University of Freiburg – Mathematical Institute

# Summer term 2025 Comments on the course catalogue

Version February 5, 2025

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# Study Planning

Dear Students of Mathematics,

The course catalogue provides information about the course offers of the Mathematical Institute for the current semester. Information on the course of study, which courses you can and which courses you have to pick, can be found on the information pages for each programme at <a href="https://www.math.uni-freiburg.de/nlehre/">https://www.math.uni-freiburg.de/nlehre/</a>. Please note that there may be different versions of examination regulations for a study programme.

You are welcome to use the advisory services of the Mathematical Institute if needed: study counseling by the programme coordinator, study counseling by the individual departments, and counseling by lecturers (office hours are listed on the personal webpages linked in the Institute's directory).

#### Please note:

- The two Bachelor programmes as well as the Master of Education as additional subject programmes begin with the basic lectures Analysis I and II and Linear Algebra I and II, on which most other mathematics courses are based on. In case you can only start with one of the two basic lectures due to the combination of subjects in the two-major Bachelor programme, you can find variants for the course of study on the programme's information page.
- As an orientation requirement, the two exams for Analysis I and Linear Algebra I must be passed by the end of the 3rd semester in the **B.Sc. programme**, and at least one of the two in the **two-major Bachelor programme**.
- There are no further regulations regarding the structure of your individual course of study and no formal prerequisites for attending courses (except for the limited number of places in each seminar or proseminar). However, you must ensure that you have the necessary prior knowledge independently.
- In the M.Sc. Mathematics programme, please note that you may take a maximum of two of the four oral exams with the same examiner.
- To what extent the material of advanced lectures is sufficient as **preparation for final theses and exams** or should be supplemented must be agreed upon soon enough with the supervisor of the thesis or the examiners. This applies in particular to the oral exam in the specialization module of the M.Sc. programme.

# Language

Courses with the abbreviation "D" are offered in German, courses with the abbreviation "E" in English. Exercises for English lectures can often also be completed in German, and vice versa.

Presentations in seminars can usually be given in German and English; the abbreviation " $\mathrm{D}/\mathrm{E}$ " indicates this possibility.

# **Usability of Courses**

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For each course, the "Usability" section indicates in which modules from which degree programmes it can be used. For the degree programmes, the following abbreviations are used:

2HfB21	Two-Major Bachelor Programme
BSc21	Bachelor of Science in Mathematics, regulations of 2021
BScInfo19	Bachelor of Science in Computer Science, regulations of 2019
BScPhys22	Bachelor of Science in Physics, regulations of of 2022
MEd18	Master of Education in Mathematics
MEdual24	Master of Education – "dual study programme"
MEH21	Master of Education, Mathematics as an additional subject with 120 ECTS points
MEB21	Master of Education, Mathematics as an additional subject with 90 ECTS points
MSc14	Master of Science in Mathematics
MScData24	Master of Science in Mathematics in Data and Technology

As a general rule, no courses that were already used in the underlying Bachelor programme may be completed in a Master programme. If you have any questions, please contact the programme coordination.

#### Please note:

• It is allowed to use higher-level lectures, typically offered for the M.Sc. Mathematics programme, for electives in other study programmes; however, due to the required prior knowledge, they will only be suitable in exceptional cases. If a course can be used for a module, does not necessarily mean that it is suitable for a module. Conversely,

- extreme cases are not listed though possible (e.g. a lecture such as "Differential Geometry II" as a specialisation module in the M.Ed. study proramme).
- In the B.Sc. Mathematics according to the regulations of 2021, in addition to the compulsory area, at least three 4-hour lectures with 2-hour exercises (9 ECTS points each) must be completed. At least one of these must be from the field of pure mathematics. You can see which of the lectures count as pure mathematics by checking whether it is approved for the module "Pure Mathematics" in the M.Sc. Mathematics programme.

# Examination Requirements and Pass/Fail Assessments

The section "Usability" will be supplemented at the beginning of the lecture period with information on which graded examinations and pass/fail assessments are required for its use in the respective module or study area. This information supplements the module handbooks in terms of examination and accreditation law and will be approved by the Mathematics Study Commission.

#### Please note:

- Deviations from the specified type of graded examination are permitted if, due to circumstances beyond the examiner's control, the intended type of examination is not suitable or would involve disproportionate effort. The same applies to pass/fail assessments.
- If a course is approved as an elective module in a non-listed study programme, the requirements follow
  - the elective module of the B.Sc. programme, if examination achievements are required,
  - the elective module of the M.Sc. programme, if only pass/fail assessments are required.

If the corresponding modules are not offered, please contact the programme coordination of the Mathematical Institute.

- If written exercise assignments are required as a study achievement, these are usually weekly exercise assignments, or bi-weekly in the case of a one-hour exercise. Depending on the start, end, rhythm, and individual breaks, there can be between 5 and 14 exercise sheets. The number of points achievable per exercise sheet can vary.
- For practical exercises, this applies analogously to programming assignments.

# Research Areas for Final Theses

Information on Bachelor and Master theses in Mathematics can be found here:

https://www.math.uni-freiburg.de/nlehre/en/studiendekanat/faq/stu\_kat\_66ae8e6510f040b07f8c7f62

The following list gives you an overview of the fields from which the professors and lecturers of the Mathematical Institute typically assign topics for final theses. If you are interested in a thesis, please arrange an appointment early!

Prof.Dr. Sören Bartels	Applied Mathematics, Partial Differential Equations, and Numerics
Prof.Dr. Harald Binder	Medical Biometrics and Applied Statistics
JProf. Dr. David Criens	Stochastic Analysis, Probability Theory, and Financial Mathematics
Prof. Dr. Moritz Diehl	Numerics, Optimization, Optimal Control
Prof. Dr. Patrick W. Dondl	Applied Mathematics, Calculus of Variations, Partial Differential Equations, and Numerics
Prof. Dr. Sebastian Goette	Differential Geometry, Topology, and Global Analysis
Prof. Dr. Nadine Große	Differential Geometry and Global Analysis
Prof.Dr. Annette Huber-Klawitter	Algebraic Geometry and Number Theory
PDDr. Markus Junker	Mathematical Logic, Model Theory
Prof. Dr. Stefan Kebekus	Algebra, Complex Analysis, Complex and Algebraic Geometry
Prof. Dr. Ernst Kuwert	Partial Differential Equations, Calculus of Variations
Prof. Dr. Eva Lütkebohmert-Holtz	Financial Mathematics, Risk Management and Regulation
Prof. Dr. Amador Mart'ın Pizarro	Mathematical Logic, especially Model Theory
Prof. Dr. Heike Mildenberger	Mathematical Logic, especially Set Theory and Infinite Combinatorics
JProf. Dr. Abhishek Oswal	Algebra
Prof. Dr. Peter Pfaffelhuber	Stochastics, Biomathematics
Prof. Dr. <b>Angelika Rohde</b>	Mathematical Statistics, Probability Theory
Prof. Dr. Michael Růžička	Applied Mathematics and Partial Differential Equations
JProf. Dr. <b>Diyora Salimova</b>	Applied Mathematics, Partial Differential Equations, Machine Learning, and Numerics
Prof. Dr. Thorsten Schmidt	Financial Mathematics, Machine Learning
Prof. Dr. Wolfgang Soergel	Algebra and Representation Theory
Prof. Dr. Guofang Wang	Partial Differential Equations, Calculus of Variations

On https://www.math.uni-freiburg.de/forschung/index.html, the research areas are described in more detail.

# Offers from EUCOR Partner Universities

As part of the EUCOR cooperation, you can attend courses at the partner universities *University of Basel, Karlsruhe Institute of Technology, Université Haute-Alsace* in Mulhouse, and the *Université de Strasbourg*. The procedure is explained in detail on this information page.

In particular, Basel and Strasbourg offer interesting additions to our lecture programme at the master's level. Credits can be recognized within the framework of the respective examination regulations, especially in the elective (required) area of the B.Sc. and M.Sc. programmes. Please discuss possible credits in advance with the programme coordination! The costs for travel by train, bus, and tram can be subsidized by EUCOR.

# Basel

Institute: The Department of Mathematics and Computer Science at the University of Basel offers eight research groups in mathematics: Algebraic Geometry, Number Theory, Analysis, Numerics, Computational Mathematics, Probability Theory, Mathematical Physics, and Statistical Science.

**Lecture Offerings:** The pages with the lecture offerings for the Bachelor and the lecture offerings for the Master seem to correspond most closely to our mathematics lecture directory. The general lecture directory of the university can be found here: https://vorlesungsverzeichnis.unibas.ch/de/semester-planung

**Dates:** In Basel, the autumn semester begins in mid-September and ends at the end of December, the spring semester runs from mid-February to the end of May.

**Travel:** The University of Basel is best reached by train: The train ride to the Badischer Bahnhof takes about 45-60 minutes by local transport, 30 minutes by ICE. Then take tram 6 towards *Allschwil Dorf* to the *Schifflände* stop (about 10 minutes).

# Strasbourg

Institute: In Strasbourg, there is a large *Institut de recherche mathématique avancée* (IRMA), which is divided into seven *équipes*: Analyse; Arithmétique et géométrie algébrique; Algèbre, représentations, topologie; Géométrie; Modélisation et contrôle; Probabilités und Statistique. Seminars and working groups (*groupes de travail*) are announced on the institute's website.

**Lecture Offerings:** Participation of Freiburg students in the offers of the second year of the Master M2 is highly welcome. Depending on prior knowledge, the lectures are suitable for our students from the 3rd year of study onwards. The lecture language is a priori French, but a switch to English is gladly possible upon request, please arrange in advance. In Strasbourg, a different focus topic is offered in the M2 annually, in 2024/25 it is: Algebra and Topology. General lecture directories typically do not exist in France.

**Dates:** In France, the 1st semester runs from early September to late December and the 2nd semester from late January to mid-May. A more precise schedule will only be available in September. The timetables are flexible; they can usually accommodate the needs of Freiburg students.

**Travel:** The *Université de Strasbourg* is best reached by car (about one hour). Alternatively, there is a very affordable connection with Flixbus to the *Place de l'Étoile*. The train ride to the main station in Strasbourg takes about 1h40 by local transport, 1h10 by ICE. Then take tram line C towards *Neuhof*, *Rodolphe Reuss* to the *Universités* stop.

For further information and organizational assistance, please contact:

in Freiburg: Prof. Dr. Annette Huber-Klawitter annette.huber@math.uni-freiburg.de

in Strasbourg: Prof. Carlo Gasbarri, Coordinator of the M2 gasbarri@math.unistra.fr

or the respective course coordinators.

# 1a. Mandatory Lectures of the Study Programmes

# Analysis II

Michael Růžička, Assistant: Maximilian Stegemeyer Lecture: Mon, Wed, 8–10 h, HS Rundbau, Albertstr. 21

Tutorial: 2 hours, various dates

in German

# **Content:**

Analysis II is the continuation of Analysis I from the winter semester and one of the basic lectures of the study programmes in Mathematics. Central concepts of Analysis I (limits and derivations) will be generalized to the case of higher dimension.

Central topics are the topology of  $\mathbb{R}^n$ , metrics and norms, differential calculs in several variables, ordinary differential equations and in particular linear differential equations.

# Prerequisites:

Analysis I, Linear Algebra I (or bridge course linear algebra)

# Usable in the following modules:

Analysis (2HfB21, BSc21, MEH21, MEB21) Analysis II (BScInfo19, BScPhys20)

# Linear Algebra II

Stefan Kebekus, Assistant: Christoph Brackenhofer Lecture: Tue, Thu, 8–10 h, HS Rundbau, Albertstr. 21

Tutorial: 2 hours, various dates

in German

# **Content:**

Linear algebra II is the continuation of the lecture linear algebra I from the winter semester and one of the basic courses of math studies. Central topics are: Jordan's normal form of endomorphisms, symmetrical bilinear forms with especially the Sylvester's theorem, Euclidian and Hermitian vector spaces, skalar products, orthonormal bases, orthogonal and (self-) adjugated, spectral theorem, principal axis theorem.

# Prerequisites:

Linear Algebra I

# Usable in the following modules:

Linear Algebra (2HfB21, BSc21, MEH21) Linear Algebra (MEB21) Linear Algebra II (BScInfo19, BScPhys20)

# **Elementary Geometry**

Nadine Große, Assistant: Jonah Reuß in German

Lecture: Wed, 10-12 h, HS Weismann-Haus, Albertstr. 21a

Tutorial: 2 hours, various dates 29.07., HS Rundbau, Albertstr. 21

# **Content:**

The lecture gives an introduction to elementary geometry in Euclidian and non-Euclidian space and its mathematical foundations. We get to know Euclidean, hyperbolic, and projective geometry as examples of incidence geometries, and study their symmetry groups.

The next main topic is the axiomatic characterization of the Euclidean plane. The focus is on the story of the fifth Euclidian axiom (and the attempts to get rid of it).

# Prerequisites:

Linear Algebra I

# Usable in the following modules:

Elementary Geometry (2HfB21, MEH21, MEB21, MEdual24) Compulsory Elective in Mathematics (BSc21)

# Numerics II

Sören Bartels, Assistant: Vera Jackisch

Lecture: Wed, 14-16 h, HS Weismann-Haus, Albertstr. 21a

Tutorial: 2 hours, every other week, various dates

31.07., 10:00-12:00

# Content:

Numerics is a discipline of mathematics that deals with the practical solution of mathematical problems. As a rule, problems are not precisely solved but approximated, for which a sensible compromise of accuracy and computing effort has to be found. In the second part of the two-semester course, questions of the analysis such as the approximation of functions by polynomials, the approximately solution of non-linear equations and the practical calculation of integrals are treated. Attendance at the accompanying computer exercise sessions is recommended. These take place fortnightly, alternating with the tutorial for the lecture.

in German

# Prerequisites:

necessary: Linear Algebra I and Analysis I

useful: Linear Algebra II, Analysis II

# Remarks:

Computer exercises accompanying the lecture are offered.

# Usable in the following modules:

Numerics (2HfB21, MEH21) Numerics (BSc21)

# Elementary Probability Theory II

Johannes Brutsche in German

Lecture: Fri, 10–12 h, HS Weismann-Haus, Albertstr. 21a

Tutorial: 2 hours, every other week, various dates

Sir-in Exam: Date to be announced

#### **Content:**

Nach dem in der Vorlesung Stochastik I erhaltenen Einblick in die Grundlagen sowie in verschiedene Methoden und Fragestellungen der Stochastik bzw. Wahrscheinlichkeitstheorie wird sich diese Vorlesung hauptsächlich statistischen Themen widmen, insbesondere solchen, die für Studierende des Lehramts an Gymnasien relevant sind. Aber auch für Studierende im B.Sc. Mathematik mit Interesse an Stochastik kann die Vorlesung eine (hoffentlich) nützliche Ergänzung und gute Grundlage für den späteren Besuch der Kursvorlesung Mathematische Statistik" sein. Nach der Präzisierung des Begriffes statistisches Modell" werden Methoden zur Konstruktion von Schätzern (z.B. Maximum-Likelihood-Prinzip, Momentenmethode) und Gütekriterien für diese (Erwartungstreue, Konsistenz) besprochen. Außerdem werden Konfidenzintervalle und Hypothesentests eingeführt. Als weitere Anwendungen werden lineare Modelle betrachtet und falls die Zeit es erlaubt, weitere statistische Verfahren. Dabei werden auch die für viele Test- und Schätzverfahren nützlichen Eigenschaften von exponentiellen Familien und multivariaten Normalverteilungen vorgestellt.

# Prerequisites:

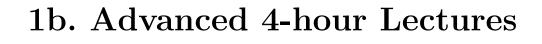
Linear Algebra I+II and Analysis I+II

#### Remarks:

If you are interested in a practical, computer-supported realisation of individual lecture contents, participation in the 'Practical Exercise Stochastics' is recommended (in parallel or subsequently).

# Usable in the following modules:

Elementary Probability Theory (2HfB21, MEH21) Elementary Probability Theory II (MEdual24) Compulsory Elective in Mathematics (BSc21)



# Differential Geometry II – Geometry of Submanifolds

Guofang Wang in German

Lecture: Mon, Wed, 12–14 h, SR 404, Ernst-Zermelo-Str. 1

Tutorial: 2 hours, date to be determined

#### Content:

In the lecture, we discuss the geometry of the sub-manifolds of Euclidian spaces. Examples of such sub-manifolds are curves in the plane and surfaces in the 3-dimensional space. In the 1st part we introduce the external geometry of the sub-manifold, e.g. the second fundamental form, the mean curvature, the first variation of the area, the equations of Gauss, Codazzi and Ricci. In the 2nd part we examine tminimal hypersurfaces (minimal surfaces), the hypersurface with constant mean curvature and the geometric inequalities, the isoperimetric inequality and its generalisations.

# Prerequisites:

Analysis III and "Differential Geometry" or "Curves and Surfaces"

# Usable in the following modules:

Elective (Option Area) (2HfB21) Compulsory Elective in Mathematics (BSc21) Pure Mathematics (MSc14) Mathematics (MSc14) Concentration Module (MSc14) Elective (MSc14) Elective (MScData24)

# **Functional Analysis**

Patrick Dondl in German

Lecture: Mon, Wed, 10–12 h, HS II, Albertstr. 23b

Tutorial: 2 hours, date to be determined Sir-in Exam: Date to be announced

#### Content:

Linear functional analysis, which is the subject of the lecture, uses concepts of linear algebra such as vector space, linear operator, dual space, scalar product, adjoint map, eigenvalue, spectrum to solve equations in infinite-dimensional function spaces, especially linear differential equations. The algebraic concepts have to be extended by topological concepts such as convergence, completeness and compactness.

This approach was developed at the beginning of the 20th century by Hilbert, among others, and is now part of the methodological foundation of analysis, numerics and mathematical physics, in particular quantum mechanics, and is also indispensable in other mathematical areas.

# Prerequisites:

Linear Algebra I+II, Analysis I-III

# Usable in the following modules:

Elective (Option Area) (2HfB21) Compulsory Elective in Mathematics (BSc21) Mathematical Concentration (MEd18, MEH21) Applied Mathematics (MSc14) Pure Mathematics (MSc14) Elective (MSc14) Elective in Data (MScData24)

# Commutative Algebra and Introduction to Algebraic Geometry

Wolfgang Soergel, Assistant: Xier Ren in German

Lecture: Tue, Thu, 8–10 h, HS II, Albertstr. 23b

Tutorial: 2 hours, date to be determined

#### **Content:**

In linear algebra you studied linear systems of equations. In commutative algebra, we study polynomial equation systems such as  $x^2 + y^2 = 1$  and their solution sets, the algebraic varieties. It will turn out that such a variety is closely related to the ring of the restrictions of polynomial functions on that variety, and that we can extrapolate this relationship to a geometric understanding of any commutative rings, in particular the ring of the integers. Commutative algebra, algebraic geometry, and number theory grow together in this conceptual building. The lecture aims to introduce into this conceptual world. We will especially focus on the dimension of algebraic varieties and their cutting behavior, which generalizes the phenomena known from the linear algebra on the case of polynomial equation systems.

# Prerequisites:

necessary: Linear Algebra I+II

useful: Algebra and Number Theory

# Usable in the following modules:

Elective (Option Area) (2HfB21) Compulsory Elective in Mathematics (BSc21) Mathematical Concentration (MEd18, MEH21) Pure Mathematics (MSc14) Mathematics (MSc14) Concentration Module (MSc14) Elective (MSc14) Elective (MScData24)

# Mathematical Logic

Amador Martín Pizarro, Assistant: Stefan Ludwig Lecture: Tue, Thu, 12–14 h, HS II, Albertstr. 23b

Tutorial: 2 hours, date to be determined Sir-in Exam: Date to be announced

in German

# **Content:**

This introductory course in mathematical logic consists of several parts. It the basics of predicate logic and a brief introduction to model theory and the axiom system

as well as the axiom system of set theory. The aim of the lecture is to explain the recursion-theoretical content of the predicate calculus, in particular the so-called Peano-arithmetic and Gödel's incompleteness theorems.

# Prerequisites:

Basic knowledge of mathematics from first semester lectures

# Usable in the following modules:

Elective (Option Area) (2HfB21) Compulsory Elective in Mathematics (BSc21) Mathematical Concentration (MEd18, MEH21) Pure Mathematics (MSc14) Elective (MSc14) Elective (MScData24)

# **Topology**

Heike Mildenberger, Assistant: Hannes Jakob Lecture: Tue, Thu, 10–12 h, HS II, Albertstr. 23b

Tutorial: 2 hours, date to be determined Sir-in Exam: Date to be announced

in German

#### Content:

A topological space consists of a basic set X and a family of open subsets of the basic set, which is called topology on X. Examples over the basic sets  $\mathbb{R}$  and  $\mathbb{R}^n$  are given in the analysis lectures. The mathematical subject Topology is the study of topological spaces and the investigation of topological spaces. Our lecture is an introduction to set-theoretic and algebraic topology.

# Literature:

- Ryszard Engelking: General Topology, Warschau, 1977.
- Marvin Greenberg: Lectures on Algebraic Topology, Amsterdam, 1967.
- Allen Hatcher: Algebraic Topology, Cambridge 2002.
- Klaus Jänich: Topologie, Spinger, 8. Auflage, 2005.
- John Kelley: General Topology, New York, 1969.
- Casimir Kuratowski: Topologie, Warschau 1958.
- James Munkres: Elements of Algebraic Topology, Cambridge, Massachusetts 1984
- Boto von Querenburg: Mengentheoretische Topologie, Springer, 3. Auflage 2001.

# Prerequisites:

Analysis I and II, Linear Algebra I

# Usable in the following modules:

Elective (Option Area) (2HfB21) Compulsory Elective in Mathematics (BSc21) Mathematical Concentration (MEd18, MEH21) Pure Mathematics (MSc14) Elective (MSc14) Elective (MScData24)

# **Probability Theory**

Angelika Rohde, Assistant: Johannes Brutsche in English

Lecture: Tue, Thu, 10-12 h, HS Weismann-Haus, Albertstr. 21a

Tutorial: 2 hours, date to be determined Sir-in Exam: Date to be announced

#### Content:

The problem of axiomatising probability theory was solved by Kolmogorov in 1933: a probability is a measure of the set of all possible outcomes of a random experiment. From this starting point, the entire modern theory of probability develops with numerous references to current applications.

The lecture is a systematic introduction to this area based on measure theory and includes, among other things, the central limit theorem in the Lindeberg-Feller version, conditional expectations and regular versions, martingales and martingale convergence theorems, the strong law of large numbers and the ergodic theorem as well as Brownian motion.

# Prerequisites:

necessary: Analysis I+II, Linear Algebra I, Elementary Probability Theory I

useful: Analysis III

# Usable in the following modules:

Elective (Option Area) (2HfB21) Compulsory Elective in Mathematics (BSc21) Mathematical Concentration (MEd18, MEH21) Applied Mathematics (MSc14) Elective (MSc14) Advanced Lecture in Stochastics (MScData24) Elective in Data (MScData24)

# Probability Theory III: Stochastic Integration

David Criens, Assistant: Samuel Adeosun in English

Lecture: Wed, 14–16 h, HS II, Albertstr. 23b, Thu, 10–12 h, SR 404, Ernst-Zermelo-Str. 1

Tutorial: 2 hours, date to be determined

#### Content:

This lecture builds the foundation of one of the key areas of probability theory: stochastic analysis. We start with a rigorous construction of the Itô integral that integrates against a Brownian motion (or, more generally, a continuous local martingale). In this connection, we learn about Itô's celebrated formula, Girsanov's theorem, representation theorems for continuous local martingales and about the exciting theory of local times. Then, we discuss the relation of Brownian motion and Dirichlet problems. In the final part of the lecture, we study stochastic differential equations, which provide a rich class of stochastic models that are of interest in many areas of applied probability theory, such as mathematical finance, physics or biology. We discuss the main existence and uniqueness results, the connection to the martingale problem of Stroock-Varadhan and the important Yamada-Watanabe theory.

# Literature:

- O. Kallenberg: Foundartions of Modern Probability, 3rd ed., Springer Nature Switzerland, 2021.
- I. Karatzas and S. E. Shreve: Brownian Motion and Stochastic Calculus, 2nd ed., Springer New York, 1991.

# Prerequisites:

Probability Theory I and II (Stochastic Processes)

# Usable in the following modules:

Elective (Option Area) (2HfB21) Compulsory Elective in Mathematics (BSc21) Applied Mathematics (MSc14) Mathematics (MSc14) Concentration Module (MSc14) Elective (MSc14) Advanced Lecture in Stochastics (MScData24) Elective in Data (MScData24)

# Reading courses

All professors and 'Privatdozenten' of the Mathematical Institute Talk/participation possible in German and English

# Content:

In a reading course, the material of a four-hour lecture is studied in supervised self-study. In rare cases, this may take place as part of a course; however, reading courses are not usually listed in the course catalog. If you are interested, please contact a professor or a private lecturer before the start of the course; typically, this will be the supervisor of your Master's thesis, as the reading course ideally serves as preparation for the Master's thesis (both in the M.Sc. and the M.Ed. programs).

The content of the reading course, the specific details, and the coursework requirements will be determined by the supervisor at the beginning of the lecture period. The workload should be equivalent to that of a four-hour lecture with exercises.

# Usable in the following modules:

Reading Course (MEd18, MEH21) Mathematics (MSc14) Concentration Module (MSc14) Elective (MSc14)

1c. Advanced 2-hour Lectures

# Algorithmic Aspects of Data Analytics and Machine Learning

Sören Bartels, Assistant: Tatjana Schreiber in English

Lecture: Wed, 10–12 h, SR 226, Hermann-Herder-Str. 10

Tutorial: 2 hours, date to be determined

#### Content:

The lecture addresses algorithmic aspects in the practical realization of mathematical methods in big data analytics and machine learning. The first part will be devoted to the development of recommendation systems, clustering methods and sparse recovery techniques. The architecture and approximation properties as well as the training of neural networks are the subject of the second part. Convergence results for accelerated gradient descent methods for nonsmooth problems will be analyzed in the third part of the course. The lecture is accompanied by weekly tutorials which will involve both, practical and theoretical exercises.

# Literature:

- B. Bohn, J. Garcke, M. Griebel: Algorithmic Mathematics in Machine Learning, SIAM, 2024
- P. Petersen: Neural Network Theory, Lecture Notes, TU Vienna, 2022
- V. Shikhman, D. Müller: Mathematical Foundations of Big Data Analytics, Springer, 2021
- N. Walkington: Nesterov's Method for Convex Optimization, SIAM Review, 2023

# Prerequisites:

Lectures "Numerik I, II" or lecture "Basics in Applied Mathematics"

# Usable in the following modules:

# Differential Topology

Mikhail Tëmkin in English

Lecture: Mon, 14–16 h, SR 404, Ernst-Zermelo-Str. 1

Tutorial: 2 hours, date to be determined

#### Content:

The notion of a manifold is fundamental importance. On one hand, it is a common ground for many branches of pure and applied mathematics, as well as mathematical physics. On the other hand, it itself is a lush source of elegant, unexpected and structural results. Next, algebraic topology is to mathematics what the periodic table is to chemistry: it offers order to what seems to be chaotic (more precisely, to topological spaces of which manifolds is an important example). Finally, differential topology studies smooth manifolds using topological tools. As it turns out, narrowing the scope to manifolds provides many new beautiful methods, structure and strong results, that are applicable elsewhere – as we will see in the course. Necessary notions from algebraic topology will be covered in the beginning.

#### Literature:

Hatcher: Algebraic topology Hirsch: Differential topology

• Milnor: Topology from the Differentiable Viewpoint

• Kosinski: Differential manifolds

# Prerequisites:

Point-set topology (e.g. "Topology" from summer semester of 2024)

# Usable in the following modules:

Elective (Option Area) (2HfB21) Compulsory Elective in Mathematics (BSc21) Pure Mathematics (MSc14) Mathematics (MSc14) Concentration Module (MSc14) Elective (MSc14) Elective (MScData24)

# Finite Simple Groups

Amador Martín Pizarro, Assistant: Charlotte Bartnick Lecture: Wed, 14–16 h, SR 125, Ernst-Zermelo-Str. 1

Tutorial: 2 hours, date to be determined

in German

#### Content:

Groups without any non-trivial normal subgroup are called simple groups. Similar to prime numbers for the natural numbers, simple groups form the building blocks for finite groups. It is easy to see that Abelian finite simple groups are cyclic. Non-Abelian examples are alternating groups and Lie-type groups.

The classification of finite simple groups is far beyond the scope of this course. However, we will illustrate some of the recurring ideas of classification and, in particular, prove the following result of Brauer and Fowler:

Theorem: Let G be a finite group of even order such that the centre is of odd order. Then there is an element  $g \neq 1_G$  with  $|G| < |C_G(g)|^3$ .

This theorem had a particularly large impact on the classification of finite simple groups, as it suggests that these could be classified by examining the centralisers of elements of order 2.

#### Literature:

- J. S. Rose: A course on Group Theory, Cambridge University Press, 1978.
- J. J. Rotman: An introduction to the Theory of Groups, Springer-Verlag, 1999.
- R. Solomon: A brief history of the classification of the finite simple groups, Bulletin American Mathematical Society 38 (2001), no. 3, 315–352.

# Prerequisites:

Algebra and Number Theory

# Usable in the following modules:

# Lévy Processes and Financial Applications

Ernst August v. Hammerstein in English

Lecture: Mon, 14–16 h, HS II, Albertstr. 23b Tutorial: 2 hours, date to be determined

#### Content:

Lévy processes are the continuous-time analogues of random walks in discrete time as they possess, by definition, independent and stationary increments. They form a fundamental class of stochastic processes which has widespread applications in financial and insurance mathematics, queuing theory, physics and telecommunication. The Brownian motion and the Poisson process, which may already be known from other lectures, also belong to this class. Despite their richness and flexibility, Lévy processes are usually analytically and numerically very tractable because their distributions are generated by a single univariate distribution which has the property of infinite divisibility.

The lecture starts with an introduction into infinitely divisible distributions and the derivation of the famous Lévy-Khintchine formula. Then it will be explained how the Lévy processes emerge from these distributions and how the characteristics of the latter influence the path properties of the corresponding processes. Finally, after a short look at the method of subordination, option pricing in Lévy-driven financial models will be discussed.

#### Literature:

- D. Applebaum: Lévy processes and Stochastic Calculus, Cambridge University Press, 2005.
- J. Bertoin: Lévy Processes, Cambridge University Press, 2005.
- R. Cont, P. Tankov: Financial Modelling with Jump Processes, Chapman & Hall/CRC, 2004.
- E. Eberlein, J. Kallsen: Mathematical Finance, Springer, 2019.
- P. E. Protter: Stochastic Integration and Differential Equations (Second Edition, Version 2.1), Springer, 2005.
- K.-I. Sato: Lévy Processes and Infinitely Divisible Distributions, Cambridge University Press, 1999.

# Prerequisites:

necessary: Probability Theory I

useful: Probability Theory II (Stochastic Processes)

# Usable in the following modules:

# Machine Learning for Stochastics

Thorsten Schmidt in English

Lecture: Wed, 12–14 h, HS II, Albertstr. 23b Tutorial: 2 hours, date to be determined

#### Content:

In this lecture we will study new and highly efficient tools from machine learning which are applied to stochastic problems. This includes neural SDEs as a generalisation of stochastic differential equations relying on neural networks, transformers as a versatile tool not only for languages but also for time series, transformers and GANs as generator of time series and a variety of applications in Finance and insurance such as (robust) deep hedging, signature methods and the application of reinforcement learning.

# Prerequisites:

The prerequisites are stochastics, for some parts we will require a good understanding of stochastic processes. A (very) short introduction will be given in the lectures – so for fast learners it would be possible to follow the lectures even without the courses on stochastic processes.

# Usable in the following modules:

# Numerical Optimization

Moritz Diehl, Assistant: Florian Messerer in English

Tutorial / flipped classroom: Tue, 14–16 h, HS II, Albertstr. 23b

Sir-in Exam: Date to be announced

#### Content:

The aim of the course is to give an introduction into numerical methods for the solution of optimization problems in science and engineering. The focus is on continuous nonlinear optimization in finite dimensions, covering both convex and nonconvex problems. The course divided into four major parts:

- 1. Fundamental Concepts of Optimization: Definitions, Types of Optimization Problems, Convexity, Duality, Computing Derivatives
- 2. Unconstrained Optimization and Newton-Type Algorithms: Exact Newton, Quasi-Newton, BFGS, Gauss-Newton, Globalization
- 3. Equality Constrained Optimization: Optimality Conditions, Newton-Lagrange and Constrained Gauss-Newton, Quasi-Newton, Globalization
- 4. Inequality Constrained Optimization Algorithms: Karush-Kuhn-Tucker Conditions, Active Set Methods, Interior Point Methods, Sequential Quadratic Programming

The course is organized as inverted classroom based on lecture recordings and a lecture manuscript, with weekly alternating Q&A sessions and exercise sessions. The lecture is accompanied by intensive computer exercises offered in Python (6 ECTS) and an optional project (3 ECTS). The project consists in the formulation and implementation of a self-chosen optimization problem or numerical solution method, resulting in documented computer code, a project report, and a public presentation. Please check the website for further information.

# Literature:

- S. Boyd, L. Vandenberghe: *Convex Optimization*, Cambridge University Press, 2004.
- M. Diehl: Lecture Notes Numerical Optimization
- J. Nocedal, S. J. Wright: *Numerical Optimization*, second edition, Springer, 2006.

# **Prerequisites:**

necessary: Analysis I–II, Linear Algebra I–II

useful: Introduction to Numerics

#### Remarks:

Together with the optional programming project, the course counts as a 9 ECTS lecture.

# Usable in the following modules:

# Lecture: tba

Patrick Dondl in English

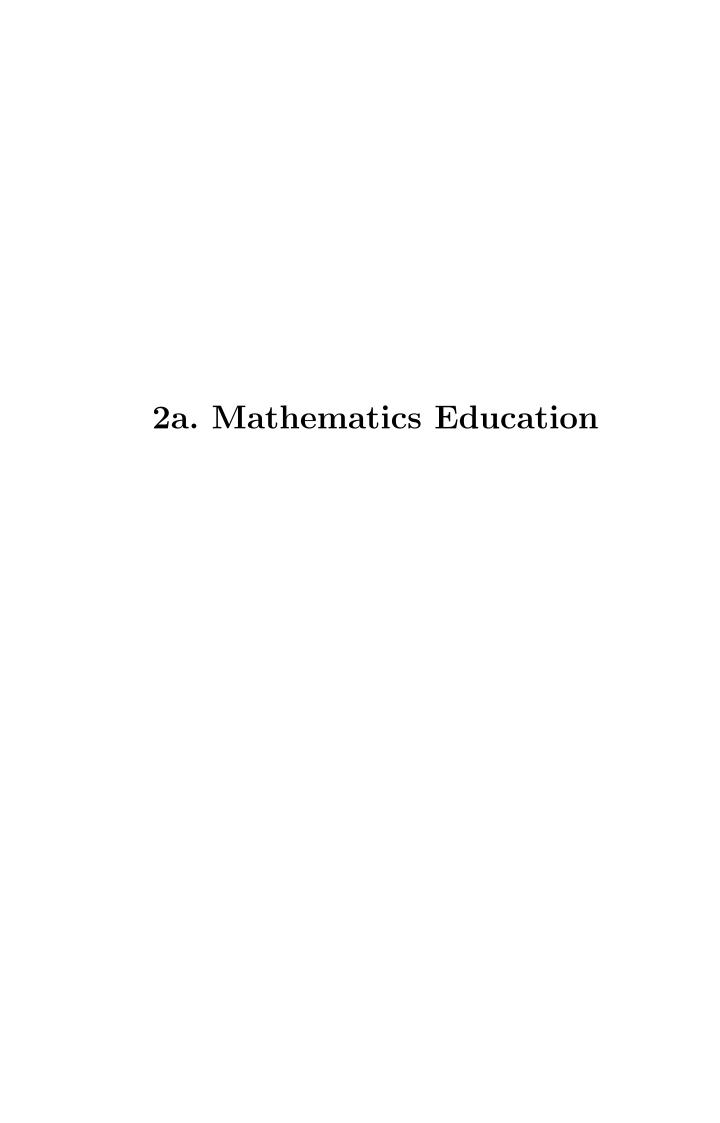
Lecture: Wed, 14–16 h, SR 226, Hermann-Herder-Str. 10

Tutorial: 2 hours, date to be determined

# **Content:**

More information will follow at the end of January!

# Usable in the following modules:



# Introduction to Mathematics Education

Katharina Böcherer-Linder
Man, 10, 12 b, SB, 226, Harmonn Hander Stn. 10

Mon, 10–12 h, SR 226, Hermann-Herder-Str. 10

Tutorial: 2 hours, various dates Sir-in Exam: Date to be announced

#### in German

#### **Content:**

Mathematics didactic principles and their learning theory foundations and possibilities of teaching implementation (also e.g. with the help of digital media).

Theoretical concepts on central mathematical thinking activities such as concept formation, modeling, problem solving and reasoning.

Mathematics didactic constructs: Barriers to understanding, pre-concepts, basic ideas, specific difficulties with selected mathematical content.

Concepts for dealing with heterogeneity, taking into account subject-specific characteristics particularities (e.g. dyscalculia or mathematical giftedness).

Levels of conceptual rigour and formalization as well as their age-appropriate implementation.

# Prerequisites:

Required: Analysis I, Linear Algebra I

# Remarks:

The course is compulsory in the teaching degree option of the two-main-subject Bachelor's degree program. It is made up of lecture components and parts with exercise and seminar character. The three forms of teaching cannot be not be completely separated from each other. Attendance at the "Didactic Seminar" (approximately fortnightly, tuesday evenings, 19:30) is expected!

This course is only offered in German.

# Usable in the following modules:

(Introduction to) Mathematics Education (2HfB21, MEH21, MEB21, MEdual24)

# Mathematics Education – Functions and Analysis

Jürgen Kury in German

Seminar: Wed, 14-17 h, SR 404, Ernst-Zermelo-Str. 1

# Content:

Exemplary implementations of the theoretical concepts of central mathematical thought processes such as concept formation, modeling, problem solving and reasoning for the content areas of functions and analysis.

Barriers to understanding, pre-concepts, basic ideas, specific difficulties for the content areas of functions and analysis. Fundamental possibilities and limitations of media, in particular of computer-aided mathematical tools mathematical tools and their application for the content areas of functions and analysis. Analysis of individual mathematical learning processes and errors as well as development individual support measures for the content areas of functions and analysis.

# Literature:

- R. Dankwerts, D. Vogel: Analysis verständlich unterrichten. Heidelberg: Spektrum, 2006.
- G. Greefrath, R. Oldenburg, H.-S. Siller, V. Ulm, H.-G. Weigand: *Didaktik der Analysis. Aspekte und Grund-vorstellungen zentraler Begriffe.* Berlin, Heidelberg: Springer 2016.

# Prerequisites:

Required: Introduction to Mathematics Education, Knowledge about analysis and numerics

# Remarks:

The two parts can be completed in different semesters semesters, but have a joint final exam, which is offered every semester and written after completing both parts.

# Usable in the following modules:

Mathematics Education for Specific Areas of Mathematics (MEd18, MEH21, MEB21)

# Mathematics Education – Probability Theory and Algebra

Frank Reinhold in German

Seminar: Mon, 13–16 h, SR 125, Ernst-Zermelo-Str. 1

# Content:

Exemplary implementations of the theoretical concepts of central mathematical thought processes such as concept formation, modeling, problem solving and reasoning for the content areas of stochastics and algebra.

Barriers to understanding, pre-concepts, basic ideas, specific difficulties for the content areas of stochastics and algebra. Basic possibilities and limitations of media, especially computer-based mathematical tools and their mathematical tools and their application for the content areas of stochastics and algebra. and algebra.

Analysis of individual mathematical learning processes and errors as well as development individual support measures for the content areas of stochastics and algebra.

# Literature:

- G. Malle: Didaktische Probleme der elementaren Algebra. Braunschweig, Wiesbaden: Vieweg 1993.
- A. Eichler, M. Vogel: Leitidee Daten und Zufall. Von konkreten Beispielen zur Didaktik der Stochastik. Wiesbaden: Vieweg 2009.

# Prerequisites:

Required: Introduction to Mathematics Education, knowledge from stochastics and algebra.

#### Remarks:

The two parts can be completed in different semesters semesters, but have a joint final exam, which is offered every semester and written after completing both parts.

# Usable in the following modules:

Mathematics Education for Specific Areas of Mathematics (MEd18, MEH21, MEB21)

# Mathematics education seminar: High School Maths = University Maths $\pm$ x

Holger Dietz in German

Seminar: Fri, 8-11 h, Seminar für Ausbildung und Fortbildung der Lehrkräfte Freiburg,

# Content:

As a high school student, you have no idea what it means to study mathematics. While studying mathematics at the university, the imagination of what it means to teach mathematics at school is similarly vague. This seminar would like to provide concrete insights into the practice of math teaching and tries to build on experiences e.g. B. from the practical semester.

Selected contents and aspects of mathematics lessons (from worksheet to the extension of number systems) are analyzed and questioned – not only from the point of view of the scientist, but also from the point of view of the lecturers, teachers, pupils. Mathematically simple topics often hide unexpected didactic challenges. Therefore, in addition to dealing with existing content and framework conditions, teaching should also be planned and – if possible – carried out at the school.

# Prerequisites:

Basic lectures

# Usable in the following modules:

Supplementary Module in Mathematics Education (MEd18, MEH21, MEB21)

# Mathematics education seminars at Freiburg University of Education

Lecturers of the University of Education Freiburg

in German

# Remarks:

For the module "Fachdidaktische Entwicklung", suitable suitable courses can also be completed at the PH Freiburg if places are available there. To find out whether courses are suitable are suitable, please discuss in advance with Ms. Böcherer-Linder in advance; you must check whether places are available if you are interested in a course from the lecturers if you are interested in a course.

Most suitable courses will be offered in German.

# Usable in the following modules:

Supplementary Module in Mathematics Education (MEd18, MEH21, MEB21)

## Module "Research in Mathematics Education"

Lecturers of the University of Education Freiburg

in German

Part 1: Seminar 'Development Research in Mathematics Education – Selected Topics'

Dates and rooms: see the course catalogue of the PH Freiburg

Part 2: Seminar 'Research Methods in Mathematics Education'

Dates and rooms: see the course catalogue of the PH Freiburg

Part 3: Master's thesis seminar: Development and Optimisation of a Research Project in Mathematics Education

Appointments by arrangement

#### **Content:**

The three related courses of the module prepare students for an empirical Master thesis in mathematics didactics. The course is jointly designed by all professors at the PH with mathematics didactics research projects at secondary levels 1 and 2 and is carried out by one of these researchers. Afterwards, students have the opportunity to start Master thesis with one of these supervisors - usually integrated into larger ongoing research projects.

The first course of the module provides an introduction to strategies of empirical didactic research (research questions, research status, research designs). Students deepen their skills in scientific research and the evaluation of subject-specific didactic research. In the second course (in the last third of the semester) students are introduced to central qualitative and quantitative research methods through concrete work with existing data (interviews, student products, experimental data), students are introduced to central qualitative and quantitative research methods. The third course is an accompanying seminar for the Master thesis.

The main objectives of the module are the ability to receive mathematics didactic research in order to didactic research to clarify questions of practical relevance and to plan an empirical mathematics didactics Master thesis. It will be held as a mixture of seminar, development of research topics in groups and active work with research data. Recommended literature will be depending on the research topics offered within the respective courses. The parts can also be attended in different semesters, for example part 1 in the second Master semester and part 2 in the compact phase of the third Master semester after the practical semester.

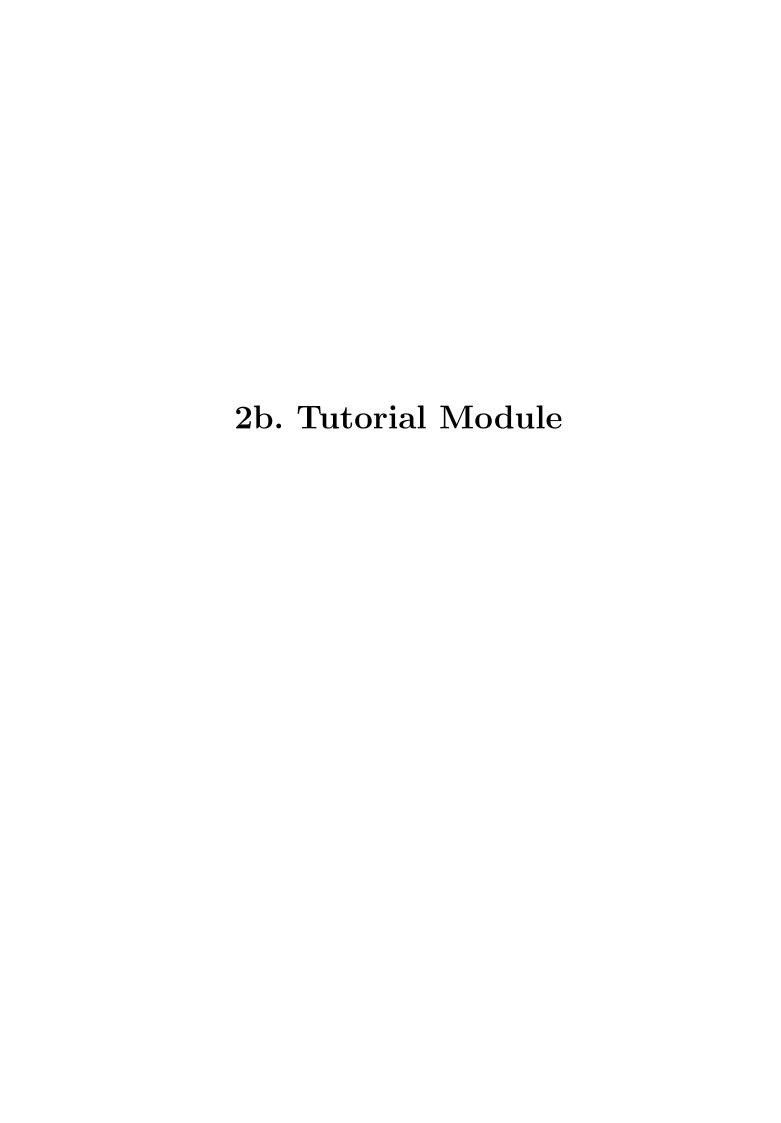
#### Remarks:

Three-part module for M.Ed. students who would like to write a didactic Master thesis in mathematics. Participation only after personal registration by the end of the lecture period of the previous semester in the Department of Didactics. Admission capacity is limited.

Pre-registration: If you would like to take part in this module, please register by by 28.02.2025 by e-mail to didaktik@math.uni-freiburg.de and to Ralf Erens. This course will only be offered in German.

### Usable in the following modules:

Research in Mathematics Education (MEd18, MEH21, MEB21)



## Learning by Teaching

in German

### Content:

What characterizes a good tutorial? This question will be discussed in the first workshop and tips and suggestions will be given. Experiences will be shared in the second workshop.

### Remarks:

Prerequisite for participation is a tutoring position for a lecture of the Institute of Mathematics in the current semester (at least one two-hour or two one-hour tutorial groups over the whole semester).

Can be used twice in the M.Sc. program in Mathematics.

This course is only offered in German.

## Usable in the following modules:

Elective (Option Area) (2HfB21) Elective (BSc21) Elective (MSc14) Elective (MScData24) 2c. Computer Exercises

## Introduction to Programming for Science Students

Ludwig Striet in German

Lecture: Mon, 16–18 h, HS Weismann-Haus, Albertstr. 21a

Tutorial: 2 hours, various dates

#### **Content:**

Die Veranstaltung bietet eine Einführung in die Programmierung mit theoretischen und praktischen Einheiten. Schwerpunkte der Veranstaltung sind

- logische Grundlagen der Programmierung
- elementares Programmieren in C
- Felder, Zeiger, abgeleitete Datentypen, (Datei-)Ein- und -ausgabe
- Algorithmik
- Programmieren und Visualisieren in MATLAB/GNU Octave
- paralleles und objektorientiertes Programmieren.

Die praktischen Inhalte werden in der Programmiersprache C++ sowie in MATLAB/GNU Octave erarbeitet. Die erworbenen Kenntnisse werden anhand von Übungen erprobt und vertieft.

### Literature:

- S. Bartels, C. Palus, L. Striet: Einführung in die Programmierung für Studierende der Naturwissenschaften, Vorlesungsskript.
- G. Küveler, D. Schwoch: C/C++ für Studium und Beruf, Springer Vieweg 2017.
- M. v. Rimscha: Algorithmen kompakt und verständlich, 3. Auf''—lage, Springer Vieweg, 2017.

### Prerequisites:

none

### Remarks:

Dieser (oder ein inhaltlich äquivalenter) Kurs ist verpflichtender BOK-Kurs im B.Sc.-Studiengang Mathematik. Bitte beachten Sie im B.Sc.-Studiengang die Belegfristen des ZfS! Studierende im Zwei-Hauptfächer-Bachelor oder M.Ed. belegen den Kurs dagegen nicht über das ZfS.

### Usable in the following modules:

Computer Exercise (2HfB21, MEH21, MEB21) Elective (Option Area) (2HfB21) BOK course (BSc21) Supplementary Module in Mathematics (MEd18)

## Computer exercises in Numerics

Sören Bartels, Assistant: Vera Jackisch in German

Tutorial: 2 hours, every other week, various dates

#### **Content:**

In den begleitenden praktischen Übungen zur Vorlesung Numerik II werden die in der Vorlesung entwickelten und analysierten Algorithmen praktisch umgesetzt und experimentell getestet. Die Implementierung erfolgt in den Programmiersprachen Matlab, C++ und Python. Elementare Programmierkenntnisse werden dabei vorausgesetzt.

### Prerequisites:

Siehe bei der Vorlesung Numerik II. Zusätzlich elementare Programmierkenntnisse.

## Usable in the following modules:

Computer Exercise (2HfB21, MEH21, MEB21) Elective (Option Area) (2HfB21) Numerics (BSc21) Supplementary Module in Mathematics (MEd18)

## Computer exercises in Statistics

Sebastian Stroppel Mon, 14–16 h, SR 226, Hermann-Herder-Str. 10 in German

#### **Content:**

This computer exercise course is aimed at students who have already attended the lectures Elementary Probability Theory I and II or are attending the second part this semester. Computer-based methods will be discussed to deepen the understanding of the lecture material and demonstrate further application examples. The freely available open source statistics programme R will be used for this purpose. After an introduction to R, methods of descriptive statistics and graphical analysis of data will be considered, the numerical generation of random numbers will be explained and parametric and non-parametric tests and linear regression methods will be discussed. Previous knowledge of R and/or programming skills are not required.

## Prerequisites:

Analysis I+II, Linear Algebra I+II, Elementary Probability Theory I+II (part II can be followed in parallel).

## Usable in the following modules:

Computer Exercise (2HfB21, MEH21, MEB21) Elective (Option Area) (2HfB21) Supplementary Module in Mathematics (MEd18) Elective (MScData24)

## Computer exercises in Machine Learning

Carola Heinzel, Assistant: Samuel Adeosun Thu, 14–16 h, PC-Pool Raum -100, Hermann-Herder-Str. 10 in English

#### Content:

This course introduces the foundational concepts and practical skills necessary for understanding and implementing machine learning models, with a particular focus on deep learning and neural networks. Students will progress from basic programming skills in Python , with a focus on the PyTorch library, to advanced topics such as training multilayer perceptrons, optimization techniques, and transformer architectures. By the end of the course, participants will have the ability to implement and analyze neural networks, apply optimization strategies, and understand modern transformer-based models for tasks such as text generation and time series analysis.

### Prerequisites:

Basic knowledge in programming and basic knowledge in stochastics.

### Usable in the following modules:

Computer Exercise (2HfB21, MEH21, MEB21) Elective (Option Area) (2HfB21) Supplementary Module in Mathematics (MEd18) Elective (MSc14) Elective (MScData24)

## Computer exercises in Formal Proofs

Peter Pfaffelhuber in English

Tue, 12–14 h, SR 404, Ernst-Zermelo-Str. 1

### Content:

Lean4 is both, a programming language and an interactive theorem prover. By the latter, we mean software that is able to check mathematical proofs. It is interactive since the software tells you what remains to be proven after every line of code. The course is an introduction to this technique, with examples from various fields of mathematics. Lean4 is special since researchers all over the world are currently building a [library of mathematical theories](https://github.com/leanprover-community/mathlib4), which contains at the moment around 1.5 million lines of code. I aim to cover basics from calculus, algebra, topology and measure theory in Lean4.

#### Literature:

You can treat Lean4 as a game. If you only want to try it out, please visit the natural number game.

Other literature is:

- Mathematics in Lean4
- Theorem Proving in Lean4
- Kevin Buzzard: Formalizing Mathematics

## Prerequisites:

Analysis 1, 2, Linear algebra 1

### Usable in the following modules:

Computer Exercise (2HfB21, MEH21, MEB21) Elective (Option Area) (2HfB21) Supplementary Module in Mathematics (MEd18) Elective (MSc14) Elective (MScData24)

3a.	Undergradua	ate Seminars

## Undergraduate seminar: One-Dimensional Maximum Principle

Guofang Wang in German

Seminar: Wed, 16–18 h, SR 125, Ernst-Zermelo-Str. 1

Preliminary Meeting 05.02., 16:00, SR 125, Ernst-Zermelo-Str. 1

Preparation meetings for talks: Dates by arrangement

#### Content:

Das Proseminar behandelt eindimensionale Maximumprinzipien. Das Prinzip basiert auf den notwendigen Bedingungen für Extremstellen. Nimmt eine zweimal differenzierbare Funktion  $f:(a,b)\to\mathbb{R}$  an einem Punkt  $x_0\in\mathbb{R}$  ein lokales Maximum an, so erfüllt die erste Ableitung  $f'(x_0)=0$  und die zweite Ableitung  $f''(x_0)\leqslant 0$ . Dies impliziert, dass die Funktion f ihr Maximum am Rand  $\partial(a,b)=\{a,b\}$  des Intervalls annehmen muss, falls man weiß, dass f''>0 in ganz (a,b) gilt. Diese Schlussfolgerung nennt man in der Theorie der partiellen Differentialgleichungen das schwache Maximumprinzip. In dem Proseminar verwenden wir es hauptsächlich für gewöhnliche Differentialgleichungen.

#### Literature:

Murray H. Protter, Hans F. Weinberger:

#### Prerequisites:

Analysis I and II

## Usable in the following modules:

## Undergraduate seminar: Infinte Dimensional Vector Spaces

Susanne Knies in German

Seminar: Thu, 14-16 h, SR 404, Ernst-Zermelo-Str. 1

Preregistration:

until 30.01. to Vivien Vogelmann

Preliminary Meeting 04.02., 12:00, SR 318, Ernst-Zermelo-Str. 1

Preparation meetings for talks: Dates by arrangement

#### **Content:**

The seminar deals with statements that are known from finite-dimensional vector spaces but no longer apply in the infinite-dimensional case. What are the consequences of this? What applies instead? With which additional conditions can one possibly save oneself?

For more detailed information see the webpage!

## Prerequisites:

Analysis I, II and Linear Algebra I, II

#### Remarks:

Das Proseminar ist insbesondere für Studierende im Polyvalenten Zwei-Hauptfächer-Bachelor-Studiengang geeignet!

## Usable in the following modules:

## Undergraduate seminar: Lattice Theory

Markus Junker in German

Seminar: Mon, 14–16 h, SR 127, Ernst-Zermelo-Str. 1

Preregistration:

until 31.01. to Markus Junker

Preliminary Meeting 07.02., 11:15, SR 318, Ernst-Zermelo-Str. 1

Preparation meetings for talks: Dates by arrangement

#### Content:

Verbände sind ähnlich grundlegende mathematische Strukturen wie Ordnungen oder Gruppen. Ein Verband ist eine Menge mit zwei assoziativen und kommutativen Verknüpfungen  $\cap$  und  $\cup$ , die die Absorptionsgesetze  $a \cap (a \cup b) = a$  und  $a \cup (a \cap b) = a$  erfüllen. Zum Beispiel bilden die Teilmengen einer festen Menge einen Verband; oder die Untervektorräume eines festen Vektorraums, wenn man für  $\cup$  den von der mengentheoretischen Vereinigung erzeugten Untervektorraum bildet. Verbände mit speziellen Zusatzeigenschaften sind beispielsweise Boole'sche Algebren,

Im Proseminar wollen wir uns einerseits anschauen, was man grundlegend über beliebige Verbände sagen kann, und dann einige Ergebnisse über speziellere Verbände.

#### Literature:

- Hans Hermes: Einführung in die Verbandstheorie, 2. Auf"—lage, Springer 1967.
- Garrett Birkhoff: Lattice Theory, AMS.

Weitere Literatur wird dann zu den einzelnen Vorträgen angegeben.

## Prerequisites:

Linear Algebra I and II, Analysis I

## Usable in the following modules:

## Undergraduate seminar: Counter-Examples in Probability Theory

David Criens Seminar

The talks will take place as blocks in the middle of the semester; the exact dates will be agreed in the preliminary discussion.

Preliminary Meeting 06.02., 10:00, Raum 232, Ernst-Zermelo-Str. 1, further meeting on 23.04.2025, 10:00; in SR 232

#### **Content:**

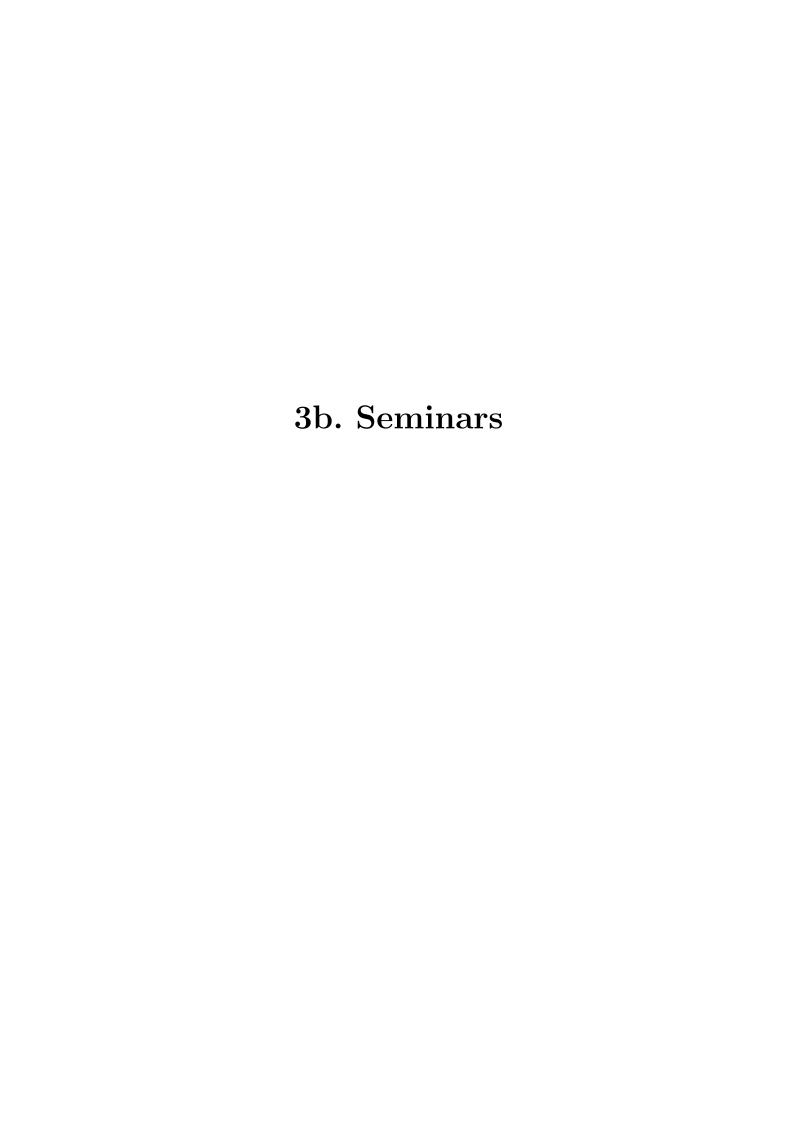
It is not only theorems, proofs or illustrative examples, but also counterexamples that show the depth and beauty of a theory. Natural questions are: (a) are the requirements of a theorem necessary and not only sufficient; (b) are the requirements sufficient and not just necessary; (c) is an implication an equivalence, i.e. does the implication in the other direction also hold.

In this undergraduate seminar we deal with counter-examples from probability theory. Possible topics range from classic questions such as measurability, independence of random variables, expectations or conditional probabilities, to more advanced topics such as limit value rates, martingals or Markov processes. A suitable topic can be found for any interested student.

## Prerequisites:

Elementary Probability Theory I (topics from probability theory I-III can also be assigned)

## Usable in the following modules:



## Seminar: Approximation Properties of Deep Learning

Diyora Salimova Talk/participation possible in German and English

Seminar: Mon, 12-14 h, online,

Preregistration:

by e-mail to Diyora Salimova Preliminary Meeting 14.04., 15:00,

via zoom (please write the lecturer in case the time slot does not fit you)

Preparation meetings for talks: Dates by arrangement

#### **Content:**

In recent years, deep learning have been successfully employed for a multitude of computational problems including object and face recognition, natural language processing, fraud detection, computational advertisement, and numerical approximations of differential equations. Such simulations indicate that neural networks seem to admit the fundamental power to efficiently approximate high-dimensional functions appearing in these applications.

The seminar will review some classical and recent mathematical results on approximation properties of deep learning. We will focus on mathematical proof techniques to obtain approximation estimates on various classes of data including, in particular, certain types of PDE solutions.

## Prerequisites:

Basics of functional analysis, numerics of differential equations, and probability theory

## Usable in the following modules:

## Seminar on representation theory

Wolfgang Soergel, Assistant: Damian Sercombe Talk/participation possible in German and English

Seminar: Thu, 10–12 h, SR 125, Ernst-Zermelo-Str. 1

Preregistration:

by e-mail to hrefmailto:wolfgang.soergel@math.uni-freiburg.deWolfgang Soergel

Preliminary Meeting 28.01., 14:15, SR 127, Ernst-Zermelo-Str. 1

Preparation meetings for talks: Dates by arrangement

#### Content:

This seminar is intended to introduce to the theory of linear algebraic groups. Linear algebraic groups are generalizations of the matrix groups known from linear algebra.

I imagine a format in which I or Sercombe lecture and in between the seminar participants give talks. The seminar is a reasonable addition to the commutative algebra lecture. Reference to that lecture increase in the course of the seminar.

#### Literature:

• A. Borel: Linear Algebraic Groups

 $\bullet~$  J. Humphreys: Linear Algebraic Groups

• T. A. Springer: Linear Algebraic Groups

• W. Soergel *Lecture Notes* (partially)

## Prerequisites:

Algebra and Number Theory (where the details of Galois theory and field theory are less relevant than the general theory of groups and rings) and Linear Algebra.

### Usable in the following modules:

## Seminar: Geometric Analysis

Ernst Kuwert Seminar: Tue, 14–16 h, SR 125, Ernst-Zermelo-Str. 1 Preliminary Meeting 04.02., 12:15, SR 218, Ernst-Zermelo-Str. 1

Preparation meetings for talks: Dates by arrangement

#### Content:

Thema des Seminars ist der curve shortening flow. Danach bewegt sich eine geschlossene Kurve c im  $\mathbb{R}^2$  nach dem Gesetz

$$\frac{\partial c}{\partial t} = \varkappa \nu.$$

Als Hauptergebnis wollen wir zeigen, dass eine eingebettete Kurve c nach endlicher Zeit auf einen Punkt kontrahiert und dabei asymptotisch rund wird (Satz von Grayson). Dies ist erstaunlich, da eingebettete Kurven in der Ebene sehr kompliziert sein können.

Als Techniken werden wir u.a. das Maximumprinzip, eine Monotonieformel, Krümmungsabschätzungen, Blow-up-Argumente kennenlernen. Der *curve shortening flow* ist ein einfacher Prototyp für Evolutionsgleichungen mit zentralen Anwendungen in der Geometrie.

#### Literature:

- Robert Haslhofer: Lectures on curve shortening flow, 2016,
- Ben Andrews et al.: Extrinsic geometric flows, AMS Graduate Studies in Mathematics.

## **Prerequisites:**

Analysis I-III

#### Remarks:

The seminar can be followed by Bachelor theses.

## Usable in the following modules:

## Seminar: Principal Fiber Bundles, Holonomy, and Characteristic Classes

Nadine Große, Assistant: Maximilian Stegemeyer Talk/participation possible in German and English

Seminar: Tue, 8–10 h, SR 404, Ernst-Zermelo-Str. 1

Preliminary Meeting 04.02., 10:00, SR 318, Ernst-Zermelo-Str. 1

Preparation meetings for talks: Dates by arrangement

#### **Content:**

Auf einer Riemannschen Mannigfaltigkeit (M,g) kann durch Paralleltransport entlang einer Kurve eine Isometrie zwischen den Tangentialräumen an verschiedenen Punkten gefunden werden. Beschränkt man sich auf geschlossene Kurven, so erhält man eine Gruppe von linearen Isometrien des Tangentialraums eines Punktes. Diese Gruppe hängt bis auf Isomorphismus nur von der Riemannschen Metrik und der Mannigfaltigkeit – nicht aber vom gewählten Punkt – ab. Man bezeichnet diese Gruppe als die Holonomie-Gruppe von (M,g). Die Holonomie-Gruppe enthält wichtige Informationen über die Metrik und über zusätzliche geometrische Strukturen der Mannigfaltigkeit.

Im ersten Teil dieses Seminars wollen wir das Konzept der Holonomiegruppe verstehen. Dafür werden wir Zusammenhänge auf Hauptfaserbündeln benutzen und zunächst noch allgemeiner den Begriff der Holonomiegruppe eines Zusammenhangs auf einem Hauptfaserbündel betrachten.

Mit den erlernten Methoden über Zusammenhänge auf Hauptfaserbündeln lassen sich dann auch *charakteristische Klassen* behandeln. Dies sind Kohomologieklassen in der de-Rham-Kohomologie einer Mannigfaltigkeit, die für ein gegebenes Vektorbündel über der Mannigfaltigkeit konstruiert werden können. Im letzten Teil des Seminars werden wir daher auch diese charakteristischen Klassen, ihre Konstruktion und deren Anwendungen kennen lernen.

## Prerequisites:

Differential Geometry I

### Usable in the following modules:

## Seminar: Mathematics without the Axiom of Choice

Heike Mildenberger, Assistant: Maxwell Levine Talk/participation possible in German and English

Seminar: Tue, 16–18 h, SR 127, Ernst-Zermelo-Str. 1

Preliminary Meeting 29.01., 13:15, Room 313, Ernst-Zermelo-Str. 1

Preparation meetings for talks: Dates by arrangement

#### Content:

Das Auswahlaxiom gehört zu den akzeptierten unbeweisbaren Grundannahmen. Es sagt, dass jede Menge M nicht leerer Mengen eine Auswahlfunktion hat, das ist eine Funktion  $f:M\to\bigcup M$  mit der Eigenschaft  $\forall x\in M,\, f(x)\in x.$  Auf der Basis der anderen Axiome ZF von Zermelo und Fraenkel gibt es zahlreiche zum Auswahlaxiom äquivalente Aussagen, zum Beispiel die Wohlordenbarkeit jeder Menge und das Lemma von Zorn. Wir studieren in diesem Seminar Modelle der Axiome ZF, in denen das Auswahlaxiom explizit negiert wird. Am Anfang stehen Modelle zum Beweis des folgenden Satzen von Cohen aus dem Jahre 1963: Wenn ZF konsistent ist, so auch ZF und das Negat von AC. Ein Jahr später zeigte Solovay: Es gibt ZF-Modelle, in denen jede Teilmenge der reellen Zahlen Lebesgue-messbar ist, es also keine Vitali-Menge gibt. Zwanzig Jahre später fand man: Eine stark unerreichbare Kardinalzahl ist zur Konstruktion eines solchen Modells unerlässlich. Zahlreiche Fragen nach Abstufungen und besonderen Formen der Negation des Auswahlaxioms sind offen.

#### Literature:

- Thomas Jech: Set Theory, The Third Millenium Edition, revised and expanded, Springer, 2003.
- Lorenz Halbeisen: Combinatorial Set Theory, Springer, 2012.
- Jacques Raisonnier, Jacques Stern: *The strength of measurability hypotheses*, Israel Journal of Mathematics 50, 1985, 337-349.
- Ralf Schindler: Set theory, Exploring independence and truth, Springer, 2014.

### Prerequisites:

Set Theory

#### Usable in the following modules:

## Seminar: Medical Data Science

Harald Binder Talk/participation possible in German and English

Seminar: Wed, 10–12 h, HS Medizinische Biometrie, 1. OG, Stefan-Meier-Str. 26

Preregistration:

to bemb.imbi.sek@list.uniklinik-freiburg.de

Preliminary Meeting 28.01., 11:30–12:30, HS Medizinische Biometrie, 1. OG, Stefan-Meier-Str. 26

#### **Content:**

To answer complex biomedical questions from large amounts of data, a wide range of analysis tools is often necessary, e.g. deep learning or general machine learning techniques, which is often summarized under the term "Medical Data Science". Statistical approaches play an important rôle as the basis for this. A selection of approaches is to be presented in the seminar lectures that are based on recent original work. The exact thematic orientation is still to be determined.

#### Literature:

Information on introductory literature is given in the preliminary meeting.

## Prerequisites:

Good knowledge of probability theory and mathematical statistics.

#### Remarks:

The seminar can serve as basis for a bachelor's or master's thesis.

## Usable in the following modules:

## Seminar: Numerics of Partial Differential Equations

Sören Bartels, Assistant: Vera Jackisch, Tatjana Schreiber Seminar: Mon, 12–14 h, SR 226, Hermann-Herder-Str. 10

Preliminary Meeting 04.02., 12:00, Room 209,

Or registration by e-mail to Sören Bartels

Talk/participation possible in German and English

#### **Content:**

The seminar will cover advanced topics in the theory and numerics of partial differential equations. This includes the iterative solution of the resulting linear systems of equations with multigrid and domain decomposition methods, the adaptive refinement of finite element grids, the derivation of an approximation theory with explicit constants, and the solution of nonlinear problems.

#### Literature:

- S. Bartels: Numerical Approximation of Partial Differential Equations, Springer, 2016.
- D. Braess: Finite Elements, Cambridge University Press, 2007.
- S. Brenner, R. Scott: The Mathematical Theory of Finite Element Methods, Springer, 2008.
- M. Dobrowolski: Angewandte Funktionalanalysis, Springer, 2010.

#### Prerequisites:

Introduction to Theory and Numerics of Partial Differential Equations

## Usable in the following modules:

## Seminar on p-adic Geometry

Abhishek Oswal, Assistant: Ben Snodgrass

Seminar: Mon, 10–12 h, SR 404, Ernst-Zermelo-Str. 1

Preliminary Meeting 13.02., 14:30, SR 404, Ernst-Zermelo-Str. 1

in English

#### **Content:**

It has become clear over the last several decades that p-adic techniques play an indispensable role in arithmetic geometry. At an elementary level, p-adic numbers provide a compact and convenient language to talk about congruences between integers. Concretely, just as the field of real numbers  $\mathbb{R}$  arise as the completion of the field  $\mathbb{Q}$  of rational numbers with respect to the usual notion of distance on  $\mathbb{Q}$ , the field  $\mathbb{Q}_p$  of p-adic numbers arise as the completion of  $\mathbb{Q}$  with respect to an equally natural p-adic metric. Roughly, in the p-adic metric, an integer n is closer to 0, the larger the power of the prime number p that divides it. A general philosophy in number theory is then to treat all these completions  $\mathbb{R}$ ,  $\mathbb{Q}_p$  of the field  $\mathbb{Q}$  on an equal footing. As we shall see in this course, familiar concepts from real analysis (i.e. notions like analytic functions, derivatives, measures, integrals, Fourier analysis, real and complex manifolds, Lie groups...), have completely parallel notions over the p-adic numbers.

While the Euclidean topology of  $\mathbb{R}^n$  is rather well-behaved (so one may talk meaningfully about paths, fundamental groups, analytic continuation, ...), the p-adic field  $\mathbb{Q}_p$  on the other hand is totally disconnected. This makes the task of developing a well-behaved notion of global p-adic analytic manifolds/spaces rather difficult. In the 1970s, John Tate's introduction of the concept of rigid analytic spaces, solved these problems and paved the way for several key future developments in p-adic geometry.

The broad goal of this course will be to introduce ourselves to this world of p-adic analysis and rigid analytic geometry (due to Tate). Along the way, we shall see a couple of surprising applications of this circle of ideas to geometry and arithmetic. Specifically, we plan to learn Dwork's proof of the fact that the zeta function of an algebraic variety over a finite field is a rational function.

### Literature:

- Neal Koblitz: p-adic numbers, p-adic analysis, and zeta-functions.
- Jean-Pierre Serre: A Course in arithmetic.
- Siegfried Bosch: Lectures on formal and rigid geometry.
- John Tate: Rigid analytic spaces.
- Bernard Dwork: On the rationality of the zeta function of an algebraic variety over a finite field.

### Prerequisites:

Field theory, Galois theory and Commutative algebra.

Some willingness to accept unfamiliar concepts as black boxes. Prior experience with algebraic number theory, or algebraic geometry will be beneficial but not necessary.

## Usable in the following modules:

## Seminar: The Wiener Chaos Decomposition and (Non-)Central Limit Theorems

Angelika Rohde

Talk/participation possible in German and English

Seminar

planned as a block seminar in the Black Forest, probably in the 2nd half of July

Preregistration:

by e-mail to Gabriele Bellerino

Preliminary Meeting 30.01., 14:00, Raum 232, Ernst-Zermelo-Str. 1

Preparation meetings for talks: Dates by arrangement

#### **Content:**

Wohingegen lineare Transformationen von Gaußprozessen ihre gaußsche Eigenschaft bewahren, gilt dies für nichtlineare Funktionale, beispielsweise additive Funktionale der Form

$$\int_0^T f(X_s)ds \qquad oder \qquad \sum_{k=1}^n f(X_{k/n}),$$

im Allgemeinen nicht. Die Wiener-Chaos-Zerlegung bietet einen Rahmen zur Analyse nichtlinearer Funktionale von Gaußprozessen. Es handelt sich hierbei um eine orthogonale Zerlegung des Raumes

$$L^2(\mathbb{P}) = \bigoplus_{k=1}^{\infty} \mathcal{H}_k$$

der bezüglich  $\mathbb{P}$  quadratintegrierbaren Zufallsvariablen, wobei  $\mathbb{P}$  ein gaußsches Wahrscheinlichkeitsmaß ist. Dieses Konzept verallgemeinert dabei die Eigenschaften orthogonaler Polynome bezüglich eines Wahrscheinlichkeitsmaßes auf der reellen Achse auf ein (potentiell) unendlichdimensionales Szenario. Es stellt sich heraus, dass Elemente eines Wiener-Chaos  $\mathcal{H}_k$  als mehrfache Wiener-Itô-Integrale dargestellt werden können, welche wiederum gut verstandene Objekte sind.

In diesem Seminar werden wir die grundlegenden Eigenschaften des Wiener-Chaos untersuchen, beginnend mit der Hermite-Polynombasis. Anschließend wenden wir uns fortgeschrittenen Themen wie Anwendungen im Malliavin-Kalkül zu, einem unendlichdimensionalen Differential-Kalkül auf gaußschen Wahrscheinlichkeitsräumen (stochastische Variationsrechnung). Des Weiteren werden zentrale und nichtzentrale Grenzwertsätze für nichtlineare Funktionale von gaußschen und nicht-gaußschen Prozessen sowie Invarianzprinzipien behandelt.

## Prerequisites:

Notwendige Vorkenntnisse bestehen nur aus Kenntnissen der Wahrscheinlichkeitstheorie I.

Für einige Vorträge sind Vorkenntnisse der Wahrscheinlichkeitstheorie II (Stochastische Prozesse) nützlich. Ihre individuellen Vorkenntnisse können bei der Vergabe der Themen jedoch selbstverständlich berücksichtigt werden.

#### Usable in the following modules:

## Seminar: Data-Driven Medicine from Routine Data

 $Nadine\ Binder$ 

Talk/participation possible in German and English

Seminar

The weekly seminar date is still to be determined!

Preregistration:

by e-mail to Nadine Binder

Preliminary Meeting 06.02., 13:30, HS Medizinische Biometrie, 1. OG, Stefan-Meier-Str. 26,

Alternative date: 9 April 2025, same time and room

#### **Content:**

Imagine being able to use routine data such as diagnoses, lab results, and medication plans to answer medical questions in innovative ways and improve patient care. In this seminar, we will learn to identify relevant data, understand suitable analysis methods, and what to consider when applying them in practice. Together, we will analyze scientific studies on routine data and discuss clinical questions, the methods used, and their feasibility for implementation.

What makes this seminar special: Medical and mathematics students collaborate to understand scientific studies from both perspectives. When possible, you will work in pairs (or individually if no pair can be formed) to analyze a study from your respective viewpoints and prepare related presentations. You may test available programming code or develop your own approaches to replicate the methods and apply them to your own questions. The pairs can be formed during the preliminary meeting.

## Prerequisites:

necessary: Basics in Appiled Mathematics

useful: Probability Theory I

### Usable in the following modules:

Mathematical Seminar (MScData24) Elective in Data (MScData24)

# Graduate Student Speaker Series

Tue, 14–16 h, SR 226, Hermann-Herder-Str. 10

in English

## Content:

More information will follow at the end of January!

## Usable in the following modules:

Graduate Student Speaker Series (MScData24)