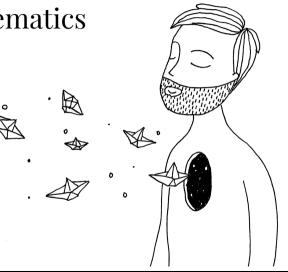
4509 - Bridging Mathematics

Sets

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A **set** is a collection of **elements**. We say that element a belongs to the set A, if a is contained in A. We denote that: $a \in A$. If a does not belong to A, we write $a \notin A$.

There are two very important sets: The Universal set \mathcal{U} and the empty set \emptyset .



 ${\cal U}$ is defined as the set that contains all the ${\it relevant}$ elements.

 \emptyset is defined as the set that does not contain any element.



Definition

Two sets A and B are said to be **equal** if

$$\forall x, x \in A \Leftrightarrow x \in B$$

Definition

B is a **subset** (\subseteq) of A if:

$$\forall b \in B \Rightarrow b \in A$$



Quick Quiz - 5 minutes

Show that if A and B are equal, then $A \subseteq B$ and $B \subseteq A$.



Definition

The **power** of a set is the set of all its subsets.

$$\mathcal{P}(A) := \{B|B \subseteq A\}$$

Note that the elements of A do not belong to $\mathcal{P}(A)$, however, the sets containing a single element do. In other words the **set** $\{a\}$ is different from the **element** a.

Definition

The **cardinality** of a set is the number of elements it contains. It is denoted as #A.



Let's consider an example. Let $A = \{1, 2, 3\}$.

- \blacksquare #A = 3, and # $\mathcal{P}(A) = 8$.

In general, if #A is finite, then $\#\mathcal{P}(A) = 2^{\#A}$.



Definition

A set A is to be called finite, if #A is finite, and infinite otherwise.

Consider the set \mathbb{N} , the natural numbers. $\#\mathbb{N}=\infty$. This infinite set is special, because you can count it, from zero to infinity. We call this cardinality *aleph zero*, \aleph_0 , and the set is **countable**.

Now consider the set \mathbb{R} , which you know as the real numbers. Again, $\#\mathbb{R} = \infty$. This cardinal is called *continuum*, and denoted with c. The sets with this cardinality are considered **uncountable**.



Quick Quiz - 5 minutes

Classify the cardinality of the following sets:

Set	Cardinality
$\mathbb Z$	\aleph_0
$\mathbb Q$	\aleph_0
$f(x) = a + bx, x \in \mathbb{R}^+$	С



Definition

Consider A and B to be subsets of \mathcal{U} .

1. The union between A and B:

$$A \cup B := \{c | c \in A \lor c \in B\}$$

2. The intersection between A and B:

$$A \cap B := \{c | c \in A \land c \in B\}$$

3. The difference between A and B:

$$A \setminus B := \{c | c \in A \land c \notin B\}$$

4. The complement of A,



If two sets (A and B) are such that there is a valid operation between their elements, say "ope", then:

A ope
$$B := \{c | c = a \text{ ope } b, a \in A b \in B\}$$

Example: $A = \{1, 2\}$, $B = \{3, 4, 7\}$,

- $A + B = \{4, 5, 6, 8, 9\}$
- $A B = \{-6, -5, -3, -2, -1\}$



Definition

A and B are **disjoint** if

$$A \cap B = \emptyset$$

Definition

A **partition** P of a set A, is a set of k nonempty subsets of A, $\{A_i\}_{i=1}^k$, such that:

- A_i and A_j are disjoint for any $i \neq j$.



Definition

Let $a \in A$, and $b \in B$ (nothing prevents A being equal to B), the **ordered pair** (a, b) is defined as:

$$(a,b) := \{a, \{a,b\}\}$$

Note that while $\{a,b\}=\{b,a\}$ (both sets have the same elements), $(a,b)\neq (b,a)$.



Definition

The **product** of two sets is defined as:

$$A \times B := \{(a, b) | a \in A \mid b \in B\}$$

Definition

The ordered **n-tuple** of *A* is:

$$A^n := \underbrace{A \times A \times A \times \ldots \times A}_{\text{n times}}$$



Numbers and Sets

- The naturals, $\mathbb{N} := \{0, 1, 2, 3, ...\}$
- lacksquare The integers, $\mathbb{Z}:=\{\ldots,-2,-1,0,1,2,\ldots\}$
- lacksquare The rationals, $\mathbb{Q}:=\left\{rac{p}{q}|p\in\mathbb{Z},q\in\mathbb{Z}\setminus\{0\}
 ight\}$
- The reals, $\mathbb{R} := \mathbb{Q} \cup I$, with I being the irrationals.

It holds that

$$\mathbb{N}\subseteq\mathbb{Z}\subseteq\mathbb{Q}\subseteq\mathbb{R}$$



Let a < b, and $a, b \in \mathbb{R}$.

- 1. [a, b] is called closed interval. This set contains all the real numbers between a and b, with a and b included.
- 2. $(a, b] := [a, b] \setminus \{a\}$.
- 3. $[a, b) := [a, b] \setminus \{b\}.$
- 4. $(a,b) := [a,b] \setminus \{a,b\}$. This is called an open interval. It is also the *interior* of [a,b].



If a subset A of $\mathbb R$ is such that $A:=\{x\in\mathbb R|x\leq a\}$ we can write it as $(-\infty,a]$.

The side where the ∞ is, is always "open".



Definition

Let $x \in \mathbb{R}$, the module of x is,

$$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{otherwise} \end{cases}$$



Definition

Let $A \subseteq \mathbb{R}$,

1. A is said to be bounded from above if

$$\exists M \in \mathbb{R}$$
 s.th. $\forall a \in A, a \leq M$

M is an upper bound of A.

2. A is said to be bounded from below if

$$\exists M \in \mathbb{R}$$
 s.th. $\forall a \in A, a \geq M$

M is a lower bound of A.

3. A is said to be bounded if it is bounded from above and from below.



Let $A \subseteq \mathbb{R}$ be bounded.

Definition

- The **supremum** of A (sup(A)) is the lowest of its upper bounds.
- The **infimum** of A (inf(A)) is the highest of its lower bounds.

If $sup(A) \in A$, it is called the **maximum** of A (max(A)). If $inf(A) \in A$, it is called the **minimum** of A (min(A)).

