

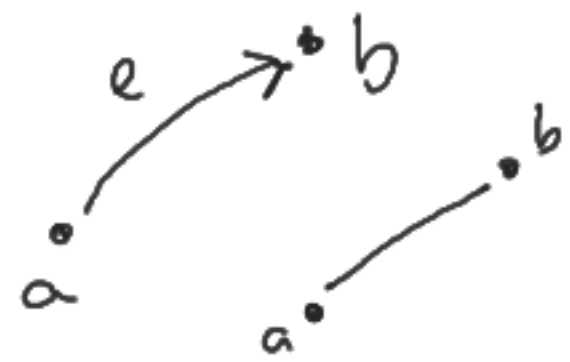
1) Shortest path

2) Minimum Spanning Tree

$$G = (V, E)$$

V = conjunto de vértices (o nodos)

E = conjunto de aristas $\subseteq V \times V$



$e = (a, b)$
 $(a, b), (b, a)$

$$n = |V| \quad m = |E|$$

Formas de Representar Grafos

1) Matriz de adyacencia

$$A_G = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \dots & & & & & \\ & & & & & 0 \end{bmatrix}_{n \times n} \quad O(n^2)$$

G no dirigido A es simétrica

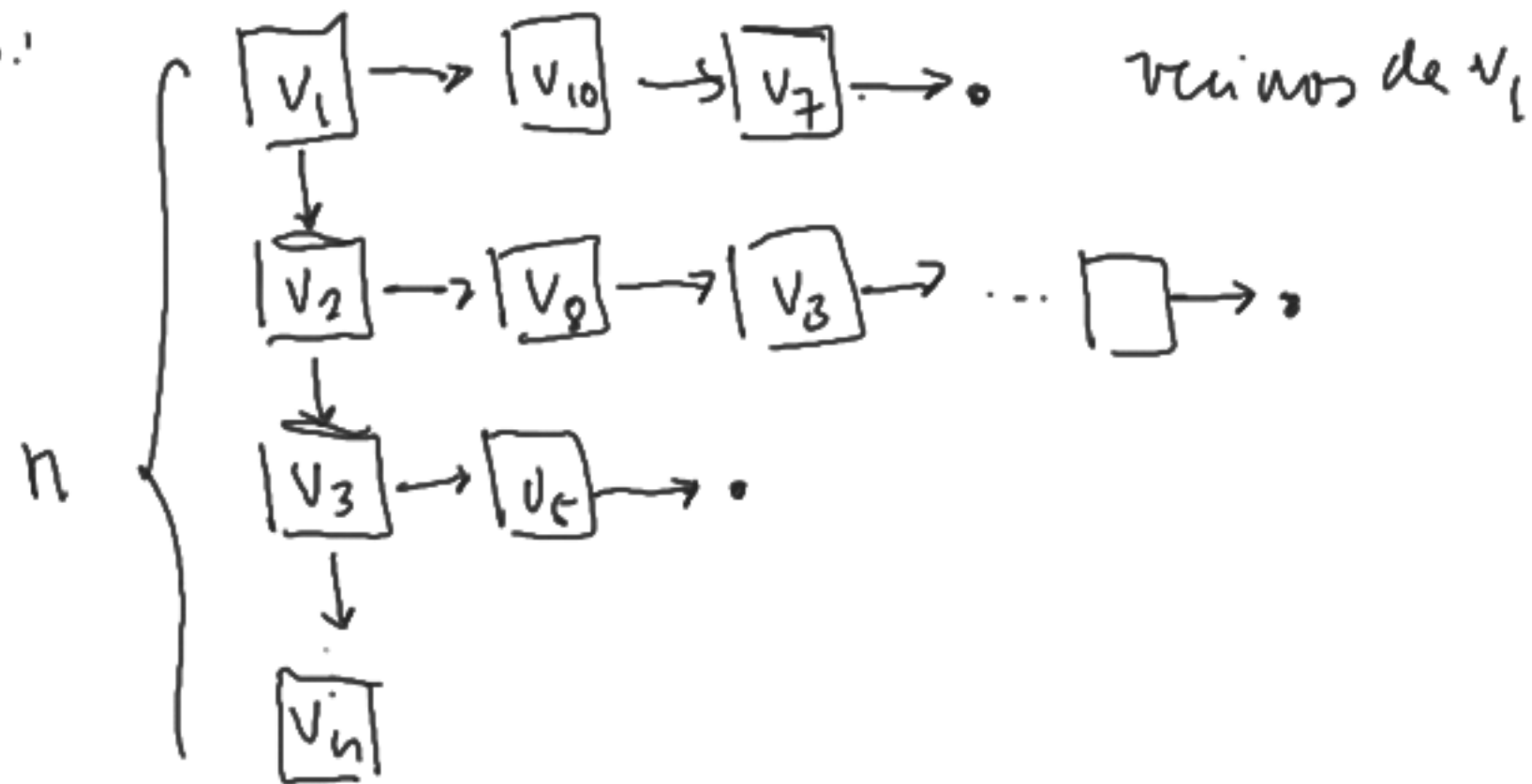
G es dirigido A no es simétrica

2) Matriz de Incidencia

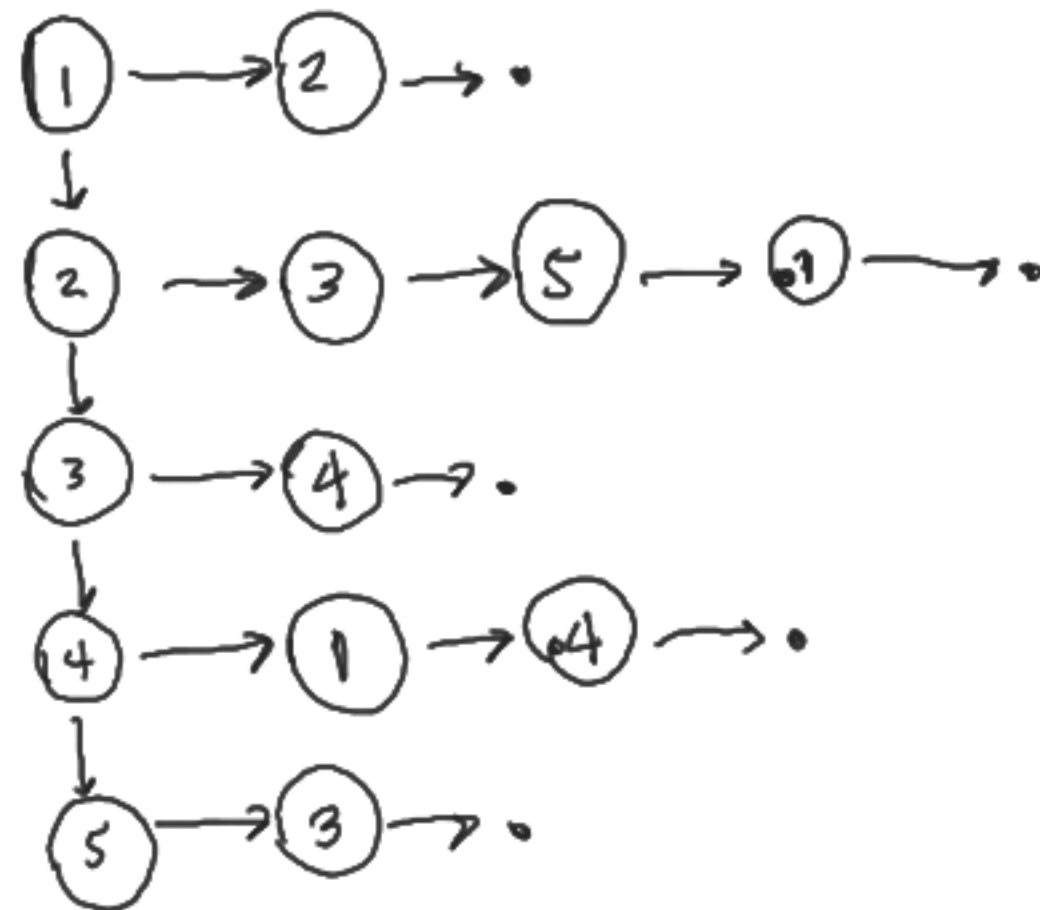
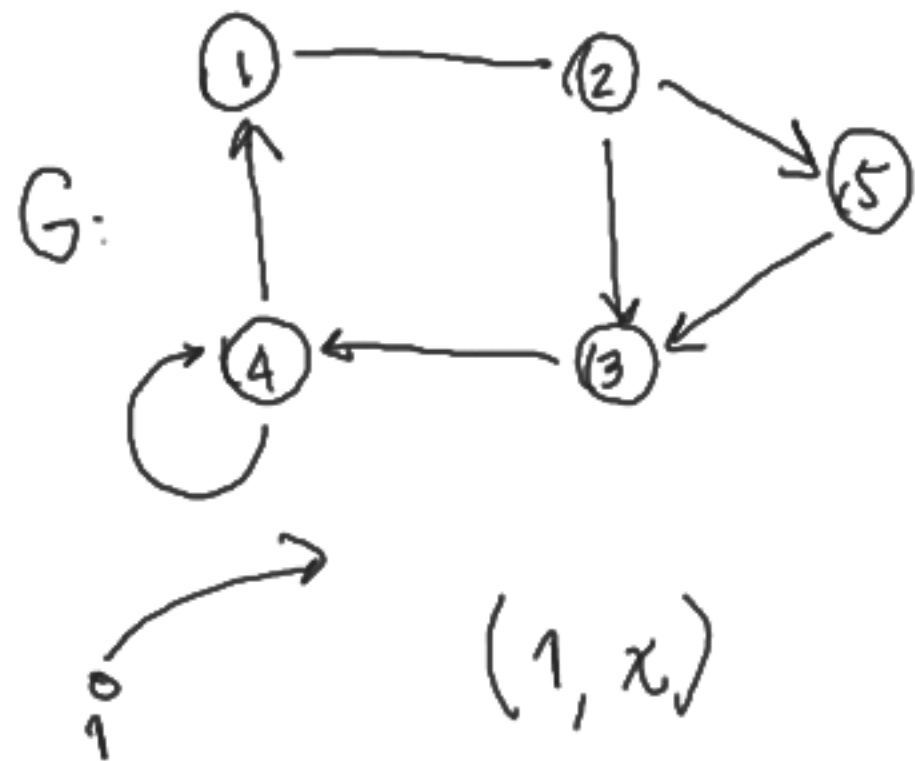
$$I_G = \begin{bmatrix} \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{matrix} & | & | & | & | & | & 1 & 0 \end{bmatrix}_{m \times n} \quad O(mn)$$

e_j

3) Lista de Listas:



Ejemplo:

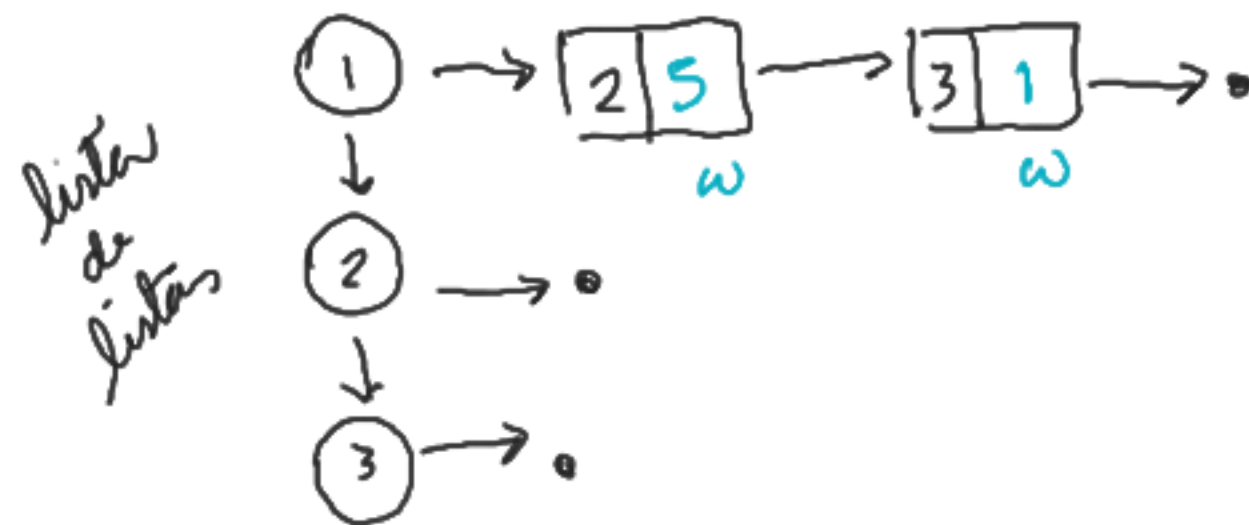
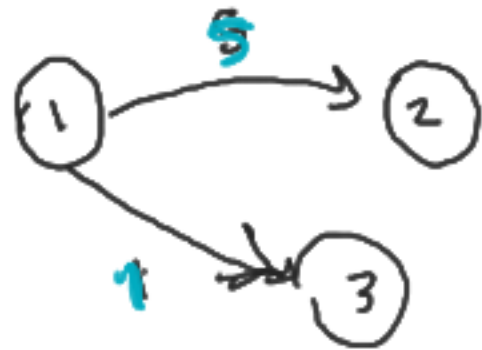


Grafo ponderado o grafo con pesos
(Weighted graphs)

$$G = (V, E, w)$$

$$w: E \rightarrow \mathbb{R}$$

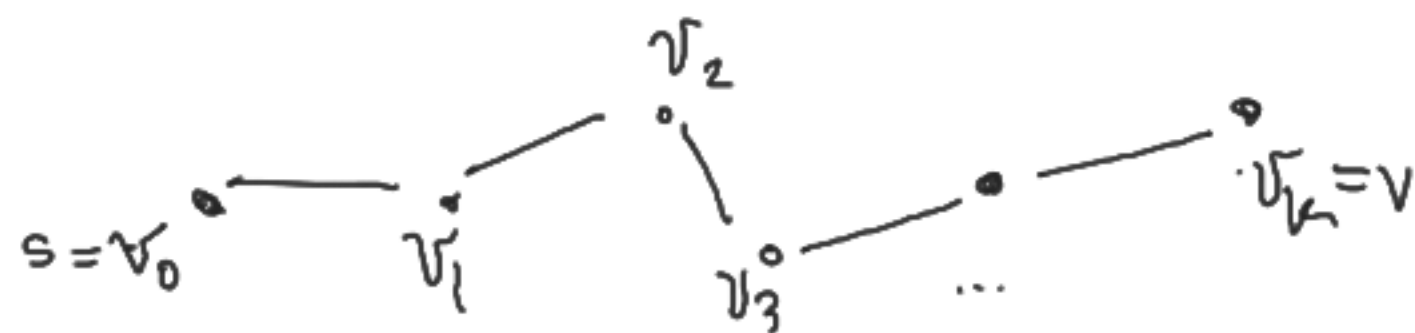
distancia, tiempo, flujo, costo, ...



1) Shortest Path:

Dado $G=(V,E,d)$ y dado $s \in V$ (source).

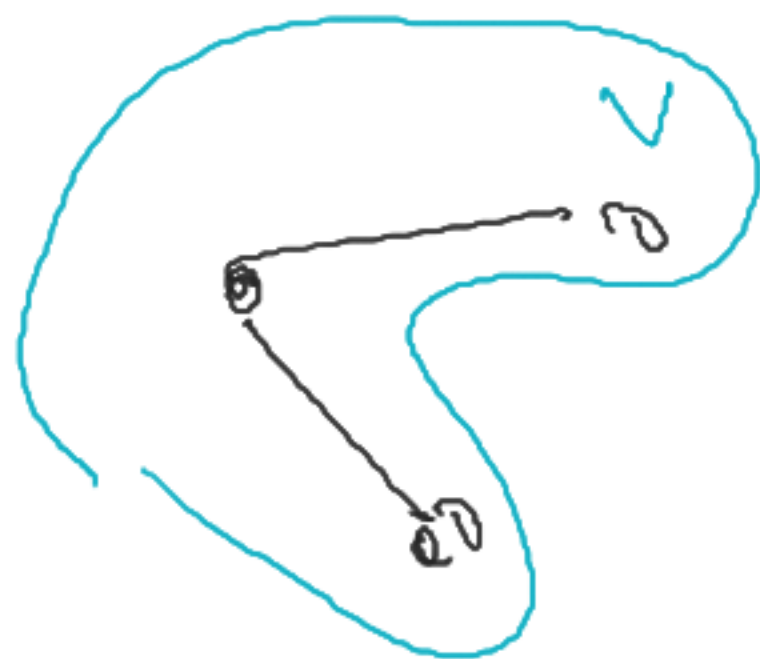
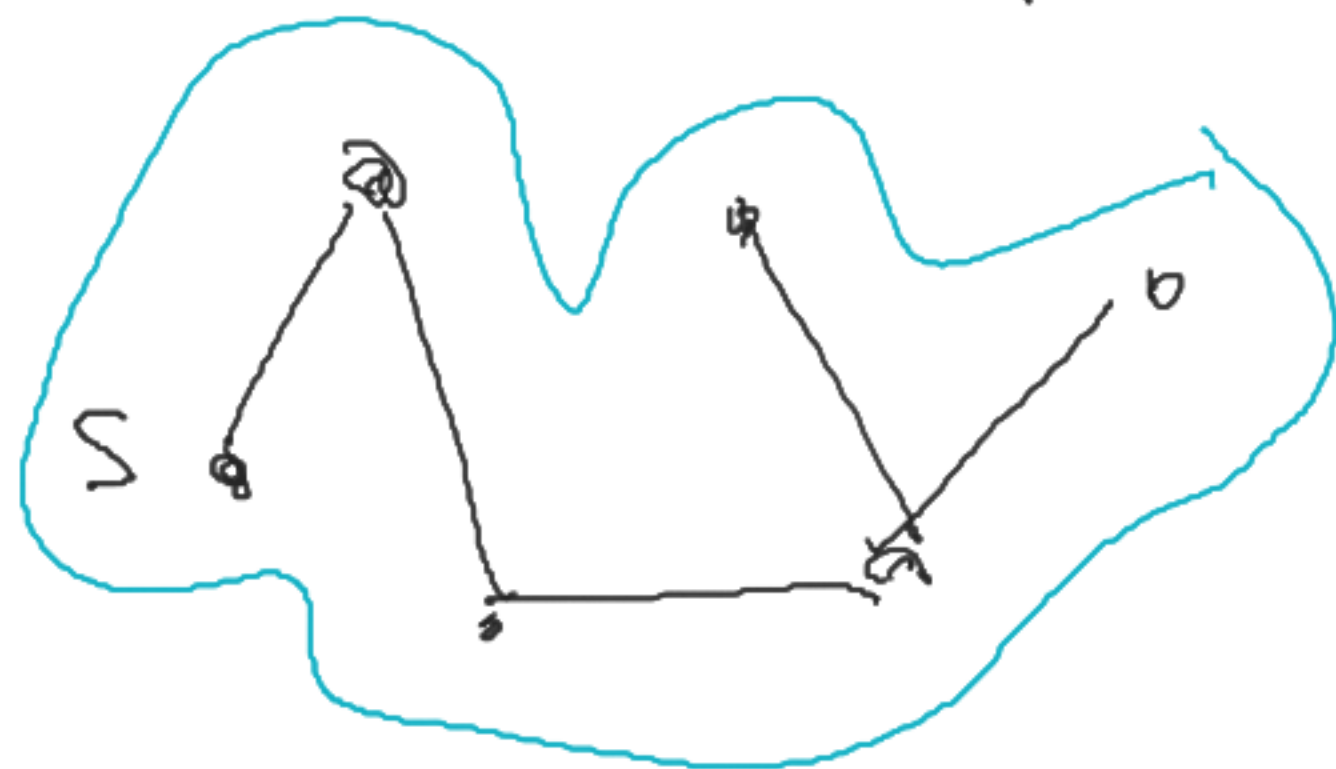
queremos hallar la ruta de menor distancia de s a $v \in V$.



$$d(\gamma) = \sum_{i=1}^k d(v_{i-1}, v_i)$$

camino $\gamma = \{ (v_0, v_1), (v_1, v_2), \dots, (v_{k-1}, v_k) \} \subseteq E$ con

$v_0 = s$ y $v_k = v$



G es conexo

Dijkstra: (1959). $O(n^2) = O(V^2)$

dist =

∞	∞	∞	...	∞
v_1	v_2	v_3	...	v_n

prev =

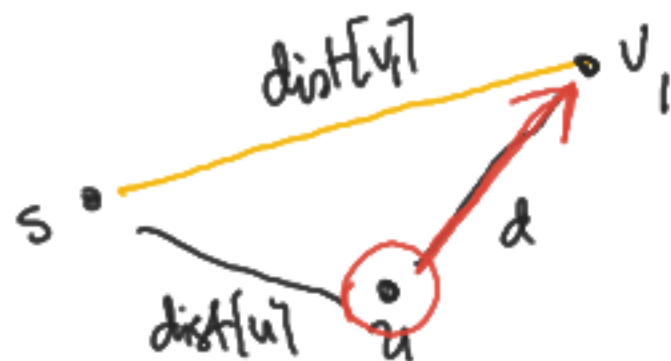
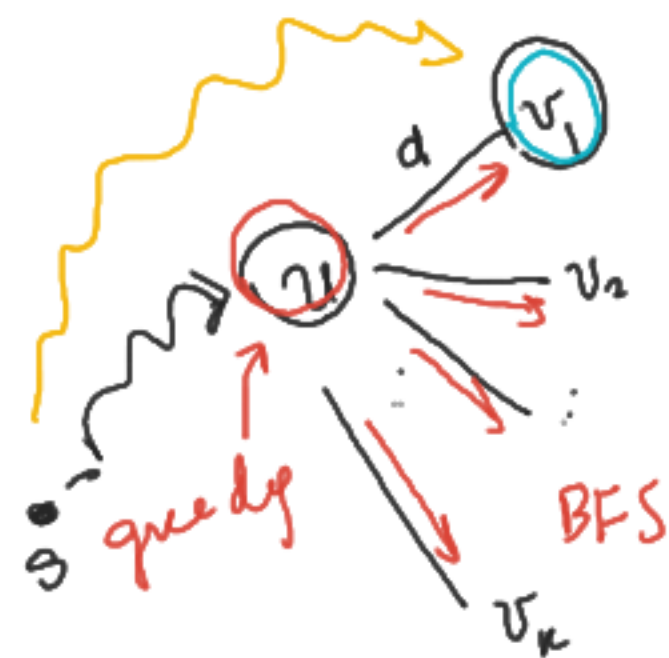
-	-	-	...	-
v_1	v_2	v_3	...	v_n

$d(s, v_i)$



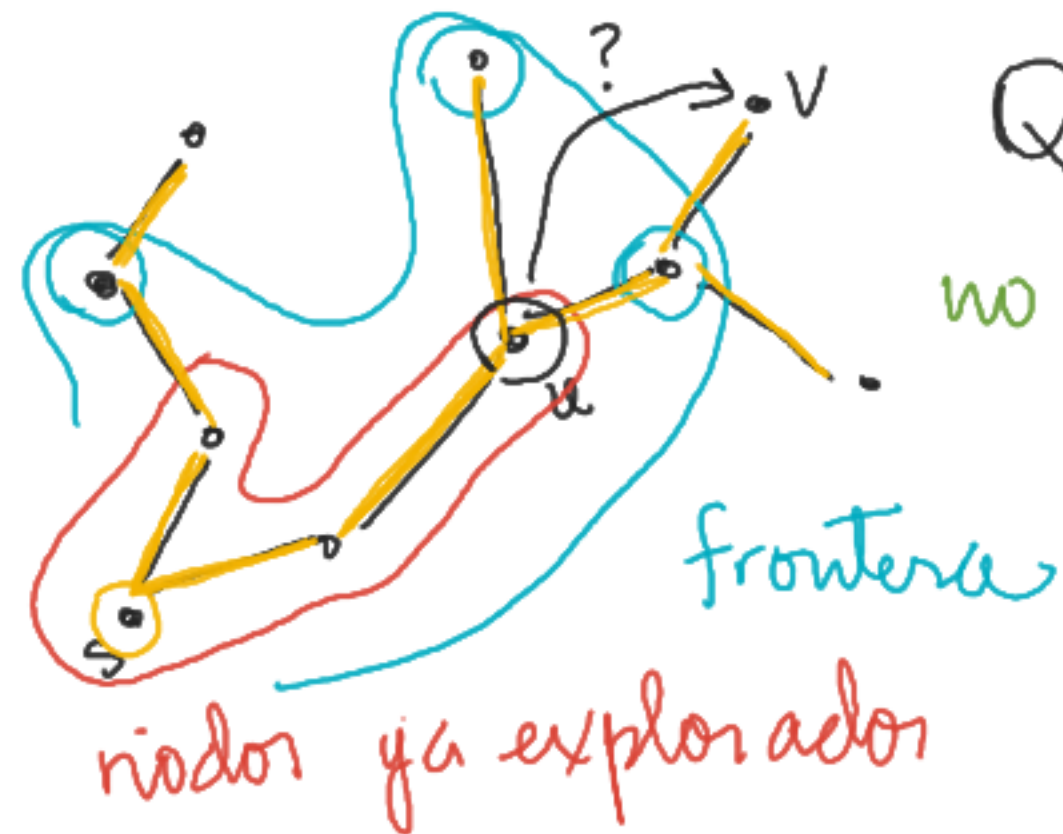
heap
Fibonacci heap
Bucket heap

$$alt = dist[u] + d(u, v_i)$$

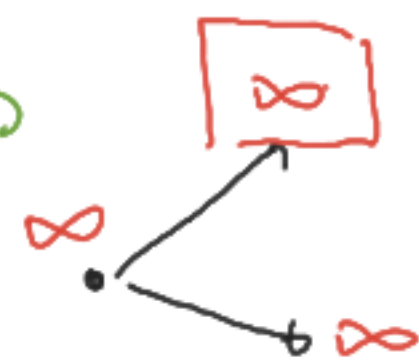


$$dist[v_i] \leftarrow alt$$

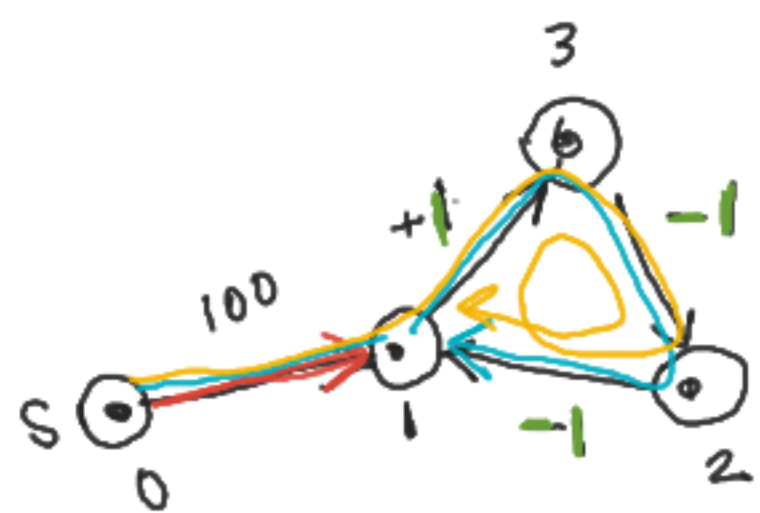
$$prev[v_i] \leftarrow u$$



no explorados



no informados
si informados



$$\sum d = +1 - 1 - 1 = -1$$

ciclo con peso negativo.

$$d = 100$$

$$d = 100 + \textcircled{+1 - 1 - 1} = 99$$

$$d = 98$$

$$d = 97 \dots$$

$$\begin{matrix} 0 \\ -1 \\ -2 \\ \vdots \end{matrix} \downarrow \rightarrow \infty$$

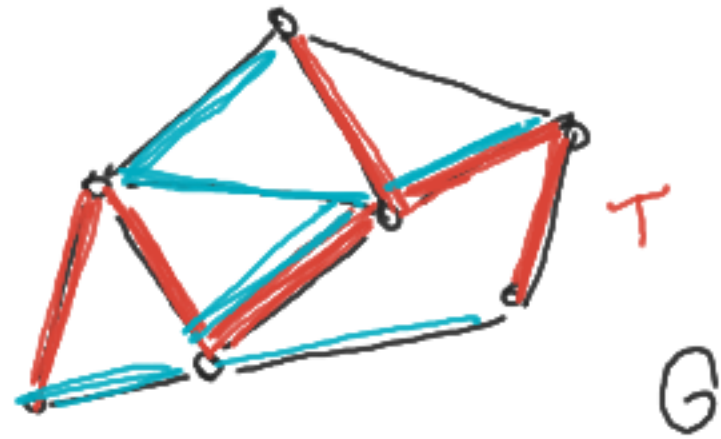
2) Minimum Spanning Tree:

$G = (V, E, d)$ no-dirigido y conexo.

Queremos hallar el árbol $T \subseteq G$ con $\sum_{e \in T} d(e)$ es mínima.

T árbol

- conexo ✓
- no ciclos ✓



$$T = (V, E_T)$$

$$E_T \subseteq E.$$

Prim: (Jarník '56, Prim '57, Dijkstra 59)

Kruskal ('56)

Borůvka ('26)