Improving Schools Through School Choice: A Market Design Approach

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Overview

1. Introduction
2. Literature Review
3. Conventional Mechanisms
   - Boston Mechanism
   - TTC Mechanism
4. Main Results
   - Improvement for School Quality
     - General Markets
     - Large Markets
5. Domain Restriction
6. Conclusion

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Motivation

- School choice is a policy that substantially improves educational outcomes.
- The main body of the school choice literature investigates how to assign school seats to students efficiently and fairly.
- Prior works consider the school quality as given and fixed.

This paper:

- Studies how the design of school choice mechanism affects competitive pressure on schools to improve.
- Introduces a criterion to evaluate conventional mechanisms in sense of incentives they provide for school improvement.

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Literature Review

- School choice literature has been started by the seminal paper of Abdulkadiroglu, Sonmez (2003).
- The concept of improvement in matchings has been first introduced in Balinski, Sonmez (1999).
- This paper uses the large market approach previously studied in Roth, Penarson (1999), and Immorlica, Mahdian (2005) among many others.
- This paper introduces domain restrictions on the class of preferences such that the desirable properties hold simultaneously. Domain restriction has been studied by many papers. Some of which are Ergin (2002), Kesten (2002), and Haeringer, Klijn (2009).

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Notations

- A finite set S of students each with strict preferences $\forall s \in S : \succ_s$.
- A finite set C of schools with responsive preferences $\forall c \in C : \succ_c$.
- Students' preferences are: $\succ_S \equiv (\succ_s)_{s \in S}$. And schools' preferences are: $\succ_C \equiv (\succ_c)_{c \in C}$.

The Boston Mechanism

The Boston Mechanism (ϕ^B) is defined by the following algorithm:

- Step 1: Each student $s \in S$ applies to her most preferred acceptable school (if any). Each school accepts its most-preferred students up to its quota and rejects every other student.
- Step $t \geq 2$: Each student who has not been accepted so far, applies to her most preferred school that has not rejected her (if any). Each school accepts its most-preferred students up to its remaining capacity and rejects every other student.

The algorithm terminates at the first step in which no student applies to a school.

* The Boston Mechanism is Pareto efficient for students.

The Top Trading Cycles Mechanism

The Top Trading Cycles (TTC) Mechanism (ϕ^{TTC}) is defined as follows:

- Step $t \geq 1$:
  - Each student $s \in S$ points to her most preferred school (if any); students who do not point at any school are assigned to $\emptyset$.
  - Each school $c \in C$ points to its most preferred student.
  - As there are a finite number of schools and students, there exists at least one cycle of length K. Every student s_k $(k = 1, ..., K)$ is assigned to the school she is pointing at. Any student who has been assigned a school seat or the outside option as well as any school $c \in C$ which has been assigned students such that the number of them is equal to its capacity q_c is removed.
  - If no student remains, the algorithm terminates; otherwise, it proceeds to the next step.

This algorithm terminates in a finite number of steps.

* The TTC Mechanism is Pareto efficient and group strategy-proof for students.

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Defining Improvement

Definition 1

We say that a preference relation $\succ'_s$ is an **improvement for school c** over the preference relation $\succ_s$ if:

1. For all $\hat{c} \in C \cup \{\emptyset\}$, if $c \succ_s \hat{c}$, then $c \succ'_s \hat{c}$, and
2. For all $\bar{c}, \hat{c} \in (C \cup \{\emptyset\}) \setminus \{c\}, \bar{c} \succ'_s \hat{c}$ iff $\bar{c} \succ_s \hat{c}$.

Definition 2

A mechanism ϕ **respects improvements of school quality** at the school preference profile $\succ_C$ if, for all $c \in C$ and student preference profiles $\succ_S$ and $\succ'_S$, if $\succ'_S$ is an improvement over the preference relation $\succ_S$ for school c, then: $\phi(\succ'_S, \succ_C)(c) \succeq_c \phi(\succ)(c)$.

Respecting Improvements in Stable Mechanisms

Proposition 1

There exists no stable mechanism that respects improvements of school quality at every school preference profile.

Proof:

Let $S = \{s_1, s_2\}, C = \{c_1, c_2\}$. Consider the preferences $(\succ)$:

$$\begin{array}{lll} \succ_{s_1}: c_2, c_1 & \succ_{c_1}: s_1, s_2 & q_{c_1} = 2 \\ \succ_{s_2}: c_2, c_1 & \succ_{c_2}: s_2, s_1 & q_{c_2} = 1 \end{array}$$

The unique stable matching is: $\phi(\succ) = \{(s_1, c_1), (s_2, c_2)\}$.

Now, suppose that the preference of s_2 changes to $\succ'_{s_2}: c_1, c_2$.
The unique stable matching is: $\phi(\succ') = \{(s_1, c_2), (s_2, c_1)\}$.
Note that $\phi(\succ)(c_1) = s_1 \succ_{c_1} s_2 = \phi(\succ')(c_1)$ although $\succ'_{s_2}$ is an improvement for c_1 over $\succ_{s_2}$. Thus, the theorem is prooved.

Respecting Improvements in Pareto Efficient Mechanisms

Proposition 2

There exists no mechanism that is Pareto efficient for students and respects improvements of school quality at every school preference profile.

Corollary

Neither the Boston mechanism nor the TTC mechanism respect such improvements.

Large Market Environment

- Now, we study the same problem in the large markets context. (Why?)

Definition 3

A **radnom market** is a tuple $\tilde{\Gamma} = (C, S, k, D)$, where k is a positive integer and D is a pair (D_C, D_S) of probability distributions over schools and students, respectively.

Definition 4

A **sequence of radnom markets** is denoted by $\{\tilde{\Gamma}^n\}_{n \in \mathbb{N}}$, where $\tilde{\Gamma}^n = (C^n, S^n, k^n, D^n)$ is a random market in which $|C^n| = n$.

Large Market Environment - Continued

Definition 5

A sequence of random markets $\{\tilde{\Gamma}^n\}_{n\in\mathbb{N}}$ is **regular** if there exist positive integers $k, \tilde{q}$, and $\hat{q}$ such that:

1. $k^n \leq k$ for all n,
2. $q_c \leq \hat{q}$ for all n and $c \in C^n$,
3. $|S^n| \leq \tilde{q}n$ for all n, and
4. for all n and $c \in C^n$, every $s \in S^n$ is acceptable to c at any realization of preferences at D_{C^n}.

Definition 6

Let $V_T(n) \equiv \{c \in C^n : \frac{\max_{\bar{c}\in C^n}\{p^n_{\bar{c}}\}}{p^n_c} \leq T, |\{s \in S^n : c \succ_c \emptyset\}| < q_c\}$.
A sequence of random markets is **sufficiently thick** if there exists $T \in \mathbb{R}$ such that $\mathbb{E}[|V_T(n)|] \to \infty$ as $n \to \infty$.

Large Market Environment - Continued

Let $\alpha_c(\tilde{\Gamma}, \phi)$ be the probability that the preference profile $\succ$ has the property that there exists a student preference profile $\succ'_S$ such that $\succ'_S$ is a disimprovement over $\succ_S$ for c while $\phi(\succ'_S, \succ_C)(c) \succ_c \phi(\succ)(c)$.

Definition 7

We say that a mechanism ϕ **approximately respects improvements of school quality in large markets** if, for any sequence of random markets $(\tilde{\Gamma}^n)_{n \in \mathbb{N}}$ that is regular and sufficiently thick, $\max_{c \in C^n} \alpha_c(\tilde{\Gamma}^n, \phi) \to 0$ as $n \to \infty$.

Improvements In Large Markets

Theorem 1

Any stable mechanism approximately respects improvements of school quality in large markets.

Theorem 2

Neither the Boston mechanism nor the TTC mechanism approximately respects improvements of school quality in large markets.

- The Boston and TTC mechanisms give incentives to schools to demote themselves in students' preferences.

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Preferences' Domain Restriction

- Under what conditions on the school preference profile $\succ_C$ does a stable or Pareto efficient mechanism respect improvements?
- Let $r^h(\succ_c)$ be the student who is h-th ranked in $\succ_c$.

Definition 8

A school preference profile $\succ_C$ is **virtually homogeneous** if $r^h(\succ_c) = r^h(\succ_{\hat{c}})$ for all $c, \hat{c} \in C$ and $h > \min\{q_{\bar{c}} : \bar{c} \in C\}$.

Conditions for Preferences to Respect Improvements

Proposition 3

There exists a stable mechanism that respects improvements of school quality at $\succ_C$ if and only if the school preference profile $\succ_C$ is virtually homogenous. (for $\min\{q_c : c \in C\} > 1$)

Proposition 4

There exists a mechanism that is Pareto efficient for students and respects improvements of school quality at $\succ_C$ if and only if the school preference profile $\succ_C$ is virtually homogenous.

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Conclusion

- We studied whether and how the choice of school choice mechanism affects schools incentives to improve their quality.
- A mild criterion was used to show that no stable mechanism or Pareto efficient (for students) exists that respects improvements over school quality. Similarly, The Boston and the TTC mechanism do not have this property.
- We showed that in large markets, any stable mechanism approximately respects improvements of school quality. However, the Boston and the TTC mechanisms fail to do so.
- We introduced virtual homogeneity over school preferences as a sufficient condition for existence of both a stable and a Pareto efficient mechanism to respect improvements of school quality.
- Extensions: considering quality improvement costs explicitly, study how often schools behave if data is available, quantitatively measure which mechanism performs better in this particular aspect, etc.

Thanks for your attention!