Matrix Square Roots

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} = \begin{bmatrix} & & \\ & ? & \\ & & \end{bmatrix}^2$$

Not unique (if it exists);

Principal Square Root $A^{1/2}$:

- Usually the one needed in practice
- Unique
- Exists if A has no eigenvalues on the closed negative real line [1]

Schur method of Bjorck and Hammarling [2]

- The most numerically stable
- Reduces A to upper triangular

$$\underset{T}{\begin{bmatrix} 1 & 8 & 38 & 129 & 350 \\ & 9 & 45 & 149 & 393 \\ & & 36 & 144 & 399 \\ & & & 100 & 350 \\ & & & & 225 \end{bmatrix}} = \underset{U}{\begin{bmatrix} 1 & 2 & 4 & 7 & 11 \\ & 3 & 5 & 8 & 12 \\ & & 6 & 9 & 13 \\ & & & 10 & 14 \\ & & & & 15 \end{bmatrix}}^2$$

$$\underset{U}{\begin{bmatrix} 1 & 2 & 4 & 7 & 11 \\ & 3 & 5 & 8 & 12 \\ & & 6 & 9 & 13 \\ & & & 10 & 14 \\ & & & & 15 \end{bmatrix}}$$

Element dependencies: left and below

Two methods:

- Point
- Block [3]

Approaches:

A row at a time

$$\underset{U}{\begin{bmatrix} 1 & 2 & 4 & 7 & 11 \\ & 3 & 5 & 8 & 12 \\ & & 6 & 9 & 13 \\ & & & 10 & 14 \\ & & & & 15 \end{bmatrix}}$$

A column at a time

$$\underset{U}{\begin{bmatrix} 1 & 2 & 4 & 7 & 11 \\ & 3 & 5 & 8 & 12 \\ & & 6 & 9 & 13 \\ & & & 10 & 14 \\ & & & & 15 \end{bmatrix}}$$

A super-diagonal at a time

$$\underset{U}{\begin{bmatrix} 1 & 2 & 4 & 7 & 11 \\ & 3 & 5 & 8 & 12 \\ & & 6 & 9 & 13 \\ & & & 10 & 14 \\ & & & & 15 \end{bmatrix}}$$