Statistical Inference

Inference for Numerical Variables

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Study 1

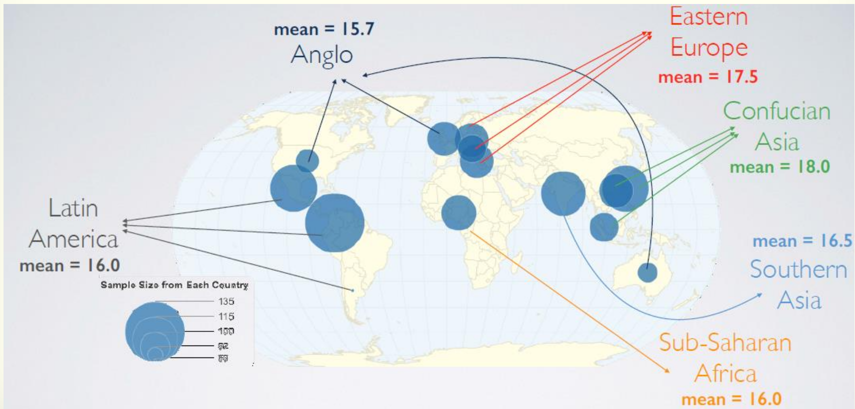
ACCEPTABILITY OF WORKPLACE BULLYING

culture and acceptability of workplace bullying across the globe

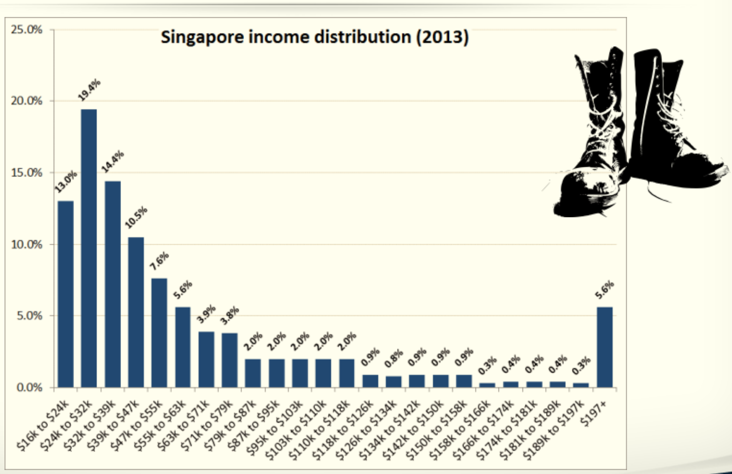
sample: 1484 alumni and current MBA students from 14 countries on six continents

study design: questionnaire on acceptability of work related bullying (giving tasks with unreasonable deadlines, exposing workers to an unreasonable workload)

Study 1



Study 2



Review

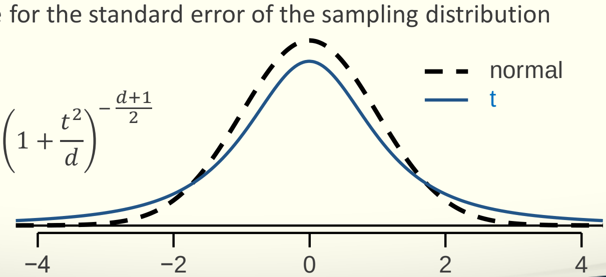
What purpose does a large sample serve?

- As long as observations are independent, and the population distribution is not extremely skewed, a large sample would ensure that...
  - the sampling distribution of the mean is nearly normal
  - the estimate of the standard error is reliable: $\frac{s}{\sqrt{n}}$

Student's t-distribution

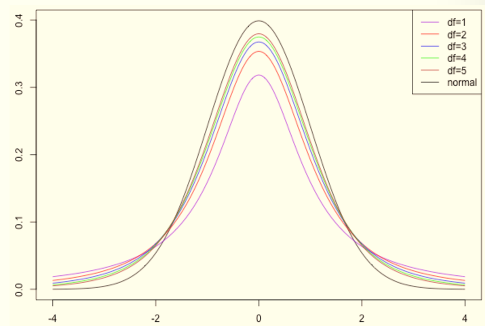
- When σ is unknown (which is almost always), use the t-distribution to address the uncertainty of the standard error estimate
- Bell shaped but thicker tails than the normal
  - Observations more likely to fall beyond 2 SDs from the mean
  - Extra thick tails helpful for mitigating the effect of a less reliable estimate for the standard error of the sampling distribution

$$f(t) = \frac{\Gamma\left(\frac{d+1}{2}\right)}{\sqrt{\pi d}\,\Gamma\left(\frac{d}{2}\right)}\left(1+\frac{t^2}{d}\right)^{-\frac{d+1}{2}}$$



Student's t-distribution

- Always centered at 0 (like the standard normal)
- Has one parameter: degrees of freedom (df) – which determines thickness of tails
- Remember the normal distribution has two parameters: mean and SD



- What happens to the shape of the t-distribution as degrees of freedom increases?

t-statistic

- t-statistic is used for inference on a mean where:
  - σ unknown, which is almost always
- It is calculated the same way as z-statistic:

$$T = \frac{observation - null}{SE}$$

- p-value has also the same definition:
  - one or two tail area, based on H_A
  - using R, or table

Example

- Find the following probabilities:

a. $P(|Z| > 2)$ 0.0455 → reject

b. $P(|t_{df=50}| > 2)$ 0.0509 → fail to reject?

c. $P(|t_{df=10}| > 2)$ 0.0734 → fail to reject

```
R
> pnorm(2, lower.tail = FALSE) * 2
[1] 0.0455
> pt(2, df = 50, lower.tail = FALSE) * 2
[1] 0.0509
```

- Suppose you have a two sided hypothesis test, and your test statistic is 2. Under which of these scenarios would you be able to reject the null hypothesis at the 5% significance level?

Student's *t*-Distribution

- This name comes from a pseudonym **Student**, used by William Gosset, in papers where he developed much of the foundation for this distribution.
- This required sometimes working with small samples because maybe he would just have a few batches to try.
- Since his company was worried about their trade secrets getting out, Gosset was asked to publish any work that he's doing under a pseudonym and **Student** was a name that he chose for himself.

William Gosset (1876 - 1937)

Experiment

PLAYING A COMPUTER GAME DURING LUNCH AFFECTS FULLNESS, MEMORY FOR LUNCH, AND LATER SNACK INTAKE

distraction and recall of food consumed and snacking

- **Sample:** 44 patients: 22 men and 22 women
- **Study design:**
  - randomized into two groups:
    (1) play solitaire while eating – "win as many games as possible"
    (2) eat lunch without distractions
  - both groups provided same amount of lunch
  - offered biscuits to snack on after lunch

Biscuit intake	$\bar{x}$	s	n
solitaire	52.1g	45.1g	22
No distraction	27.1g	26.4g	22

Estimating the Mean

- point estimate ± margin of error

$$\bar{x} \pm t^{\star}_{df} SE_{\bar{x}}$$

$$\bar{x} \pm t^{\star}_{df} \frac{s}{\sqrt{n}}$$

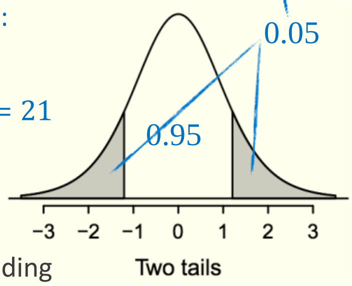
$$\bar{x} \pm t^{\star}_{n-1} \frac{s}{\sqrt{n}}$$

- Degrees of freedom for t statistic for inference on one sample mean: $df = n - 1$

Finding the Critical t-score

a) Using the table:

1. determine df:
   $df = 22 - 1 = 21$



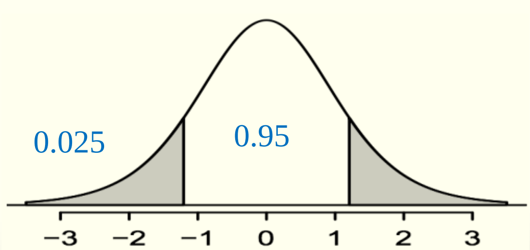
2. Find corresponding tail area for desired confidence level

one tail		0.100	0.050	0.025	0.010	0.005
two tails		0.200	0.100	0.050	0.020	0.010
df	1	3.08	6.31	12.71	31.82	63.66
	2	1.89	2.92	4.30	6.96	9.92
	3	1.64	2.35	3.18	4.54	5.84
	4	1.53	2.13	2.78	3.75	4.60
	5	1.48	2.02	2.57	3.36	4.03
	6	1.44	1.94	2.45	3.14	3.71
	7	1.41	1.89	2.36	3.00	3.50
	8	1.40	1.86	2.31	2.90	3.36
	9	1.38	1.83	2.26	2.82	3.25
	10	1.37	1.81	2.23	2.76	3.17
	11	1.36	1.80	2.20	2.72	3.11
	12	1.36	1.78	2.18	2.68	3.05
	13	1.35	1.77	2.16	2.65	3.01
	14	1.35	1.76	2.14	2.62	2.98
	15	1.34	1.75	2.13	2.60	2.95
	16	1.34	1.75	2.12	2.58	2.92
	17	1.33	1.74	2.11	2.57	2.90
	18	1.33	1.73	2.10	2.55	2.88
	19	1.33	1.73	2.09	2.54	2.86
	20	1.33	1.72	2.09	2.53	2.85
	21	1.32	1.72	2.08	2.52	2.83
	22	1.32	1.72	2.07	2.51	2.82
	23	1.32	1.71	2.07	2.50	2.81
	24	1.32	1.71	2.06	2.49	2.80
	25	1.32	1.71	2.06	2.49	2.79
	26	1.31	1.71	2.06	2.48	2.78
	27	1.31	1.70	2.05	2.47	2.77

Finding the Critical t-score

- Using R:

```
R
> qt(0.025, df = 21)
[1] -2.079614
```



Example

- Estimate the average after-lunch snack consumption (in grams) of people who eat lunch **distracted** using a 95% confidence interval.

$\bar{x} = 52.1\ g$

$s = 45.1\ g$

$n = 22$

$t^\star_{21} = 2.08$

$$\bar{x} \pm t^{*}SE = 52.1 \pm 2.08 \times \frac{45.1}{\sqrt{22}}$$
$$= 52.1 \pm 2.08 \times 9.62$$
$$= 52.1 \pm 20 = (32.1, 72.1)$$

- We are 95% confident that distracted eaters consume between 32.1 to 72.1 grams of snacks post-meal.

Example

- Suppose the suggested serving size of these biscuits is 30 g. Do these data provide convincing evidence that the amount of snacks consumed by distracted eaters post-lunch is different than the suggested serving size?

$\bar{x} = 52.1\ g$

$s = 45.1\ g$

$n = 22$

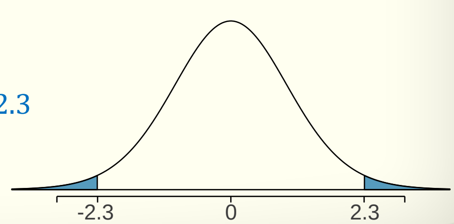
$SE = 9.62$

$H_0: \mu = 30$

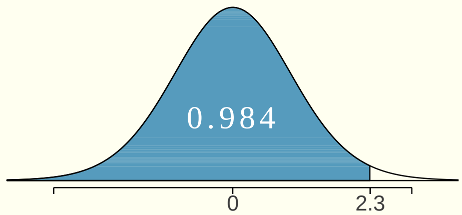
$H_A: \mu \neq 30$

$$T = \frac{52.1 - 30}{9.62} = 2.3$$

$df = 22 - 1 = 21$



Finding the p-value using R



R
```
> pt(2.3, df = 21)
[1] 0.984
> 2 * pt(2.3, df = 21, lower.tail = FALSE)
[1] 0.03180228
```


Finding the p-value using the table

1. Determine df

2. Locate the calculated T score in the df row

$$df = 21$$
$$T = 2.3 \;\rightarrow\; 2.08 < T < 2.52$$

3. Grab the one or two tail p-value from the top row

$$0.02 < \text{p-value} < 0.05$$

one tail	0.100	0.050	0.025	0.010	0.005
two tails	0.200	0.100	0.050	0.020	0.010
df 1	3.08	6.31	12.71	31.82	63.66
2	1.89	2.92	4.30	6.96	9.92
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18	1.33	1.73	2.10	2.55	2.88
19	1.33	1.73	2.09	2.54	2.86
20	1.33	1.72	2.09	2.53	2.85
21	1.32	1.72	2.08	2.52	2.83
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Recap

$\bar{x} = 52.1\ g$

$s = 45.1\ g$

$n = 22$

95% confidence interval: (32.1 g, 72.1 g)

$H_0 : \mu = 30$

$H_A : \mu \neq 30$

agree

p-value ≈ 0.0318 Reject H_0

Conditions

- Independent observations
  - random assignment
  - 22 < 10% of all distracted eaters
- Sample size / skew

$\bar{x} = 52.1\ g$

$s = 45.1\ g$

$n = 22$