Computer Engineering Department

Inverse Reinforcement Learning

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Slides are adopted from CS 285, UC Berkeley.

Lecture Outline

1. So far: manually design reward function to define a task
2. What if we want to *learn* the reward function from observing an expert, and then use reinforcement learning?
3. Apply approximate optimality model from last time, but now learn the reward!

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2. What if we want to *learn* the reward function from observing an expert, and then use reinforcement learning?
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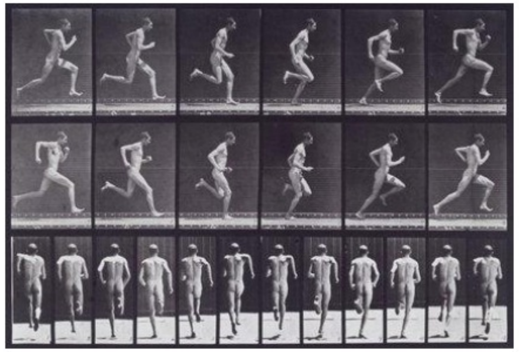
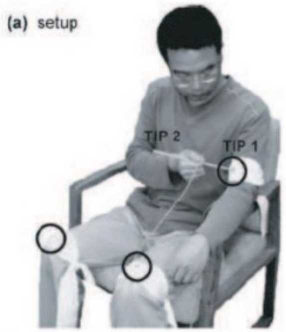
- Goals:
  - Understand the inverse reinforcement learning problem definition
  - Understand how probabilistic models of behavior can be used to derive inverse reinforcement learning algorithms
  - Understand a few practical inverse reinforcement learning algorithms we can use

Modeling Human Behavior with Optimal Control



Muybridge (c. 1870)

Mombaur et al. '09



Li & Todorov '06

Ziebart '08

$$\mathbf{a}_1, \ldots, \mathbf{a}_T = \arg \max_{\mathbf{a}_1, \ldots, \mathbf{a}_T} \sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t)$$

$$\mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{a}_t)$$

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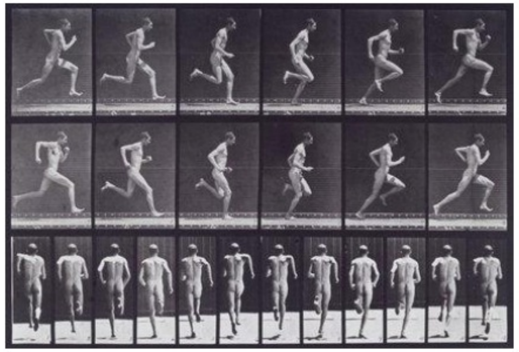

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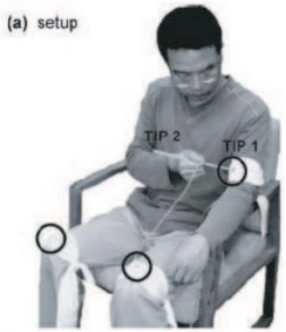

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optimize this to explain the data

$$\pi = \arg\max_{\pi} E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t), \mathbf{a}_t \sim \pi(\mathbf{a}_t|\mathbf{s}_t)}[r(\mathbf{s}_t, \mathbf{a}_t)]$$

$$\mathbf{a}_t \sim \pi(\mathbf{a}_t|\mathbf{s}_t)$$

Imitation learning vs RL perspective

The imitation learning perspective

Standard imitation learning:

- copy the *actions* performed by the expert
- no reasoning about outcomes of actions

Human imitation learning:

- copy the *intent* of the expert
- might take very different actions!

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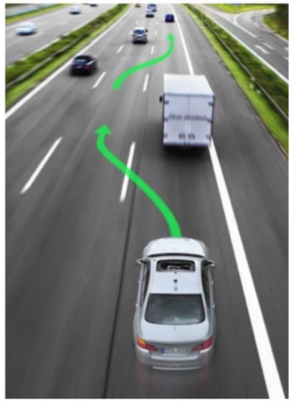
The reinforcement learning perspective

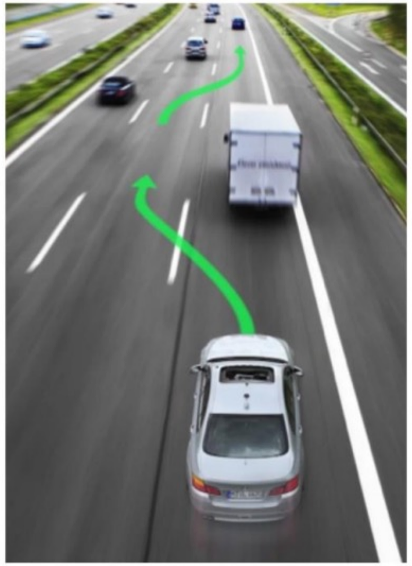
reward



what is the reward?

Inverse Reinforcement Learning

Infer **reward functions** from **demonstrations**



$$\rightarrow r(\mathbf{s}, \mathbf{a})$$

Inverse Reinforcement Learning

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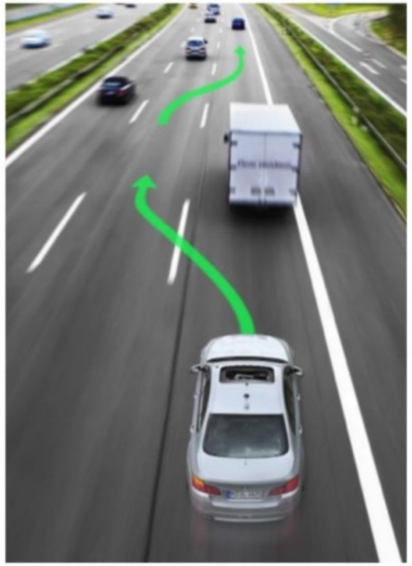
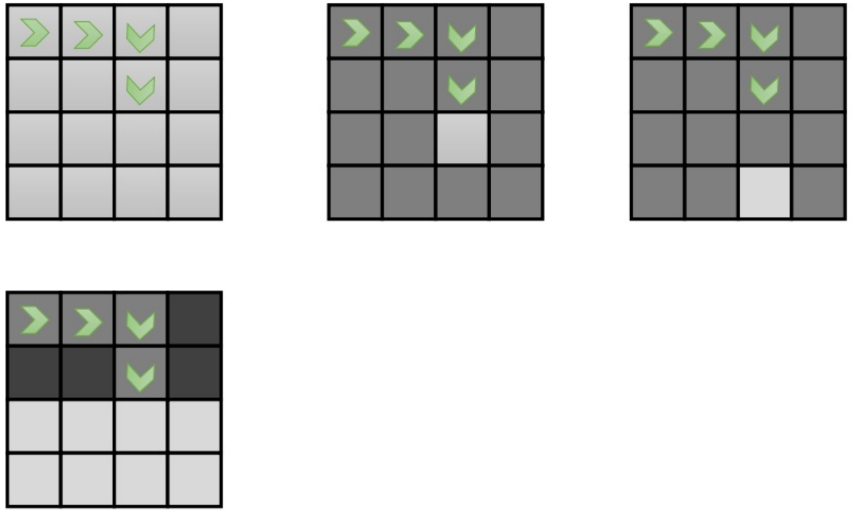


$$r(\mathbf{s}, \mathbf{a})$$

by itself, this is an **underspecified** problem

many reward functions can explain the **same** behavior



Inverse Reinforcement Learning Formulation

"forward" reinforcement learning

inverse reinforcement learning

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"forward" reinforcement learning

given:

states $\mathbf{s} \in \mathcal{S}$, actions $\mathbf{a} \in \mathcal{A}$

(sometimes) transitions $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$

inverse reinforcement learning

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reward parameters

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reward parameters

linear reward function:

$$r_\psi(\mathbf{s}, \mathbf{a}) = \sum_i \psi_i f_i(\mathbf{s}, \mathbf{a}) = \psi^T \mathbf{f}(\mathbf{s}, \mathbf{a})$$

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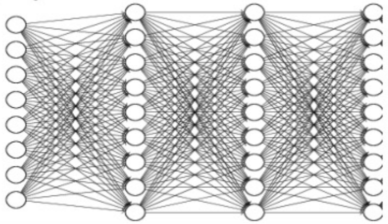
neural net reward function:

$\mathbf{s}$

$\mathbf{a}$



$r_\psi(\mathbf{s}, \mathbf{a})$

parameters ψ

Inverse Reinforcement Learning Formulation

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learn $\pi^\star(\mathbf{a}|\mathbf{s})$

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learn $r_\psi(\mathbf{s}, \mathbf{a})$

reward parameters

...and then use it to learn $\pi^\star(\mathbf{a}|\mathbf{s})$

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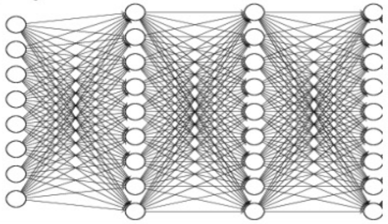

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Feature Matching Inverse RL

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if features $\mathbf{f}$ are important, what if we match their expectations?

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maximum margin principle:

$$\max_{\psi, m} m \qquad \text{such that } \psi^T E_{\pi^\star}[\mathbf{f}(\mathbf{s}, \mathbf{a})] \geq \max_{\pi \in \Pi} \psi^T E_\pi[\mathbf{f}(\mathbf{s}, \mathbf{a})] + m$$

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need to somehow "weight" by similarity between $\pi^\star$ and π

Feature Matching IRL & Maximum Margin

remember the "SVM trick":

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e.g., difference in feature expectations!

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Further reading:
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- Ratliff et al: Maximum margin planning

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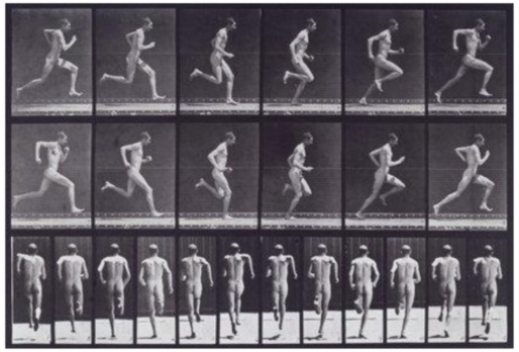
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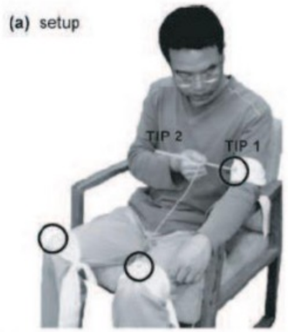

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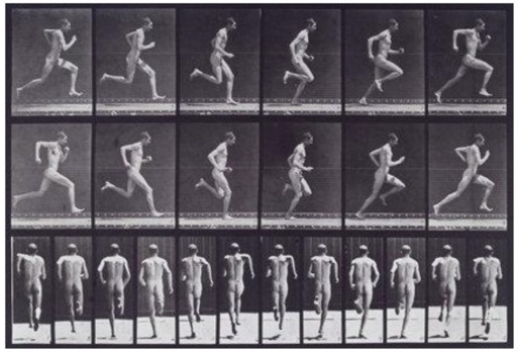

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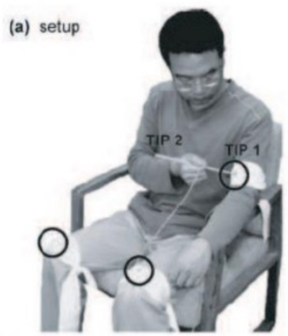

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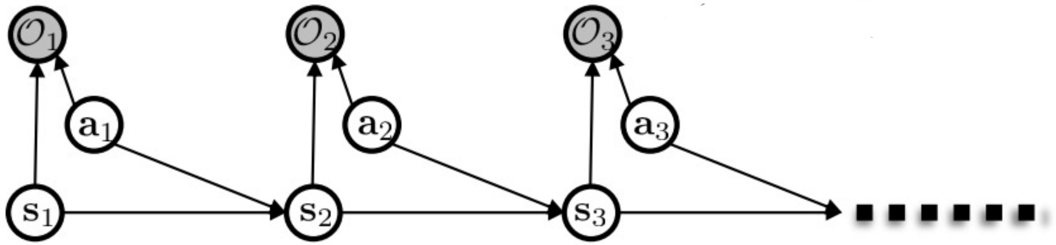

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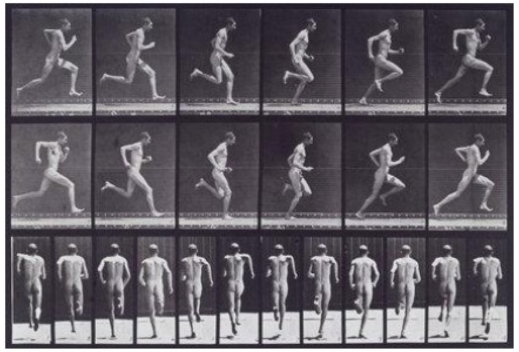

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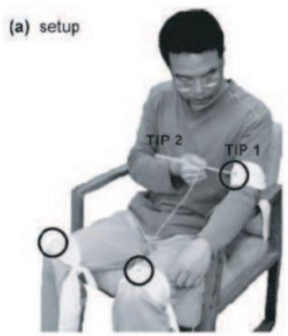

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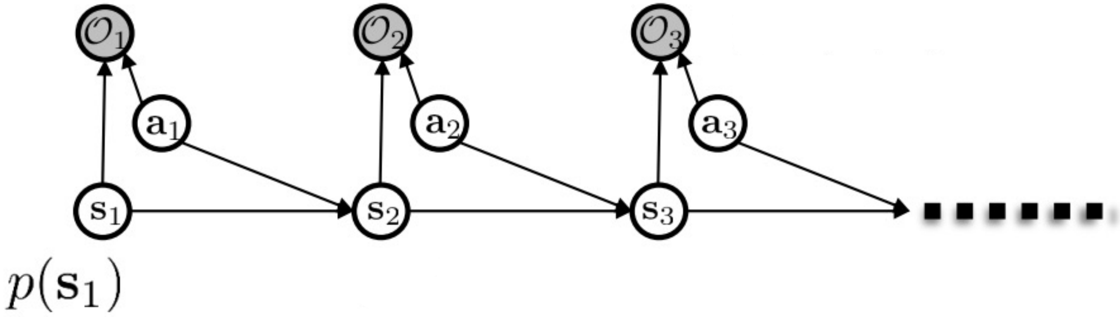
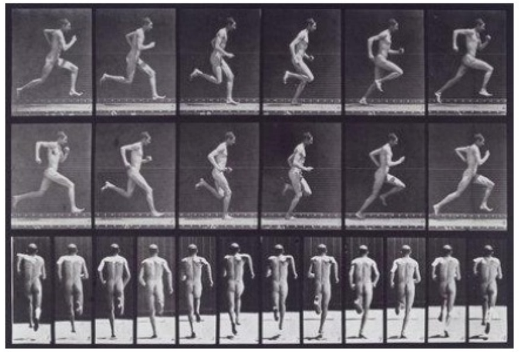
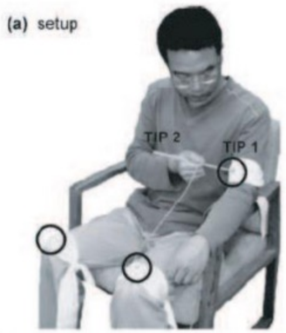


Modeling Human Behavior with Optimal Control

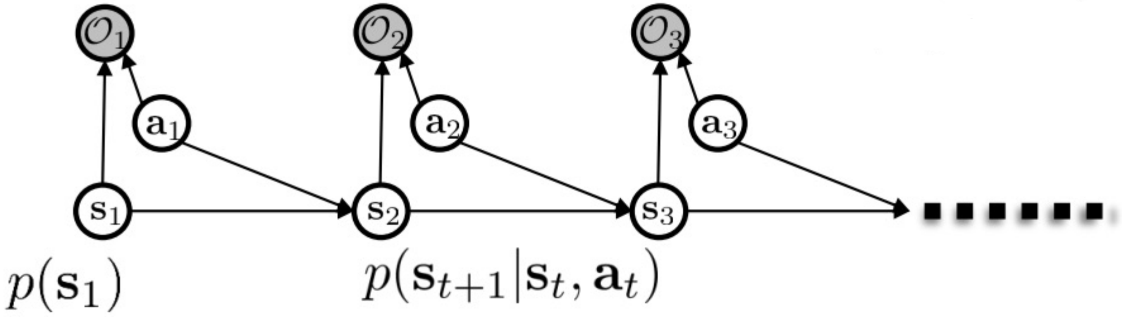

Muybridge (c. 1870)


Mombaur et al. ‘09

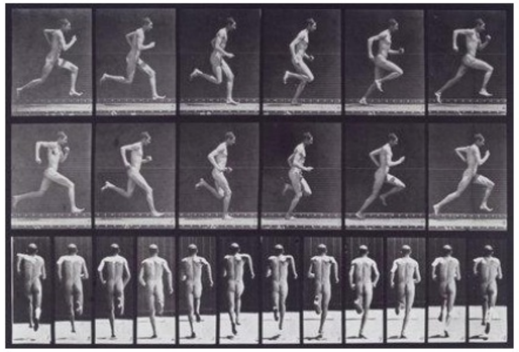


Li & Todorov ‘06


Ziebart ‘08



Modeling Human Behavior with Optimal Control



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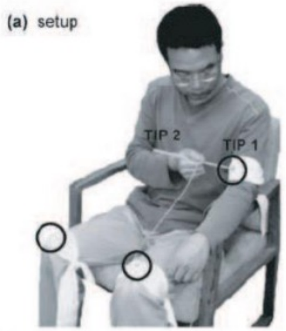

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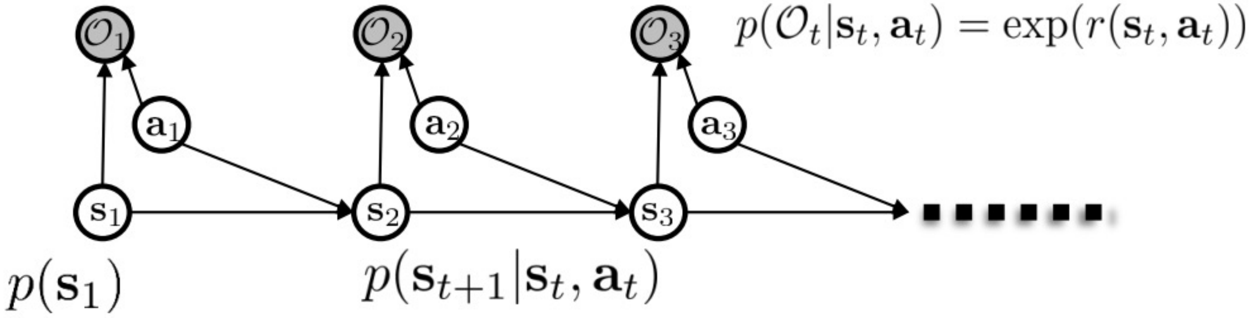
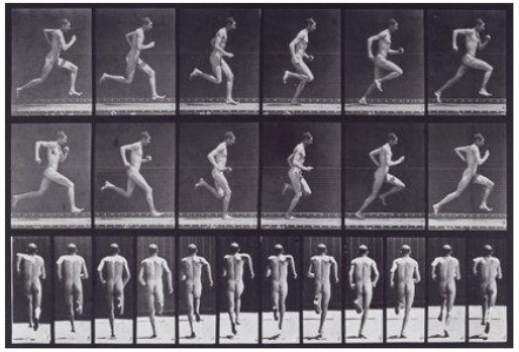


Modeling Human Behavior with Optimal Control



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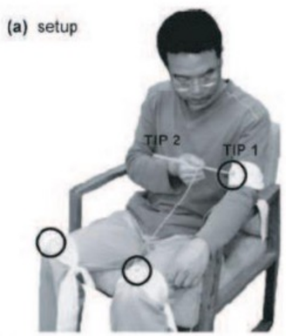
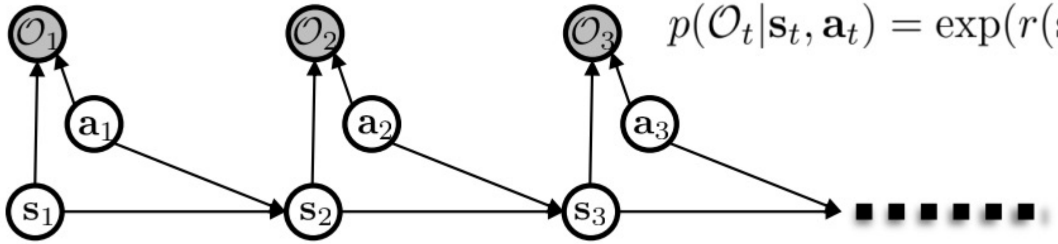


Li & Todorov '06

Ziebart '08

$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$$

$$p(\mathbf{s}_{1:T}, \mathbf{a}_{1:T}|\mathcal{O}_{1:T})$$

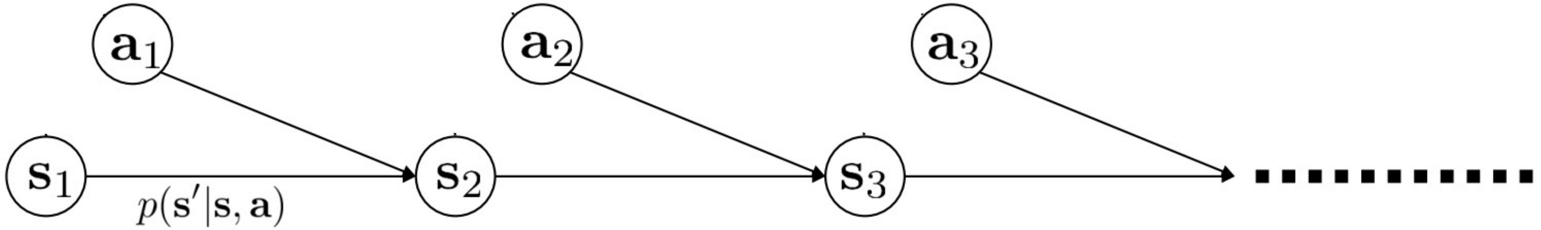


$$p(\mathbf{s}_1)$$

$$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

A probabilistic graphical model of decision making

$$p(\underbrace{\mathbf{s}_{1:T}, \mathbf{a}_{1:T}}_{\tau}) = ??$$

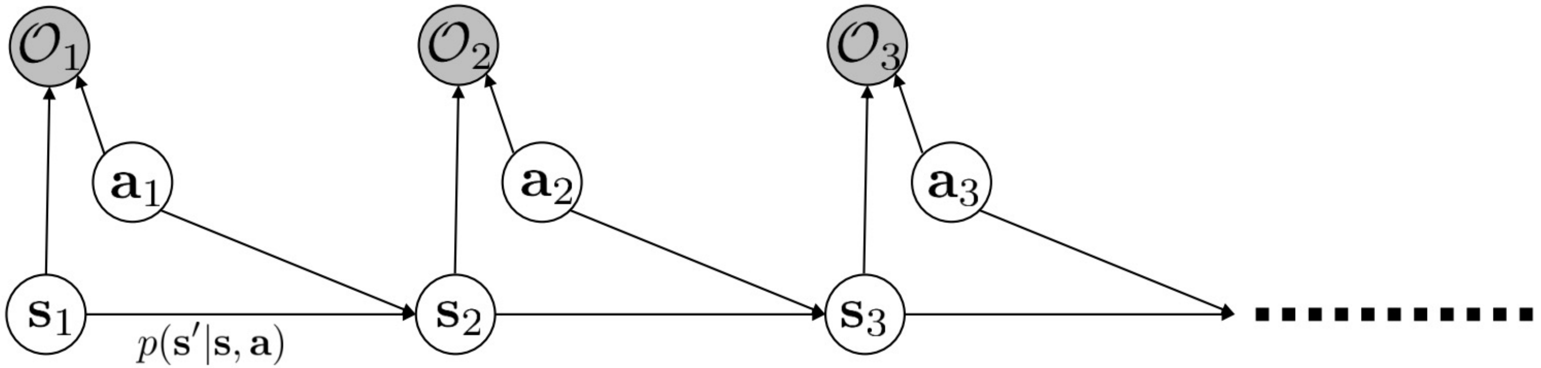


A probabilistic graphical model of decision making

$p(\underbrace{\mathbf{s}_{1:T}, \mathbf{a}_{1:T}}_{\tau}) = ??$ no assumption of optimal behavior!

$p(\tau | \mathcal{O}_{1:T})$

$p(\mathcal{O}_t | \mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$

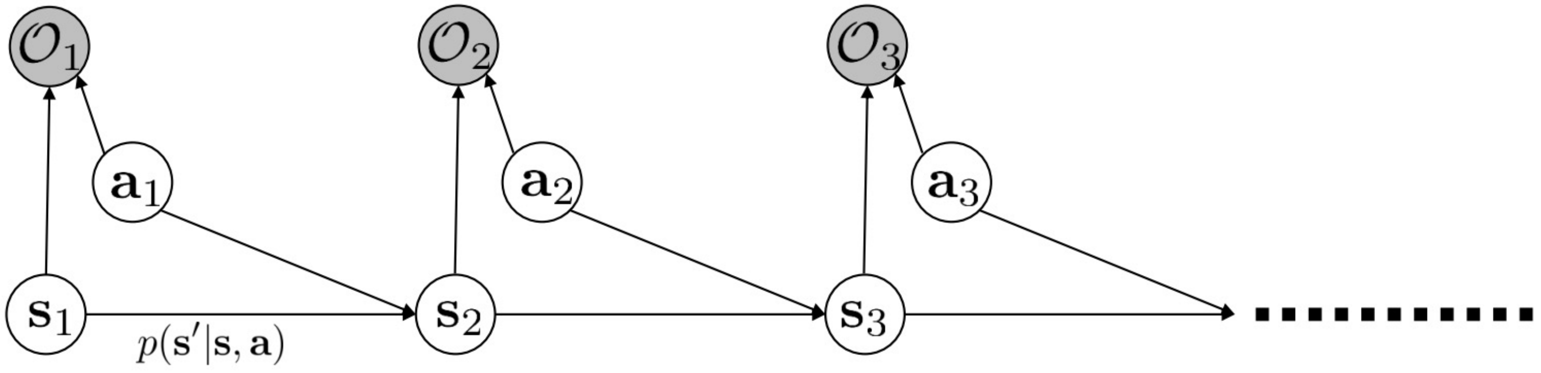


A probabilistic graphical model of decision making

$p(\underbrace{\mathbf{s}_{1:T}, \mathbf{a}_{1:T}}_{\tau}) = ??$ no assumption of optimal behavior!

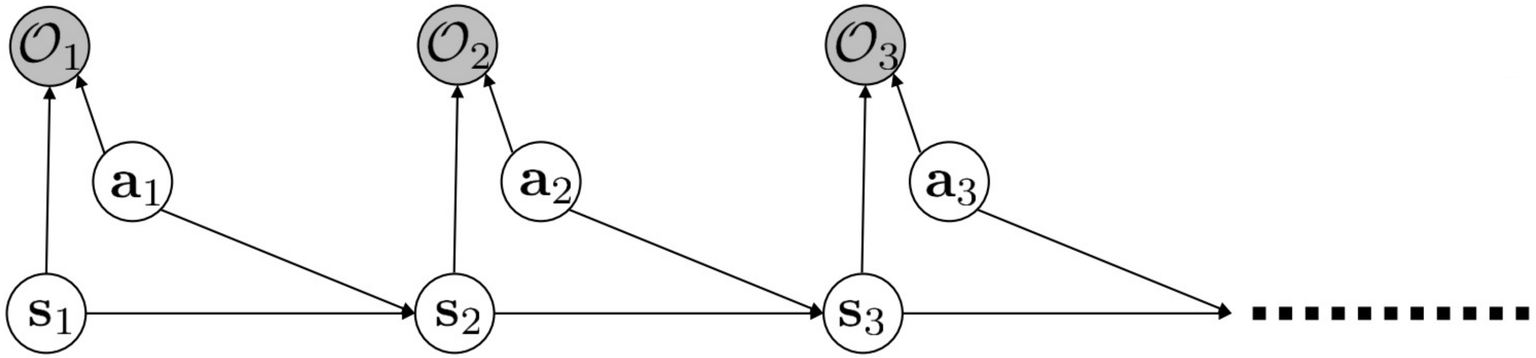
$p(\tau | \mathcal{O}_{1:T})$ $\quad p(\mathcal{O}_t | \mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$

$$p(\tau | \mathcal{O}_{1:T}) = \frac{p(\tau, \mathcal{O}_{1:T})}{p(\mathcal{O}_{1:T})}$$
$$\propto p(\tau) \prod_t \exp(r(\mathbf{s}_t, \mathbf{a}_t)) = p(\tau) \exp\left(\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right)$$



Learning the optimality variable

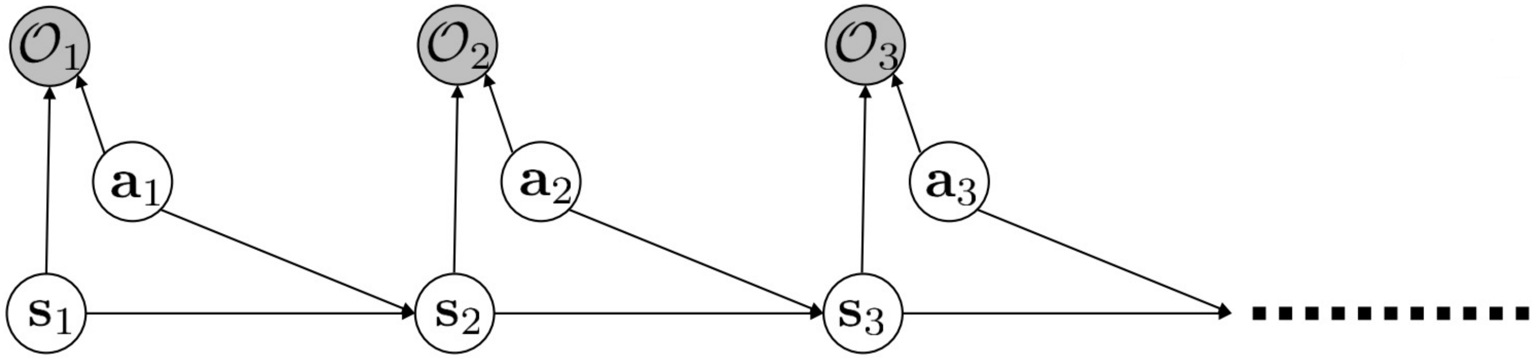
$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))$$



Learning the optimality variable

$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))$$

reward parameters



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$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t, \psi) = \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))$$

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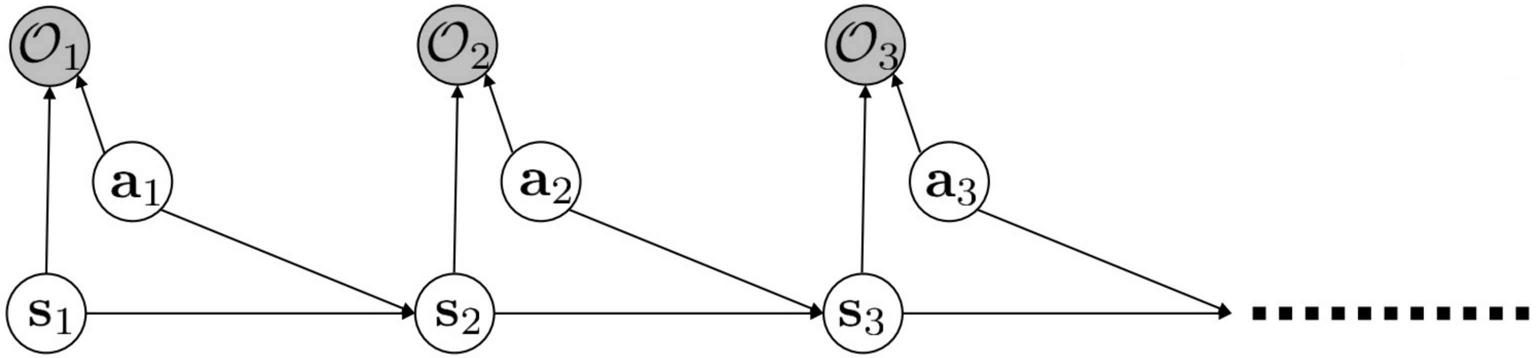

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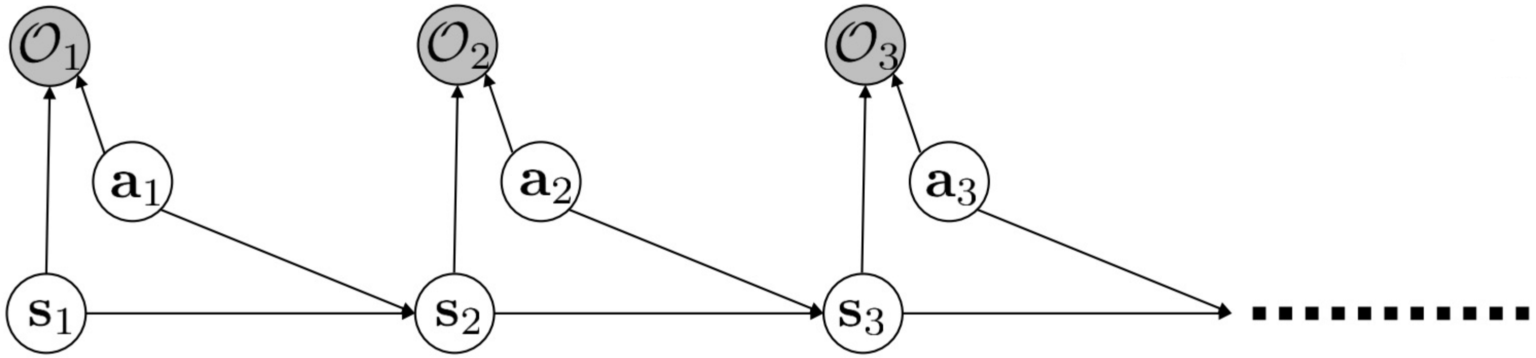

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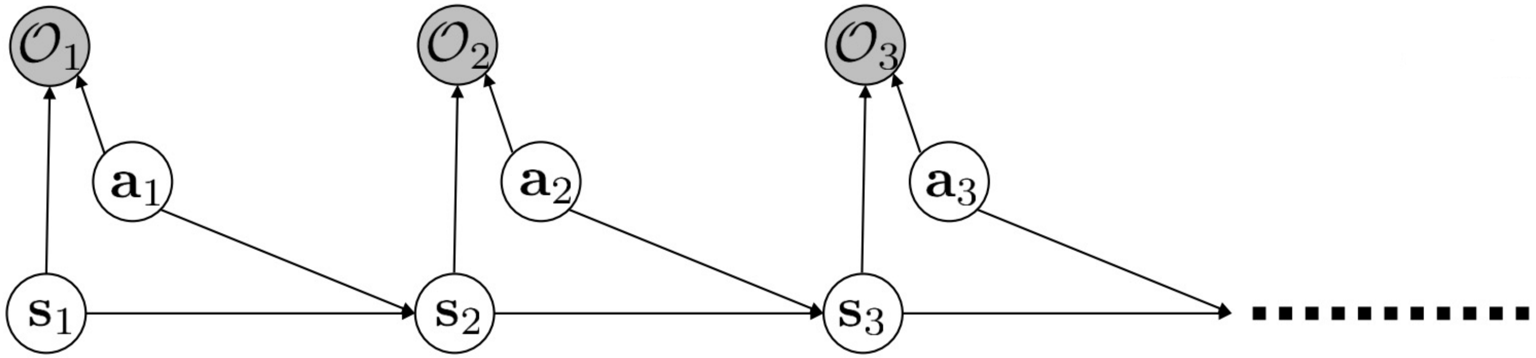
given:

samples $\{\tau_i\}$ sampled from $\pi^\star(\tau)$



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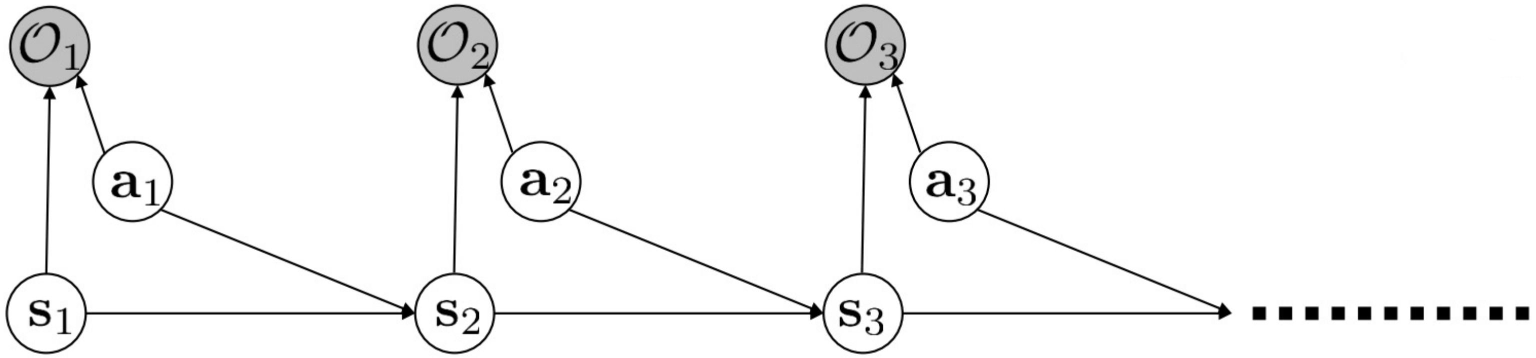
maximum likelihood learning



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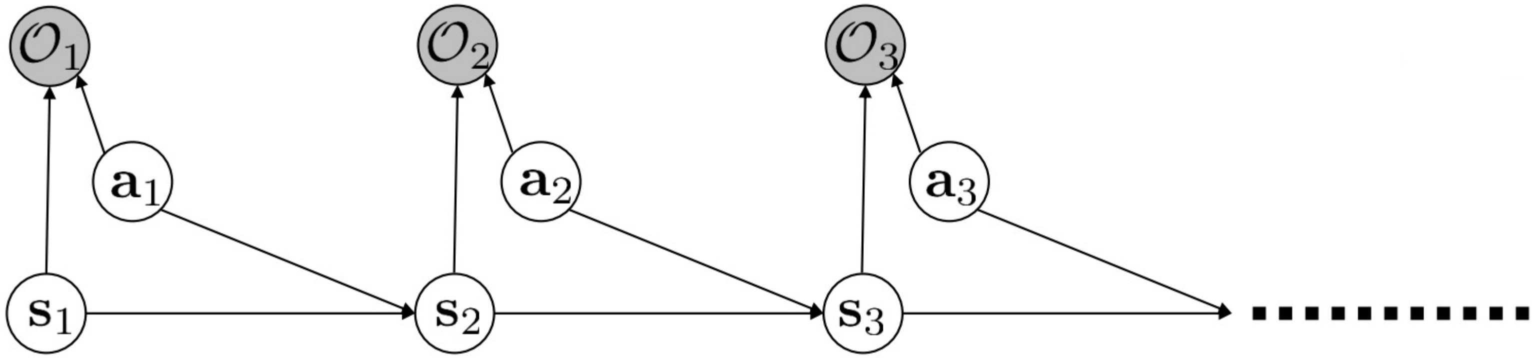
maximum likelihood learning: $\max_\psi \frac{1}{N} \sum_{i=1}^{N} \log p(\tau_i|\mathcal{O}_{1:T}, \psi)$



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can ignore (independent of ψ)

given:

samples $\{\tau_i\}$ sampled from $\pi^\star(\tau)$

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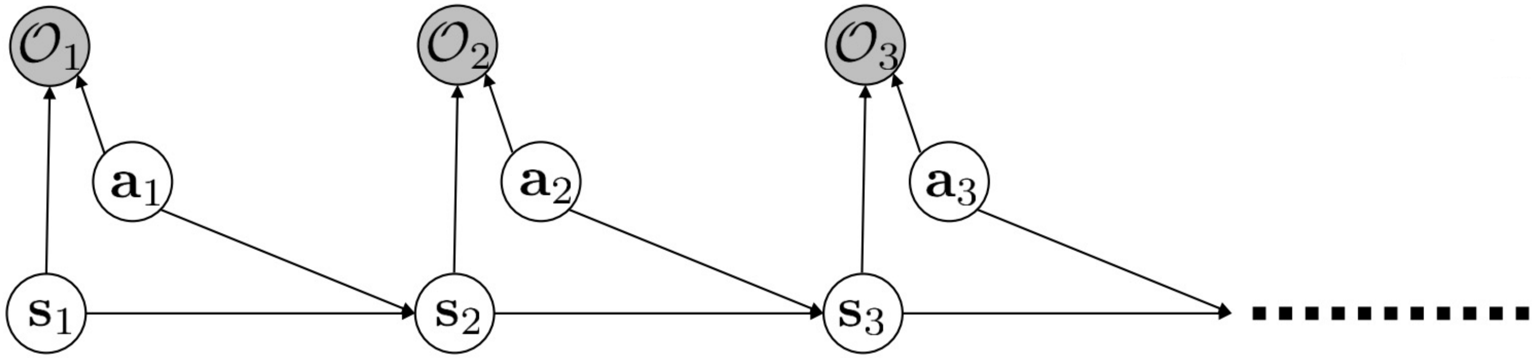

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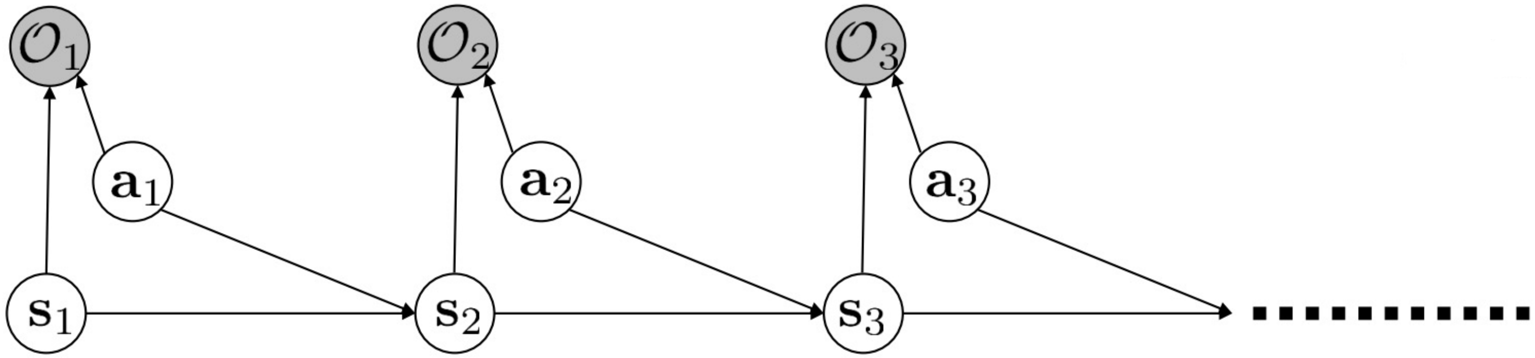

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$$p(\tau|\mathcal{O}_{1:T}, \psi) \propto \cancel{p(\tau)} \exp\left(\sum_t r_\psi(\mathbf{s}_t, \mathbf{a}_t)\right)$$

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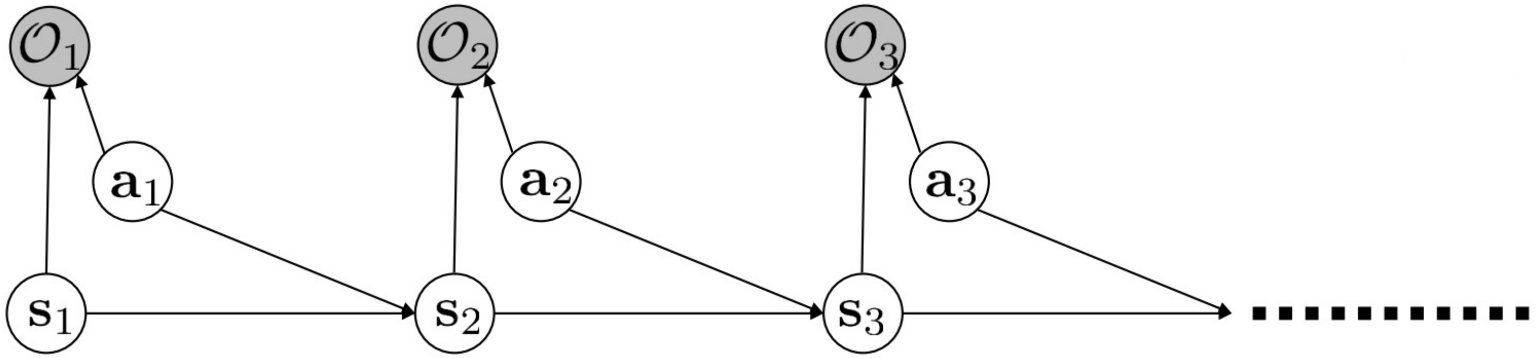
partition function
(the hard part)



The IRL Partiotion Function

$$\max_{\psi} \frac{1}{N} \sum_{i=1}^{N} r_\psi(\tau_i) - \log Z$$

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estimate with expert samples

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estimate with expert samples

soft optimal policy under current reward

Estimating The Expectation

$$\nabla_\psi \mathcal{L} = E_{\tau \sim \pi^\star(\tau)}[\nabla_\psi r_\psi(\tau_i)] - E_{\tau \sim p(\tau|\mathcal{O}_{1:T}, \psi)}[\nabla_\psi r_\psi(\tau)]$$

Estimating The Expectation

$$\nabla_\psi \mathcal{L} = E_{\tau \sim \pi^\star(\tau)}[\nabla_\psi r_\psi(\tau_i)] - \underbrace{E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)}[\nabla_\psi r_\psi(\tau)]}_{E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)}\left[\nabla_\psi \sum_{t=1}^{T} r_\psi(\mathbf{s}_t, \mathbf{a}_t)\right]}$$

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$$\nabla_\psi \mathcal{L} = E_{\tau \sim \pi^\star(\tau)}[\nabla_\psi r_\psi(\tau_i)] - \underbrace{E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)}[\nabla_\psi r_\psi(\tau)]}$$

$$E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)}\left[\nabla_\psi \sum_{t=1}^{T} r_\psi(\mathbf{s}_t, \mathbf{a}_t)\right]$$

$$= \sum_{t=1}^{T} E_{(\mathbf{s}_t,\mathbf{a}_t) \sim p(\mathbf{s}_t,\mathbf{a}_t|\mathcal{O}_{1:T},\psi)}\left[\nabla_\psi r_\psi(\mathbf{s}_t, \mathbf{a}_t)\right]$$

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$$p(\mathbf{a}_t|\mathbf{s}_t,\mathcal{O}_{1:T},\psi)p(\mathbf{s}_t|\mathcal{O}_{1:T},\psi)$$

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$$\nabla_\psi \mathcal{L} = E_{\tau\sim\pi^\star(\tau)}[\nabla_\psi r_\psi(\tau_i)] - \underbrace{E_{\tau\sim p(\tau|\mathcal{O}_{1:T},\psi)}[\nabla_\psi r_\psi(\tau)]}$$

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$$p(\mathbf{a}_t|\mathbf{s}_t,\mathcal{O}_{1:T},\psi)p(\mathbf{s}_t|\mathcal{O}_{1:T},\psi)$$

where have we seen this before?

Estimating The Expectation

$$\nabla_\psi \mathcal{L} = E_{\tau \sim \pi^\star(\tau)}[\nabla_\psi r_\psi(\tau_i)] - \underbrace{E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)}[\nabla_\psi r_\psi(\tau)]}$$

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$$p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi) p(\mathbf{s}_t|\mathcal{O}_{1:T}, \psi)$$

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$$= \frac{\beta(\mathbf{s}_t, \mathbf{a}_t)}{\beta(\mathbf{s}_t)}$$

Estimating The Expectation

$$\nabla_\psi \mathcal{L} = E_{\tau \sim \pi^\star(\tau)}[\nabla_\psi r_\psi(\tau_i)] - \underbrace{E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)}[\nabla_\psi r_\psi(\tau)]}$$

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$$p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi) p(\mathbf{s}_t|\mathcal{O}_{1:T}, \psi)$$

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backward message   forward message

Estimating The Expectation

$$\nabla_\psi \mathcal{L} = E_{\tau \sim \pi^\star(\tau)}[\nabla_\psi r_\psi(\tau_i)] - E_{\tau \sim p(\tau|\mathcal{O}_{1:T}, \psi)}[\nabla_\psi r_\psi(\tau)]$$

let $\mu_t(\mathbf{s}_t, \mathbf{a}_t) \propto \beta(\mathbf{s}_t, \mathbf{a}_t)\alpha(\mathbf{s}_t)$

Estimating The Expectation

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Estimating The Expectation

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state-action visitation probability for each $(\mathbf{s}_t, \mathbf{a}_t)$

Maximum Entropy Inverse Reinforcement Learning

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Ziebart et al. 2008: Maximum Entropy Inverse Reinforcement Learning

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in the case where $r_\psi(\mathbf{s}_t, \mathbf{a}_t) = \psi^T \mathbf{f}(\mathbf{s}_t, \mathbf{a}_t)$, we can show that it optimizes

$\max_\psi \mathcal{H}(\pi^{r_\psi})$ such that $E_{\pi^{r_\psi}}[\mathbf{f}] = E_{\pi^\star}[\mathbf{f}]$

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unknown expert policy
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as random as possible while matching features

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Maximum Entropy Inverse Reinforcement Learning

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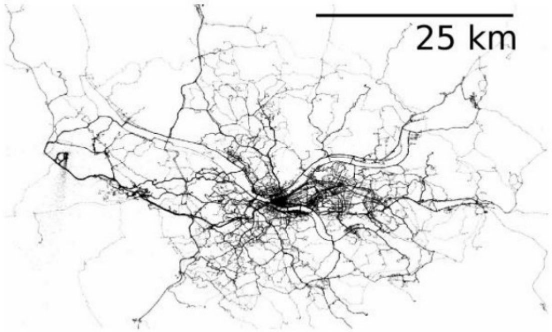

Feature	Value
Highway	3.3 miles
Major Streets	2.0 miles
Local Streets	0.3 miles
Above 55mph	4.0 miles
35-54mph	1.1 miles
25-34 mph	0.5 miles
Below 24mph	0 miles
3+ Lanes	0.5 miles
2 Lanes	3.3 miles
1 Lane	1.8 miles

Feature	Value
Hard left turn	1
Soft left turn	3
Soft right turn	5
Hard right turn	0
No turn	25
U-turn	0

Approximations in High Dimensions

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Assume we don't know the dynamics, but we can sample, like in standard RL

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looks expensive! what if we use "lazy" policy optimization?

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$$= \frac{\exp(\sum_t r_\psi(\mathbf{s}_t, \mathbf{a}_t))}{\prod_t \pi(\mathbf{a}_t|\mathbf{s}_t)}$$

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$$\nabla_\psi \mathcal{L} \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{\sum_j w_j}\sum_{j=1}^{M} w_j \nabla_\psi r_\psi(\tau_j)$$

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$$\frac{\cancel{p(\mathbf{s}_1)}\prod_t \cancel{p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}\exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))}{\cancel{p(\mathbf{s}_1)}\prod_t \cancel{p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}\pi(\mathbf{a}_t|\mathbf{s}_t)}$$

$$= \frac{\exp(\sum_t r_\psi(\mathbf{s}_t, \mathbf{a}_t))}{\prod_t \pi(\mathbf{a}_t|\mathbf{s}_t)}$$

each policy update w.r.t. r_ψ brings us closer to the target distribution!

Guided Cost Learning Algorithm (Finn et al. ICML '16)

initial
policy π

human
demonstrations

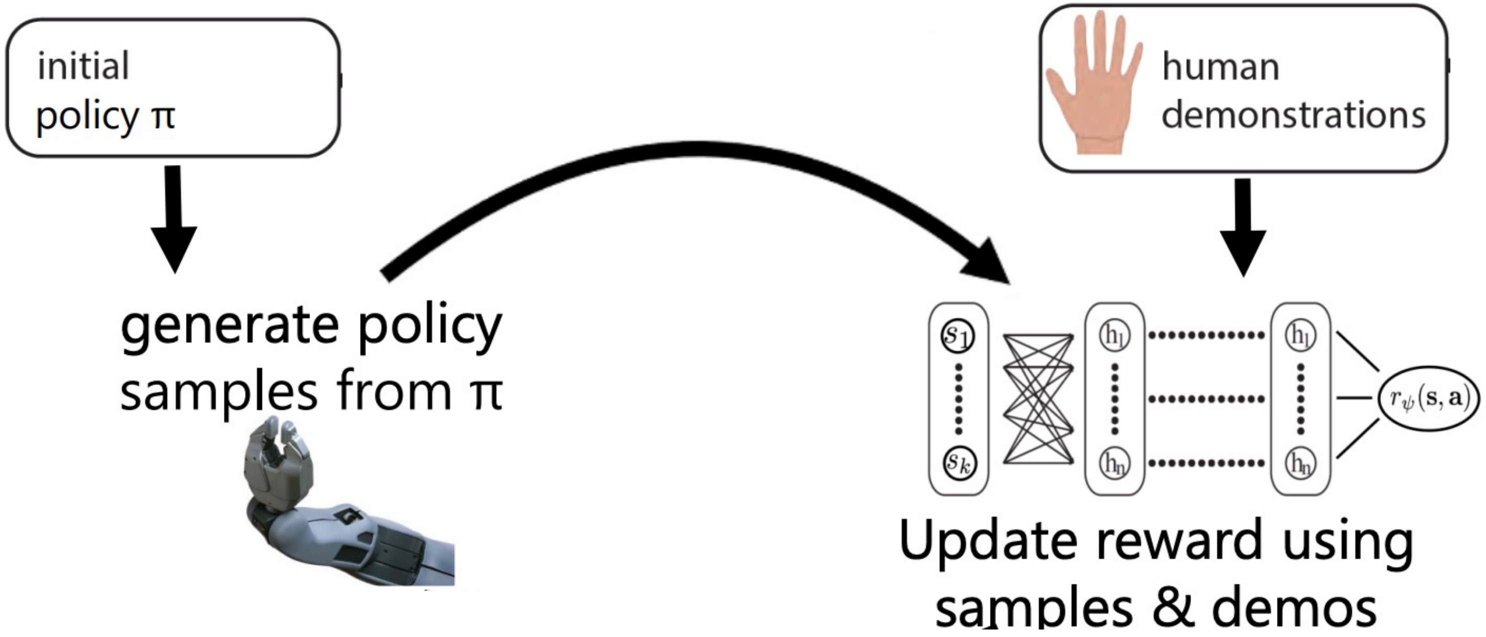
Guided Cost Learning Algorithm (Finn et al. ICML '16)

initial
policy π

generate policy
samples from π

human
demonstrations

Guided Cost Learning Algorithm (Finn et al. ICML ’16)



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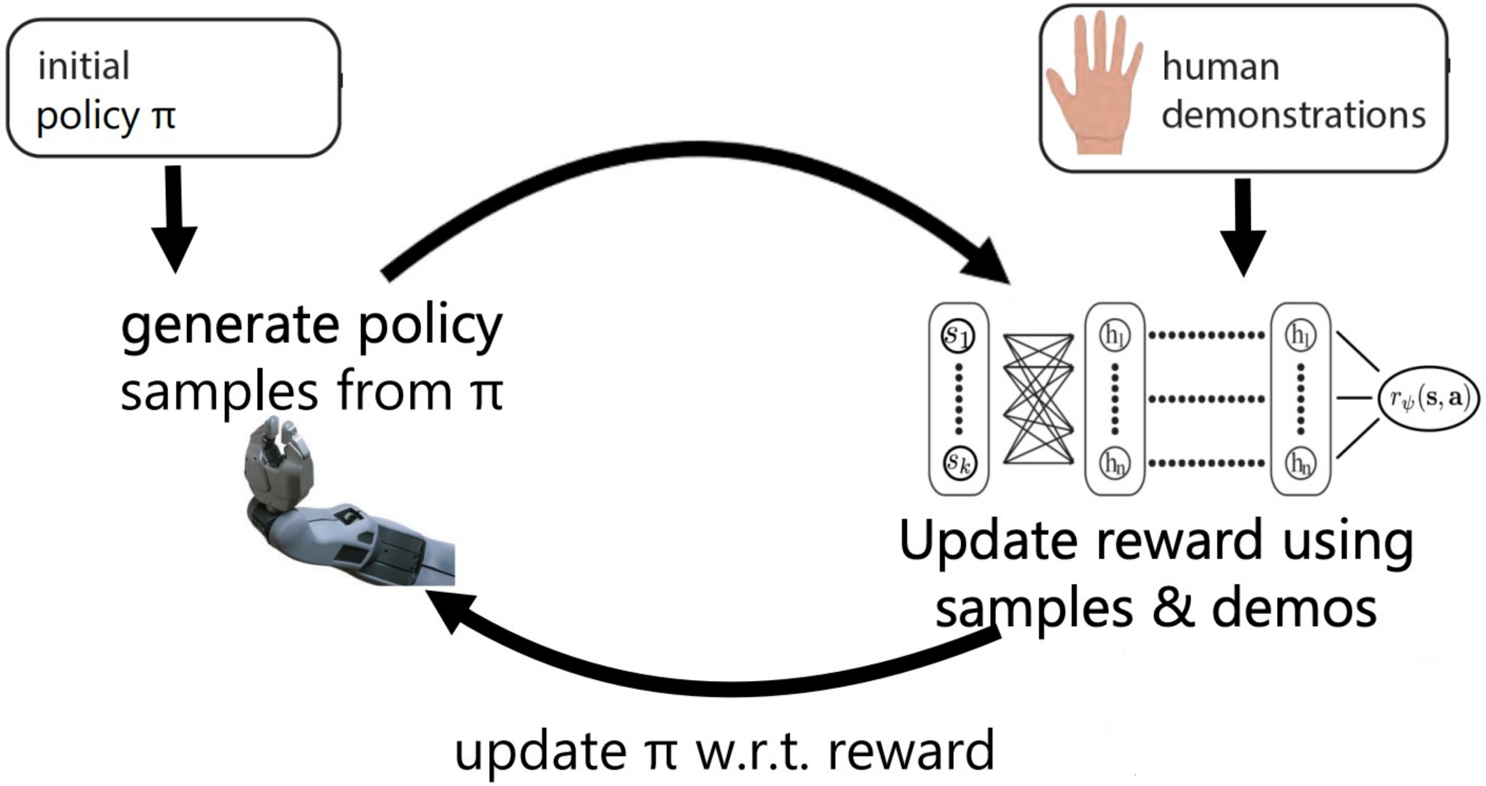

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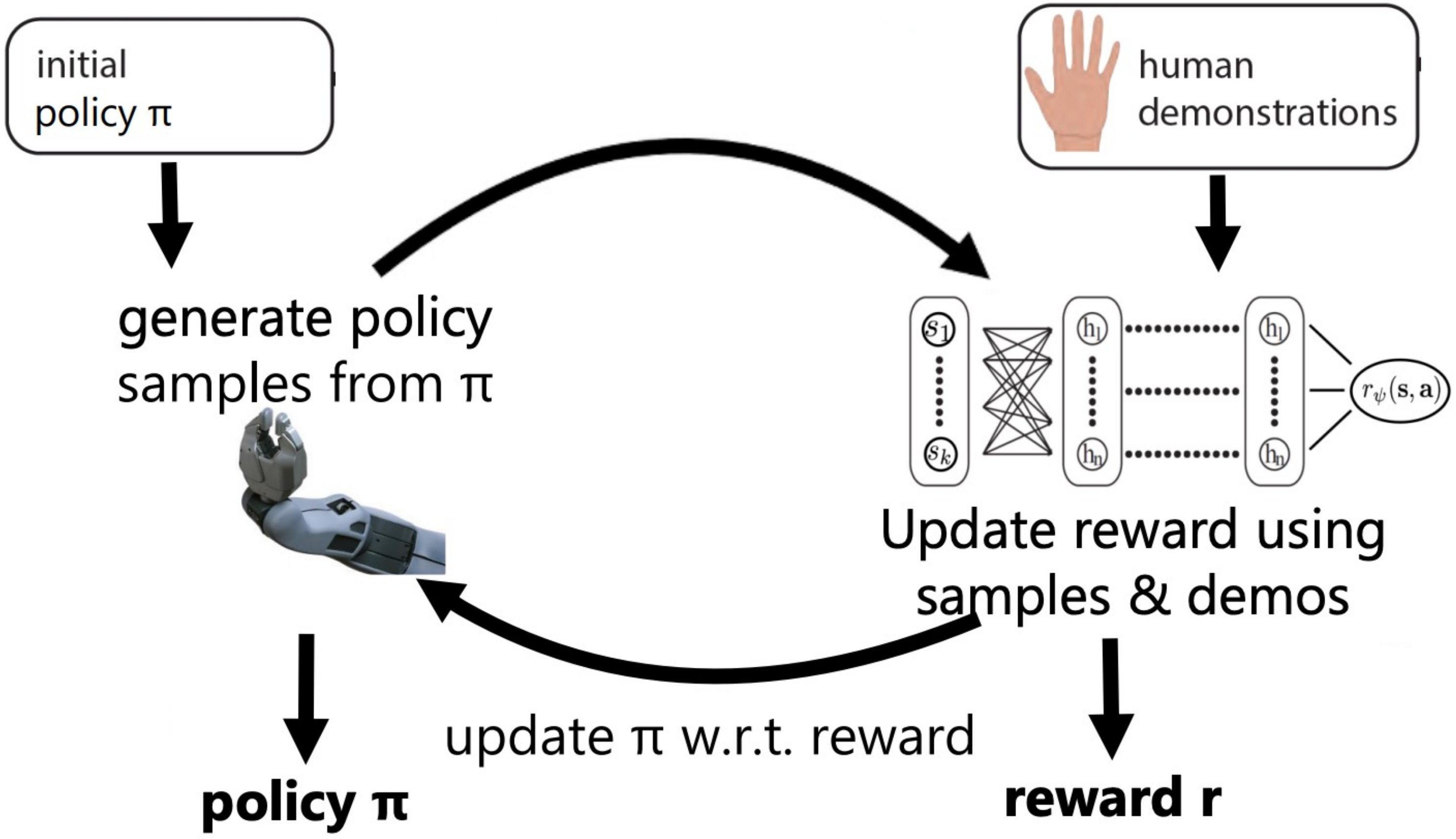

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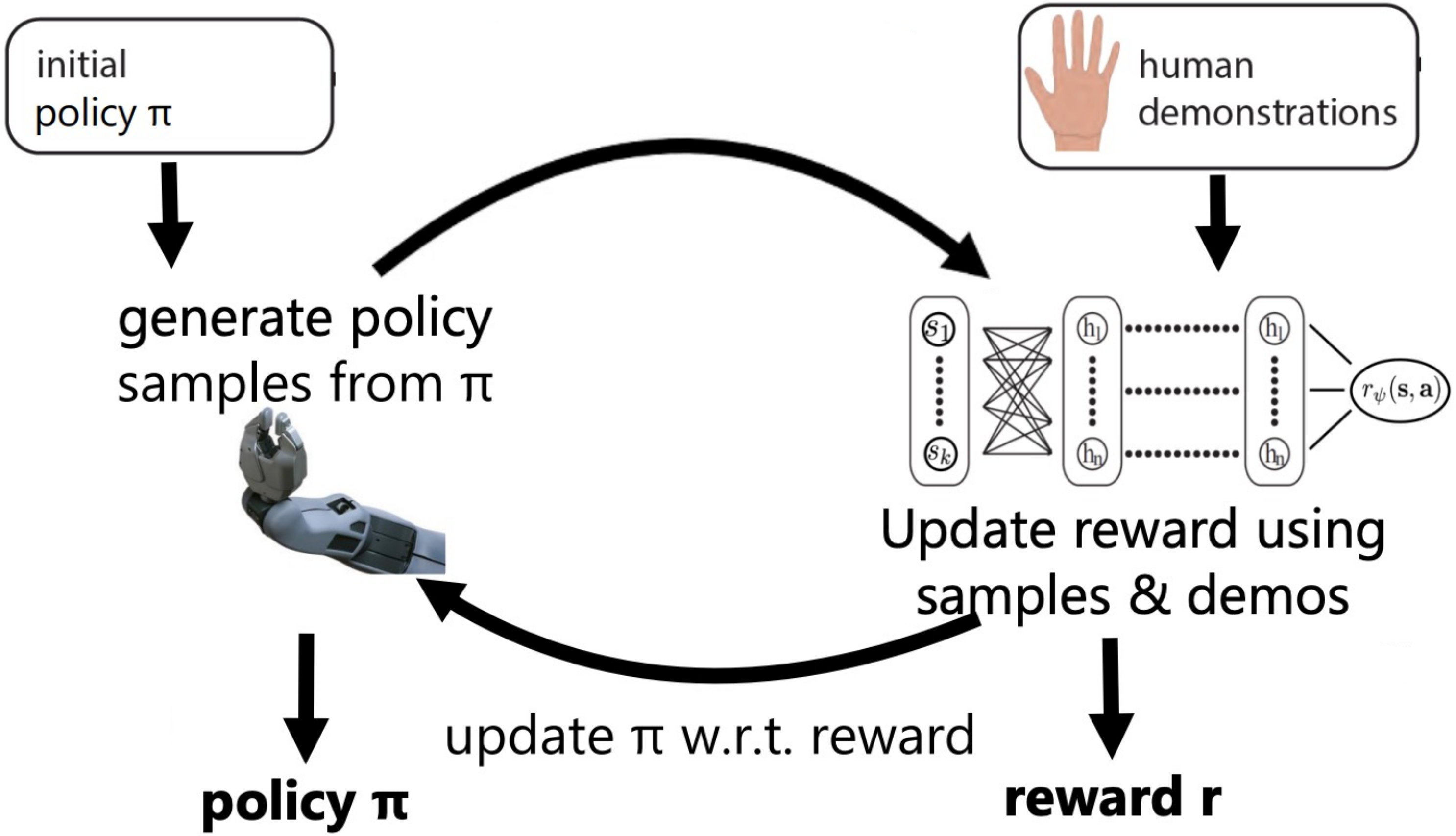

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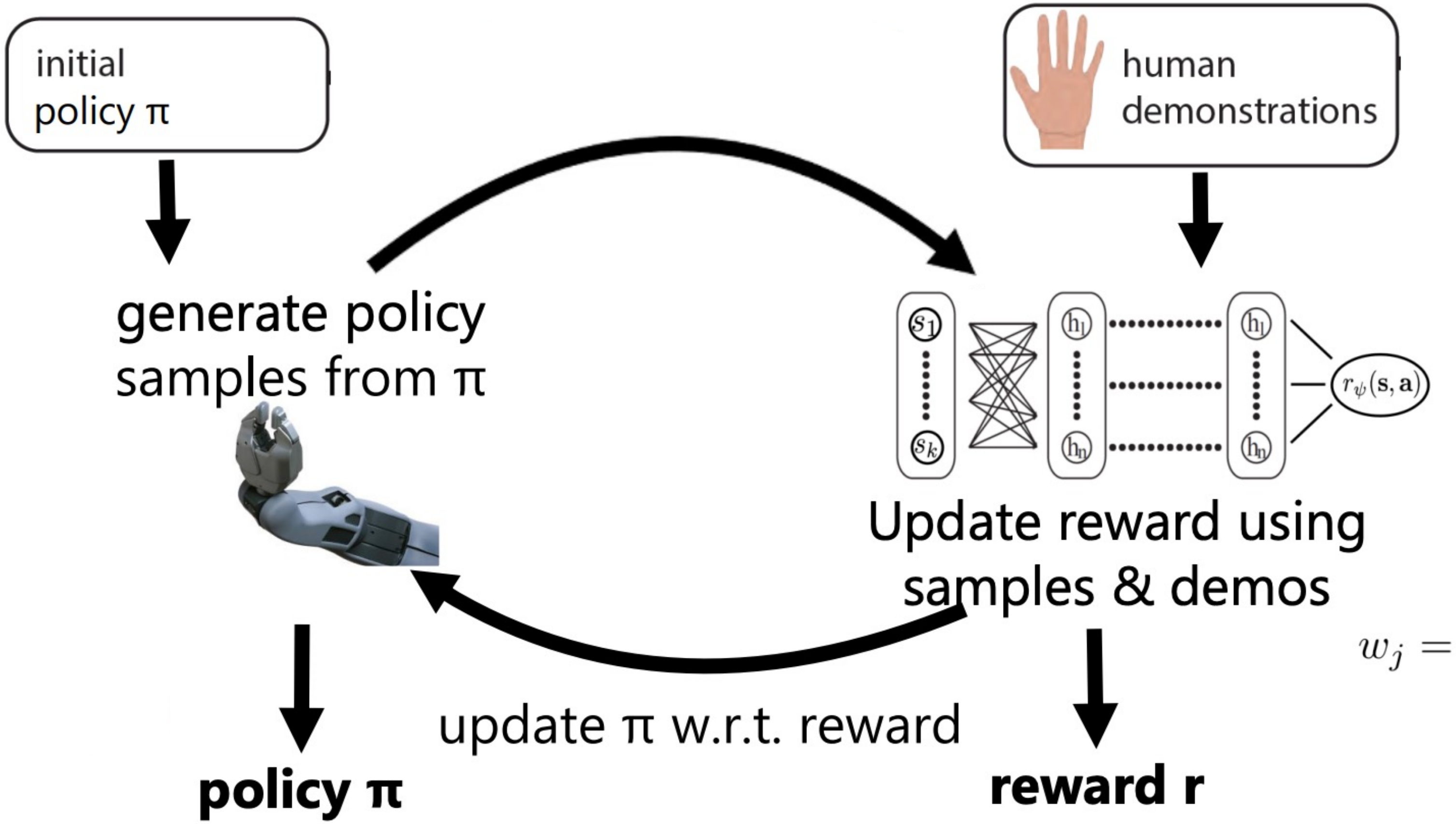

$$w_j = \frac{\exp(r_\psi(\tau_j))}{\pi(\tau_j)}$$

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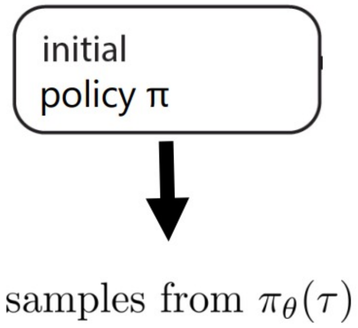
Inverse RL and GANs

It looks a bit like a game

initial
policy π

Inverse RL and GANs

It looks a bit like a



Inverse RL and GANs

It looks a bit like a game...

Inverse RL and GANs

It looks a bit like a game...

initial
policy π

samples from $\pi_\theta(\tau)$

human
demonstrations

samples from $\pi^\star(\tau)$

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demos are made more likely, samples less likely

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demos are made more likely, samples less likely

$$\nabla_\theta \mathcal{L} \approx \frac{1}{M}\sum_{j=1}^{M} \nabla_\theta \log \pi_\theta(\tau_j) r_\psi(\tau_j)$$

policy changed to make it *harder* to distinguish from demos

Generative Adversarial Networks

Zhu et al. '17

Generative Adversarial Networks

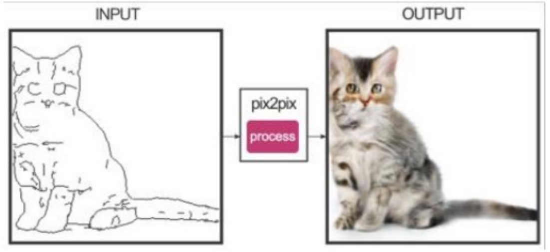
Zhu et al. '17

Arjovsky et al. '17

Generative Adversarial Networks

Zhu et al. '17

Arjovsky et al. '17

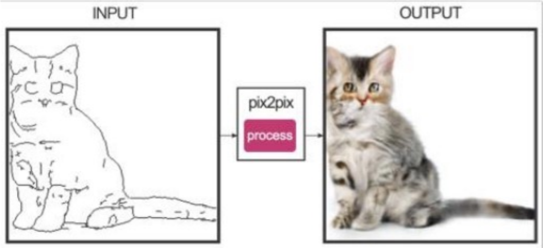


Isola et al. '17

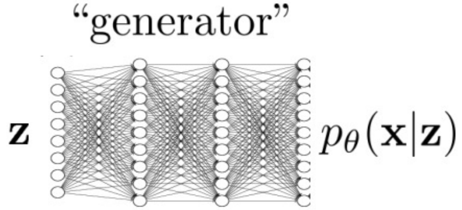
Generative Adversarial Networks

Zhu et al. '17

Arjovsky et al. '17



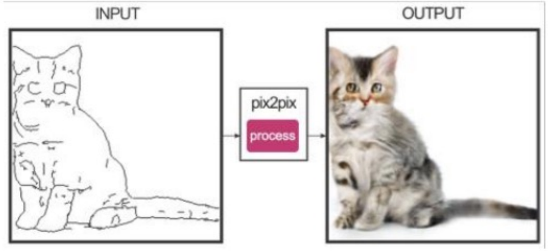
Isola et al. '17



Generative Adversarial Networks

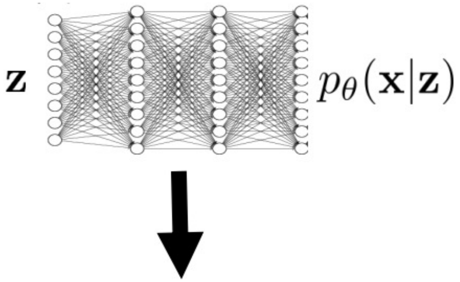
Zhu et al. '17

Arjovsky et al. '17



Isola et al. '17

"generator"

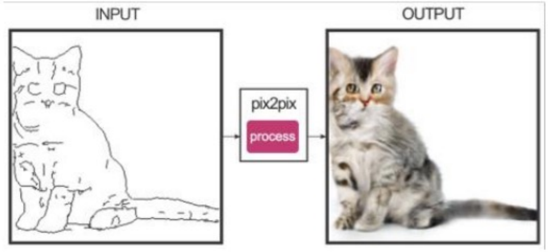


samples from $p_\theta(\mathbf{x})$

Generative Adversarial Networks

Zhu et al. '17

Arjovsky et al. '17



Isola et al. '17

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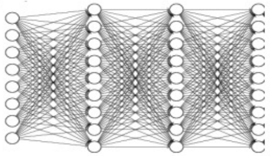


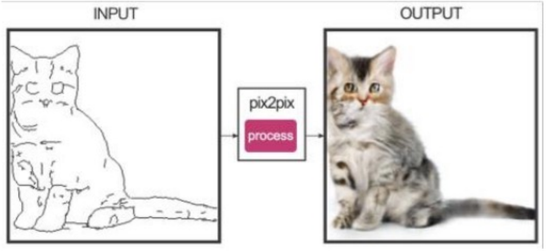
$\mathbf{z}$ $p_\theta(\mathbf{x}|\mathbf{z})$

samples from $p_\theta(\mathbf{x})$

data ("demonstrations")

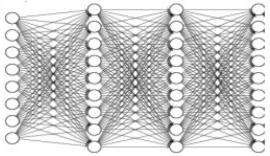
samples from $p^\star(\mathbf{x})$

Generative Adversarial Networks



Zhu et al. '17 Arjovsky et al. '17 Isola et al. '17

"generator"

$\mathbf{z}$  $p_\theta(\mathbf{x}|\mathbf{z})$

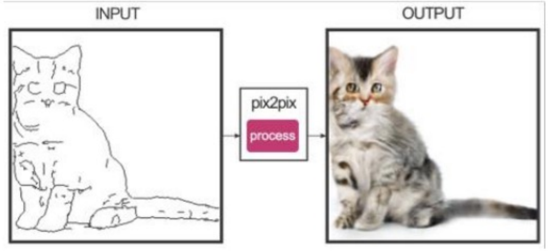
data ("demonstrations") 

samples from $p^\star(\mathbf{x})$

samples from $p_\theta(\mathbf{x})$

$$\psi = \arg\max_\psi \frac{1}{N} \sum_{\mathbf{x} \sim p^\star} \log D_\psi(\mathbf{x}) + \frac{1}{M} \sum_{\mathbf{x} \sim p_\theta} \log(1 - D_\psi(\mathbf{x}))$$

Generative Adversarial Networks



Zhu et al. '17

Arjovsky et al. '17

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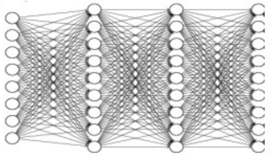
"generator"

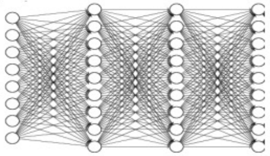
$\mathbf{z}$  $p_\theta(\mathbf{x}|\mathbf{z})$

samples from $p_\theta(\mathbf{x})$

data ("demonstrations")

samples from $p^\star(\mathbf{x})$

$D(\mathbf{x}) = p_\psi(\text{real image}|\mathbf{x})$

$\mathbf{x}$ 

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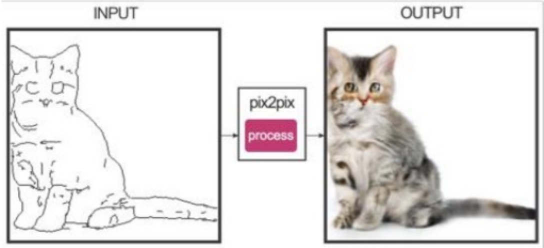
Generative Adversarial Networks



Zhu et al. '17

Arjovsky et al. '17

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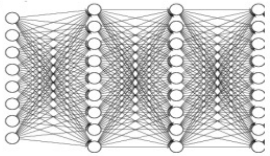
"generator"

$\mathbf{z}$  $p_\theta(\mathbf{x}|\mathbf{z})$

data ("demonstrations")

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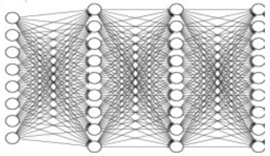
samples from $p^\star(\mathbf{x})$

$\mathbf{x}$ 

samples from $p_\theta(\mathbf{x})$

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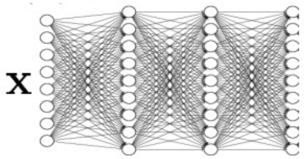
$$\theta \leftarrow \arg\max_\theta E_{\mathbf{x}\sim p_\theta} \log D_\psi(\mathbf{x})$$

Goodfellow et al. '14

Inverse RL as a GAN

$$D(\mathbf{x}) = p_\psi(\text{real image}|\mathbf{x})$$



Finn*, Christiano* et al. "A Connection Between Generative Adversarial Networks, Inverse Reinforcement Learning, and Energy-Based Models."

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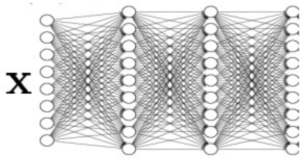

which discriminator is best?

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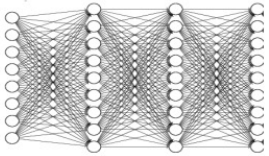
$\mathbf{x}$



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Inverse RL as a GAN

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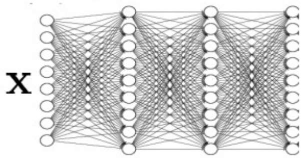

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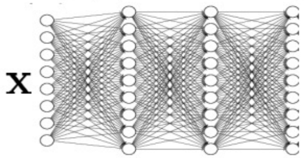

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choose this parameterization for discriminator:

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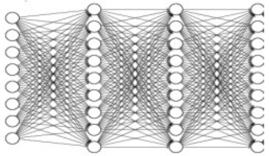
$\mathbf{x}$ 

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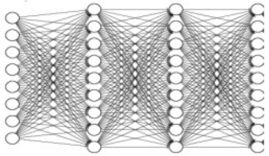

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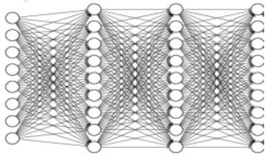
$\mathbf{x}$ 

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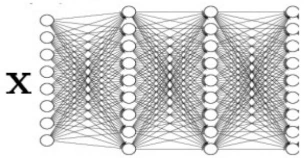

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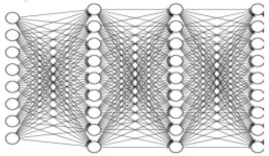
$\mathbf{x}$ 

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optimize this w.r.t. ψ

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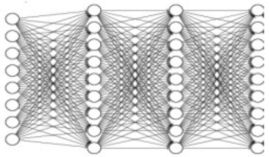
$\mathbf{x}$ 

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optimize this w.r.t. ψ

$$\psi \leftarrow \arg\max_\psi E_{\tau\sim p^\star}[\log D_\psi(\tau)] + E_{\tau\sim\pi_\theta}[\log(1 - D_\psi(\tau))]$$

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Inverse RL as a GAN

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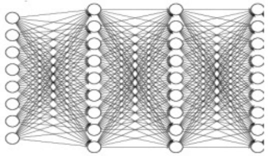
$\mathbf{x}$ 

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choose this parameterization for discriminator:

optimize Z w.r.t. same objective as ψ!

$$D_\psi(\tau) = \frac{p(\tau)\frac{1}{Z}\exp(r(\tau))}{p_\theta(\tau) + p(\tau)\frac{1}{Z}\exp(r(\tau))} = \frac{\cancel{p(\tau)}\frac{1}{Z}\exp(r(\tau))}{\cancel{p(\tau)}\prod_t \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) + \cancel{p(\tau)}\frac{1}{Z}\exp(r(\tau))} = \frac{\frac{1}{Z}\exp(r(\tau))}{\prod_t \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) + \frac{1}{Z}\exp(r(\tau))}$$

optimize this w.r.t. ψ

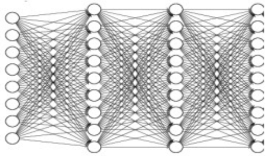
$$\psi \leftarrow \arg\max_\psi E_{\tau\sim p^\star}[\log D_\psi(\tau)] + E_{\tau\sim\pi_\theta}[\log(1 - D_\psi(\tau))]$$

Finn*, Christiano* et al. "A Connection Between Generative Adversarial Networks, Inverse Reinforcement Learning, and Energy-Based Models."

Inverse RL as a GAN

$D(\mathbf{x}) = p_\psi(\text{real image}|\mathbf{x})$

$\mathbf{x}$



which discriminator is best?

$$D^\star(\mathbf{x}) = \frac{p^\star(\mathbf{x})}{p_\theta(\mathbf{x}) + p^\star(\mathbf{x})}$$

for IRL, optimal policy approaches $\pi_\theta(\tau) \propto p(\tau)\exp(r_\psi(\tau))$

choose this parameterization for discriminator:

optimize Z w.r.t. same objective as ψ!

$$D_\psi(\tau) = \frac{p(\tau)\frac{1}{Z}\exp(r(\tau))}{p_\theta(\tau) + p(\tau)\frac{1}{Z}\exp(r(\tau))} = \frac{\cancel{p(\tau)}\frac{1}{Z}\exp(r(\tau))}{\cancel{p(\tau)}\prod_t \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) + \cancel{p(\tau)}\frac{1}{Z}\exp(r(\tau))} = \frac{\frac{1}{Z}\exp(r(\tau))}{\prod_t \pi_\theta(\mathbf{a}_t|\mathbf{s}_t) + \frac{1}{Z}\exp(r(\tau))}$$

optimize this w.r.t. ψ

$$\psi \leftarrow \arg\max_\psi E_{\tau\sim p^\star}[\log D_\psi(\tau)] + E_{\tau\sim\pi_\theta}[\log(1 - D_\psi(\tau))]$$

we don't need importance weights anymore – they are subsumed into Z

Finn*, Christiano* et al. "A Connection Between Generative Adversarial Networks, Inverse Reinforcement Learning, and Energy-Based Models."

Inverse RL as a GAN

generator/policy

$\pi_\theta(\tau)$

↓

samples from $\pi_\theta(\tau)$

data/demonstrations

↓

samples from $p^\star(\tau)$

Finn*, Christiano* et al. "A Connection Between Generative Adversarial Networks, Inverse Reinforcement Learning, and Energy-Based Models."

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Inverse RL as a GAN

Can we just use a regular discriminator?

generator/policy

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samples from $\pi_\theta(\tau)$

data/demonstrations

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policy changed to make it *harder* to distinguish from demos

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Inverse RL as a GAN

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$\downarrow$

samples from $\pi_\theta(\tau)$

data/demonstrations

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Ho & Ermon. Generative adversarial imitation learning.

Inverse RL as a GAN

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$D_\psi(\tau) =$ standard binary neural net classifier

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Pros & cons:

Ho & Ermon. Generative adversarial imitation learning.

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Pros & cons:

+ often simpler to set up optimization, fewer moving parts

Ho & Ermon. Generative adversarial imitation learning.

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Ho & Ermon. Generative adversarial imitation learning.

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policy changed to make it *harder* to distinguish from demos

Pros & cons:

+ often simpler to set up optimization, fewer moving parts

- discriminator knows *nothing* at convergence

- generally cannot reoptimize the "reward"

Ho & Ermon. Generative adversarial imitation learning.

IRL as Adversarial Optimization

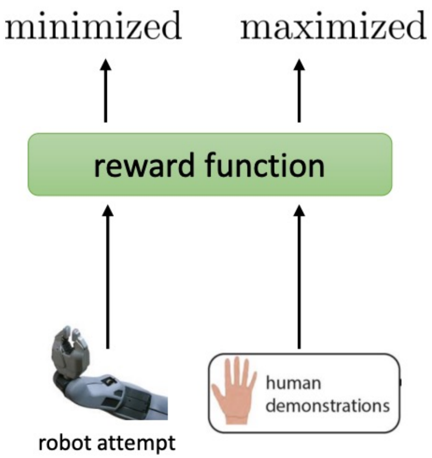
Guided Cost Learning

Finn et al., ICML 2016

IRL as Adversarial Optimization

Guided Cost Learning

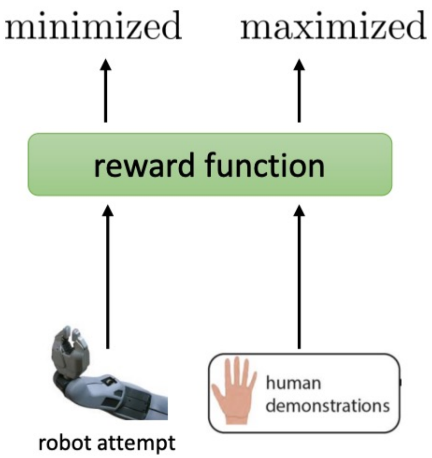
Finn et al., ICML 2016



IRL as Adversarial Optimization

Guided Cost Learning

Finn et al., ICML 2016



learns distribution $p(\tau)$ such that demos have max likelihood $p(\tau) \propto \exp(r(\tau))$ (MaxEnt model)

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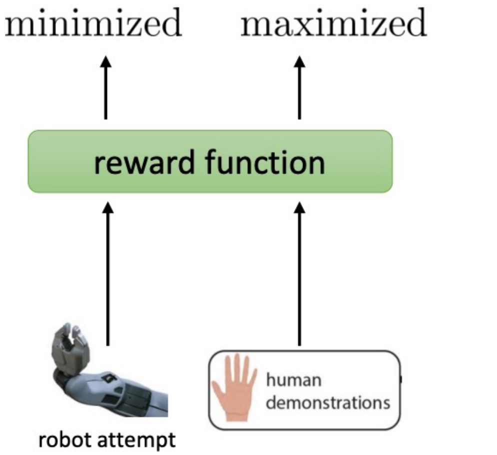

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Generative Adversarial Imitation Learning

Ho & Ermon, NIPS 2016

IRL as Adversarial Optimization

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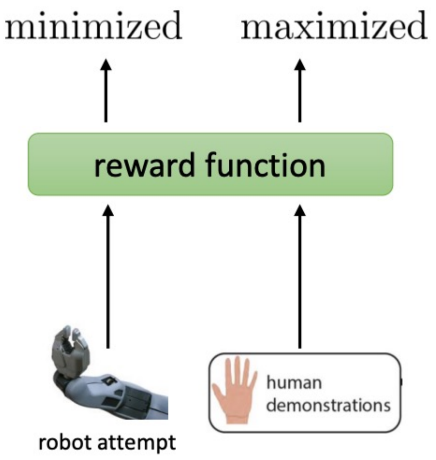

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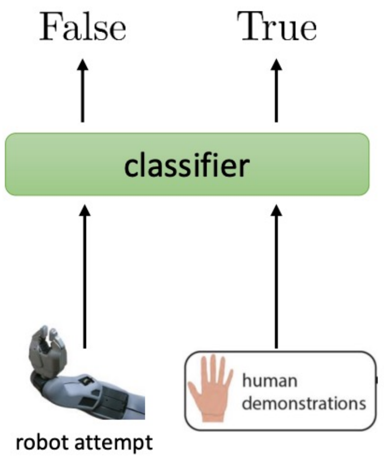

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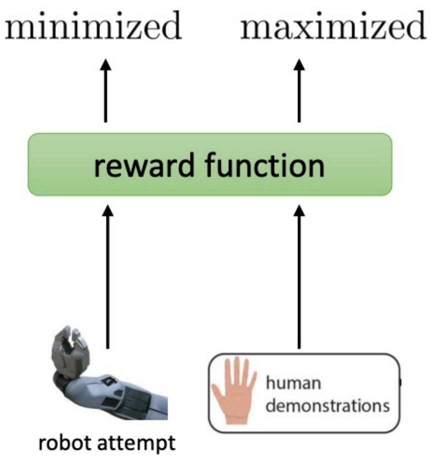

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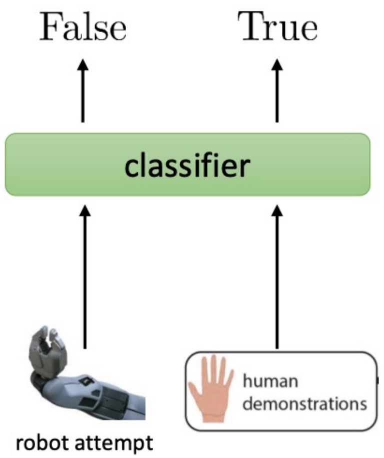

$D(\tau)$ = probability τ is a demo

use $\log D(\tau)$ as "reward"

IRL as Adversarial Optimization

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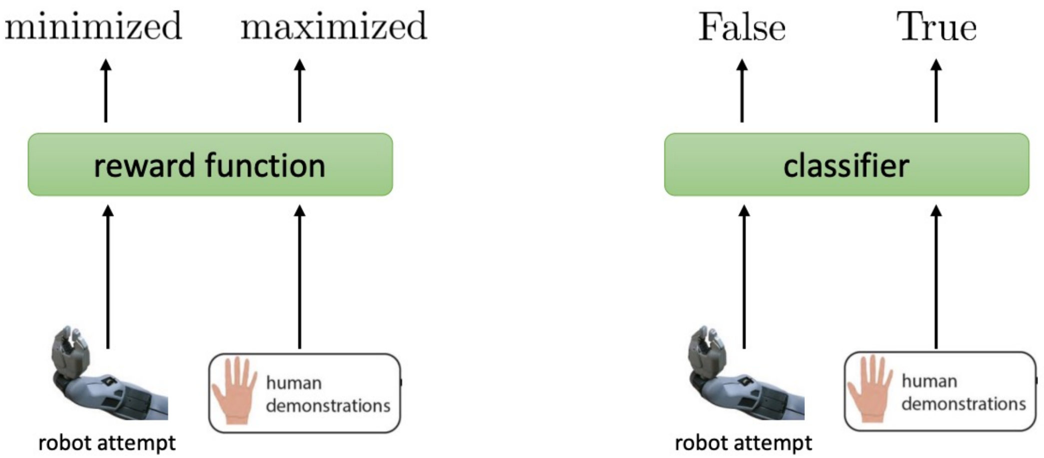

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actually the same thing!

IRL as Adversarial Optimization

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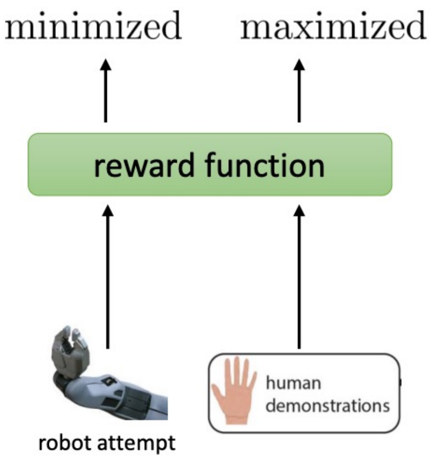


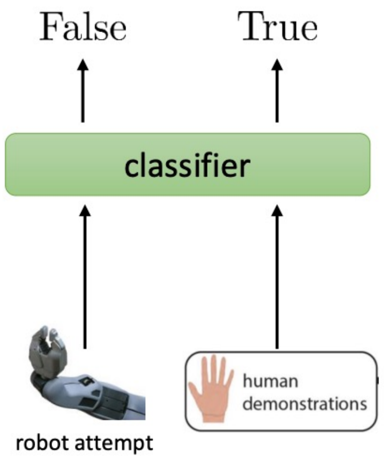
learns distribution $p(\tau)$ such that demos have max likelihood

$p(\tau) \propto \exp(r(\tau))$ (MaxEnt model)

$$D(\tau) = \frac{\frac{1}{Z}\exp(r(\tau))}{\frac{1}{Z}\exp(r(\tau)) + \pi(\tau)}$$

Generative Adversarial Imitation Learning

Ho & Ermon, NIPS 2016



$D(\tau) =$ probability τ is a demo

use $\log D(\tau)$ as "reward"

actually the same thing!

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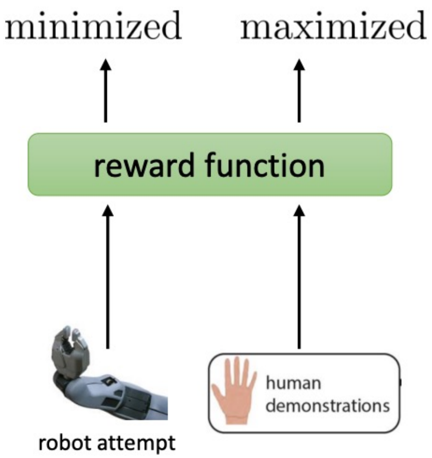

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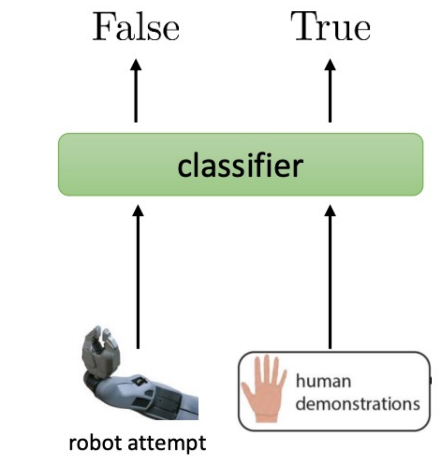
Generative Adversarial Imitation Learning

Ho & Ermon, NIPS 2016



$D(\tau) =$ probability τ is a demo

use $\log D(\tau)$ as "reward"

$D(\tau) =$ some classifier

actually the same thing!

Suggested Reading on Inverse RL

Classic Papers:
Abbeel & Ng ICML '04. *Apprenticeship Learning via Inverse Reinforcement Learning*. Good introduction to inverse reinforcement learning
Ziebart et al. AAAI '08. *Maximum Entropy Inverse Reinforcement Learning*. Introduction to probabilistic method for inverse reinforcement learning

Modern Papers:
Finn et al. ICML '16. *Guided Cost Learning*. Sampling based method for MaxEnt IRL that handles unknown dynamics and deep reward functions
Wulfmeier et al. arXiv '16. *Deep Maximum Entropy Inverse Reinforcement Learning*. MaxEnt inverse RL using deep reward functions
Ho & Ermon NIPS '16. *Generative Adversarial Imitation Learning*. Inverse RL method using generative adversarial networks
Fu, Luo, Levine ICLR '18. Learning Robust Rewards with Adversarial Inverse Reinforcement Learning