Statistics tutorial 5

Sebastian Wuchterl 331453, Peter Fackeldey 330532

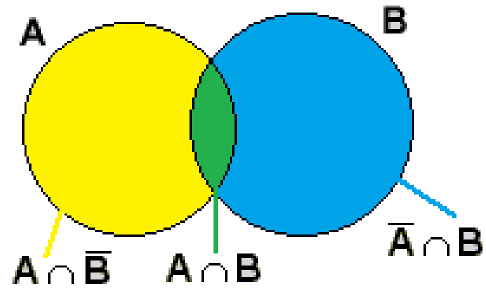
May 2017

1 task 1

1.1 first part

First we have to show:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{1}$$



Therefore look at Figure 1.1. Obviously one can make the following statements:

$$P(A) = P(A \cap \bar{B}) + P(A \cap B) \tag{2}$$

$$P(B) = P(\bar{A} \cap B) + P(A \cap B) \tag{3}$$

Rearranged one gets:

$$P(A \cap \bar{B}) = P(A) - P(A \cap B) \tag{4}$$

$$P(\bar{A} \cap B) = P(B) - P(A \cap B) \tag{5}$$

Also one can see in Figure 1.1:

$$P(A \cup B) = P(A \cap \bar{B}) + P(A \cap B) + P(\bar{A} \cap B) \tag{6}$$

By plugging equation (4) and (5) in (6) one gets:

$$P(A \cup B) = P(A) - P(A \cap B) + P(A \cap B) + P(B) - P(A \cap B) \tag{7}$$

And by simplifying finally:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{8}$$

1.2 second part

Now we have our fire alarm system with:

$$P(smoke) = 0.91; P(heat) = 0.95; P(smoke \cap heat) = 0.88 \tag{9}$$

One can use our derived equation and gets:

$$\begin{aligned} P(smoke \cup heat) &= P(smoke) + P(heat) - P(smoke \cap heat) \\ &= 0.95 + 0.91 - 0.88 \\ &= 0.98 \end{aligned} \tag{10}$$

2 task 2

2.1 b)

Analytical calculation: If the point lands inside the triangle or in the bottom part of the three parts cutted by the triangle, the chord is longer. So you can calculate the propability like that:

$$\begin{aligned} P(longer) &= \frac{\frac{1}{3} \cdot (Area_{circle} - Area_{triangle}) + Area_{triangle}}{Area_{cirlce}} \\ &= \frac{\frac{1}{3} \cdot (\pi \cdot 1^2 - \frac{\sqrt{3}}{4} \cdot (\frac{3 \cdot 1}{\sqrt{3}})^2) + \frac{\sqrt{3}}{4} \cdot (\frac{3 \cdot 1}{\sqrt{3}})^2}{\pi \cdot 1^2} \\ &= 0.609 \end{aligned} \tag{11}$$

3 task 3

$$P(e) = 0.0001;\ P(\gamma) = 0.9999;\ P(signal|e) = 0.995;\ P(signal|\gamma) = 0.003 \tag{12}$$

3.1 a)

To make the calculations clear, see Figure 1 for the tree diagram.

electron

$$P(0signal|e) = (1 - 0.995)^2 = 2.5 \cdot 10^{-5} \tag{13}$$

$$P(1signal|e) = 2 \cdot (0.995 \cdot (1 - 0.995)) = 9.95 \cdot 10^{-3} \tag{14}$$

$$P(2signal|e) = (0.995)^2 = 0.990025 \tag{15}$$

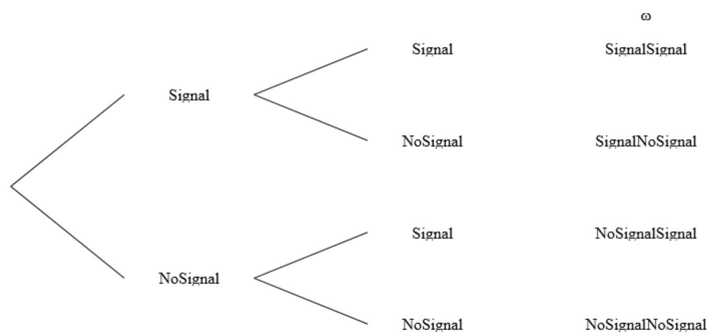


Figure 1: tree diagram

photon

$$P(0signal|\gamma) = (1-0.003)^2 = 0.994009 \tag{16}$$

$$P(1signal|\gamma) = 2\cdot(0.003\cdot(1-0.003)) = 9.982\cdot 10^{-3} \tag{17}$$

$$P(2signal|\gamma) = (0.003)^2 = 9\cdot 10^{-6} \tag{18}$$

3.2 b)

$$\begin{aligned} P(\gamma|1signal) &= \frac{P(1signal|\gamma)\cdot P(\gamma)}{P(1signal)} \\ &= \frac{P(1signal|\gamma)\cdot P(\gamma)}{P(1signal|\gamma)\cdot P(\gamma) + P(1signal|e)\cdot P(e)} \\ &= \frac{5.982\cdot 10^{-3}\cdot 0.9999}{5.982\cdot 10^{-3}\cdot 0.9999 + 9.95\cdot 10^{-3}\cdot 0.0001} \\ &= 0.99983 \end{aligned} \tag{19}$$

3.3 c)

$$\begin{aligned} P(e|2signal) &= \frac{P(2signal|e)\cdot P(e)}{P(2signal)} \\ &= \frac{P(2signal|e)\cdot P(e)}{P(2signal|\gamma)\cdot P(\gamma) + P(2signal|e)\cdot P(e)} \\ &= \frac{0.990025\cdot 0.0001}{9\cdot 10^{-6}\cdot 0.9999 + 0.990025\cdot 0.0001} \\ &= 0.916 \end{aligned} \tag{20}$$

For the second part set $P(e|2signal)$ to 0.99 and express everything in terms of $P(signal|\gamma)$, which is only $P(2signal|\gamma)$.
With the values directly plugged in, because its analogous to the calculation above, one gets:

$$0.99 = \frac{0.990025 \cdot 0.0001}{P(signal|\gamma)^2 \cdot 0.9999 + 0.990025 \cdot 0.0001} \tag{21}$$

And solved for $P(signal|\gamma)$ one gets $P(signal|\gamma) = 0.0001$.