Statistics and Data Analysis

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(derived from Jan-Frederik Schulte)

SS 2017

Computer exercise: 28.04.2017. Hand in by 04.05.2017

Week 2

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1. Statistics exercises (5 points)

(a) Show that,

$$< (x-\mu)^2 >=< x^2 > -\mu^2 \tag{1}$$

(b) The uniform distribution is given by,

$$P(x) = \begin{cases} \frac{1}{b-a} & \text{for} \quad a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases} \tag{2}$$

Derive the expectation value, $E(x) =< x >$ and the variance $Var(x) = \sigma^2$ for this distribution.

2. Properties of a distribution of random numbers (10 points)

The file `randNumbers.txt` contains ≈1000 numbers diced according to an unknown probability density function. Have a look at `properties.cc`, which reads in the numbers and already calculates the mean. Extend it to calculate the properties *median*, *mode*, *width*, *skew* and *kurtosis*.

3. Random Numbers and the Central Limit Theorem (10 points)

Check the Central Limit Theorem in practice. Use for this the framework `centralLimit.cc`. It uses ROOT and compiles with the command
`g++ centralLimit.cc -I ‘root-config --incdir‘ ‘root-config --libs‘`. The program generates 1000 random numbers according to a Gaussian distribution using ROOT's `TRandom3` class and plots a histogram of the numbers together with the PDF.

(a) Compile and execute the program. Change the parameters of the generated Gaussian to see their effects on the distribution.

(b) Among the predefined functions available in ROOT are the Binomial, Poisson, and Uniform distribution. Have a look at the ROOT documentation (links are in the code) to see what parameters they expect. Generate and draw random numbers according these distributions and compare them to their pdf.

(c) Demonstrate the CLT by summing random numbers drawn from Binomal, Poisson and Uniform distributions. Repeat this process a significant number of times and fill the results into a histogram. Compare this to a Gaussian distribution.

4. *Electron in hydrogen atom (Optional problem)

The probability that an electron is at a distance r from the centre of the nucleus of a hydrogen atom is given by

$$dP(r) = Cr^2 e^{-r/R} dr \tag{3}$$

(a) Find the mean radius $\bar{r}$ and the normalization C.