Reducing Variance

- Causality trick
- Discount factor
- Baseline
- Actor-critic
- Optimization techniques:
  - Natural gradient
  - Trust region

Reducing Variance: Causality

$\nabla_\theta \log \pi_\theta(a_{i,t}|s_{i,t})$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\underbrace{\left(\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\right)}_{\text{prob. traj}}\underbrace{\left(\sum_{t=1}^{T} r(\mathbf{s}_{i,t},\mathbf{a}_{i,t})\right)}_{\text{return}}$$

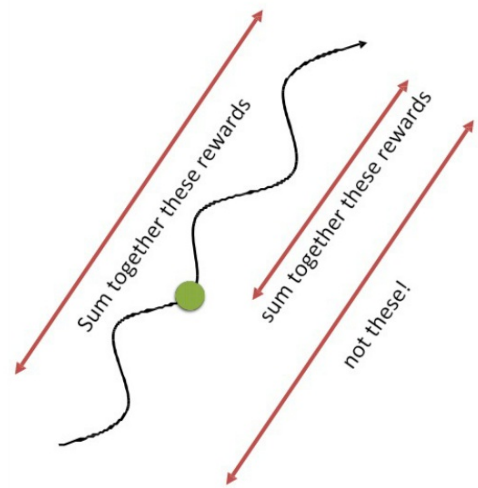
Causality: policy at time t' cannot affect reward at time t when $t < t'$

original

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(\sum_{t'=1}^{T} r(\mathbf{a}_{i,t'},\mathbf{s}_{i,t'})\right)$$

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\underbrace{\left(\sum_{t'=t}^{T} r(\mathbf{a}_{i,t'},\mathbf{s}_{i,t'})\right)}_{\text{"reward to go" } \hat{Q}_{i,t}}$$

$0 = \mathbb{E}\left(\nabla_\theta \log \pi_\theta(a_{i,t}|s_{i,t}) \sum_{t'=1}^{t-1} r(a_{i,t'}, s_{i,t'})\right)$



Reducing Variance: Discount Factor

option 1: $\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(\sum_{t'=t}^{T}\gamma^{t'-t}r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right)$

option 2: $\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\right)\left(\sum_{t=1}^{T}\gamma^{t-1}r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right)$

Not the same

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(\sum_{t=1}^{T}\gamma^{t-1}r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\boxed{\gamma^{t-1}}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(\sum_{t'=t}^{T}\gamma^{t'-t}r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right)$$

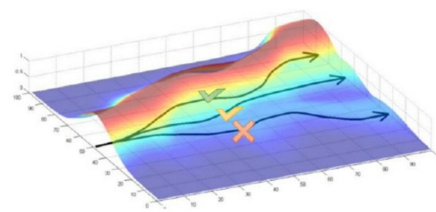
Reducing Variance: Baselines

a convenient identity

$$p_\theta(\tau)\nabla_\theta \log p_\theta(\tau) = \nabla_\theta p_\theta(\tau)$$

$$\nabla_\theta J(\theta) \approx \underbrace{\frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log p_\theta(\tau)[r(\tau) - b]}$$

$$b = \frac{1}{N}\sum_{i=1}^{N} r(\tau)$$



const(τ)

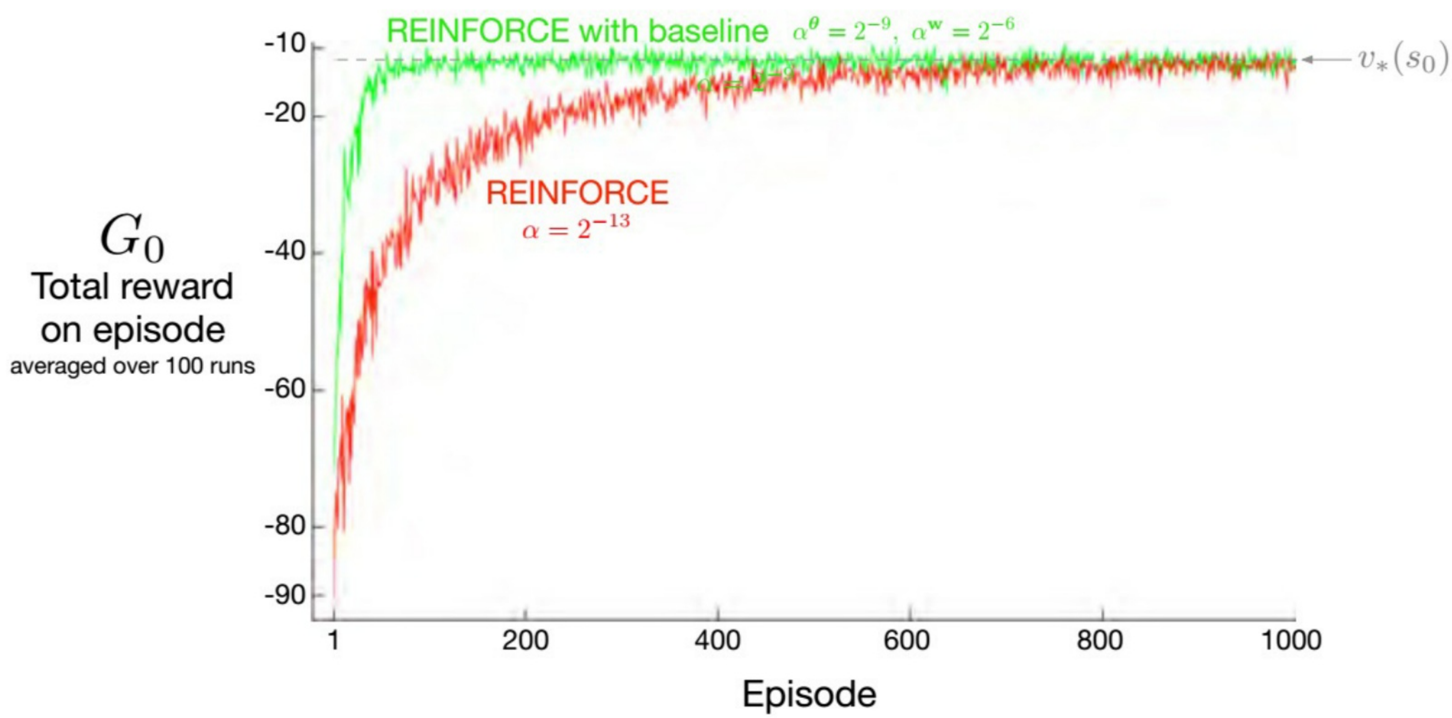
$$E[\nabla_\theta \log p_\theta(\tau) b] = \int p_\theta(\tau)\nabla_\theta \log p_\theta(\tau) b \, d\tau = \int \nabla_\theta p_\theta(\tau) b \, d\tau = b\nabla_\theta \underbrace{\int p_\theta(\tau) d\tau}_{1} = b\nabla_\theta 1 = 0$$

subtracting a baseline is *unbiased* in expectation!

average reward is *not* the best baseline, but it's pretty good!

Reducing Variance: Baselines

Faster convergence:



Analyzing Variance

min Var[f(b, ... b)]

min Var[f(b_1, ... , b_n)]

$$\mathrm{Var}[x] = E[x^2] - E[x]^2$$

$$\nabla_\theta J(\theta) = E_{\tau\sim p_\theta(\tau)}[\nabla_\theta \log p_\theta(\tau)(r(\tau)^{2} - b)]$$

Const(b)

$$\mathrm{Var} = E_{\tau\sim p_\theta(\tau)}[(\underbrace{\nabla_\theta \log p_\theta(\tau)}_{g(\tau)}(r(\tau) - b))^2] - \underbrace{E_{\tau\sim p_\theta(\tau)}[\nabla_\theta \log p_\theta(\tau)(r(\tau) - b)]^2}$$

this bit is just $E_{\tau\sim p_\theta(\tau)}[\nabla_\theta \log p_\theta(\tau) r(\tau)]$
(baselines are unbiased in expectation)

$$\frac{d\mathrm{Var}}{db} = \frac{d}{db} E[g(\tau)^2 (r(\tau) - b)^2] = \frac{d}{db}\left(\overset{0}{\cancel{E[g(\tau)^2 r(\tau)^2]}} - 2E[g(\tau)^2 r(\tau) b] + b^2 E[g(\tau)^2]\right)$$

$$= -2E[g(\tau)^2 r(\tau)] + 2bE[g(\tau)^2] = 0$$

$$b = \frac{E[g(\tau)^2 r(\tau)]}{E[g(\tau)^2]}$$

This is just expected reward, but weighted by gradient magnitudes!

Reducing Variance: Review

- Exploiting causality
  - Future doesn't affect the past
- Discount factor
  - Two different version
- Baselines
  - Analyzing variance for deriving optimal baselines
- Now: Introducing actor-critic methods!

Policy Gradients so Far

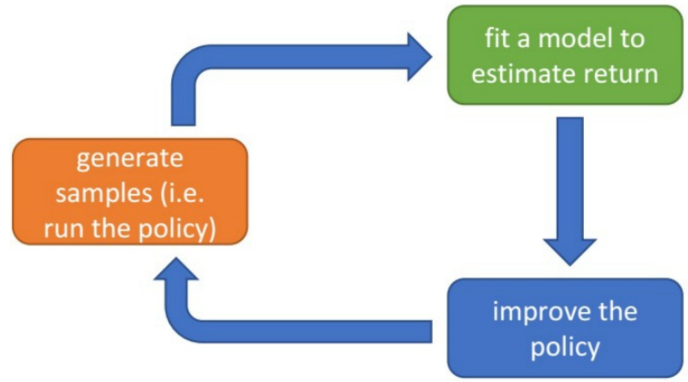
REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run the policy)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)\right)\left(\sum_{t'=t}^T r(\mathbf{s}_{t'}^i, \mathbf{a}_{t'}^i)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$Q^\pi(s,a)$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \underbrace{\hat{Q}_{i,t}^\pi}_{\text{"reward to go"}}$$

$$\hat{Q}^\pi(\mathbf{x}_t, \mathbf{u}_t) = \sum_{t'=t}^T r(\mathbf{x}_{t'}, \mathbf{u}_{t'})$$

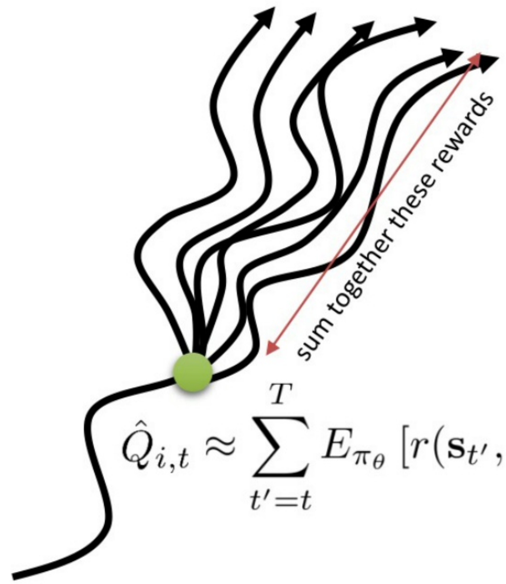


$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

Improving Estimation of Reward to Go

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\underbrace{\left(\sum_{t'=1}^{T} r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right)}$$

$\hat{Q}_{i,t}$: estimate of expected reward if we take action $\mathbf{a}_{i,t}$ in state $\mathbf{s}_{i,t}$



$$\hat{Q}_{i,t} \approx \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'},\mathbf{a}_{t'})|\mathbf{s}_t,\mathbf{a}_t\right]$$

How to make a better estimate?

$Q(\mathbf{s}_t,\mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'},\mathbf{a}_{t'})|\mathbf{s}_t,\mathbf{a}_t\right]$: true *expected* reward-to-go

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})Q(\mathbf{s}_{i,t},\mathbf{a}_{i,t})$$

much lower variance!

Improving Estimation of Reward to Go

Further improvement: Adding a baseline!

$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$: true *expected* reward-to-go

baseline

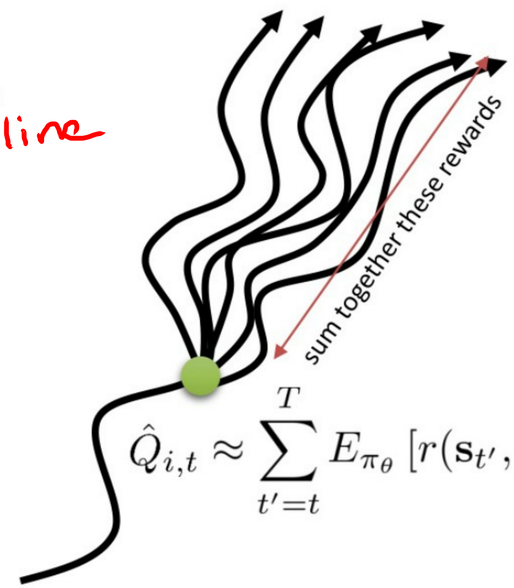
$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(Q(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) - b_t\right)$$

$$\underbrace{b_t = \frac{1}{N}\sum_{i} Q(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})}_{\hat{V}(s_{i,t})}$$

$a_{i,t} \sim \pi_\theta$

return

value



$$\hat{Q}_{i,t} \approx \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

Improving Estimation of Reward to Go

Further improvement: Adding a baseline!

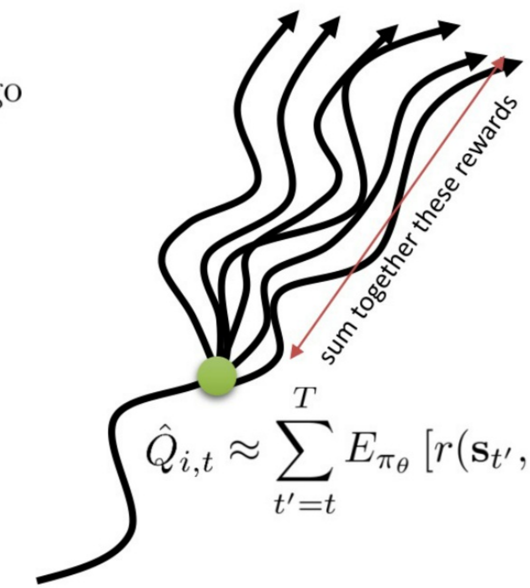
$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$: true *expected* reward-to-go

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(Q(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) - V(\mathbf{s}_{i,t})\right)$$

$$b_t = \frac{1}{N}\sum_i Q(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

$$\downarrow$$

$$V(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q(\mathbf{s}_t, \mathbf{a}_t)]$$



$$\hat{Q}_{i,t} \approx \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

Advantage Value

$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$: total reward from taking $\mathbf{a}_t$ in $\mathbf{s}_t$

$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$: total reward from $\mathbf{s}_t$

$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = \underbrace{Q^\pi(\mathbf{s}_t, \mathbf{a}_t)}_{\text{Reward to go}} - \underbrace{V^\pi(\mathbf{s}_t)}_{\text{baseline}}$: how much better $\mathbf{a}_t$ is

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

the better this estimate, the lower the variance

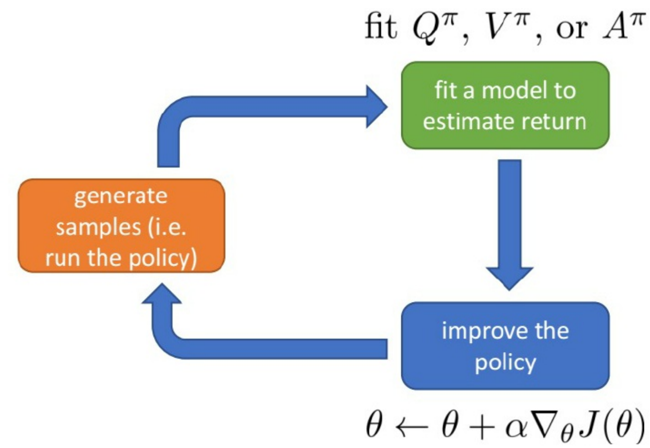
Advantage Value Approximation

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t\right]$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$



$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t\right]$$

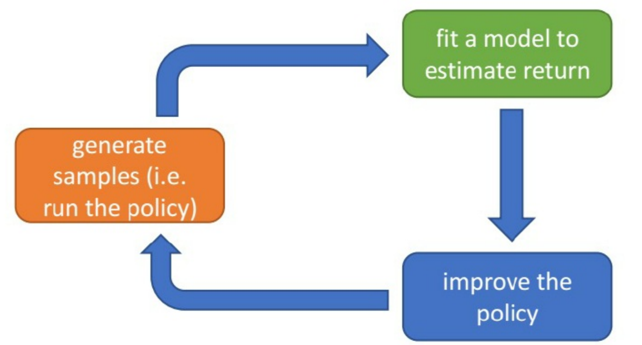
$$= r(\mathbf{s}_t, \mathbf{a}_t) + \underbrace{\sum_{t'=t+1}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t\right]}_{\approx V^\pi(\mathbf{s}_{t+1})}$$

Advantage Value Approximation

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1})$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1}) - V^\pi(\mathbf{s}_t)$$

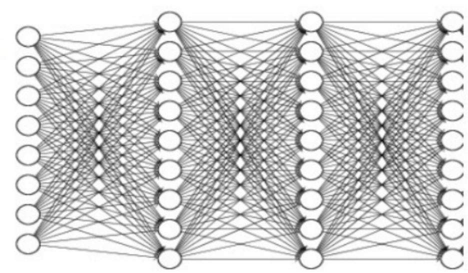
fit Q^π, V^π, or A^π



$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

let's just fit $V^\pi(\mathbf{s})$!

$\mathbf{s}$



$\hat{V}^\pi(\mathbf{s})$

parameters ϕ

Policy Evaluation

$$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t\right]$$

$$J(\theta) = E_{\mathbf{s}_1 \sim p(\mathbf{s}_1)}[V^\pi(\mathbf{s}_1)]$$

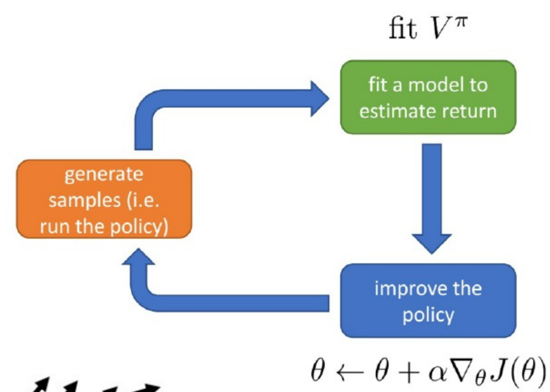
how can we perform policy evaluation?

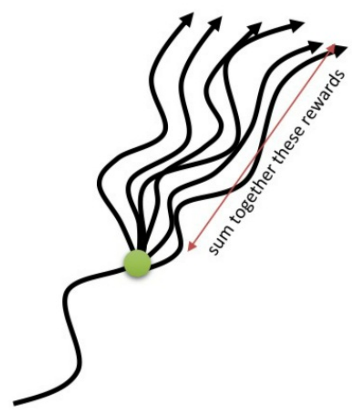
Monte Carlo policy evaluation (this is what policy gradient does)

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$$

$$V^\pi(\mathbf{s}_t) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$$

(requires us to reset the simulator)





Policy Evaluation

Monte Carlo estimation with function approximator:

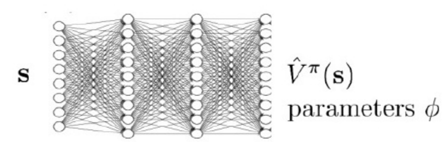
$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$$

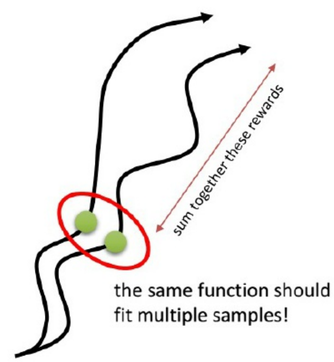
not as good as this: $V^\pi(\mathbf{s}_t) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$

but still pretty good!

training data: $\left\{\left(\mathbf{s}_{i,t}, \underbrace{\sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})}_{y_{i,t}}\right)\right\}$

supervised regression: $\mathcal{L}(\phi) = \frac{1}{2}\sum_i \left\| \hat{V}_\phi^\pi(\mathbf{s}_i) - y_i \right\|^2$





Policy Evaluation

How to make a better estimate?

$$\text{ideal target: } y_{i,t} = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_{i,t}\right] \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + V^\pi(\mathbf{s}_{i,t+1}) \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \underbrace{\hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})}_{\text{directly use previous fitted value function!}}$$

$$\text{Monte Carlo target: } y_{i,t} = \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$$

Policy Evaluation

Bootstrap Estimation with Function Approximator

ideal target: $y_{i,t} = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_{i,t}\right] \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})$

$$\text{training data: } \left\{\left(\mathbf{s}_{i,t}, \underbrace{r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})}_{y_{i,t}}\right)\right\}$$

$$\text{supervised regression: } \mathcal{L}(\phi) = \frac{1}{2}\sum_i \left\|\hat{V}_\phi^\pi(\mathbf{s}_i) - y_i\right\|^2$$

Batch Actor-Critic Algorithm

$$\min_{\phi} \left\| \hat{V}_{\phi}(s_t) - \underbrace{\sum_{i=1}^{T} r(s_{t+i})}_{MC} \right\|_2^2$$

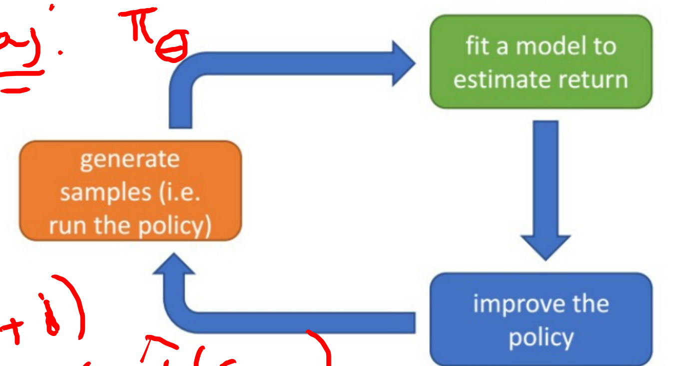
batch actor-critic algorithm:

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot) $\longrightarrow$ traj. π_θ
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums $\longleftarrow$ MC
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i)\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

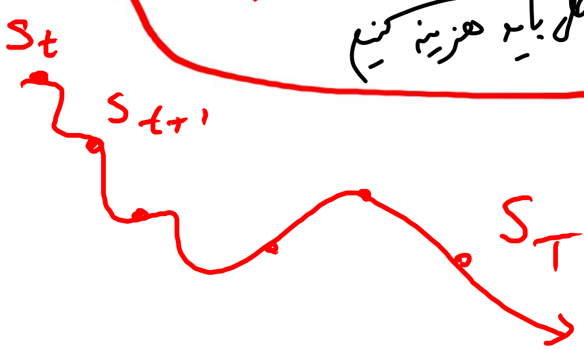
برای این آپدیت کردن کلی باید هزینه کنیم

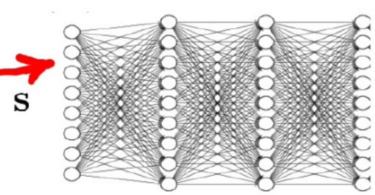
$r(s_{t+1}) + \gamma \hat{V}_{\phi}(s_{t+1})$

fit $\hat{V}^\pi$



$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$





$\hat{V}^\pi(\mathbf{s})$

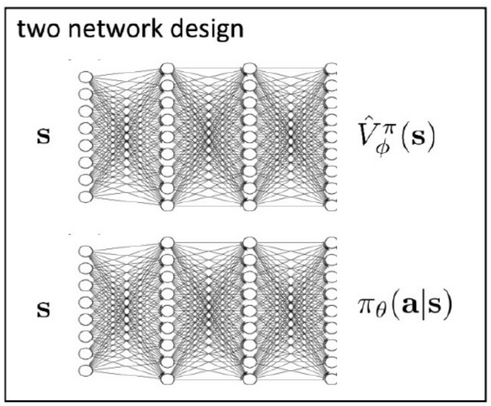
parameters ϕ

$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t]$

Actor-Critic Algorithm: Architecture Design

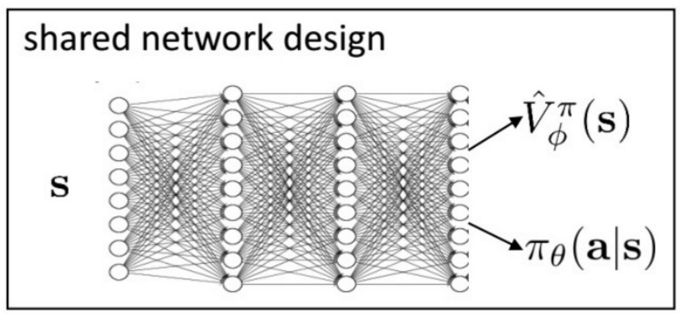
batch actor-critic algorithm:

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}_\phi^\pi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \hat{V}_\phi^\pi(\mathbf{s}_i') - \hat{V}_\phi^\pi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i)\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$



+ simple & stable

- no shared features between actor & critic



Policy Evaluation in Infinite Horizon Settings

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}^{\pi}_{\phi}(\mathbf{s}_{i,t+1})$$

$$\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}^{\pi}_{\phi}(\mathbf{s}_i) - y_i \right\|^2$$

what if T (episode length) is ∞?

$\hat{V}^{\pi}_{\phi}$ can get infinitely large in many cases

simple trick: better to get rewards sooner than later

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}^{\pi}_{\phi}(\mathbf{s}_{i,t+1})$$

$$\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}^{\pi}_{\phi}(\mathbf{s}_i) - y_i \right\|^2$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\overbrace{r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}^{\pi}_{\phi}(\mathbf{s}_{i,t+1}) - \hat{V}^{\pi}_{\phi}(\mathbf{s}_{i,t})}^{\hat{A}^{\pi}(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})} \right)$$

Actor-Critic Algorithm: Infinite Horizon

batch actor-critic algorithm:

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \boxed{\gamma}\hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i)\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Critics as Baselines

Actor-critic: $$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(r(\mathbf{s}_{i,t},\mathbf{a}_{i,t}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1}) - \hat{V}_\phi^\pi(\mathbf{s}_{i,t})\right)$$

+ lower variance (due to critic)
- not unbiased (if the critic is not perfect)

Policy gradient: $$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(\left(\sum_{t'=t}^{T}\gamma^{t'-t} r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right) - b\right)$$

+ no bias
- higher variance (because single-sample estimate)

can we use $\hat{V}_\phi^\pi$ and still keep the estimator unbiased?

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(\left(\sum_{t'=t}^{T}\gamma^{t'-t} r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right) - \hat{V}_\phi^\pi(\mathbf{s}_{i,t})\right)$$

+ no bias

+ lower variance (baseline is closer to rewards)

Eligibility Traces and N-step Returns

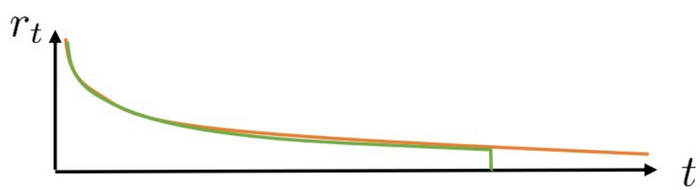
$$\hat{A}^{\pi}_{\mathrm{C}}(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \hat{V}^{\pi}_{\phi}(\mathbf{s}_{t+1}) - \hat{V}^{\pi}_{\phi}(\mathbf{s}_t)$$

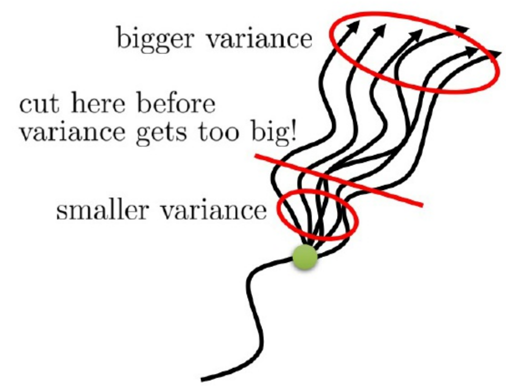
+ lower variance
- higher bias if value is wrong (it always is)

$$\hat{A}^{\pi}_{\mathrm{MC}}(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{\infty} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - \hat{V}^{\pi}_{\phi}(\mathbf{s}_t)$$

+ no bias
- higher variance (because single-sample estimate)

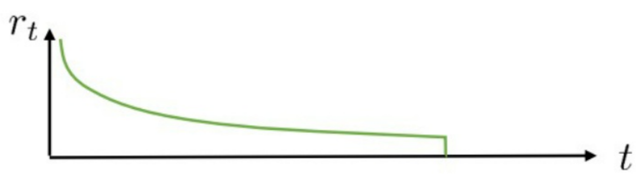
Can we combine these two, to control bias/variance tradeoff?





$$\hat{A}^{\pi}_{n}(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{t+n} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - \hat{V}^{\pi}_{\phi}(\mathbf{s}_t) + \gamma^{n} \hat{V}^{\pi}_{\phi}(\mathbf{s}_{t+n})$$

Generalized Advantage Estimation



Do we have to choose just one n?

Cut everywhere all at once!

$$\hat{A}_n^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{t+n} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - \hat{V}_\phi^\pi(\mathbf{s}_t) + \gamma^n \hat{V}_\phi^\pi(\mathbf{s}_{t+n})$$

$$\hat{A}_{\mathrm{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{n=1}^{\infty} w_n \hat{A}_n^\pi(\mathbf{s}_t, \mathbf{a}_t)$$

weighted combination of n-step returns

How to weight? Mostly prefer cutting earlier (less variance) $w_n \propto \lambda^{n-1}$ exponential falloff

$$\hat{A}_{\mathrm{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \gamma((1-\lambda)\hat{V}_\phi^\pi(\mathbf{s}_{t+1}) + \lambda(r(\mathbf{s}_{t+1}, \mathbf{a}_{t+1}) + \gamma((1-\lambda)\hat{V}_\phi^\pi(\mathbf{s}_{t+2}) + \lambda r(\mathbf{s}_{t+2}, \mathbf{a}_{t+2}) + \dots)$$

$$\hat{A}_{\mathrm{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{\infty} \underline{(\gamma\lambda)}^{t'-t} \delta_{t'} \qquad \delta_{t'} = r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{t'+1}) - \hat{V}_\phi^\pi(\mathbf{s}_{t'})$$

Policy Grad V.S Q_Learning

⇐ پیچیدگی ناشی V بهینه در مقابل پیچیدگی ناشی π بهینه

off-policy + Approximation (TD) + Approximation func representation →

Actor-Critic Algorithm: Batch vs. Online

batch actor-critic algorithm:

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Full batch

⇒ Full Batch
(تولید یک episode کامل)

vs

online actor-critic algorithm: → فقط یک اکشن

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}^\pi_\phi$ using target $r + \gamma \hat{V}^\pi_\phi(\mathbf{s}')$ → just 1 training sample
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}') - \hat{V}^\pi_\phi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$ → جمع نداریم
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

1

⇒ یک batch size
(اکشن)

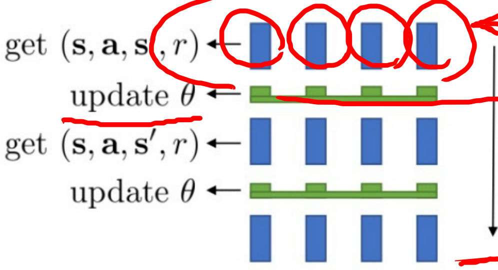
Online Actor-Critic in Practice: A2C and A3C

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}^\pi_\phi$ using target $r + \gamma \hat{V}^\pi_\phi(\mathbf{s}')$
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}') - \hat{V}^\pi_\phi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

works best with a batch (e.g., parallel workers)

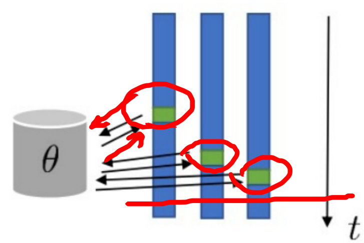
synchronized parallel actor-critic



Avg. update

k^{th}

asynchronous parallel actor-critic



حتی θ لوص نیز !

Thread k ام موازی

چرا off ← sample inefficency ← یعنی interaction تا فقط یک آپدیت ...

با ایده ⊳ replay buffer ایده ⇐ ولی نشد

From On-Policy to Off-Policy Actor-Critic

چون فرض P.G اینه ما از همان پالیسی که دیتا گرفتیم رو با همون آپدیت کنیم

باید On باشیم تا تخمین درست باشه!!

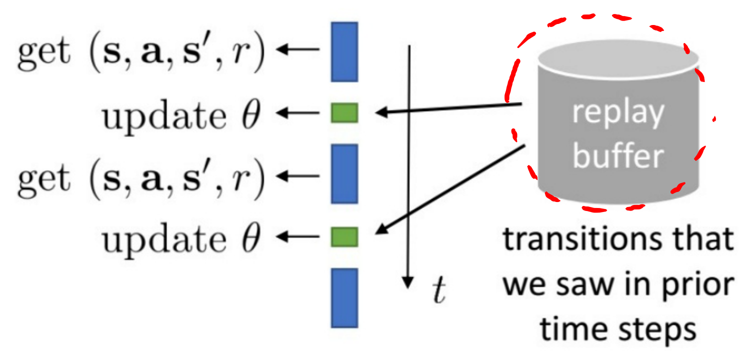
یعنی دیتا و گرادیان از همین پالیسی باشن ☺

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}^\pi_\phi$ using target $r + \gamma \hat{V}^\pi_\phi(\mathbf{s}')$
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}') - \hat{V}^\pi_\phi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Can we remove the on-policy assumption?

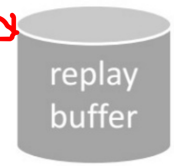
off-policy actor-critic → (s, a, s', r)



From On-Policy to Off-Policy Actor-Critic

off-policy actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$, store in $\mathcal{R}$
2. sample a batch $\{\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i\}$ from buffer $\mathcal{R}$
3. update $\hat{V}^\pi_\phi$ using targets $\boxed{y_i = r_i + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i)}$ for each $\mathbf{s}_i$
4. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
5. $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \nabla_\theta \boxed{\log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i)} \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
6. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$



$$\mathcal{L}(\phi) = \frac{1}{N} \sum_i \left\| \hat{V}^\pi_\phi(\mathbf{s}_i) - y_i \right\|^2$$

not the right target value

داری V^π رو تخمین می‌زنی درحالی که اکشن از یک π دیگه اومده

not the action π_θ would have taken!

اکشن از پالیسی π نیومده

پالیسی گرادیان روی یک اپیزود کامل گرفته میشه نه همینطور الکی چندتا s,a رو پشت هم قرار داد

This algorithm is broken!

Can you spot the problems?

ایده Q_learning ← فرار از V و توسل به Q برای اینکه a رو دلبخواه بزاریم و آپدیت کنیم

Off-Policy Actor-Critic: Fixing the Value Function

$s \xrightarrow{a} s \rightarrow$

off-policy actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$, store in $\mathcal{R}$
2. sample a batch $\{\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i\}$ from buffer $\mathcal{R}$
3. update $\hat{V}^\pi_\phi$ using targets $\boxed{y_i = r_i + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i)}$ for each $\mathbf{s}_i$
4. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
5. $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
6. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$Q(s_i, a_i)$

$$\cancel{V^\pi(\mathbf{s}_t) = \sum_{t'=t}^T E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t\right]}$$

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

not the right target value

$s, a \rightarrow \boxed{\phi} \rightarrow$

3. update $\hat{Q}^\pi_\phi$ using targets $y_i = r_i + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i)$ for each $\mathbf{s}_i, \mathbf{a}_i$

$$= r_i + \gamma \hat{Q}^\pi_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$$

$$\mathcal{L}(\phi) = \frac{1}{N} \sum_i \left\| \hat{Q}^\pi_\phi(\mathbf{s}_i, \mathbf{a}_i) - y_i \right\|^2$$

(s_t, a_t, s', r) $\quad (s_t, \hat{a}_t)$

الزاما نداره از π اومده باشه

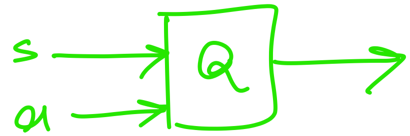
$$\boxed{\mathbf{a}'_i \sim \pi_\theta(\mathbf{a}'_i|\mathbf{s}'_i)}$$

not from replay buffer $\mathcal{R}$!

$a \rightarrow$ الزاما نداره از پالیسی باشه

$a' \rightarrow$ ولی این باید از π باشه

$$\boxed{V^\pi(\mathbf{s}_t) = \sum_{t'=t}^T E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t\right] = E_{\mathbf{a} \sim \pi(\mathbf{a}_t|\mathbf{s}_t)}\left[Q(\mathbf{s}_t, \mathbf{a}_t)\right]}$$



$Q^{\pi}(s,a) = r(s,a) + Q^{\pi}(s,a')$

a' باید از پالیسی بیاد!! ولی s, a, r, s' نه لزوما!

Off-Policy Actor-Critic: Fixing the Policy Update

off-policy actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$, store in $\mathcal{R}$
2. sample a batch $\{\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i\}$ from buffer $\mathcal{R}$
3. update $\hat{Q}^\pi_\phi$ using targets $y_i = r_i + \gamma \hat{Q}^\pi_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$ for each $\mathbf{s}_i, \mathbf{a}_i$
4. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = Q(\mathbf{s}_i, \mathbf{a}_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
5. $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \nabla_\theta \boxed{\log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i)} \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
6. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$\pi(s|a)$ از شبکه

$\int P_\theta(S)\, P(\theta)\, d\theta$

next_action نداریم

not the action π_θ would have taken!

use the same trick, but this time for $\mathbf{a}_i$ rather than $\mathbf{a}'_i$!

sample $\mathbf{a}^\pi_i \sim \pi_\theta(\mathbf{a}|\mathbf{s}_i)$ $\longrightarrow$ $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}^\pi_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}^\pi_i)$

π از شبکه

in practice: $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}^\pi_i|\mathbf{s}_i) \hat{Q}^\pi(\mathbf{s}_i, \mathbf{a}^\pi_i)$

higher variance, but convenient
why is higher variance OK here?

s, a, r, s, a' است

Off-Policy Actor-Critic

$Q(s,a)$

Sim.

off-policy actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$, store in $\mathcal{R}$
2. sample a batch $\{\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i\}$ from buffer $\mathcal{R}$
3. update $\hat{Q}^\pi_\phi$ using targets $y_i = r_i + \gamma \hat{Q}^\pi_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$ for each $\mathbf{s}_i, \mathbf{a}_i$
4. $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}^\pi_i|\mathbf{s}_i) \hat{Q}^\pi(\mathbf{s}_i, \mathbf{a}^\pi_i)$ where $\mathbf{a}^\pi_i \sim \pi_\theta(\mathbf{a}|\mathbf{s}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$a'_i \sim \pi_\theta(s_i)$

Is there any remaining problem?

$\mathbf{s}_i$ didn't come from $p_\theta(\mathbf{s})$

nothing we can do here, just accept it

intuition: we want optimal policy on $p_\theta(\mathbf{s})$
but we get optimal policy on a *broader* distribution

پالیسی گرادیان رو از p_θ مختلف داده‌ای برای میشه از Traj های مختلف

همان mini-batch

حالت min_batch با بهتره کم

حالت off-policy الگوریتم AC

Advantage بجای

① از Q به جای V استفاده کنیم

② base line حذف میشه ← جبران می‌کنیم هر چی بیشتر دیتا بگیریم

③ n ...

next-action از شبکه π می‌آید

چه برا آپدیت π چه Q