Statistical Inference

Foundations for Inference

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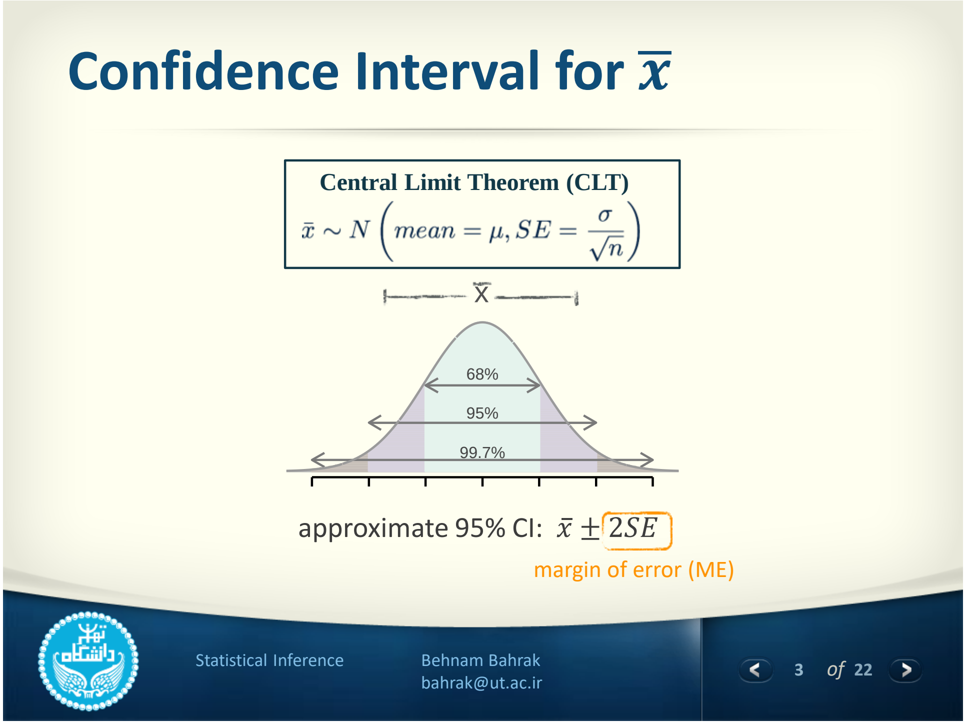
Confidence Interval

- A plausible range of values for the population parameter is called a confidence interval.

- If we report a point estimate, we probably won’t hit the exact population parameter.
- If we report a range of plausible values we have a good shot at capturing the parameter.



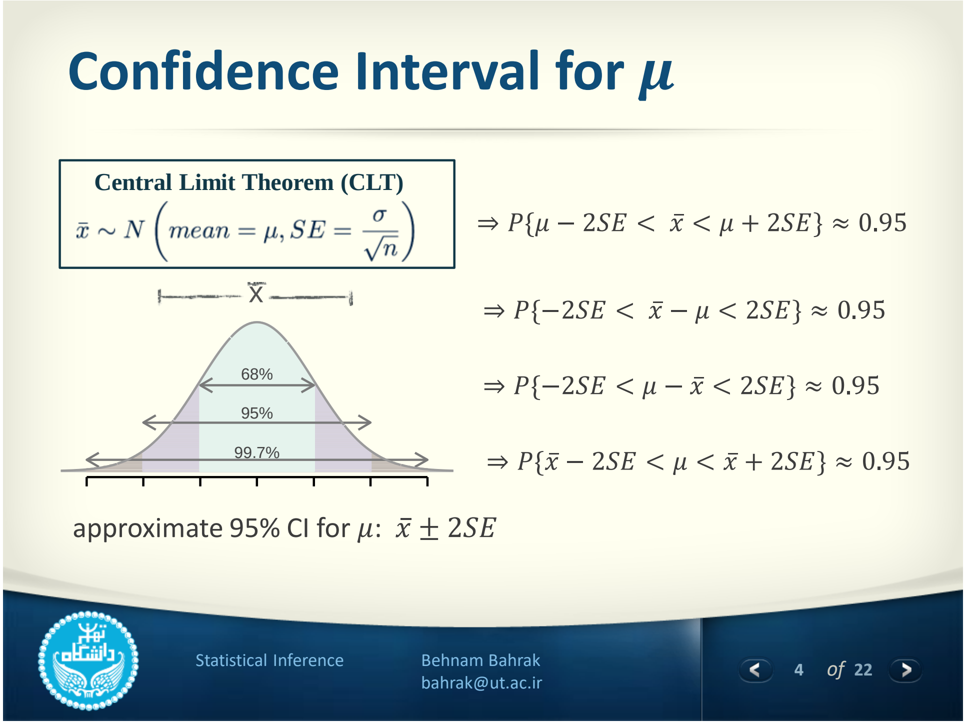
Confidence Interval for $\overline{x}$

Central Limit Theorem (CLT)

$$\bar{x} \sim N\left(mean = \mu, SE = \frac{\sigma}{\sqrt{n}}\right)$$

approximate 95% CI: $\bar{x} \pm \boxed{2SE}$

margin of error (ME)



Confidence Interval for μ

Central Limit Theorem (CLT)

$$\bar{x} \sim N\left(mean = \mu, SE = \frac{\sigma}{\sqrt{n}}\right)$$

$$\Rightarrow P\{\mu - 2SE < \bar{x} < \mu + 2SE\} \approx 0.95$$

$$\Rightarrow P\{-2SE < \bar{x} - \mu < 2SE\} \approx 0.95$$

$$\Rightarrow P\{-2SE < \mu - \bar{x} < 2SE\} \approx 0.95$$

$$\Rightarrow P\{\bar{x} - 2SE < \mu < \bar{x} + 2SE\} \approx 0.95$$

approximate 95% CI for μ: $\bar{x} \pm 2SE$

Question

- One of the earliest examples of behavioral asymmetry is a preference in humans for turning the head to the right, rather than to the left, during the first 6 months after birth. This is thought to influence subsequent development of perceptual and motor preferences. A study of 124 people found that 64.5% turned their heads to the right when someone is calling them. The standard error associated with this estimate is roughly 4%. Which of the below is **false**?

✓ (a) A higher sample size would yield a lower standard error.

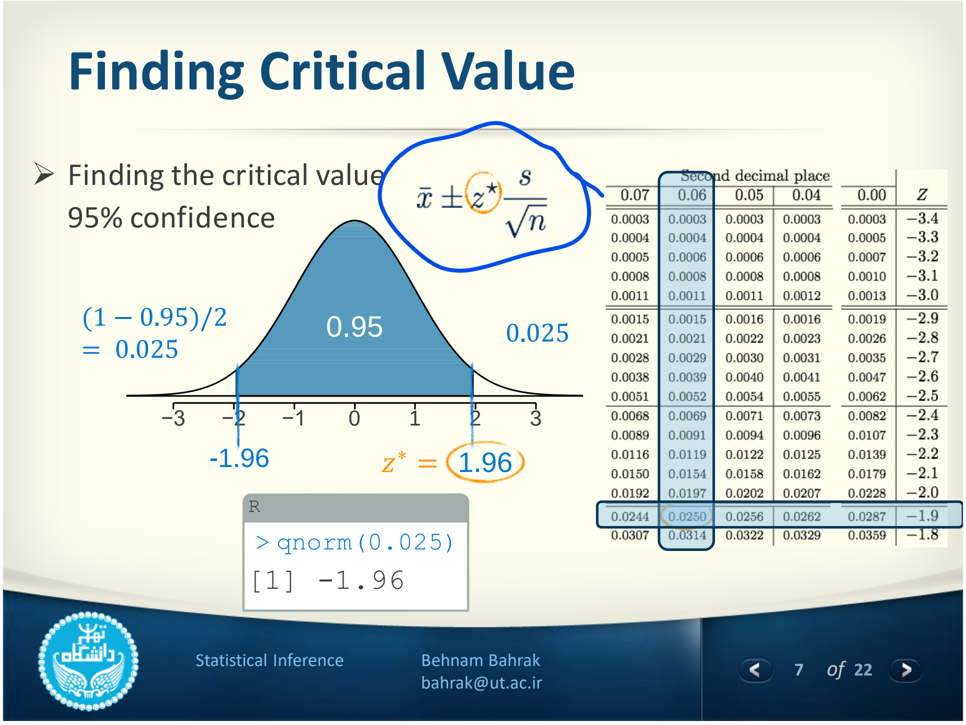
✓ (b) The margin of error for a 95% CI for the percentage of people who turn their heads to the right is roughly 8%.

[✓ (c)] The 95% CI for the percentage of people who turn their heads to the right is roughly 64.5% ± 4%.

✓ (d) The 99.7% CI for the percentage of people who turn their heads to the right is roughly 64.5% ± 12%.

Confidence Interval for a Population Mean

- Confidence interval for a population mean: Computed as the sample mean plus/minus a margin of error (critical value corresponding to the middle XX% of the normal distribution times the standard error of the sampling distribution): $\bar{x} \pm z^{\star} \frac{s}{\sqrt{n}}$
- **Conditions for this confidence interval:**
  1. **Independence*:*** Sampled observations must be independent.
     - random sample/assignment
     - if sampling without replacement, $n < 10\%$ of population
  2. **Sample size/skew*:*** $n \geq 30$, larger if the population distribution is very skewed.



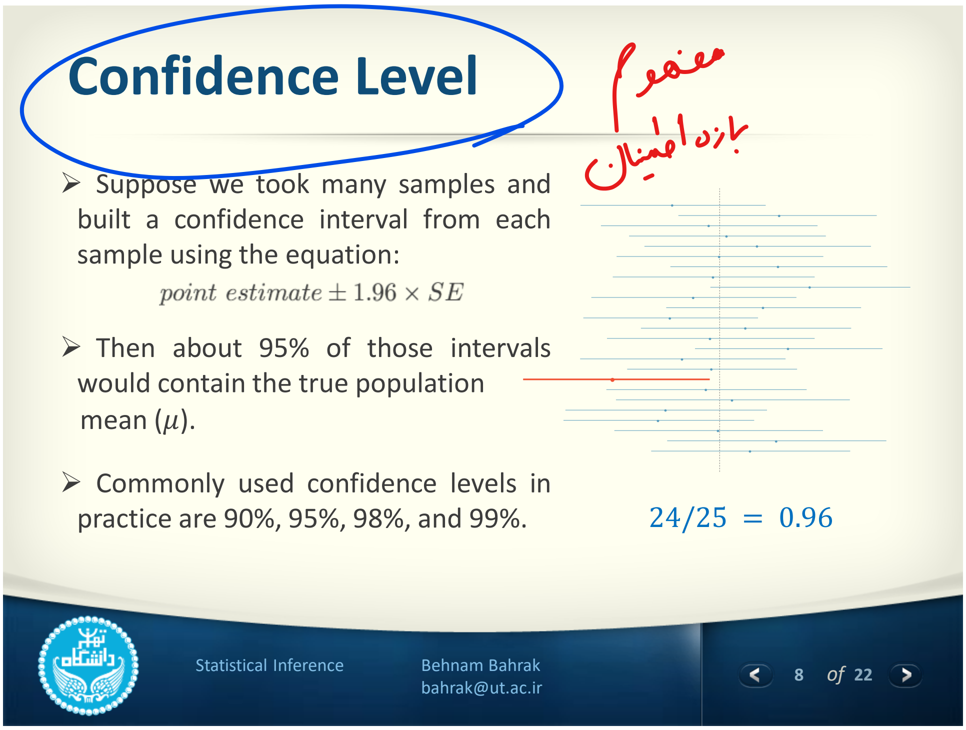
Finding Critical Value

- Finding the critical value 95% confidence

$$\bar{x} \pm z^{\star} \frac{s}{\sqrt{n}}$$

```
R
> qnorm(0.025)
[1] -1.96
```

	Second decimal place				
0.07	0.06	0.05	0.04	0.00	Z
0.0003	0.0003	0.0003	0.0003	0.0003	−3.4
0.0004	0.0004	0.0004	0.0004	0.0005	−3.3
0.0005	0.0006	0.0006	0.0006	0.0007	−3.2
0.0008	0.0008	0.0008	0.0008	0.0010	−3.1
0.0011	0.0011	0.0011	0.0012	0.0013	−3.0
0.0015	0.0015	0.0016	0.0016	0.0019	−2.9
0.0021	0.0021	0.0022	0.0023	0.0026	−2.8
0.0028	0.0029	0.0030	0.0031	0.0035	−2.7
0.0038	0.0039	0.0040	0.0041	0.0047	−2.6
0.0051	0.0052	0.0054	0.0055	0.0062	−2.5
0.0068	0.0069	0.0071	0.0073	0.0082	−2.4
0.0089	0.0091	0.0094	0.0096	0.0107	−2.3
0.0116	0.0119	0.0122	0.0125	0.0139	−2.2
0.0150	0.0154	0.0158	0.0162	0.0179	−2.1
0.0192	0.0197	0.0202	0.0207	0.0228	−2.0
0.0244	0.0250	0.0256	0.0262	0.0287	−1.9
0.0307	0.0314	0.0322	0.0329	0.0359	−1.8



Confidence Level

مفهوم
بازه اطمینان

- Suppose we took many samples and built a confidence interval from each sample using the equation:

$$point\ estimate \pm 1.96 \times SE$$

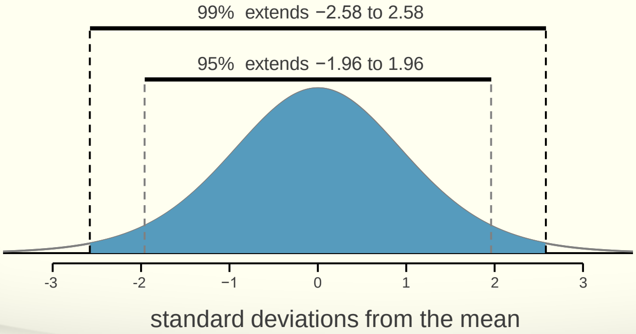
- Then about 95% of those intervals would contain the true population mean (μ).

- Commonly used confidence levels in practice are 90%, 95%, 98%, and 99%.

$24/25 = 0.96$

Width of Confidence Interval

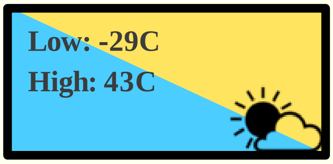
- If we want to be very certain that we capture the population parameter, should we use a wider interval or a narrower interval?



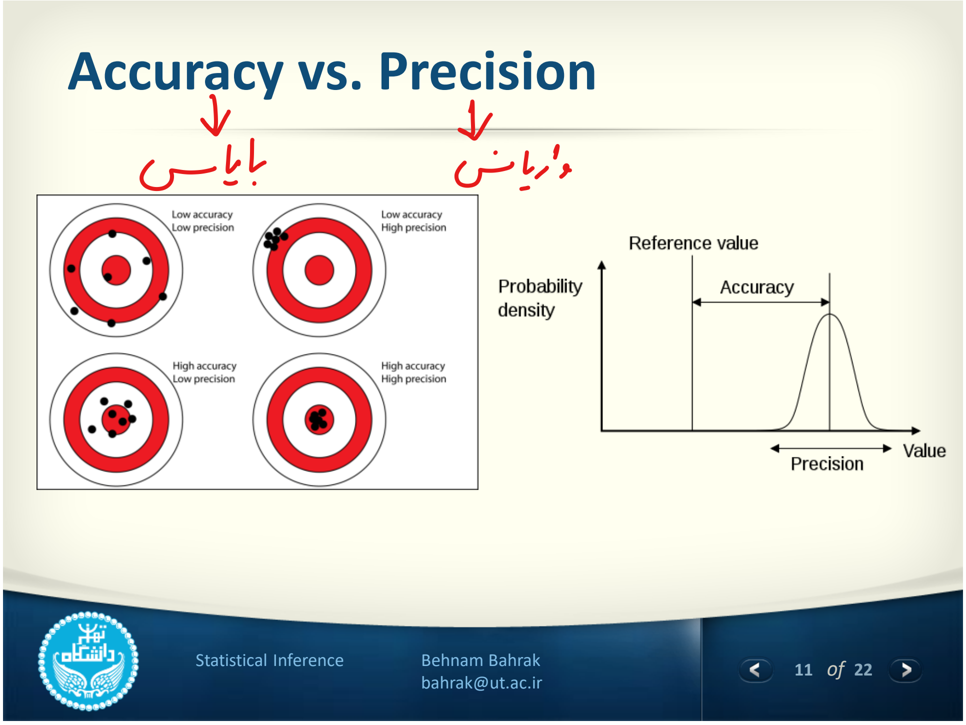
CL ↑ width ↑

Why not a wider interval?

- What drawbacks are associated with using a wider interval?



- Is this accurate? Most likely, yes.
- Is it informative, or, in other words, is it precise? Not really.



Accuracy vs. Precision

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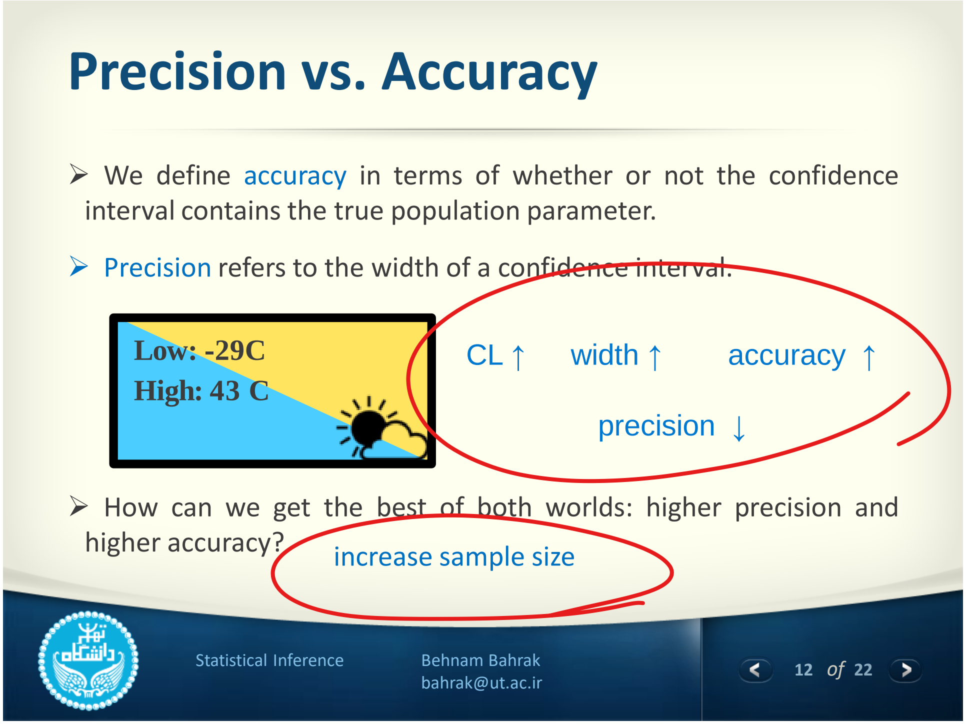


Precision vs. Accuracy

- We define accuracy in terms of whether or not the confidence interval contains the true population parameter.
- Precision refers to the width of a confidence interval.

CL ↑ width ↑ accuracy ↑

precision ↓

- How can we get the best of both worlds: higher precision and higher accuracy?

increase sample size

از نمونه‌های تصادفی با این سایز خاص ۹۵ درصد اون‌ها بازه اطمینانی می‌سازن که میانگین واقعی توش می‌افته

Question

با احتمال 95% توی بازه می‌افته میانگین واقعی جامعه

In 2010, a survey collected responses from 1,154 US residents. Based on the survey results, a 95% confidence interval for the average number of hours Americans have to relax or pursue activities that they enjoy after an average work day was found to be 3.53 to 3.83 hours. Determine if each of the following statements are true or false.

(a) 95% of Americans spend 3.53 to 3.83 hours relaxing after a work day. **F**

(b) 95% of random samples of 1,154 Americans will yield confidence intervals that contain the true average number of hours Americans spend relaxing after a work day. **T**

(c) 95% of the time the true average number of hours Americans spend relaxing after a work day is between 3.53 and 3.83 hours. **F**

(d) We are 95% confident that Americans in this sample spend on average 3.53 to 3.83 hours relaxing after a work day. **F**

Required Sample Size

- Given a target margin of error, confidence level, and information on the variability of the sample (or the population), we can determine the required sample size to achieve the desired margin of error.

$$ME = z^{\star}\frac{s}{\sqrt{n}} \rightarrow n = \left(\frac{z^{\star}s}{ME}\right)^2$$

Example

- A group of researchers want to estimate the IQ scores of three-year-old children born to mothers who were on a specific medication during pregnancy.
- Previous studies suggest that the SD of IQ scores of three-year-old children is 18 points.
- How many such children should the researchers sample in order to obtain a 90% confidence interval with a margin of error less than or equal to 4 points?

$$\left.\begin{aligned} & ME \leq 4\ pts \\ & CL = 90\% \;\; \Rightarrow \frac{1-0.9}{2} = 0.05 \;\; \Rightarrow \;\; z^* = 1.65 \\ & \sigma = 18 \end{aligned}\right\} \Rightarrow n = \left(\frac{1.65 \times 18}{4}\right)^2 = 55.13$$

- We need at least 56 such children in the sample obtain a maximum margin of error of 4 points.

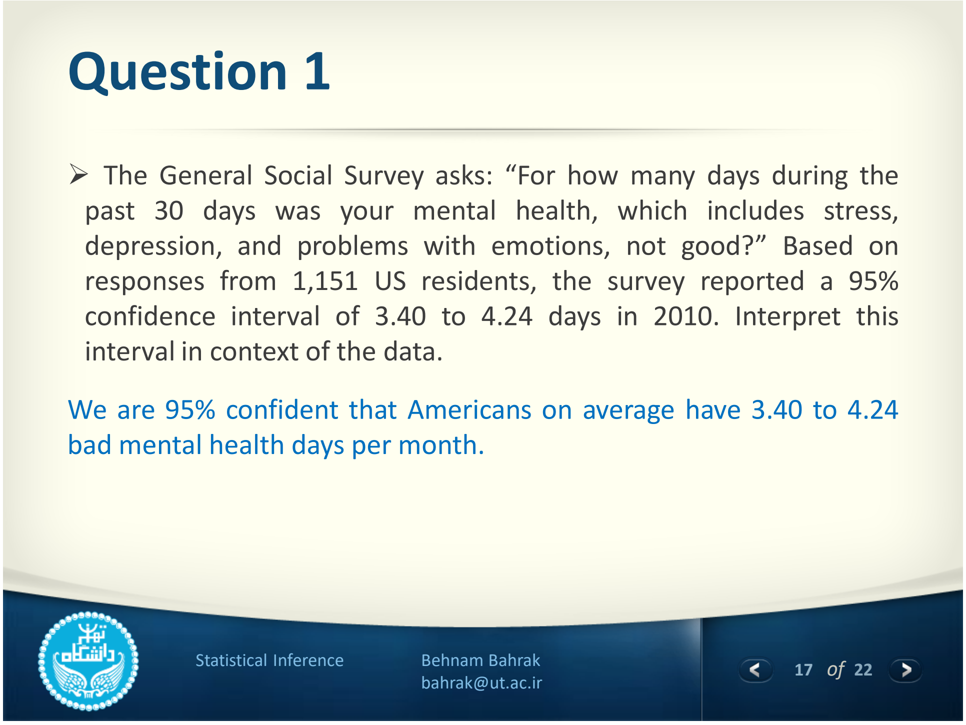
Example

- We found that we needed at least 56 children in the sample to achieve a maximum margin of error of 4 points. How would the required sample size change if we want to further decrease the margin of error to 2 points?

$$\frac{1}{2} ME = \frac{1}{2} z^* \frac{s}{\sqrt{n}}$$

$$\frac{1}{2} ME = z^* \frac{s}{\sqrt{4n}}$$

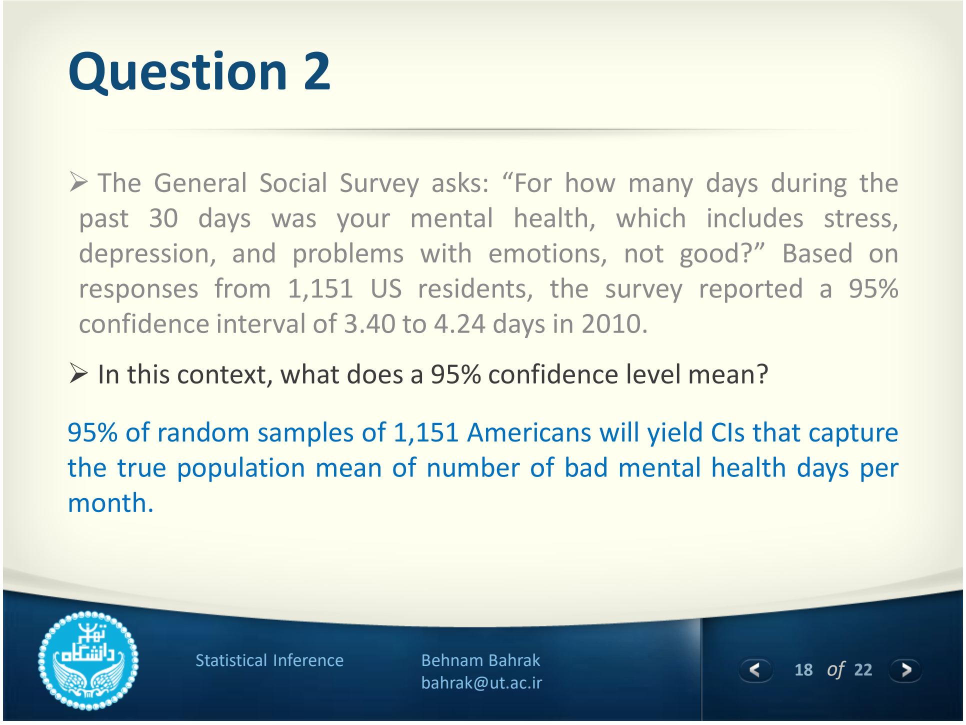
$$4n = 56 \times 4 = 224$$



Question 1

- The General Social Survey asks: "For how many days during the past 30 days was your mental health, which includes stress, depression, and problems with emotions, not good?" Based on responses from 1,151 US residents, the survey reported a 95% confidence interval of 3.40 to 4.24 days in 2010. Interpret this interval in context of the data.

We are 95% confident that Americans on average have 3.40 to 4.24 bad mental health days per month.



Question 2

- The General Social Survey asks: "For how many days during the past 30 days was your mental health, which includes stress, depression, and problems with emotions, not good?" Based on responses from 1,151 US residents, the survey reported a 95% confidence interval of 3.40 to 4.24 days in 2010.
- In this context, what does a 95% confidence level mean?

95% of random samples of 1,151 Americans will yield CIs that capture the true population mean of number of bad mental health days per month.

Question 3

- The General Social Survey asks: “For how many days during the past 30 days was your mental health, which includes stress, depression, and problems with emotions, not good?” Based on responses from 1,151 US residents, the survey reported a 95% confidence interval of 3.40 to 4.24 days in 2010.
- Suppose the researchers think a 99% confidence level would be more appropriate for this interval. Will this new interval be narrower or wider than the 95% confidence interval?

As CL increases so does the width of the confidence interval, so wider.

Example

- A sample of 50 college students were asked how much money they spend on textbooks each semester. The students in the sample paid an average of \$320 for textbooks, with a standard deviation of \$174. In addition, the sample distribution was only slightly skewed to the right. Estimate the true average textbooks cost for college students based on this sample using a 95% confidence interval.

Checking conditions:

1. random sample & 50 < 10% of all college students

We can assume that the textbooks one student in the sample has bought is independent of another.

Example

2. $n > 30$ & not so skewed sample

We can assume that the sampling distribution of average textbooks cost from samples of size 50 will be nearly normal.

$$n = 50 \ , \qquad \bar{x} = 320 \ , \qquad s = 174$$

$$SE = \frac{s}{\sqrt{n}} = \frac{174}{\sqrt{50}} \approx 24.6$$

$$\bar{x} \pm z^{*}SE = 320 \pm 1.96 \times 24.6 = (272\,, 368)$$

We are 95% confident that college students on average have paid \$272 to \$368 for textbooks.

Bar Plot with Confidence Interval