CPSC 340 tutorial 1

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Reference

- CPSC 340 Lecture Slides by Mark schmidt
- CPSC 340 Tutorial 1 Ke (Mark) Ma
- CPSC 340 Tutorial 1 Nam Hee Gordon Kim
- CPSC 340 Tutorial 0 Jonathan W. Lavington
- CPSC 340 Tutorial 1 Yancey Yang

Overview

- Linear Algebra
- Probability
- Gradients
- Python

Linear Algebra

- Matrix
  - Denoted by upper-case letters
  - n by m matrix (n rows by m columns)
- Vector
  - Denoted by lower-case letters
  - Vectors are column vectors by default (d by 1)
- Transpose
  - A (2 by 3 matrix) -> A^T (3 by 2 matrix)
  - Symmetric -> $A = A^T$

Linear Algebra

- Addition
  - Associative and commutative -> $A + (B + C) = (A + B) + C$ & $A + B = B + A$
- Inner Product
  - Is a scalar
  - Commutative and distributive across addition -> $a^Tb = b^Ta$ & $a^T(b + c) = a^Tb + a^Tc$
  - Not associative -> $a^T(b^Tc) \neq (a^Tb)^Tc$
  - Orthogonal -> $a^Tb = 0$
- Matrix Multiplication
  - Associative and distributive across addition -> $A(BC) = (AB)C$ & $A(B + C) = AB + AC$
  - Not commutative -> $AB \neq BA$
  - Transposing reverses order -> $(AB)^T = B^TA^T$
  - Powers don’t change order -> $(AB)^2 = ABAB$

Linear Algebra Tips: Sanity Check

$$\underset{3x3}{A} = \begin{matrix} 3 \\ \\ \\ \\ 3 \end{matrix}\begin{bmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} \qquad \underset{3x1}{x} = \begin{matrix} 3 \\ \\ \\ \\ 1 \end{matrix}\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

6. Ax (matrix-vector multiplication).
7. $x^T Ax$ (quadratic form).

Q: How do we know whether an expression is valid?
Q: What are the dimensions of the results?

Linear Algebra Tip: Vectors and Scalars

In CPSC 340 you'll often see expressions like this:

$$||\underset{\text{nxd}}{X}\underset{\text{dx1}}{w} - \underset{\text{nx1}}{y}||^2$$

Learn to recognize **scalars** from **vectors.**

- All functions are scalar unless otherwise specified
- Norms of vectors are scalar
- Dot products between vectors are scalar

Probability

Conditional Probability.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Bayes' Rule.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Independence.

$$A \perp B \Rightarrow P(A|B) = P(A)$$
$$P(A \cap B) = P(A)P(B)$$

Marginalization.

- 2 cases: $P(A) = P(A \cap B) + P(A \cap \bar{B}) = P(A|B)P(B) + P(A|\bar{B})P(\bar{B})$

- n cases: $P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \ldots + P(A|B_n)P(B_n)$
$= P(A \cap B_1) + P(A \cap B_2) + \ldots + P(A \cap B_n)$

Exercise (borrowed from Wikipedia)

Rolling two dice, D_1 and D_2.

- What is $P(D_1 == 2)$?
- What is $P(D_1 + D_2 \leq 5)$?
- What is $P(D_1 == 2 \cap D_1 + D_2 \leq 5)$?
- What is $P(D_1 == 2 | D_1 + D_2 \leq 5)$?

Exercise (borrowed from Wikipedia)

What is $P(D_1 == 2)$?

D1 \ D2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Exercise (borrowed from Wikipedia)

What is $P(D_1 + D_2 \leq 5)$?

D1 \ D2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Exercise (borrowed from Wikipedia)

What is $P(D_1 == 2 \cap D_1 + D_2 \leq 5)$?

D1 \ D2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Exercise (borrowed from Wikipedia)

What is $P(D_1 == 2 | D_1 + D_2 \leq 5)$? $= \frac{P(D_1 == 2 \cap D_1 + D_2 \leq 5)}{P(D_1 + D_2 \leq 5)}$

D1 \ D2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Gradients

- Define:
  - $\mathbb{R}^d, \nabla f(x)$

Suppose that $f : \mathbb{R}^{m \times n} \to \mathbb{R}$ is a function that takes as input a matrix A of size $m \times n$ and returns a real value. Then the ***gradient*** of f (with respect to $A \in \mathbb{R}^{m \times n}$) is the matrix of partial derivatives, defined as:

$$\nabla_A f(A) \in \mathbb{R}^{m \times n} = \begin{bmatrix} \frac{\partial f(A)}{\partial A_{11}} & \frac{\partial f(A)}{\partial A_{12}} & \cdots & \frac{\partial f(A)}{\partial A_{1n}} \\ \frac{\partial f(A)}{\partial A_{21}} & \frac{\partial f(A)}{\partial A_{22}} & \cdots & \frac{\partial f(A)}{\partial A_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f(A)}{\partial A_{m1}} & \frac{\partial f(A)}{\partial A_{m2}} & \cdots & \frac{\partial f(A)}{\partial A_{mn}} \end{bmatrix}$$

Gradients

- Define:
  - $\mathbb{R}^d, \nabla f(x)$
- Difference between $\nabla f(x)$ and $\frac{\partial f(x)}{\partial x_j}$
- Sanity check:
  - Check the dimensions of gradient vector and input x
  - $f(x)$ is a scalar
  - $\nabla f(x)$ is the same dimension as x
- Exercise: Find the gradient
  - $f(x) = a^T x$
  - $f(x) = log(a^T x)$

Gradients

Remember the Chain Rule (Recursion)

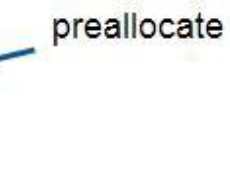
$$f(x) = \left(\exp(2x^2) - 1\right)^3$$

$$\frac{d}{dx} f(x) = \frac{d}{dx} \cdot \frac{d}{dx} \cdot \frac{d}{dx} \cdot \frac{d}{dx} \cdot \frac{d}{dx} x$$

- $f(x) = a^T x$
- $f(x) = log(a^T x)$

Gradients Tips

```
12  function func1(x)
13      f = 0;
14      for x_i in x
15          f += x_i^3;
16      end
17      return f
18  end
19
20  function grad1(x)
21      n = length(x);
22      g = zeros(n);
23      for i in 1:n
24          # Put gradient code here
25      end
26      return g
27  end
```



As long as you use correct partial derivatives you **don't** need "linear algebra" gradient form

Big-O

The notation

$$g(n) = O(f(n))$$

means "for all large n, $g(n) \leq cf(n)$ for some constant c > 0".
Examples:

- $20n + 5 = O(n)$
- $n^2 + 50n + 10000 = O(n^2)$
- $1/n + 10 = O(1)$
- $\log(n) + n = O(n)$
- $n\log(n) + n = O(n\log(n))$

Python Review Packages You Should Know

There are a few packages that you should make yourself familiar with if you want to use python efficiently in the machine learning realm.

- Numpy
- Matplotlib
- Networkx
- Sklearn
- Scipy
- Pandas
- Tensorflow

Python Review: A Not So quick Introduction

These slides are from a tutorial by Justin Johnson found here: http://cs231n.github.io/python-numpy-tutorial/

Basic data types

Like most languages, Python has a number of basic types including integers, floats, booleans, and strings. These data types behave in ways that are familiar from other programming languages.

Numbers: Integers and floats work as you would expect from other languages:

```
x = 3
print(type(x)) # Prints "<class 'int'>"
print(x)       # Prints "3"
print(x + 1)   # Addition; prints "4"
print(x - 1)   # Subtraction; prints "2"
print(x * 2)   # Multiplication; prints "6"
print(x ** 2)  # Exponentiation; prints "9"
x += 1
print(x)  # Prints "4"
x *= 2
print(x)  # Prints "8"
y = 2.5
print(type(y)) # Prints "<class 'float'>"
print(y, y + 1, y * 2, y ** 2) # Prints "2.5 3.5 5.0 6.25"
```

Python Review: A Not So quick Introduction

Booleans: Python implements all of the usual operators for Boolean logic, but uses English words rather than symbols (`&&`, `||`, etc.):

```
t = True
f = False
print(type(t)) # Prints "<class 'bool'>"
print(t and f) # Logical AND; prints "False"
print(t or f)  # Logical OR; prints "True"
print(not t)   # Logical NOT; prints "False"
print(t != f)  # Logical XOR; prints "True"
```

Strings: Python has great support for strings:

```
hello = 'hello'    # String literals can use single quotes
world = "world"    # or double quotes; it does not matter.
print(hello)       # Prints "hello"
print(len(hello))  # String length; prints "5"
hw = hello + ' ' + world  # String concatenation
print(hw)  # prints "hello world"
hw12 = '%s %s %d' % (hello, world, 12)  # sprintf style string formatting
print(hw12)  # prints "hello world 12"
```

String objects have a bunch of useful methods; for example:

```
s = "hello"
print(s.capitalize())  # Capitalize a string; prints "Hello"
print(s.upper())       # Convert a string to uppercase; prints "HELLO"
print(s.rjust(7))      # Right-justify a string, padding with spaces; prints "  hello"
print(s.center(7))     # Center a string, padding with spaces; prints " hello "
print(s.replace('l', '(ell)'))  # Replace all instances of one substring with another;
                                # prints "he(ell)(ell)o"
print('  world '.strip())  # Strip leading and trailing whitespace; prints "world"
```

Python Review: A Not So quick Introduction

Containers

Python includes several built-in container types: lists, dictionaries, sets, and tuples.

Lists

A list is the Python equivalent of an array, but is resizeable and can contain elements of different types:

```
xs = [3, 1, 2]    # Create a list
print(xs, xs[2])  # Prints "[3, 1, 2] 2"
print(xs[-1])     # Negative indices count from the end of the list; prints "2"
xs[2] = 'foo'     # Lists can contain elements of different types
print(xs)         # Prints "[3, 1, 'foo']"
xs.append('bar')  # Add a new element to the end of the list
print(xs)         # Prints "[3, 1, 'foo', 'bar']"
x = xs.pop()      # Remove and return the last element of the list
print(x, xs)      # Prints "bar [3, 1, 'foo']"
```

As usual, you can find all the gory details about lists in the documentation.

Slicing: In addition to accessing list elements one at a time, Python provides concise syntax to access sublists; this is known as *slicing*:

```
nums = list(range(5))     # range is a built-in function that creates a list of integer
print(nums)               # Prints "[0, 1, 2, 3, 4]"
print(nums[2:4])          # Get a slice from index 2 to 4 (exclusive); prints "[2, 3]"
print(nums[2:])           # Get a slice from index 2 to the end; prints "[2, 3, 4]"
print(nums[:2])           # Get a slice from the start to index 2 (exclusive); prints "
print(nums[:])            # Get a slice of the whole list; prints "[0, 1, 2, 3, 4]"
print(nums[:-1])          # Slice indices can be negative; prints "[0, 1, 2, 3]"
nums[2:4] = [8, 9]        # Assign a new sublist to a slice
print(nums)               # Prints "[0, 1, 8, 9, 4]"
```

Python Review: A Not So quick Introduction

Loops: You can loop over the elements of a list like this:

```
animals = ['cat', 'dog', 'monkey']
for animal in animals:
    print(animal)
# Prints "cat", "dog", "monkey", each on its own line.
```

If you want access to the index of each element within the body of a loop, use the built-in `enumerate` function:

```
animals = ['cat', 'dog', 'monkey']
for idx, animal in enumerate(animals):
    print('#%d: %s' % (idx + 1, animal))
# Prints "#1: cat", "#2: dog", "#3: monkey", each on its own line
```

List comprehensions: When programming, frequently we want to transform one type of data into another. As a simple example, consider the following code that computes square numbers:

```
nums = [0, 1, 2, 3, 4]
squares = []
for x in nums:
    squares.append(x ** 2)
print(squares)   # Prints [0, 1, 4, 9, 16]
```

You can make this code simpler using a **list comprehension**:

```
nums = [0, 1, 2, 3, 4]
squares = [x ** 2 for x in nums]
print(squares)   # Prints [0, 1, 4, 9, 16]
```

List comprehensions can also contain conditions:

```
nums = [0, 1, 2, 3, 4]
even_squares = [x ** 2 for x in nums if x % 2 == 0]
print(even_squares)  # Prints "[0, 4, 16]"
```

Python Review: A Not So quick Introduction

Dictionaries

A dictionary stores (key, value) pairs, similar to a `Map` in Java or an object in Javascript. You can use it like this:

```
d = {'cat': 'cute', 'dog': 'furry'}  # Create a new dictionary with some data
print(d['cat'])       # Get an entry from a dictionary; prints "cute"
print('cat' in d)     # Check if a dictionary has a given key; prints "True"
d['fish'] = 'wet'     # Set an entry in a dictionary
print(d['fish'])      # Prints "wet"
# print(d['monkey'])  # KeyError: 'monkey' not a key of d
print(d.get('monkey', 'N/A'))  # Get an element with a default; prints "N/A"
print(d.get('fish', 'N/A'))    # Get an element with a default; prints "wet"
del d['fish']         # Remove an element from a dictionary
print(d.get('fish', 'N/A')) # "fish" is no longer a key; prints "N/A"
```

You can find all you need to know about dictionaries in the documentation.

Loops: It is easy to iterate over the keys in a dictionary:

```
d = {'person': 2, 'cat': 4, 'spider': 8}
for animal in d:
    legs = d[animal]
    print('A %s has %d legs' % (animal, legs))
# Prints "A person has 2 legs", "A cat has 4 legs", "A spider has 8 legs"
```

If you want access to keys and their corresponding values, use the `items` method:

```
d = {'person': 2, 'cat': 4, 'spider': 8}
for animal, legs in d.items():
    print('A %s has %d legs' % (animal, legs))
# Prints "A person has 2 legs", "A cat has 4 legs", "A spider has 8 legs"
```

Dictionary comprehensions: These are similar to list comprehensions, but allow you to easily construct dictionaries. For example:

```
nums = [0, 1, 2, 3, 4]
even_num_to_square = {x: x ** 2 for x in nums if x % 2 == 0}
print(even_num_to_square)  # Prints "{0: 0, 2: 4, 4: 16}"
```

Python Review: A Not So quick Introduction

Sets

A set is an unordered collection of distinct elements. As a simple example, consider the following:

```
animals = {'cat', 'dog'}
print('cat' in animals)   # Check if an element is in a set; prints "True"
print('fish' in animals)  # prints "False"
animals.add('fish')       # Add an element to a set
print('fish' in animals)  # Prints "True"
print(len(animals))       # Number of elements in a set; prints "3"
animals.add('cat')        # Adding an element that is already in the set does nothing
print(len(animals))       # Prints "3"
animals.remove('cat')     # Remove an element from a set
print(len(animals))       # Prints "2"
```

As usual, everything you want to know about sets can be found in the documentation.

Loops: Iterating over a set has the same syntax as iterating over a list; however since sets are unordered, you cannot make assumptions about the order in which you visit the elements of the set:

```
animals = {'cat', 'dog', 'fish'}
for idx, animal in enumerate(animals):
    print('#%d: %s' % (idx + 1, animal))
# Prints "#1: fish", "#2: dog", "#3: cat"
```

Set comprehensions: Like lists and dictionaries, we can easily construct sets using set comprehensions:

```
from math import sqrt
nums = {int(sqrt(x)) for x in range(30)}
print(nums)  # Prints "{0, 1, 2, 3, 4, 5}"
```

Python Review: A Not So quick Introduction

Tuples

A tuple is an (immutable) ordered list of values. A tuple is in many ways similar to a list; one of the most important differences is that tuples can be used as keys in dictionaries and as elements of sets, while lists cannot. Here is a trivial example:

```
d = {(x, x + 1): x for x in range(10)}  # Create a dictionary with tuple keys
t = (5, 6)        # Create a tuple
print(type(t))    # Prints "<class 'tuple'>"
print(d[t])       # Prints "5"
print(d[(1, 2)])  # Prints "1"
```

The documentation has more information about tuples.

Functions

Python functions are defined using the `def` keyword. For example:

```
def sign(x):
    if x > 0:
        return 'positive'
    elif x < 0:
        return 'negative'
    else:
        return 'zero'

for x in [-1, 0, 1]:
    print(sign(x))
# Prints "negative", "zero", "positive"
```

Python Review: A Not So quick Introduction

Classes

The syntax for defining classes in Python is straightforward:

```
class Greeter(object):

    # Constructor
    def __init__(self, name):
        self.name = name  # Create an instance variable

    # Instance method
    def greet(self, loud=False):
        if loud:
            print('HELLO, %s!' % self.name.upper())
        else:
            print('Hello, %s' % self.name)

g = Greeter('Fred')  # Construct an instance of the Greeter class
g.greet()            # Call an instance method; prints "Hello, Fred"
g.greet(loud=True)   # Call an instance method; prints "HELLO, FRED!"
```

Python Review: A Not So quick Introduction

We can initialize numpy arrays from nested Python lists, and access elements using square brackets:

```
import numpy as np

a = np.array([1, 2, 3])   # Create a rank 1 array
print(type(a))            # Prints "<class 'numpy.ndarray'>"
print(a.shape)            # Prints "(3,)"
print(a[0], a[1], a[2])   # Prints "1 2 3"
a[0] = 5                  # Change an element of the array
print(a)                  # Prints "[5, 2, 3]"

b = np.array([[1,2,3],[4,5,6]])    # Create a rank 2 array
print(b.shape)                     # Prints "(2, 3)"
print(b[0, 0], b[0, 1], b[1, 0])   # Prints "1 2 4"
```

Numpy also provides many functions to create arrays:

```
import numpy as np

a = np.zeros((2,2))   # Create an array of all zeros
print(a)              # Prints "[[ 0.  0.]
                      #          [ 0.  0.]]"

b = np.ones((1,2))    # Create an array of all ones
print(b)              # Prints "[[ 1.  1.]]"

c = np.full((2,2), 7)  # Create a constant array
print(c)               # Prints "[[ 7.  7.]
                       #          [ 7.  7.]]"

d = np.eye(2)         # Create a 2x2 identity matrix
print(d)              # Prints "[[ 1.  0.]
                      #          [ 0.  1.]]"

e = np.random.random((2,2))  # Create an array filled with random values
print(e)                     # Might print "[[ 0.91940167  0.08143941]
                             #               [ 0.68744134  0.87236687]]"
```

Python Review: A Not So quick Introduction

Array indexing

Numpy offers several ways to index into arrays.

Slicing: Similar to Python lists, numpy arrays can be sliced. Since arrays may be multidimensional, you must specify a slice for each dimension of the array:

```
import numpy as np

# Create the following rank 2 array with shape (3, 4)
# [[ 1  2  3  4]
#  [ 5  6  7  8]
#  [ 9 10 11 12]]
a = np.array([[1,2,3,4], [5,6,7,8], [9,10,11,12]])

# Use slicing to pull out the subarray consisting of the first 2 rows
# and columns 1 and 2; b is the following array of shape (2, 2):
# [[2 3]
#  [6 7]]
b = a[:2, 1:3]

# A slice of an array is a view into the same data, so modifying it
# will modify the original array.
print(a[0, 1])   # Prints "2"
b[0, 0] = 77     # b[0, 0] is the same piece of data as a[0, 1]
print(a[0, 1])   # Prints "77"
```

Python Review: A Not So quick Introduction

Integer array indexing: When you index into numpy arrays using slicing, the resulting array view will always be a subarray of the original array. In contrast, integer array indexing allows you to construct arbitrary arrays using the data from another array. Here is an example:

```
import numpy as np

a = np.array([[1,2], [3, 4], [5, 6]])

# An example of integer array indexing.
# The returned array will have shape (3,) and
print(a[[0, 1, 2], [0, 1, 0]])  # Prints "[1 4 5]"

# The above example of integer array indexing is equivalent to this:
print(np.array([a[0, 0], a[1, 1], a[2, 0]]))  # Prints "[1 4 5]"

# When using integer array indexing, you can reuse the same
# element from the source array:
print(a[[0, 0], [1, 1]])  # Prints "[2 2]"

# Equivalent to the previous integer array indexing example
print(np.array([a[0, 1], a[0, 1]]))  # Prints "[2 2]"
```

Python Review: A Not So quick Introduction

Boolean array indexing: Boolean array indexing lets you pick out arbitrary elements of an array. Frequently this type of indexing is used to select the elements of an array that satisfy some condition. Here is an example:

```
import numpy as np

a = np.array([[1,2], [3, 4], [5, 6]])

bool_idx = (a > 2)   # Find the elements of a that are bigger than 2;
                     # this returns a numpy array of Booleans of the same
                     # shape as a, where each slot of bool_idx tells
                     # whether that element of a is > 2.

print(bool_idx)      # Prints "[[False False]
                     #          [ True  True]
                     #          [ True  True]]"

# We use boolean array indexing to construct a rank 1 array
# consisting of the elements of a corresponding to the True values
# of bool_idx
print(a[bool_idx])  # Prints "[3 4 5 6]"

# We can do all of the above in a single concise statement:
print(a[a > 2])     # Prints "[3 4 5 6]"
```

Datatypes

Every numpy array is a grid of elements of the same type. Numpy provides a large set of numeric datatypes that you can use to construct arrays. Numpy tries to guess a datatype when you create an array, but functions that construct arrays usually also include an optional argument to explicitly specify the datatype. Here is an example:

```
import numpy as np

x = np.array([1, 2])   # Let numpy choose the datatype
print(x.dtype)         # Prints "int64"

x = np.array([1.0, 2.0])   # Let numpy choose the datatype
print(x.dtype)             # Prints "float64"

x = np.array([1, 2], dtype=np.int64)   # Force a particular datatype
print(x.dtype)                         # Prints "int64"
```

Python Review: A Not So quick Introduction

Array math

Basic mathematical functions operate elementwise on arrays, and are available both as operator overloads and as functions in the numpy module:

```
import numpy as np

x = np.array([[1,2],[3,4]], dtype=np.float64)
y = np.array([[5,6],[7,8]], dtype=np.float64)

# Elementwise sum; both produce the array
# [[ 6.0  8.0]
#  [10.0 12.0]]
print(x + y)
print(np.add(x, y))

# Elementwise difference; both produce the array
# [[-4.0 -4.0]
#  [-4.0 -4.0]]
print(x - y)
print(np.subtract(x, y))

# Elementwise product; both produce the array
# [[ 5.0 12.0]
#  [21.0 32.0]]
print(x * y)
print(np.multiply(x, y))

# Elementwise division; both produce the array
# [[ 0.2         0.33333333]
#  [ 0.42857143  0.5       ]]
print(x / y)
print(np.divide(x, y))

# Elementwise square root; produces the array
# [[ 1.          1.41421356]
#  [ 1.73205081  2.        ]]
print(np.sqrt(x))
```

Python Review: A Not So quick Introduction

Note that unlike MATLAB, `*` is elementwise multiplication, not matrix multiplication. We instead use the `dot` function to compute inner products of vectors, to multiply a vector by a matrix, and to multiply matrices. `dot` is available both as a function in the numpy module and as an instance method of array objects:

```
import numpy as np

x = np.array([[1,2],[3,4]])
y = np.array([[5,6],[7,8]])

v = np.array([9,10])
w = np.array([11, 12])

# Inner product of vectors; both produce 219
print(v.dot(w))
print(np.dot(v, w))

# Matrix / vector product; both produce the rank 1 array [29 67]
print(x.dot(v))
print(np.dot(x, v))

# Matrix / matrix product; both produce the rank 2 array
# [[19 22]
#  [43 50]]
print(x.dot(y))
print(np.dot(x, y))
```

Numpy provides many useful functions for performing computations on arrays; one of the most useful is `sum`:

```
import numpy as np

x = np.array([[1,2],[3,4]])

print(np.sum(x))  # Compute sum of all elements; prints "10"
print(np.sum(x, axis=0))  # Compute sum of each column; prints "[4 6]"
print(np.sum(x, axis=1))  # Compute sum of each row; prints "[3 7]"
```