universität freiburg

Measure theory

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https://pfaffelh.github.io/hp/2025WS_measure_theory.html
https://www.stochastik.uni-freiburg.de/

Tutorial 1 - Review of metric spaces and topologies I

Exercise 1.
Let $X = \{a,b,c,d\}$. Which of the following are topologies for X?

(a) $\{\emptyset, X, \{a\}, \{b\}, \{a,c\}, \{a,b,c\}, \{a,b\}\}$

(b) $\{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{b,d\}\}$

(c) $\{\emptyset, X, \{a,c,d\}, \{b,c,d\}\}$

Can you further give an example of two sets A and B of $\mathbb{R}$ such that

$$A \cap B = \emptyset, \quad \overline{A} \cap B \neq \emptyset, \quad A \cap \overline{B} \neq \emptyset.$$

Exercise 2 (4 Points)**.**
If X is a set and $r : X \times X \to \mathbb{R}_+$ is defined by

$$r(x,y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$$

Show that r is a metric on X.
Note: r is in fact called the discrete metric on X.

Exercise 3 (4 Points)**.**
Show that every mapping from a metric space (Ω, r) to a metric space (Ω', r') is continuous if r is the discrete metric.

Exercise 4.
Given a metric space (Ω, r). Consider the topology generated by r and recall the definition of the open set in A.1. Then the following hold:

(a) the whole set and the empty set are open;

(b) the union of any collection of open subsets of Ω is open.

(c) the intersection of any two open subsets of Ω is open;

Exercise 5.
Is the set of rational numbers open or closed? Give any two examples of sets that are both open and closed.