Statistics and Data Analysis 11.05.2017

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Computer exercise: 12.05.2017, Hand in by 18.05.2017 **Week 4**

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1. The CLT in Monte Carlo (15 points)

(a) Use the Monte-Carlo method to generate random numbers according to an "M"–shaped distribution. Plot the distribution of the sum of 2,5,10,20 and 50 "M"–distributed variables to show how it converges to a Gauss-distribution.
Hint: You can use the following form for the "M"–distribution:

$$M(x) = \begin{cases} -x+1 & -1 < x < 0 \\ x+1 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

(b) We now want to generalize our previous examples of the CLT to cases where the CDF can no longer be inverted trivially. The "elephant.cc" reads the elephant bitmap ("elephant.bmp") and stores the RGB codes of every pixel in an array. Use the code to generate random numbers according to the elephant shape and plot the distribution of the sum of 2,5,10,20 and 50 elephant-distributed variables to show how it converges to a Gauss-distribution.
Hint: The elephant distribution is a uniform distribution in 2D. Therefore you need 2D histrograms like TH2F in ROOT.

2. Monte-Carlo integration (5 points)

Write a program that calculates π using the acceptance and rejection method given in the lectures.

3. Error propagation (5 points)

(a) Calculate σ_g of the trigonometric function $g(\theta)$ with the independent variables A, B, and θ with according errors σ_A, σ_B, and σ_θ.

$$g = A\sin\theta + B\cos\theta.$$

(b) Consider the functions $f = xy$, $g = x/y$, and $h = y/x$ where x is given to a precision of 3% and y is given to a precision of 4%. What is the precision of f, g, and h?
Hint: Calculate first the relative uncertainties σ_f/f, σ_g/g, and σ_h/h.

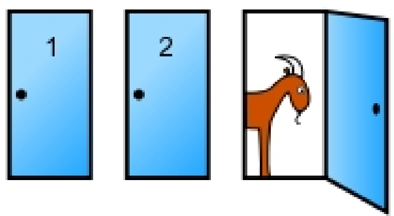
(c) Tracking chambers measure the position where a charged track traversed in cylindrical polar coordinates $(r, \phi \pm \sigma_\phi, z \pm \sigma_z)$. The uncertainty on the radius r can

be neglected. What are the errors in cartesian coordinates?
Hint: With $x = r\cos\phi$, $y = r\sin\phi$ and $z = z$ we get:

$$G = \begin{pmatrix} \cos\phi & -r\sin\phi & 0 \\ \sin\phi & r\cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

4. *Monty Hall problem (optional)

Develop a Monte Carlo simulation of the Monty Hall program. Calculate from the simulation the probability to win the car, depending on your strategy. I.e. simulate either that the candidate always switches his choice of doors after Monty opened one door to show a goat, or that he sticks to his initial choice.



Hints:

(a) Use for example the randomNumber() function to draw a uniformly distributed random integer number between 1 and 3. This is the number of the door behind which is the car.

(b) Similarly, let the candidate randomly choose a door

(c) Let Monty open a door. In the case that he has a choice, let him choose randomly.

(d) Implement the strategy, i.e. let the candidate switch the doors, or let him keep his initial choice.

(e) Check if your Monte Carlo candidate has won and repeat the process enough times to reduce the statistical uncertainties.

(f) Explain your result!