Generalization

Reinforcement Learning

Computer Engineering Department
Sharif University of Technology

$Q(s,a)$

States are continuous

Mohammad Hossein Rohban, Ph.D.

feature

$\phi(s)$ Representation

inp.

(s,a)

or there are infinitely many of them.

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Courtesy: Some slides are adopted from CS 285 Berkeley, and CS 234 Stanford, and Pieter Abbeel's compact series on RL.

Function Approximation

forgetting

مشکلات

Replay Buffer

$\pi = \epsilon\text{-greedy}\ \hat{Q}_w(s,a)$

$\rightarrow S_0, a_0, r_0, S_1, a_1, r_1 \ldots$

training sample

① اینا شامل شده ← مدل یاد ندارد

② دیتا همبستگی داره

③ داده ها جزو سون تقریبی اند

MC

$(\Phi(s_0, a_0), G_0) \rightarrow$

$(\Phi(s_1, a_1), G_1)$

TD for the network

$(\Phi(s_0, a_0), r_0 + \gamma \max_a \hat{Q}_w(s_1, a))$

$(\Phi(s, a_0), r_0 + \gamma \hat{Q}_w(s_1, a_1))$

$\hat{Q}_w(s,a)$

استقلال وجود ندارد

حتی در حالت مارکوف ← مارکوف هم که استقلال شرطی داریم

Replay Buffer

Function approximation and deep RL

- The policy, value function, model, and agent state update are all functions

شما هر مقدار value ها رو عوض کنی
که شبکه گیج میشه بنابراین یه شبکه
دیگه داریم که این قدری هست
و سریع به روز نمیشه $\hat{Q}_{w^-}(s,a)$

- We want to learn these from experience (data).

- If there are too many states, we need to approximate.

- This is often called deep reinforcement learning, when using neural networks to represent these functions.

$q(s,a) \xrightarrow{F} \hat{q}(s,a)$

داده ها با F تولید میشه

تارگت یه شبکه است
یه شبکه دیگه داریم.

Large-Scale Reinforcement Learning

- Reinforcement learning can be used to solve large problems, e.g.

  - Backgammon: 10^{20} states
  - Go: 10^{170} states
  - Helicopter: continuous state space
  - Robots: real world

- How can we apply our methods for prediction and control?

Value Function Approximation

Value Function Approximation

- So far we mostly considered lookup tables
  - Every state s has an entry $v(s)$
  - Or every state-action pair s, a has an entry $q(s, a)$
- Problem with large MDPs:
  - There are too many states and/or actions to store in memory
  - It is too slow to learn the value of each state individually
  - Individual environment states are often not fully observable

Value Function Approximation

- Solution for large MDPs:
  - Estimate value function with function approximation

$$v_{\mathbf{w}}(s) \approx v_\pi(s) \qquad \text{(or } v_*(s)\text{)}$$
$$q_{\mathbf{w}}(s,a) \approx q_\pi(s,a) \qquad \text{(or } q_*(s,a)\text{)}$$

  - Update parameter w (e.g., using MC or TD learning)
  - Generalize to unseen states

Agent state

- When the environment state is not fully observable ($S_t^{env} \neq O_t$)
- Use the agent state: (with parameters ω)

$$\mathbf{s}_t = u_\omega(\mathbf{s}_{t-1}, A_{t-1}, O_t)$$

- Henceforth, S_t or s_t denotes the agent state
- Think of this as either a vector inside the agent, or, in the simplest case, just the current observation: $S_t = O_t$

Function Classes

Classes of Function Approximation

- Tabular: a table with an entry for each MDP state
- State aggregation: Partition environment states (or observations) into a discrete set
- Linear function approximation
  - Consider fixed agent state update (e.g. $S_t = O_t$)
  - Fixed feature map $x: \mathcal{S} \to \mathbb{R}^n$
  - Values are linear function of features: $v_w(s) = w^T x(s)$
  - Note: state aggregation and tabular are special cases of linear FA
- Differentiable function approximation
  - $v_w(s)$ is a differentiable function of w, could be non-linear
  - E.g., a convolutional neural network that takes pixels as input
  - Another interpretation: features are not fixed, but learnt

Classes of Function Approximation

- In principle, any function approximator can be used, but RL has specific properties:

  - Experience is not iid — successive time-steps are correlated
  - Agent's policy affects the data it receives
  - Regression targets can be non-stationary
    - ...because of changing policies (which can change the target and the data!)
    - ...because of bootstrapping
    - ...because of non-stationary dynamics (e.g., other learning agents)
    - ...because the world is large (never quite in the same state)

Classes of Function Approximation

- Which function approximation should you choose? This depends on your goals.

  - Tabular: good theory but does not scale/generalize
  - Linear: reasonably good theory, but requires good features
  - Non-linear: less well-understood, but scales well. Flexible, and less reliant on picking good features first (e.g., by hand)
  - (Deep) neural nets often perform quite well, and remain a popular choice

Gradient-based Algorithms

Approximate Values By Stochastic Gradient Descent

- Goal: find w that minimize the difference between $v_w(s)$ and $v_\pi(s)$

$$J(\mathbf{w}) = \mathbb{E}_{S\sim d}[(v_\pi(S) - v_\mathbf{w}(S))^2]$$

- Gradient descent:

$$\Delta\mathbf{w} = -\frac{1}{2}\alpha\nabla_\mathbf{w}J(\mathbf{w}) = \alpha\mathbb{E}_d(v_\pi(S) - v_\mathbf{w}(S))\nabla_\mathbf{w}v_\mathbf{w}(S)$$

- Stochastic gradient descent (SGD), sample the gradient:

$$\Delta\mathbf{w} = \alpha(G_t - v_\mathbf{w}(S_t))\nabla_\mathbf{w}v_\mathbf{w}(S_t)$$

Note: Monte Carlo return G_t is a sample for $v_\pi(s_t)$

Linear function approximation

Feature Vectors

- Represent state by a feature vector

$$\mathbf{x}(s) = \begin{pmatrix} x_1(s) \\ \vdots \\ x_n(s) \end{pmatrix}$$

- $x: \mathcal{S} \rightarrow \mathbb{R}^n$ is a fixed mapping from state (e.g., observation) to features
- Short-hand: $x_t = x(S_t)$
- For example:
  - Distance of robot from landmarks
  - Trends in the stock market
  - Piece and pawn configurations in chess

Linear value function approximation

- Approximate value function by a linear combination of features

$$v_{\mathbf{w}}(s) = \mathbf{w}^\top \mathbf{x}(s) = \sum_{j=1}^{n} \mathbf{x}_j(s)\mathbf{w}_j$$

- Objective function (‘loss‘) is quadratic in w

$$J(\mathbf{w}) = \mathbb{E}_{S\sim d}[(v_\pi(S) - \mathbf{w}^\top \mathbf{x}(S))^2]$$

- Stochastic gradient descent converges to the global optimum
- Update rule is simple

$$\nabla_{\mathbf{w}} v_{\mathbf{w}}(S_t) = \mathbf{x}(S_t) = \mathbf{x}_t \qquad \Longrightarrow \qquad \Delta\mathbf{w} = \alpha(v_\pi(S_t) - v_{\mathbf{w}}(S_t))\mathbf{x}_t$$

Prediction algorithms

- We can't update towards the true value function $v_\pi(s)$
- We substitute a target for $v_\pi(s)$
  - For MC, the target is the return G_t

$$\Delta \mathbf{w}_t = \alpha(G_t - v_{\mathbf{w}}(s))\nabla_{\mathbf{w}} v_{\mathbf{w}}(s)$$

  - For TD, the target is the TD target $R_{t+1} + \gamma v_w(S_{t+1})$

$$\Delta \mathbf{w}_t = \alpha(R_{t+1} + \gamma v_{\mathbf{w}}(S_{t+1}) - v_{\mathbf{w}}(S_t))\nabla_{\mathbf{w}} v_{\mathbf{w}}(S_t)$$

  - TD(λ):

$$\Delta \mathbf{w}_t = \alpha(G_t^\lambda - v_{\mathbf{w}}(S_t))\nabla_{\mathbf{w}} v_{\mathbf{w}}(S_t)$$

$$G_t^\lambda = R_{t+1} + \gamma\Big((1-\lambda)v_{\mathbf{w}}(S_{t+1}) + \lambda G_{t+1}^\lambda\Big)$$

Monte-Carlo with Value Function Approximation

- The return G_t is an unbiased sample of $v_\pi(s)$
- Can therefore apply “supervised learning” to (online) “training data":

$$\{(S_0, G_0), \ldots, (S_t, G_t)\}$$

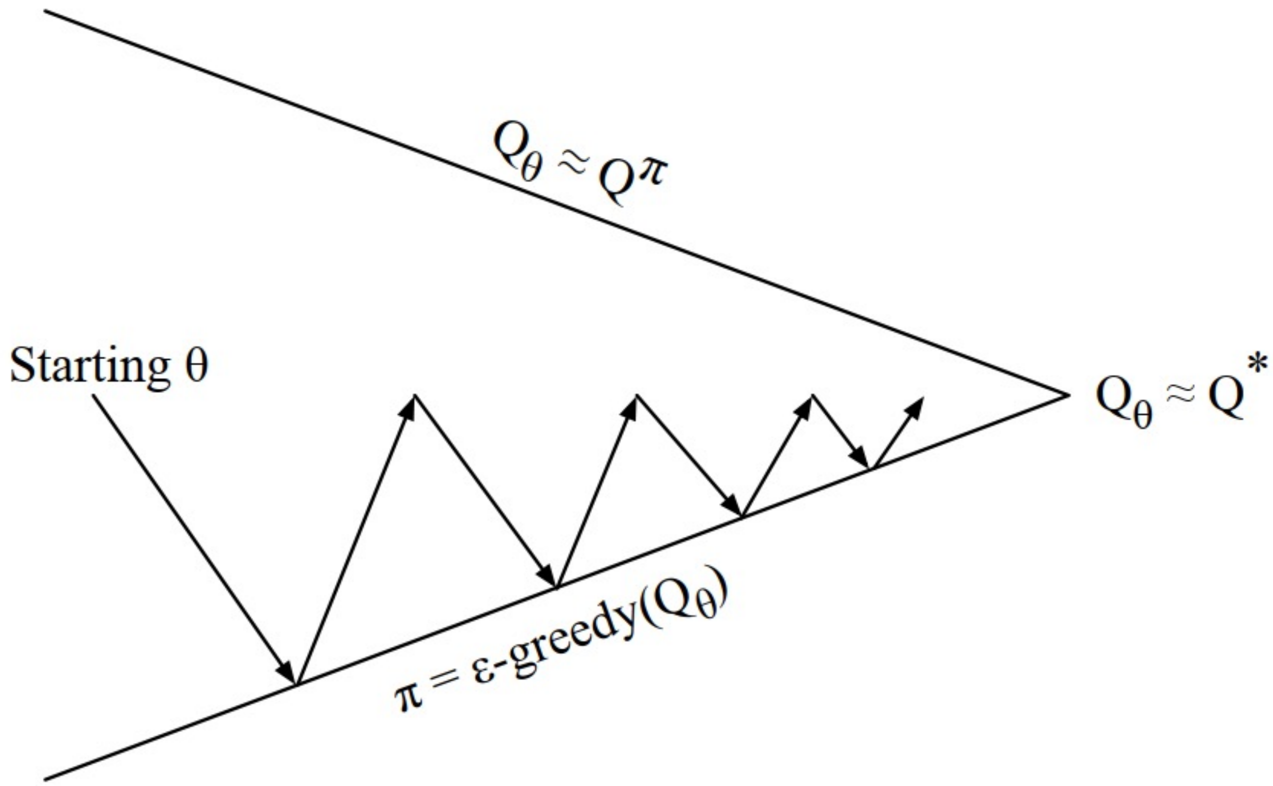
- For example, using linear Monte-Carlo policy evaluation

$$\begin{aligned}\Delta \mathbf{w}_t &= \alpha(G_t - v_{\mathbf{w}}(S_t))\nabla_{\mathbf{w}} v_{\mathbf{w}}(S_t) \\ &= \alpha(G_t - v_{\mathbf{w}}(S_t))\mathbf{x}_t\end{aligned}$$

- Linear Monte-Carlo evaluation converges to the global optimum
- Even when using non-linear value function approximation it converges (but perhaps to a local optimum)

Control with value-function approximation

Control with Value Function Approximation



Policy evaluation **Approximate** policy evaluation, $q_{\mathbf{w}} \approx q_\pi$

Policy improvement E.g., ϵ-greedy policy improvement

Action-Value Function Approximation

- Approximate the action-value function $q_w(s,a) \approx q_\pi(s,a)$
- For instance, with linear function approximation with state-action features

$$q_{\mathbf{w}}(s,a) = \mathbf{x}(s,a)^\top \mathbf{w}$$

- Stochastic gradient descent update

$$\begin{aligned}\Delta \mathbf{w} &= \alpha(q_\pi(s,a) - q_{\mathbf{w}}(s,a))\nabla_{\mathbf{w}} q_{\mathbf{w}}(s,a) \\ &= \alpha(q_\pi(s,a) - q_{\mathbf{w}}(s,a))\mathbf{x}(s,a)\end{aligned}$$

Action-Value Function Approximation (Alternative)

- Approximate the action-value function $q_w(s, a) \approx q_\pi(s, a)$
- For instance, with linear function approximation with state features

$$\mathbf{q}_\mathbf{w}(s) = \mathbf{W}\mathbf{x}(s) \qquad (\mathbf{W} \in \mathbb{R}^{m\times n},\ \mathbf{x}(s) \in \mathbb{R}^n \implies \mathbf{q} \in \mathbb{R}^m)$$

$$q_\mathbf{w}(s, a) = \mathbf{q}_\mathbf{w}(s)[a] = \mathbf{x}(s)^\top \mathbf{w}_a \qquad (\text{where } \mathbf{w}_a = \mathbf{W}_{a\cdot}^\top)$$

Action-Value Function Approximation

- Should we use action-in, or action-out?
  - Action in: $q_w(s, a) = w^T x(s, a)$
  - Action out: $q_w(s) = Wx(s)$ such that $q_w(s, a) = q_w(s)[a]$
- One reuses the same weights, the other the same features
- Unclear which is better in general
- If we want to use continuous actions, action-in is easier
- For (small) discrete action spaces, action-out is common (e.g., DQN)

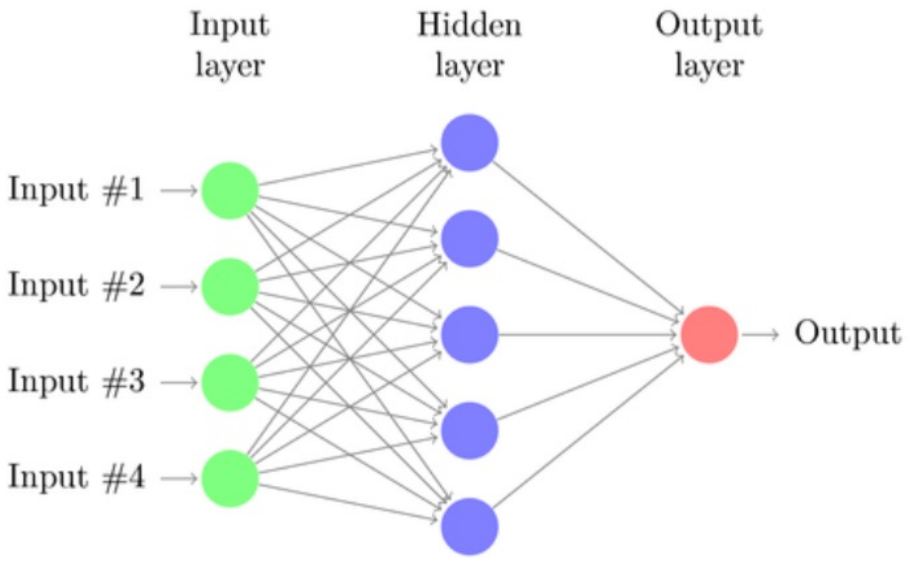
Deep reinforcement learning

RL with Function Approximation

- Linear value function approximators assume value function is a weighted combination of a set of features, where each feature a function of the state
- Linear VFA often works well given the right set of features
- But can require carefully hand designing that feature set
- An alternative is to use a much richer function approximation class that is able to directly go from states without requiring an explicit specification of features

Neural Networks as Function Approximators

- It is possible to use deep neural networks for function approximations.
- Deep networks are clearly more powerful and can model more complex environments.



Generalization

- Using function approximation to help scale up to making decisions in really large domains

Deep Reinforcement Learning

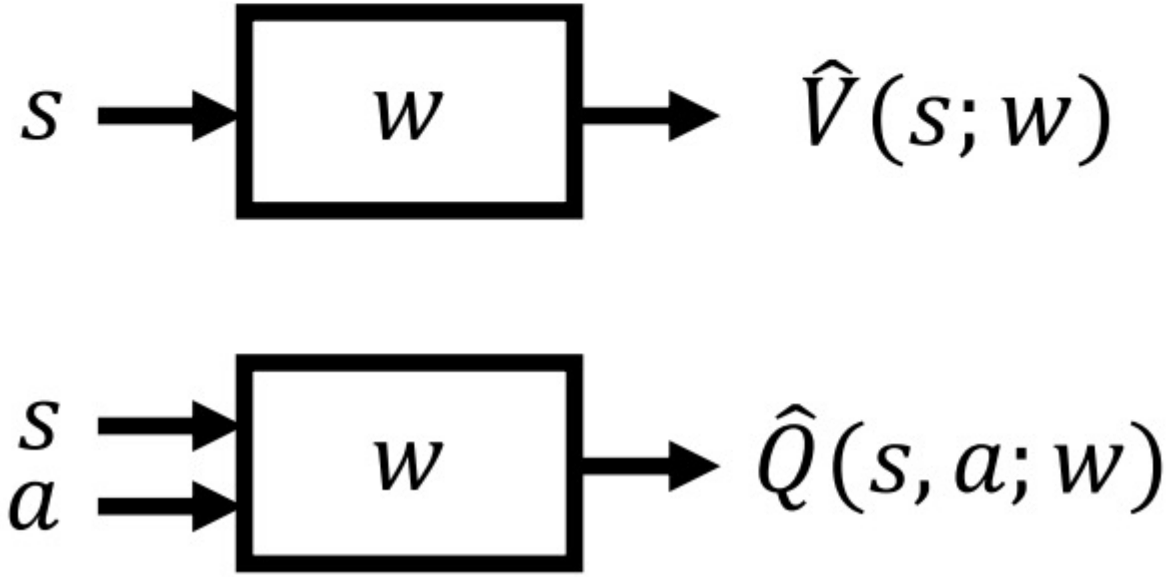
- Use deep neural networks to represent
  - Value, Q function
  - Policy
  - Model

- Optimize loss function by stochastic gradient descent (SGD)

Deep Q-Networks (DQN)

- Represent state-action value function by Q-network with weights w

$$\hat{Q}(s, a; \mathbf{w}) \approx Q(s, a)$$



Recall: Model free control

- Similar to policy evaluation, true state-action value function for a state is unknown and so substitute a target value
- In Monte Carlo methods, use a return G_t as a substitute target

$$\Delta \boldsymbol{w} = \alpha(G_t - \hat{Q}(s_t, a_t; \boldsymbol{w}))\nabla_{\boldsymbol{w}} \hat{Q}(s_t, a_t; \boldsymbol{w})$$

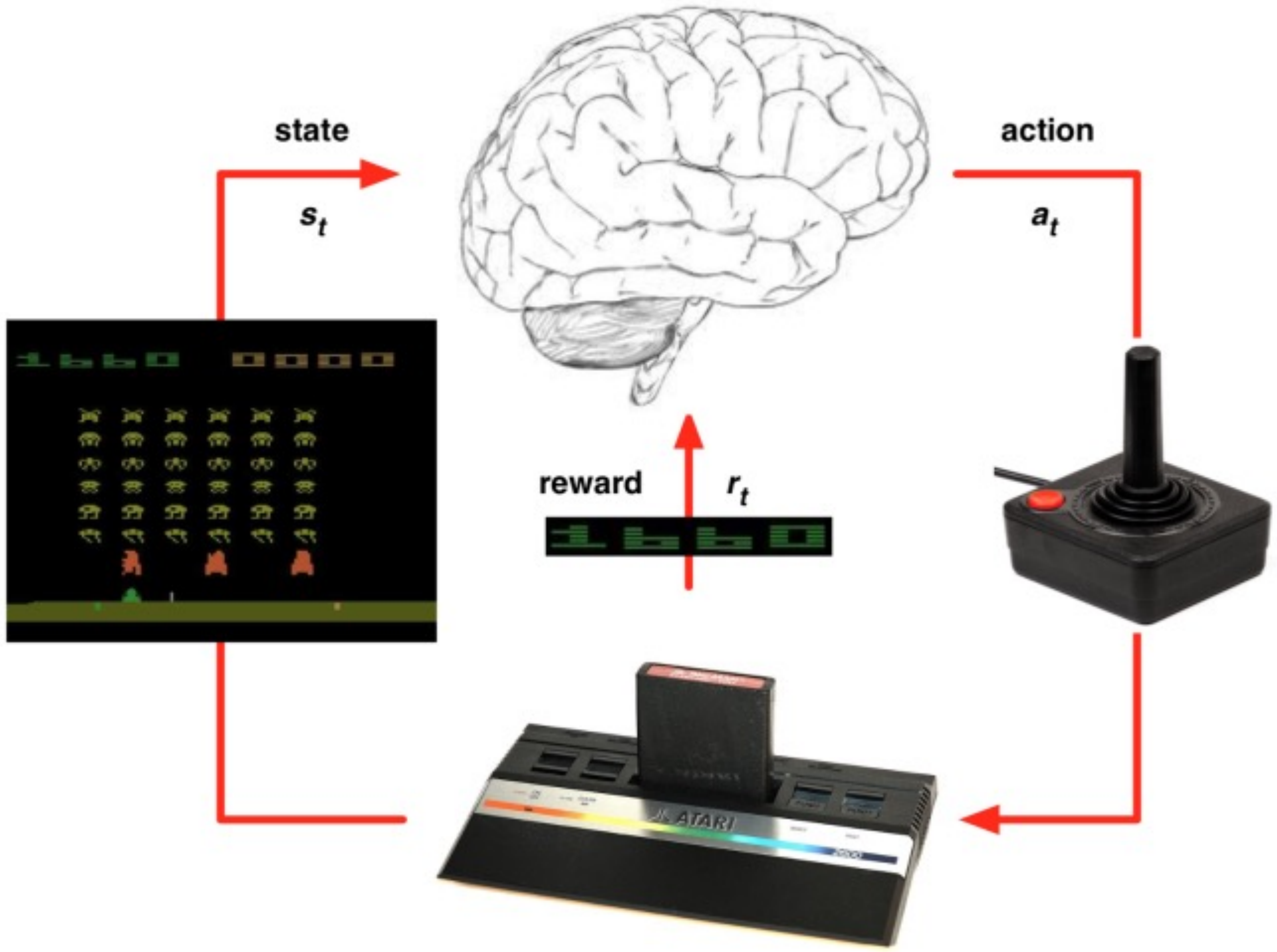
- For SARSA instead use a TD target

$$\Delta \boldsymbol{w} = \alpha(r + \gamma \hat{Q}(s_{t+1}, a_{t+1}; \boldsymbol{w}) - \hat{Q}(s_t, a_t; \boldsymbol{w}))\nabla_{\boldsymbol{w}} \hat{Q}(s_t, a_t; \boldsymbol{w})$$

- For Q-learning

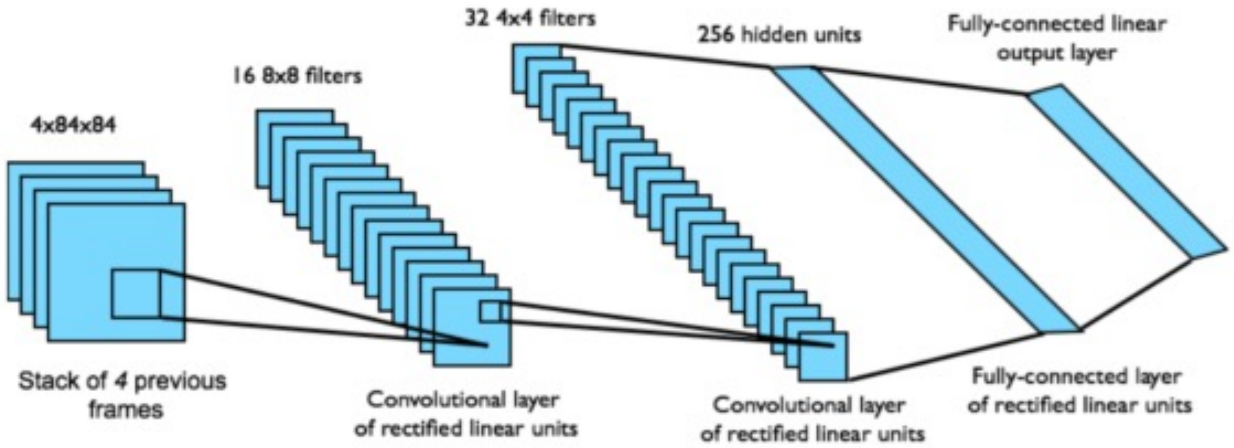
$$\Delta \boldsymbol{w} = \alpha(r + \gamma \max_a \hat{Q}(s_{t+1}, a; \boldsymbol{w}) - \hat{Q}(s_t, a_t; \boldsymbol{w}))\nabla_{\boldsymbol{w}} \hat{Q}(s_t, a_t; \boldsymbol{w})$$

Doing deep RL in Atari



DQNs in Atari

- End-to-end learning of values Q(s, a) from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is Q(s, a) for 18 joystick/button positions
- Reward is change in score for that step
- Network architecture and hyperparameters fixed across all games



DQNs in Atari

- Q-learning converges to the optimal $Q^*(s, a)$ using table lookup representation.
- In value function approximation Q-learning, we can minimize MSE loss by stochastic gradient descent using a target Q estimate instead of true Q (as we saw with linear VFA)
- But Q-learning with VFA can diverge
- Two of the issues causing problems:
  - Correlations between samples
  - Non-stationary targets
- Deep Q-learning (DQN) addresses these challenges by
  - Experience replay
  - Fixed Q-targets

DQNs: Experience Replay

- To help remove correlations, store dataset (called a replay buffer) D from prior experience

s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
s_3, a_3, r_4, s_4
$\ldots$
$s_t, a_t, r_{t+1}, s_{t+1}$

$\rightarrow \quad s, a, r, s'$

- To perform experience replay, repeat the following:
  - $(s, a, r, s') \sim D$: sample an experience tuple from the dataset
  - Compute the target value for the sampled s: $r + \gamma \max_{a'} \hat{Q}(s', a'; w)$
  - Use stochastic gradient descent to update the network weights

$$\Delta \mathbf{w} = \alpha(r + \gamma \max_{a'} \hat{Q}(s', a'; \mathbf{w}) - \hat{Q}(s, a; \mathbf{w}))\nabla_{\mathbf{w}} \hat{Q}(s, a; \mathbf{w})$$

Problem

Can treat the target as a constant scalar, but the weights will get updated on the next round, changing the target value

DQNs: Fixed Q-Targets

- To help improve stability, fix the target weights used in the target calculation for multiple updates
- Target network uses a different set of weights than the weights being updated
- Let parameters w^{-} be the set of weights used in the target, and w be the weights that are being updated
- Slight change to computation of target value:
  - $(s, a, r, s') \sim D$: sample an experience tuple from the dataset
  - Compute the target value for the sampled s: $r + \gamma \max_{a'} \hat{Q}(s', a'; w^{-})$
  - Use stochastic gradient descent to update the network weights

$$\Delta \mathbf{w} = \alpha(r + \gamma \max_{a'} \hat{Q}(s', a'; \mathbf{w}^{-}) - \hat{Q}(s, a; \mathbf{w}))\nabla_{\mathbf{w}} \hat{Q}(s, a; \mathbf{w})$$

DQN Algorithm

linear func.

Input C, α, $D = \{\}$, Initialize $\mathbf{w}$, $\mathbf{w}^- = \mathbf{w}$, $t = 0$

Get initial state s_0

loop

  Sample action a_t given ϵ-greedy policy for current $\hat{Q}(s_t, a; \mathbf{w})$

  Observe reward r_t and next state s_{t+1}

  Store transition (s_t, a_t, r_t, s_{t+1}) in replay buffer D

  Sample random minibatch of tuples (s_i, a_i, r_i, s_{i+1}) from D

  for j in minibatch **do**

    if episode terminated at step $i+1$ **then**

      $y_i = r_i$

    else

      $y_i = r_i + \gamma \max_{a'} \hat{Q}(s_{i+1}, a'; \mathbf{w}^-)$

    end if

    Do gradient descent step on $(y_i - \hat{Q}(s_i, a_i; \mathbf{w}))^2$ for parameters $\mathbf{w}$: $\Delta \mathbf{w} = \alpha(y_i - \hat{Q}(s_i, a_i; \mathbf{w}))\nabla_{\mathbf{w}} \hat{Q}(s_i, a_i; \mathbf{w})$

  end for

  $t = t + 1$

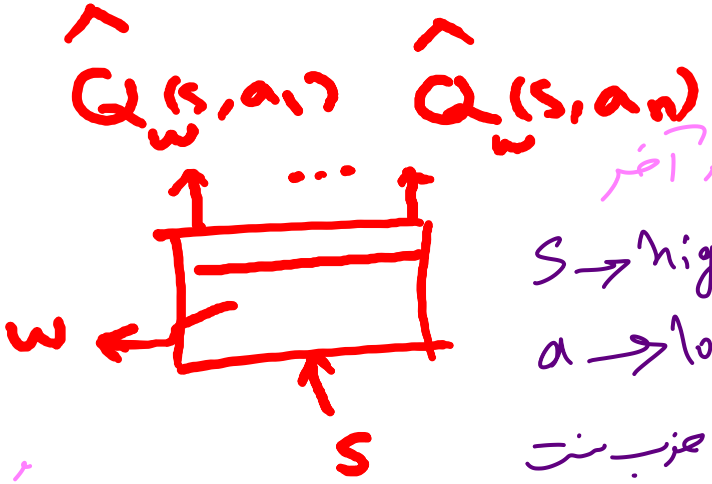
  if mod(t,C) == 0 **then**

    $\mathbf{w}^- \leftarrow \mathbf{w}$

  end if

end loop

MSE loss



action

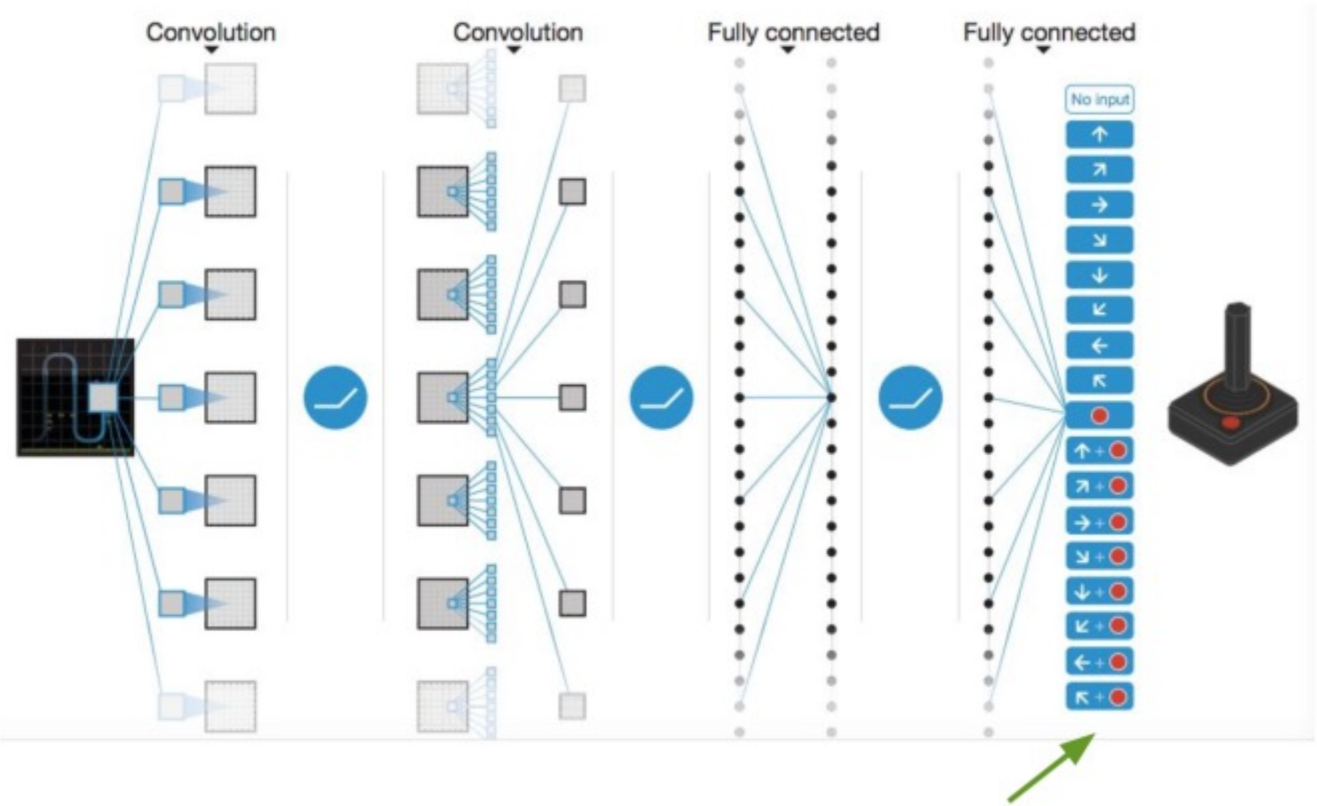
$s \to$ high-dim

$a \to$ low-dim

DQN Summary

- DQN uses experience replay and fixed Q-targets
- Store transition $(s_t, a_t, r_{t+1}, s_{t+1})$ in replay memory D
- Sample random mini-batch of transitions (s, a, r, s') from D
- Compute Q-learning targets w.r.t. old, fixed parameters w^-
- Optimizes MSE between Q-network and Q-learning targets
- Uses stochastic gradient descent

DQN



1 network, outputs Q value for each action

Results

Game	Linear	Deep Network	DQN w/ fixed Q	DQN w/ replay	DQN w/replay and fixed Q
Breakout	3	3	10	241	317
Enduro	62	29	141	831	1006
River Raid	2345	1453	2868	4102	7447
Seaquest	656	275	1003	823	2894
Space Invaders	301	302	373	826	1089