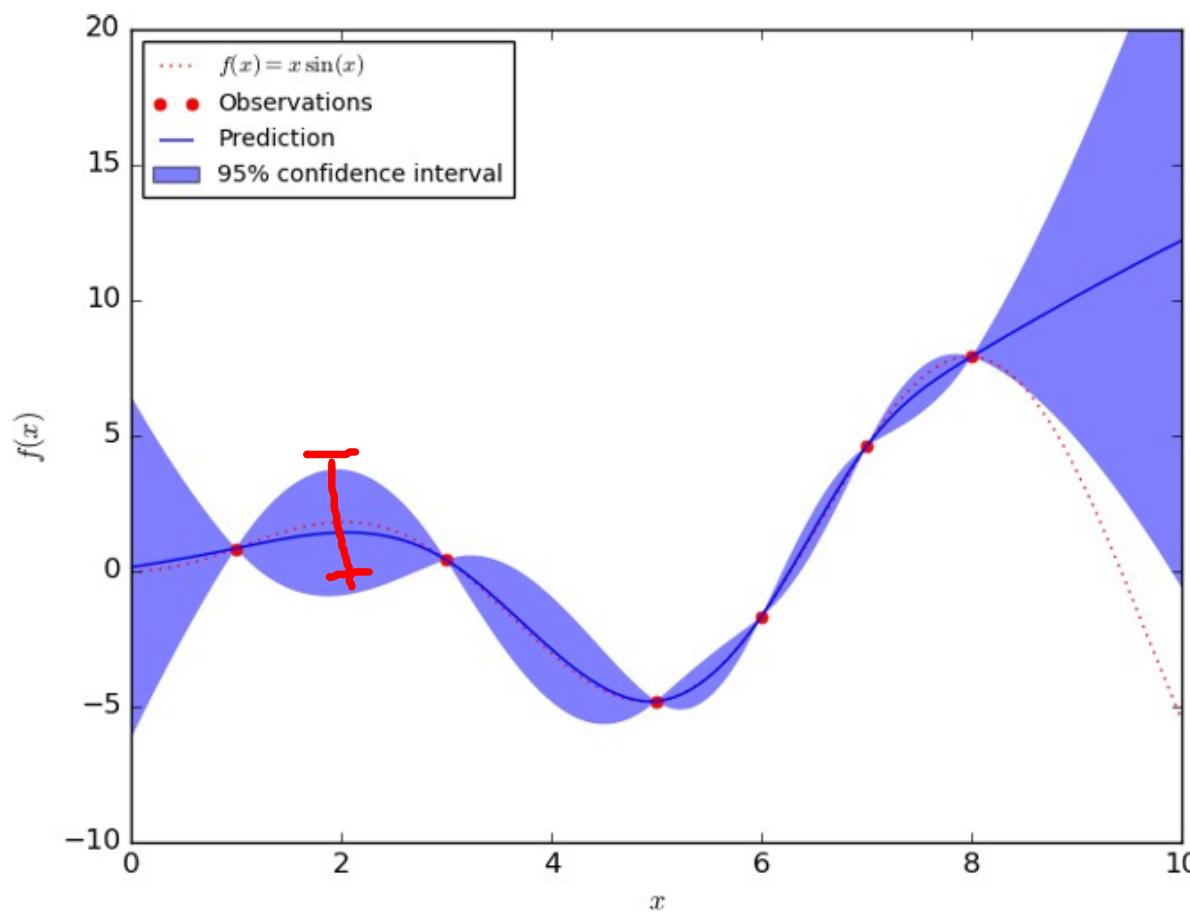
Why the performance gap?

model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
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3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute the first planned action, observe resulting state \mathbf{s}' (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset \mathcal{D}



How can uncertainty estimation help?



$$p_{\pi_f}(\mathbf{s}_t) \neq p_{\pi_0}(\mathbf{s}_t)$$

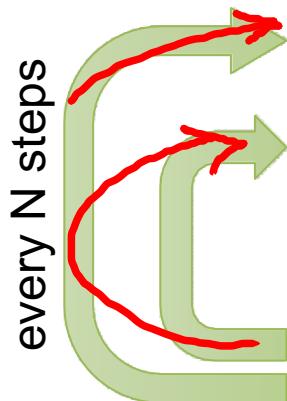


expected reward under high-variance prediction
is **very** low, even though mean is the same!

Intuition behind uncertainty-aware RL

model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t | \mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute the first planned action, observe resulting state \mathbf{s}' (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset \mathcal{D}



only take actions for which we think we'll get high reward in expectation (w.r.t. uncertain dynamics)

This avoids “exploiting” the model

The model will then adapt and get better

Uncertain dynamics
exploit
adapt
improve

There are a few caveats...



Need to explore to get better

Expected value is not the same as pessimistic value

Expected value is not the same as optimistic value

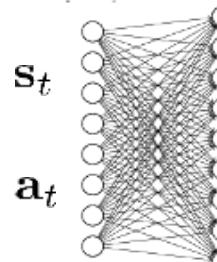
...but expected value is often a good start

Uncertainty-Aware Neural Net Models

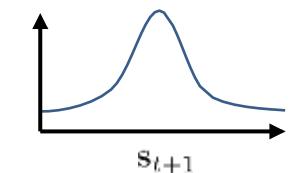
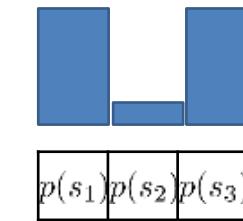
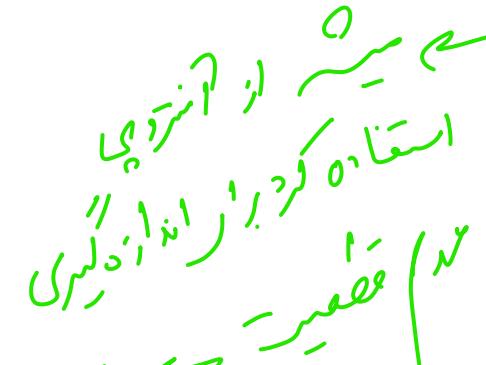
Uncertainty-aware neural net models

How can we have uncertainty-aware models?

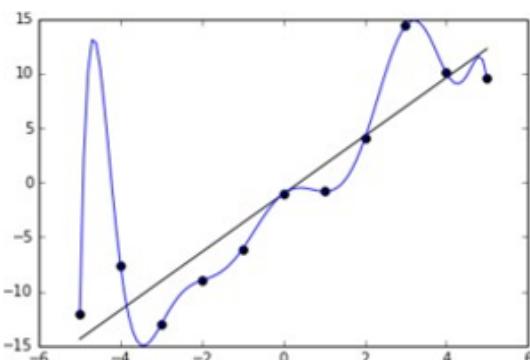
Idea 1: use output entropy



$$p(s_{t+1}|s_t, a_t)$$

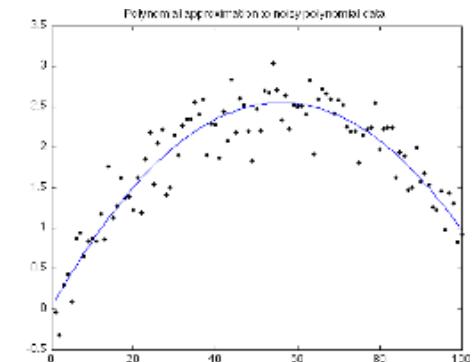


why is this not enough?



Two types of uncertainty:

aleatoric or statistical uncertainty
epistemic or model uncertainty



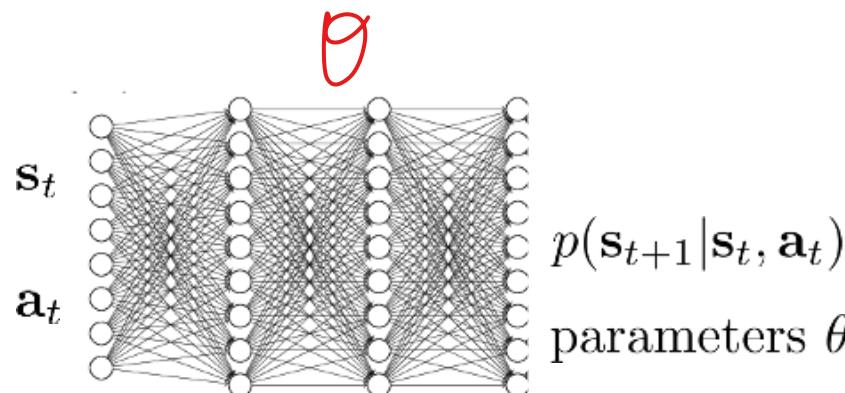
what is the variance here?

"the model is certain about the data, but we are not certain about the model"

How can we have uncertainty-aware models?

Idea 2: estimate model uncertainty

"the model is certain about the data, but we are not certain about the model"



predict according to:

$$\int p(s_{t+1}|s_t, a_t, \theta) p(\theta|\mathcal{D}) d\theta$$

usually, we estimate

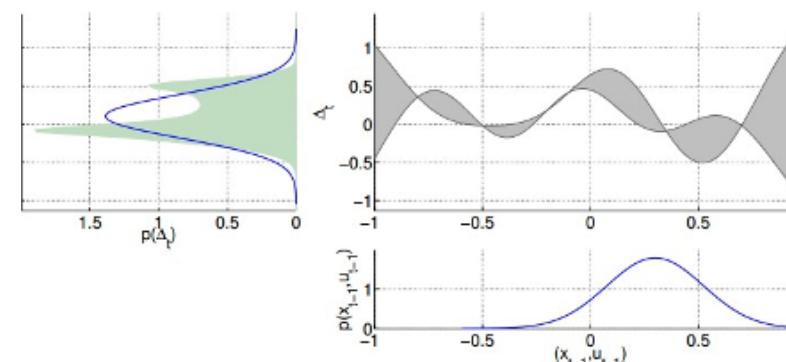
$$\arg \max_{\theta} \log p(\theta|\mathcal{D}) = \arg \max_{\theta} \log p(\mathcal{D}|\theta)$$

can we instead estimate $p(\theta|\mathcal{D})$?

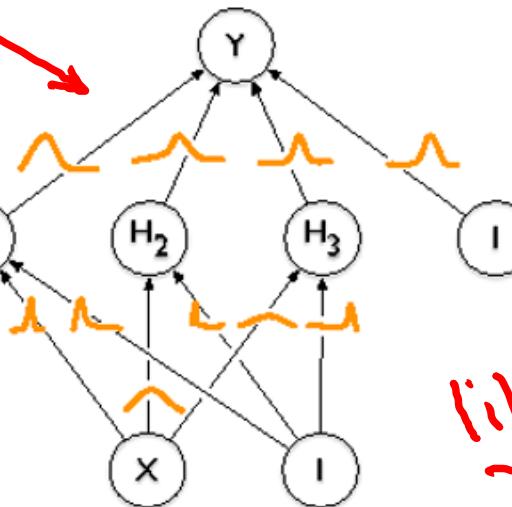
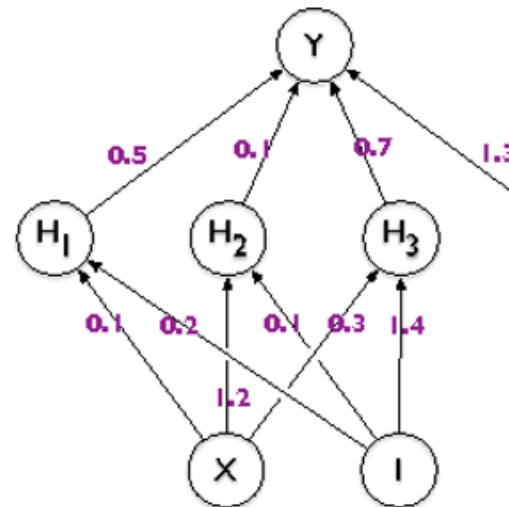
Model \rightarrow

$P(\theta|\mathcal{D})$

the entropy of this tells us
the model uncertainty!



Quick overview of Bayesian neural networks



common approximation:

$$p(\theta|\mathcal{D}) = \prod_i p(\theta_i|\mathcal{D})$$

$$p(\theta_i|\mathcal{D}) = \mathcal{N}(\mu_i, \sigma_i)$$

likelihood

expected weight

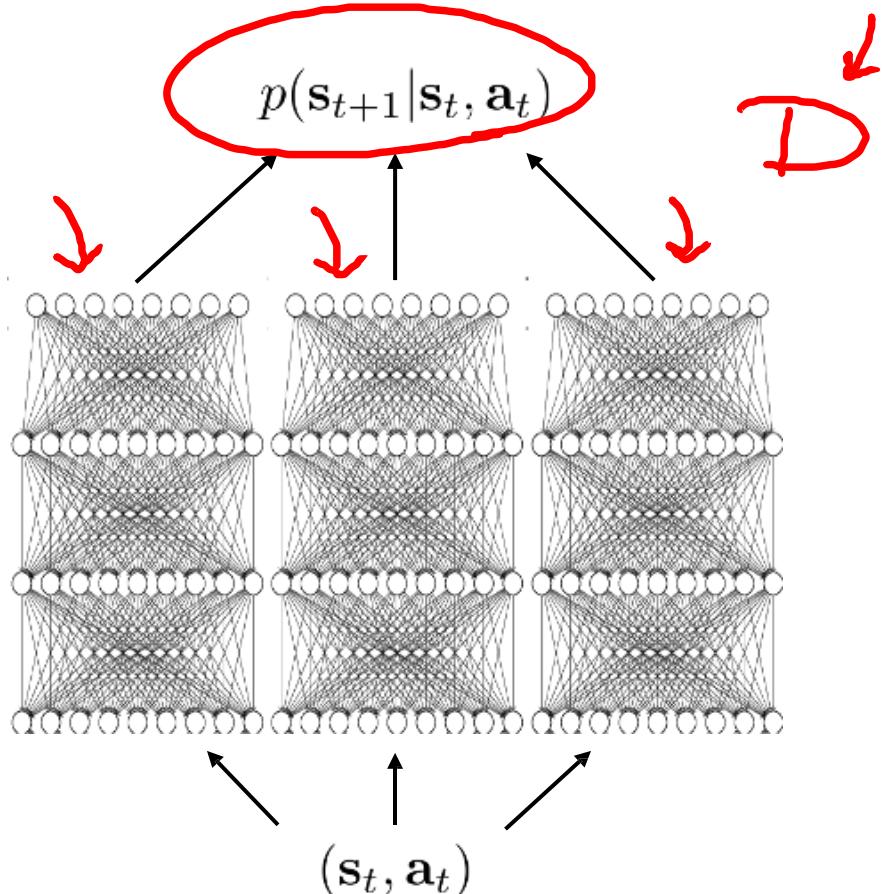
uncertainty about
the weight

$$P(\theta|\mathcal{D}) = P(D|\theta) \cdot P(\theta) \quad \begin{matrix} \nearrow \\ \text{z} \end{matrix} \quad \begin{matrix} \searrow \\ \text{Uninformative} \end{matrix}$$

For more, see:

Blundell et al., Weight Uncertainty in Neural Networks Gal et al., Concrete Dropout

Bootstrap ensembles



Train multiple models and see if they agree!

formally: $p(\theta|\mathcal{D}) \approx \frac{1}{N} \sum_i \delta(\theta_i)$

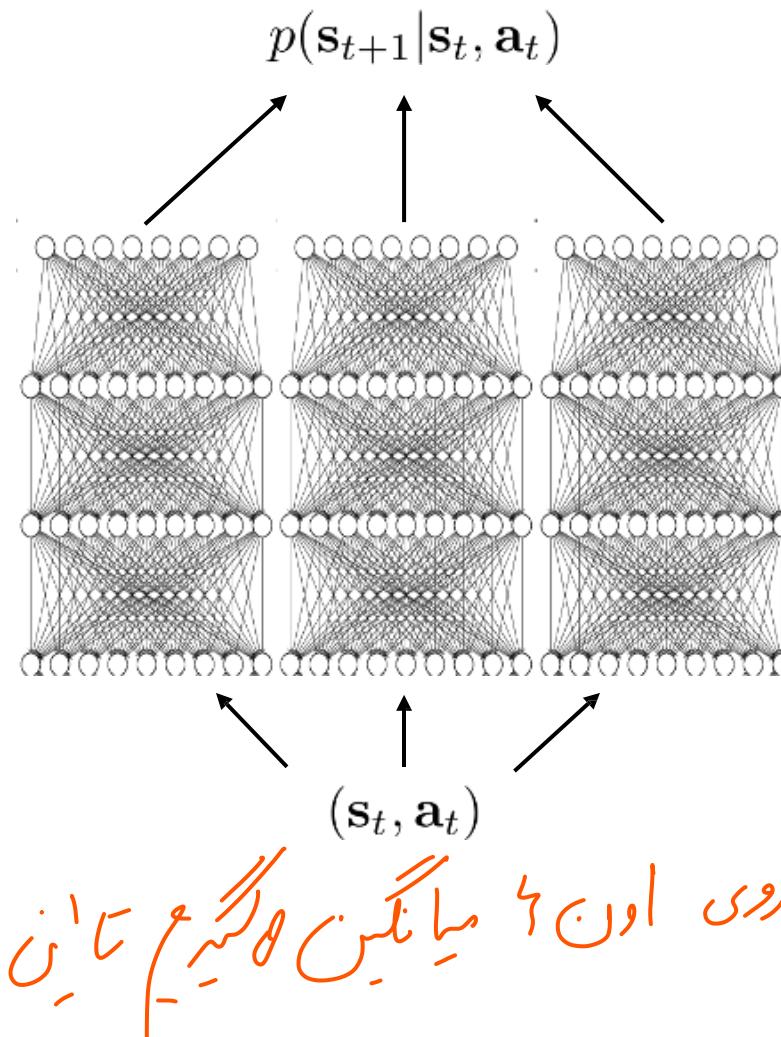
$$\int p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t, \theta) p(\theta|\mathcal{D}) d\theta \approx \frac{1}{N} \sum_i p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t, \theta_i)$$

How to train?

Main idea: need to generate “independent” datasets to get “independent” models

θ_i is trained on \mathcal{D}_i , sampled *with replacement* from \mathcal{D}

Bootstrap ensembles in deep learning



This basically works

$$P(\theta|D)$$

Very crude approximation, because the number of models is usually small (< 10)

Resampling with replacement is usually unnecessary, because SGD and random initialization usually makes the models sufficiently independent

جایزه ای این میانگین و میانگین اولیه میتواند باشد لیکن نیز $\mathcal{L} - N$
برای درستی فر

Planning with Uncertainty, Examples

How to plan with uncertainty

Before: $J(\mathbf{a}_1, \dots, \mathbf{a}_H) = \sum_{t=1}^H r(\mathbf{s}_t, \mathbf{a}_t)$, where $\mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{a}_t)$

Now: $J(\mathbf{a}_1, \dots, \mathbf{a}_H) = \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^H r(\mathbf{s}_{t,i}, \mathbf{a}_t)$, where $\mathbf{s}_{t+1,i} = f_i(\mathbf{s}_{t,i}, \mathbf{a}_t)$

In general, for candidate action sequence $\mathbf{a}_1, \dots, \mathbf{a}_H$:

Step 1: sample $\theta \sim p(\theta|\mathcal{D})$

Step 2: at each time step t , sample $\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t, \theta)$

Step 3: calculate $R = \sum_t r(\mathbf{s}_t, \mathbf{a}_t)$

Step 4: repeat steps 1 to 3 and accumulate the average reward

$\rightarrow p(\cdot | \mathcal{D}, \theta)$

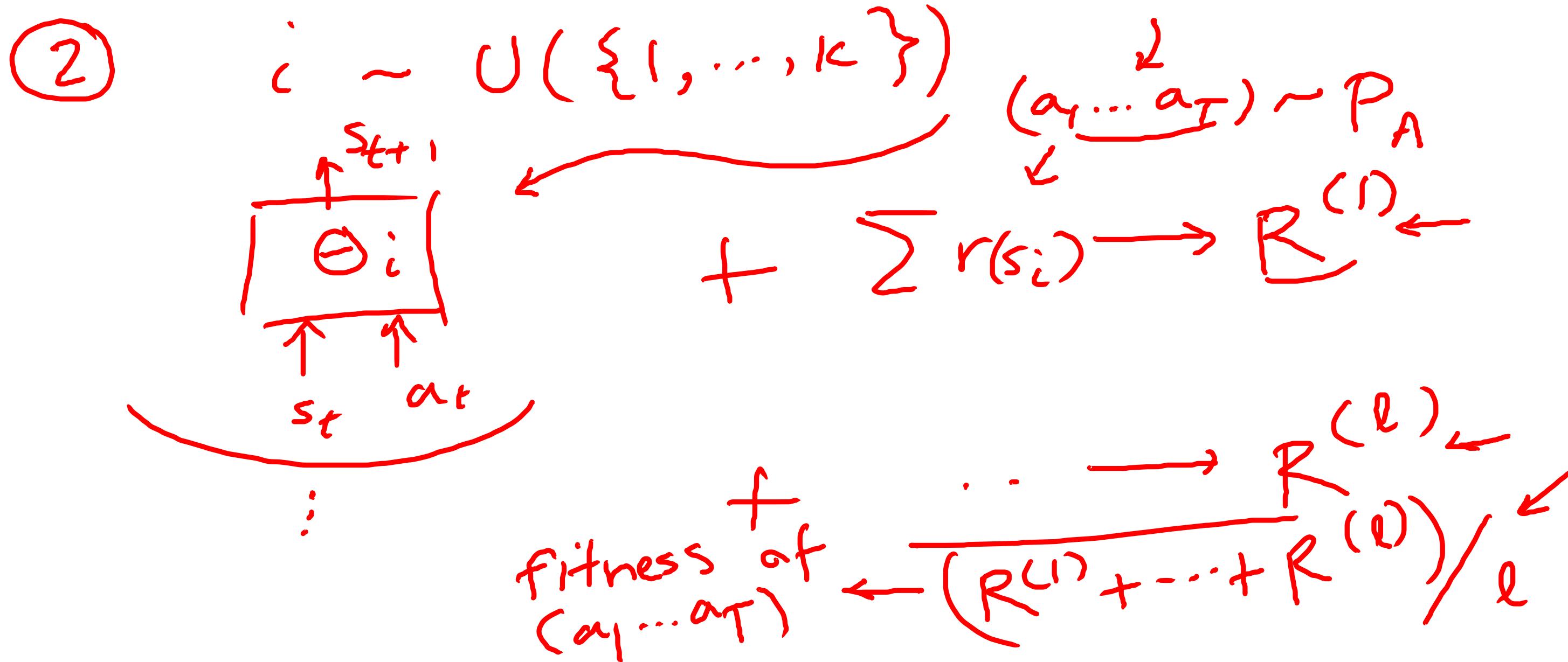
distribution over
deterministic models

$$\frac{(R_1 + R_2 + \dots + R_K)}{K}$$

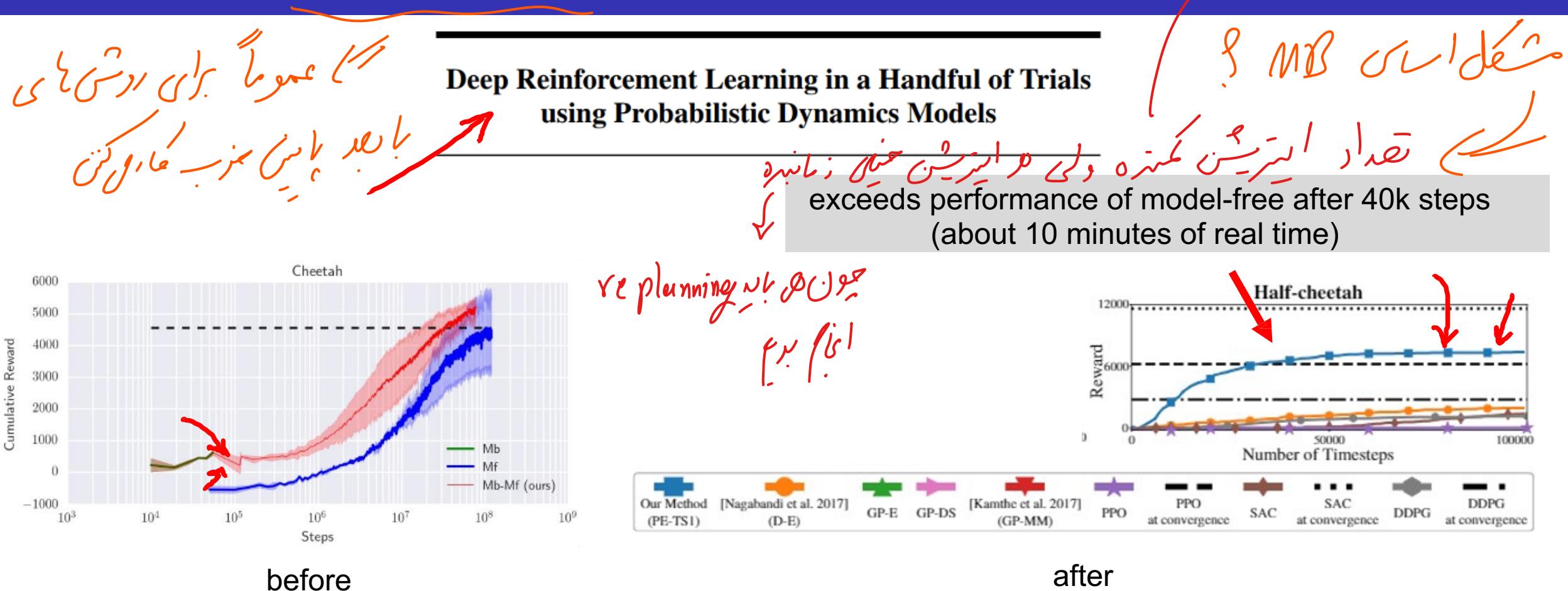
Other options: moment matching, more complex posterior estimation with BNNs, etc.



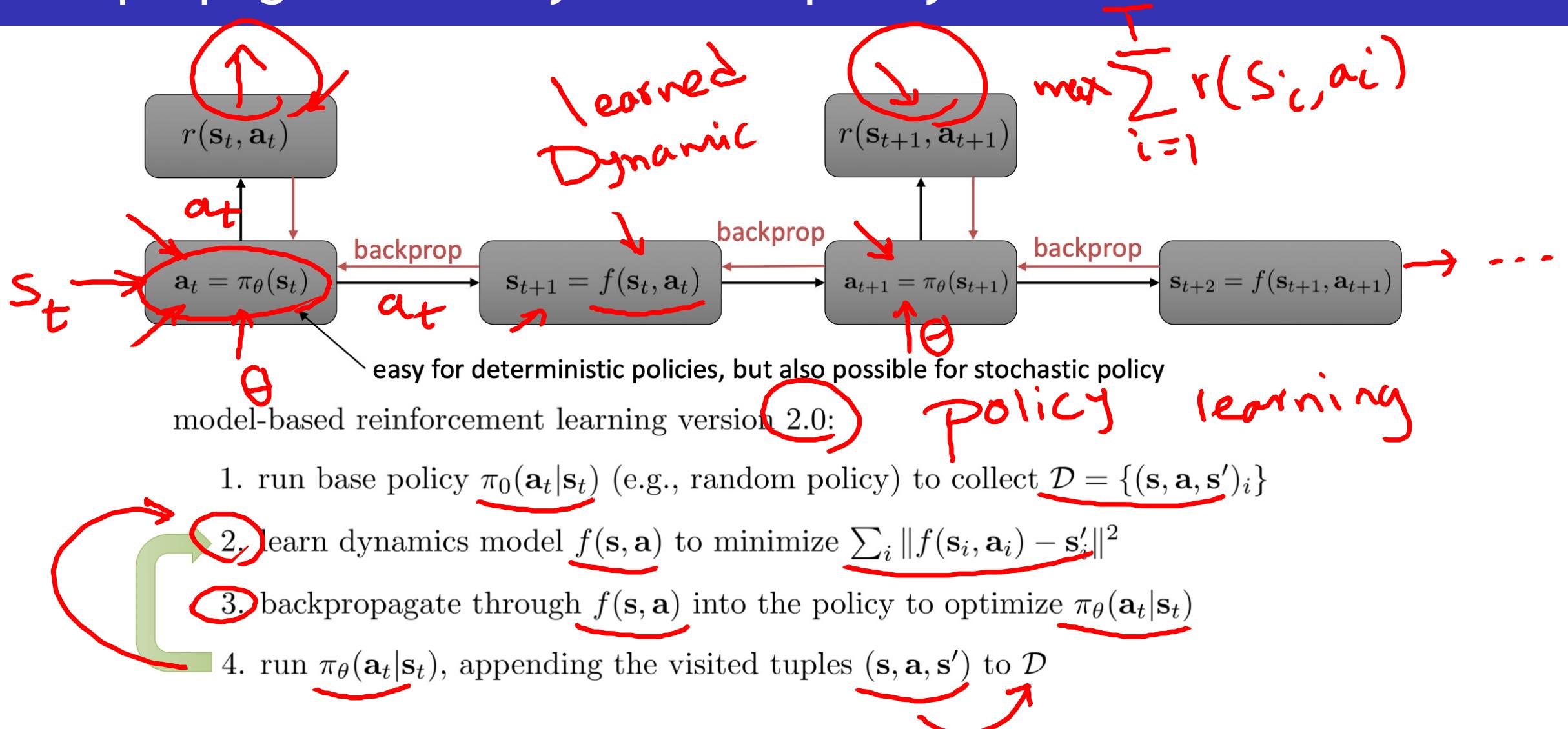
$$D \sim \pi_0$$



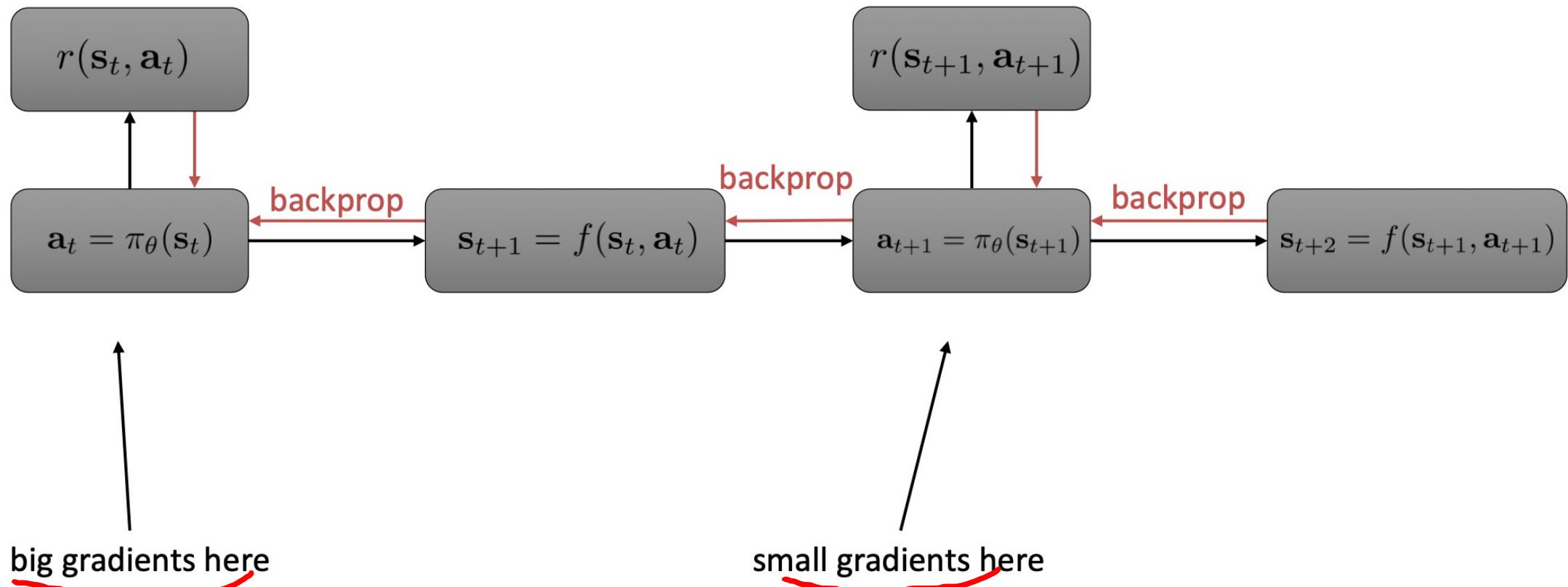
Example: model-based RL with ensembles



Backpropagate directly into the policy?



What's the problem with backprop into policy?



What's the problem with backprop into policy?

- Similar parameter sensitivity problems as shooting methods
 - But no longer have convenient second order LQR-like method, because policy parameters **couple** all the time steps, so no dynamic programming
- Similar problems to training long RNNs with BPTT
 - **Vanishing** and **exploding** gradients
 - Unlike LSTM, we can't just "choose" a simple dynamics, **dynamics are chosen by nature**

What's the problem with backprop into policy?

چیزی نیست

- Use derivative-free (“model-free”) RL algorithms, with the model used to generate synthetic samples
- Seems weirdly backwards
 - Actually works very well
 - Essentially “model-based acceleration” for model-free RL

$$f(s, a)$$

s₀ a₀ s₁ a₁ s₂

MF $\sim \{s_n, a_n\}$ \rightarrow MB

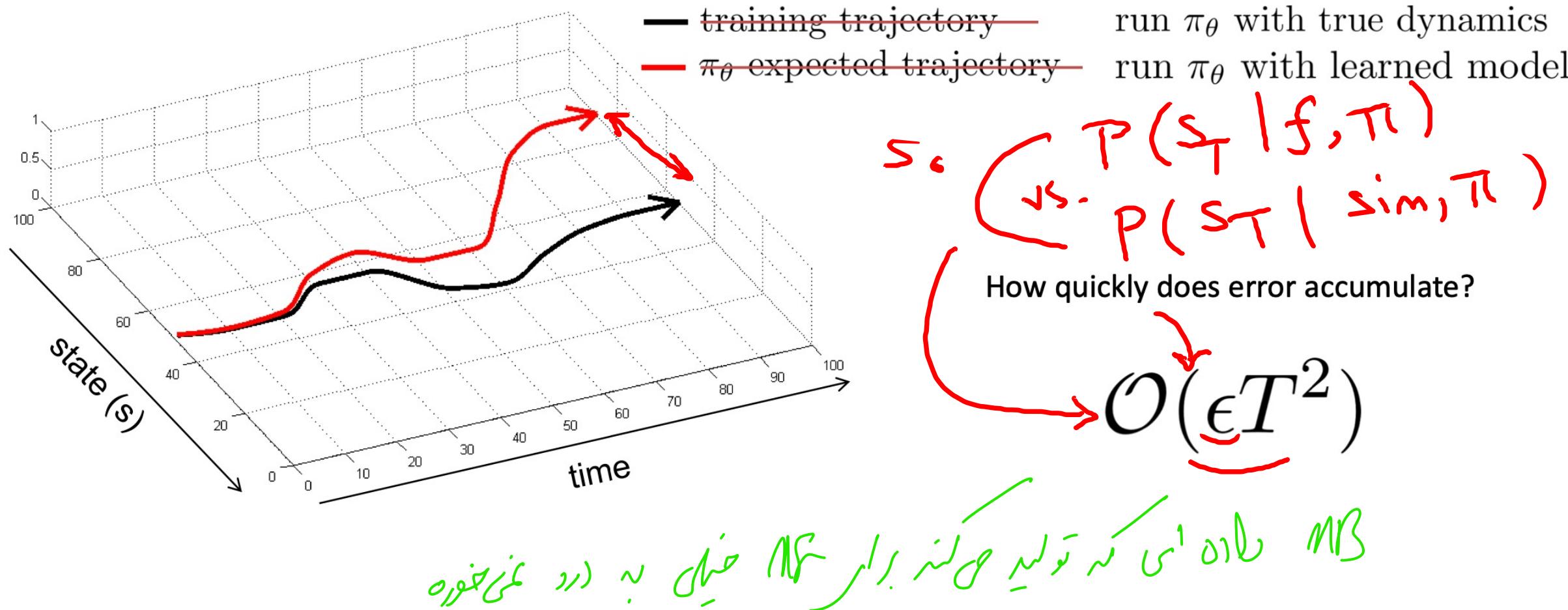
Model-based RL via policy gradient

model-based reinforcement learning version 2.5:

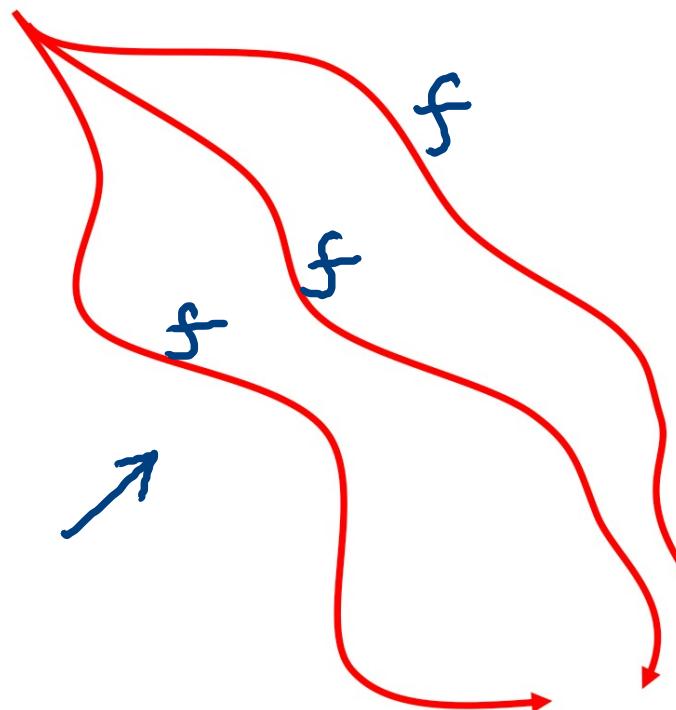
1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. use $f(\mathbf{s}, \mathbf{a})$ to generate trajectories $\{\tau_i\}$ with policy $\pi_\theta(\mathbf{a}|\mathbf{s})$
4. use $\{\tau_i\}$ to improve $\pi_\theta(\mathbf{a}|\mathbf{s})$ via policy gradient
5. run $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$, appending the visited tuples $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to \mathcal{D}

What's a potential **problem** with this approach?

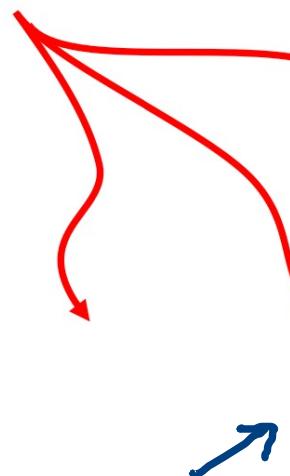
The curse of long model-based rollouts



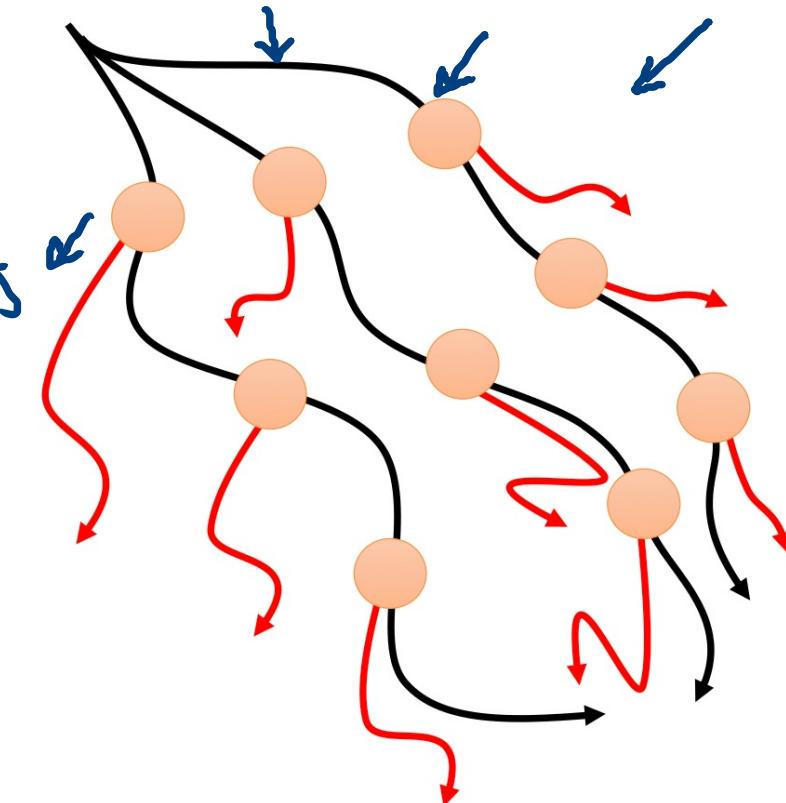
How to get away with accumulated errors?



- huge accumulating error



+ much lower error
- never see later time steps



+ much lower error
+ see all time steps
- wrong state distribution

Model-based RL with short rollouts

model-based reinforcement learning version 3.0:

1. run base policy $\pi_0(\mathbf{a}_t | \mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. pick states \mathbf{s}_i from \mathcal{D} , use $f(\mathbf{s}, \mathbf{a})$ to make *short* rollouts from them
4. use *both* real and model data to improve $\pi_\theta(\mathbf{a} | \mathbf{s})$ with *off-policy RL*
5. run $\pi_\theta(\mathbf{a}_t | \mathbf{s}_t)$, appending the visited tuples $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to \mathcal{D}

(Dyna Style)

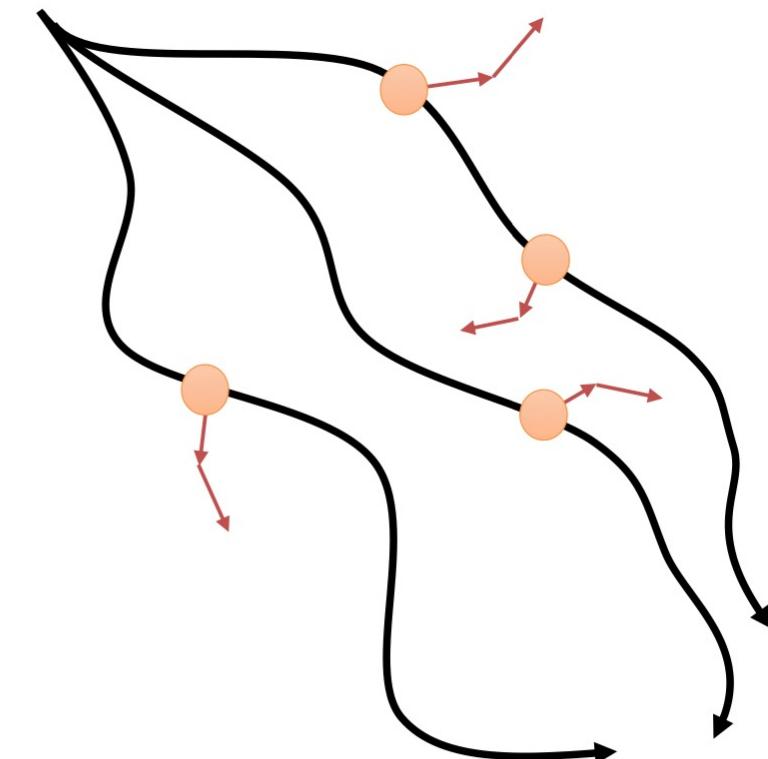
1. given state s , pick action a using exploration policy
2. observe s' and r , to get transition $(s, a, s', r) \rightarrow \mathcal{B}$
3. update model $\hat{p}(s'|s, a)$ and $\hat{r}(s, a)$ using (s, a, s')
4. Q-update: $Q(s, a) \leftarrow Q(s, a) + \alpha E_{s', r}[r + \max_{a'} Q(s', a') - Q(s, a)]$
5. repeat K times:
 6. sample $(s, a) \sim \mathcal{B}$ from buffer of past states and actions
 7. Q-update: $Q(s, a) \leftarrow Q(s, a) + \alpha E_{s', r}[r + \max_{a'} Q(s', a') - Q(s, a)]$

Richard S. Sutton. Integrated architectures for learning, planning, and reacting based on approximating dynamic programming.

General “Dyna-style” model-based RL recipe

Sim

1. collect some data, consisting of transitions (s, a, s', r)
2. learn model $\hat{p}(s'|s, a)$ (and optionally, $\hat{r}(s, a)$)
3. repeat K times:
 4. sample $s \sim \mathcal{B}$ from buffer
 5. choose action a (from \mathcal{B} , from π , or random)
 6. simulate $s' \sim \hat{p}(s'|s, a)$ (and $r = \hat{r}(s, a)$)
 7. train on (s, a, s', r) with model-free RL
 8. (optional) take N more model-based steps



+ only requires short (as few as one step) rollouts from model

+ still sees diverse states

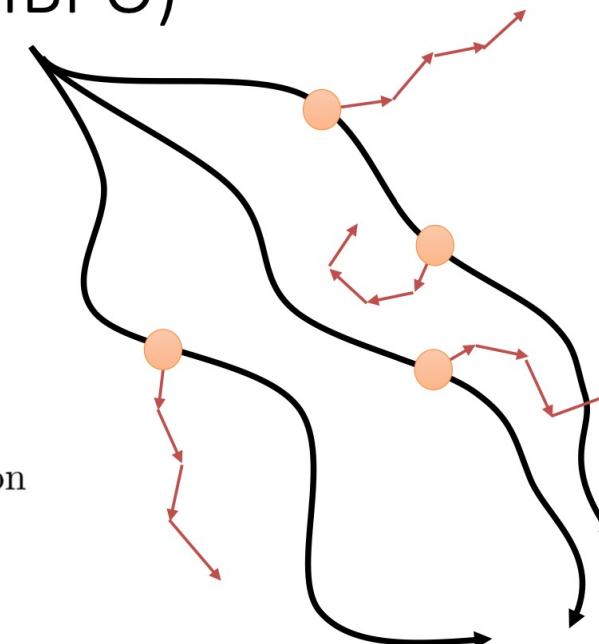
Instantiations

- { Model-Based Acceleration (MBA)
- Model-Based Value Expansion (MVE)
- Model-Based Policy Optimization (MBPO)

1. take some action \mathbf{a}_i and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to \mathcal{B}
2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from \mathcal{B} uniformly
3. use $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j\}$ to update model $\hat{p}(\mathbf{s}'|\mathbf{s}, \mathbf{a})$
4. sample $\{\mathbf{s}_j\}$ from \mathcal{B}
5. for each \mathbf{s}_j , perform model-based rollout with $\mathbf{a} = \pi(\mathbf{s})$
6. use all transitions $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$ along rollout to update Q-function

+ why is this a *good idea*?

- why is this a *bad idea*?



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