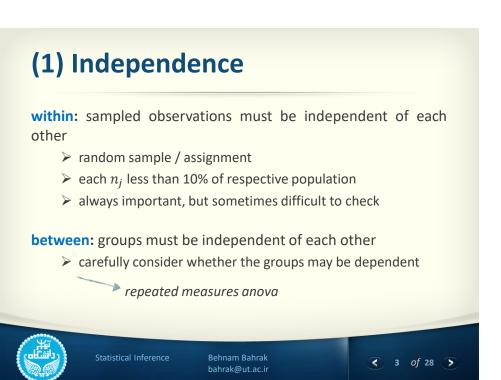
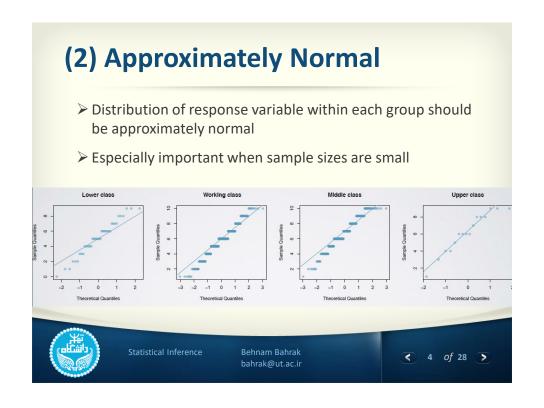
Statistical Inference Inference for Numerical Variables Behnam Bahrak Spring 2020 1 of 28

Conditions for ANOVA

- 1. Independence:
 - √ within groups: sampled observations must be independent
 - √ between groups: the groups must be independent of each
 other (non-paired)
- 2. Approximate normality: distributions should be nearly normal within each group
- 3. Equal variance: groups should have roughly equal variability

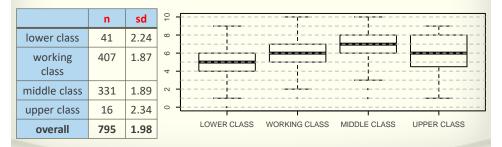






(3) Constant Variance

- Variability should be consistent across groups: homoscedastic groups
- Especially important when sample sizes differ between groups





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Misconducting Statistical Tests

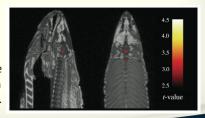
Neural Correlates of Interspecies Perspective Taking in the Post-Mortem Atlantic Salmon: An Argument For Proper Multiple Comparisons Correction

Craig M. Bennett 1* , Abigail A. Baird 2 , Michael B. Miller 1 and George L. Wolford 3

¹Department of Psychology, University of California at Santa Barbara, Santa Barbara, CA 93106 ²Department of Psychology, Blodgett Hall, Vassar College, Poughkeepsie, NY 12604

³Department of Psychological and Brain Sciences, Moore Hall, Dartmouth College, Hanover, NH

- The salmon was shown the same social perspective taking task that was later administered to a group of human subjects.
- > Statistics that were uncorrected for multiple comparisons showed active voxel clusters in the salmon's brain cavity and spinal column.

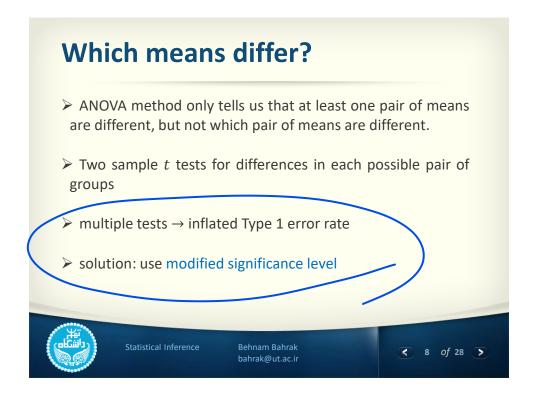




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Multiple Comparisons

- > Testing many pairs of groups is called multiple comparisons
- The Bonferroni correction suggests that a more stringent significance level is more appropriate for these tests
 - \triangleright Adjust α by the number of comparisons being considered:

$$\alpha^* = \alpha/K$$

 $\triangleright K$: number of comparisons

$$K = k(k-1)/2$$



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Example

 \triangleright The social class variables has 4 levels. If $\alpha = 0.05$ for the original ANOVA, what should the modified significance level be for two sample t tests for determining which pairs of groups have significantly different means?

$$k = 4$$

$$K = \frac{4 \times 3}{2} = 6$$

$$a^* = \frac{0.05}{6} = 0.0083$$



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Pairwise Comparisons

- ➤ Constant variance → re-think standard error and degrees of freedom:
 - use consistent standard error and degrees of freedom for all pairwise comparison tests
- Compare the p-values from each test to the modified significance level α^*



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Pairwise Comparisons

> Standard error for multiple pairwise comparisons:

$$SE = \sqrt{\frac{MSE}{n_1} + \frac{MSE}{n_2}}$$

> Independent groups test:

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

> Degrees of freedom for multiple pairwise comparisons:

$$df = df_E = n - k$$

$$df = \min(n_1 - 1, n_2 - 1)$$



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Example

> Is there a difference between the average vocabulary scores between middle and lower class Americans?

	Df	Sum Sq	Mean Sq	F value	Pr (>F)
Class	3	236.56	78.855	21.735	<0.0001
Residuals	791	2869.80	3.628		
Total	794	3106.36			

	n	mean
lower class	41	5.07
middle class	331	6.76

$$H_0: \mu_{\text{middle}} - \mu_{\text{lower}} = 0$$
 , $H_A: \mu_{\text{middle}} - \mu_{\text{lower}} \neq 0$

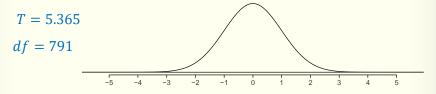
$$T = \frac{(\bar{x}_{\text{middle}} - \bar{x}_{\text{lower}}) - 0}{\sqrt{\frac{MSE}{n_{\text{middle}}}} + \frac{MSE}{n_{\text{lower}}}} = \frac{6.76 - 5.07}{\sqrt{\frac{3.628}{331} + \frac{3.628}{41}}} = \frac{1.69}{0.315} = 5.365$$



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Example



$$\begin{array}{l} \alpha^* = 0.0083 \\ p-value \, < \, \alpha^* \, \rightarrow \, \mathrm{Reject} \, H_0 \end{array}$$

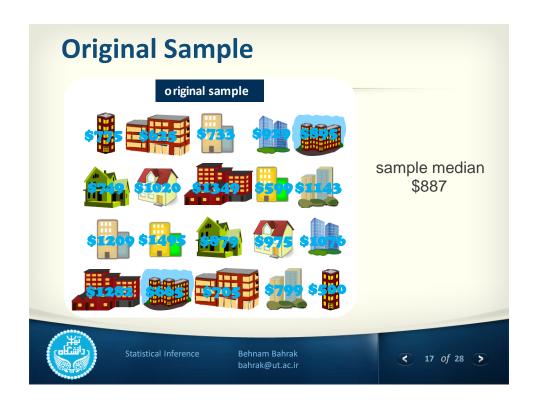


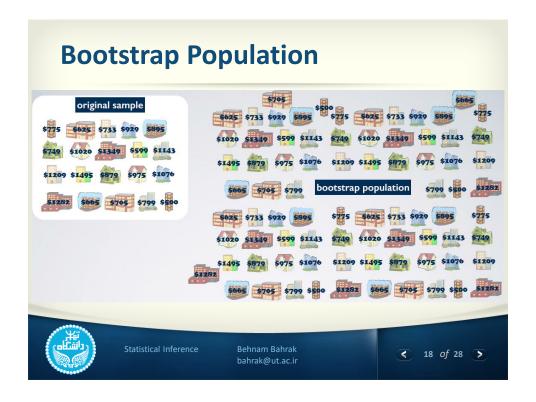
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Bootstrapping > Twenty 1+ bedroom apartments Rent in Durham, NC were randomly selected in Durham, NC. Is the mean or the median a better measure of typical rent in Durham? > Can we apply CLT based methods we have learned so far 600 800 1000 1200 to construct confidence intervals for both? Rent Statistical Inference Behnam Bahrak 15 of 28



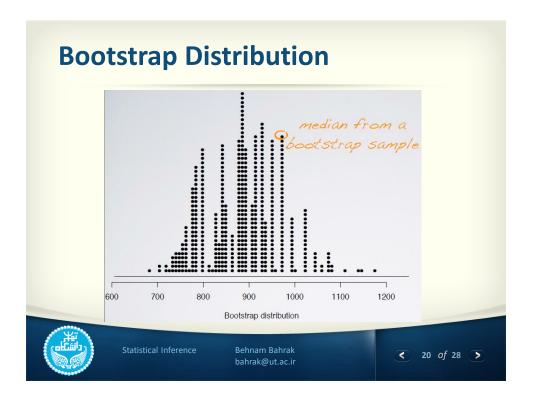




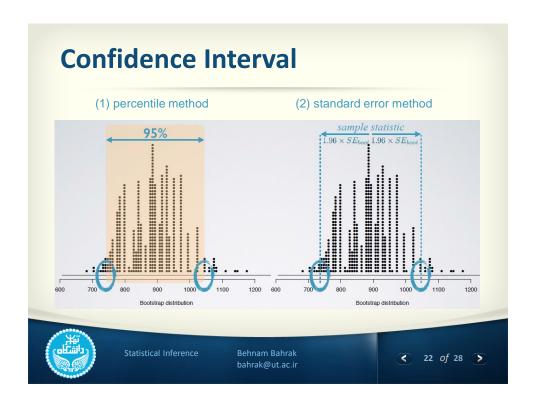
Bootstrapping Scheme

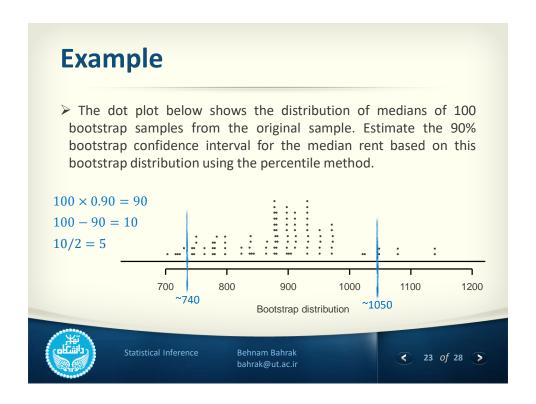
- (1) Take a bootstrap sample a random sample taken **with replacement** from the original sample, of the same size as the original sample
- (2) Calculate the bootstrap statistic a statistic such as mean, median, proportion, etc. computed on the bootstrap samples
- (3) Repeat steps (1) and (2) many times to create a bootstrap distribution a distribution of bootstrap statistics

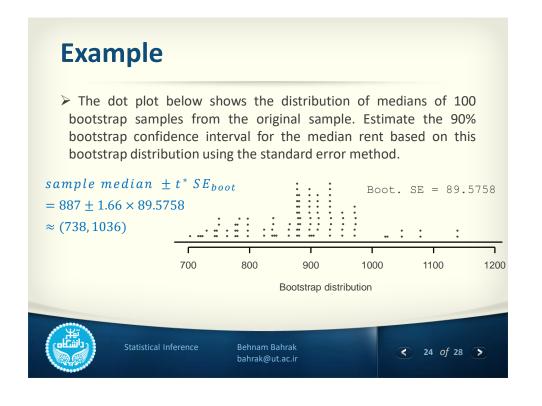


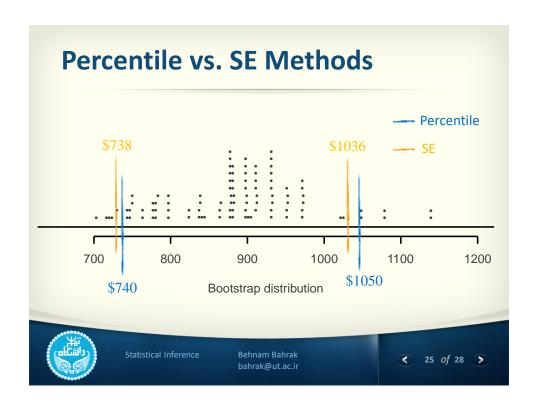


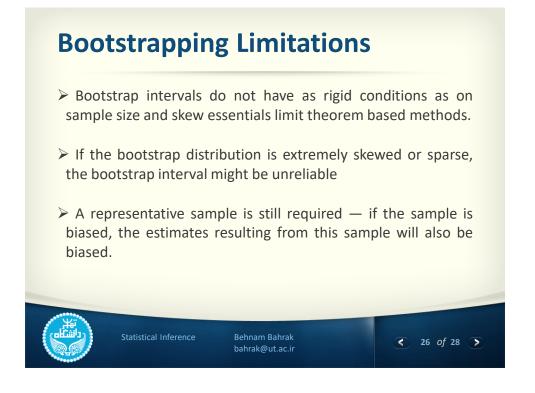
Confidence Interval Percentile Method: Simply as the middle 95% of the bootstrap distribution. So the bounds of the interval are the 2.5th and the 97.5th percentiles of the bootstrap distribution. Standard error method: We calculate the interval as the sample statistic plus or minus t* times the standard error of the bootstrap distribution The critical T-score (t*) will have (n − 1) as its degrees of freedom, where n is the number of bootstrap samples.











Bootstrap vs. Sampling Distribution

- > Sampling distribution created using sampling (with replacement) from the population
- > Bootstrap distribution created using sampling (with replacement) from the sample
- > Both are distributions of sample statistics



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When to use bootstrapping?

- ➤ When **to** use bootstrapping?
 - ➤ When the theoretical distribution of a statistic of interest is complicated or unknown.
 - ➤ When the sample size is insufficient for straightforward statistical inference.
 - ➤ When sample size calculations have to be performed.
- > When **not to** use bootstrapping?
 - when the underlying population lacks a finite variance (e.g. a power law or Cauchy distribution), then the bootstrap distribution will not converge to the same limit as the sample statistic.
 - Unless one is reasonably sure that the underlying distribution is not heavy tailed, one should hesitate to use the naive bootstrap.



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bahrak@ut.ac.ir

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