universitätfreiburg

Measure theory for probabilists   Winter semester 2024

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https://pfaffelh.github.io/hp/2024WS_measure_theory.html
https://www.stochastik.uni-freiburg.de/

Tutorial 8 - Measurable functions and the integral II

Exercise 1 (4 Points).
Let $(\Omega,\mathcal{F})$,$(\Omega',\mathcal{F}')$,$(\Omega'',\mathcal{F}'')$ be measurable spaces and $f:\Omega\to\Omega'$ measurable and $Z:\Omega\to\Omega''$. Then, Z is $\sigma(X)$-measurable if and only if there is a $\mathcal{F}'/\mathcal{F}''$-measurable mapping $\varphi:\Omega'\to\Omega''$ with $\varphi\circ X=Z$.

Exercise 2 (4 Points).
Prove Theorem 3.25

Exercise 3 (4 Points).
Let λ be Lebesgue measure on $(\mathbb{R},\mathcal{B}(\mathbb{R}))$. Find $f,f_1,f_2,...\in\mathcal{L}^1(\lambda)$ with $f_n\xrightarrow{n\to\infty}f$ almost everywhere, with $\int f_n d\lambda\xrightarrow{n\to\infty}\int f d\mu$, but the corresponding Riemann integrals do not converge.

Exercise 4 (4 Points).
Let $f:\mathbb{R}\to\mathbb{R}$ be differentiable with derivative f'. Show that f' is $\mathcal{B}(\mathbb{R})-\mathbb{B}(\mathbb{R})-$measurable.