Consumer Theory

Lecture 6: Consumer Preferences and Axioms of Rationality

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Recap: Lecture 5 ⏪

Budget Set & Budget Constraint:

- Budget line: $p_1x_1 + p_2x_2 = M$
- Budget set: all affordable bundles ($\leq M$)
- Slope $= -p_1/p_2$: market rate of exchange
- Income changes ➡️ parallel shift; Price changes ➡️ pivot

We answered: *What can the consumer afford?*

Today: *What does the consumer actually **want**?* 🤔

The Consumer’s Full Problem 🧩

The consumer must solve **two** questions:

1️⃣ **What is feasible?**

✅ Budget constraint (Lecture 5)

The set of bundles you *can* buy.

2️⃣ **What is desirable?**

👉 **Preferences** (Today!)

How you *rank* bundles — which ones you like more.

💡 The optimal choice (Lecture 7) will be where these two meet: the *best* bundle you can *afford*.

What Are Preferences?

Bundles and Rankings 📦

A **bundle** (or **basket**) is a specific combination of goods:

$$\text{Bundle } A = (x_1^A,\ x_2^A)$$

Example: A tourist choosing between activities in Lisbon:

Bundle	Meals (x_1)	Museums (x_2)
A	3	4
B	5	2
C	3	4

Preferences describe how a consumer **ranks** these bundles — without needing prices or income!

Three Preference Relations ↔

For any two bundles A and B, the consumer can say:

Strictly Prefers ❤️

$$A \succ B$$

"I **prefer** A to B"

Example: 4 nights in the Algarve is **better than** 2 nights

Indifferent ⚖️

$$A \sim B$$

"A and B are **equally good**"

Example: 3 beach days + 2 city days **is as good as** 2 beach days + 3 city days

Weakly Prefers 👍

$$A \succsim B$$

"A is **at least as good** as B"

Combines the two: either prefers A or is indifferent

The Axioms of Rationality

Why Do We Need Axioms? 🏛️

People's preferences can be anything — messy, emotional, contradictory.

Economists assume preferences satisfy certain **axioms** (basic rules) so that we can:

- **Model** consumer behavior mathematically
- **Predict** how choices change when prices or income change
- Build the concept of **utility functions** (Lecture 7)

👉 These axioms don't say *what* people prefer — just that preferences are **logically consistent**.

Think of them as the "rules of the game" for rational choice.

Axiom 1: Completeness ✅

COMPLETENESS

For **any** two bundles A and B, the consumer can always rank them:

$$A \succsim B \quad \text{or} \quad B \succsim A \quad (\text{or both, meaning } A \sim B)$$

In plain language: You are never "stuck" — you can always decide which bundle is at least as good.

✅ **Satisfies completeness**:

"I prefer a beach holiday to a city break"

❌ **Violates completeness**:

"I literally cannot compare a spa weekend to a ski trip — I have no opinion at all"

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Axiom 2: Transitivity 🔗

TRANSITIVITY

If $A \succsim B$ and $B \succsim C$, then $A \succsim C$

In plain language: Rankings must be **logically consistent** — no cycles.

✅ **Satisfies transitivity**:

"I prefer Paris over Rome, and Rome over Berlin. So I prefer Paris over Berlin."

❌ **Violates transitivity**:

"I prefer Paris over Rome, Rome over Berlin, but Berlin over Paris."

😵 This is a **preference cycle** — it makes consistent choice impossible!

Why Transitivity Matters ⚠️

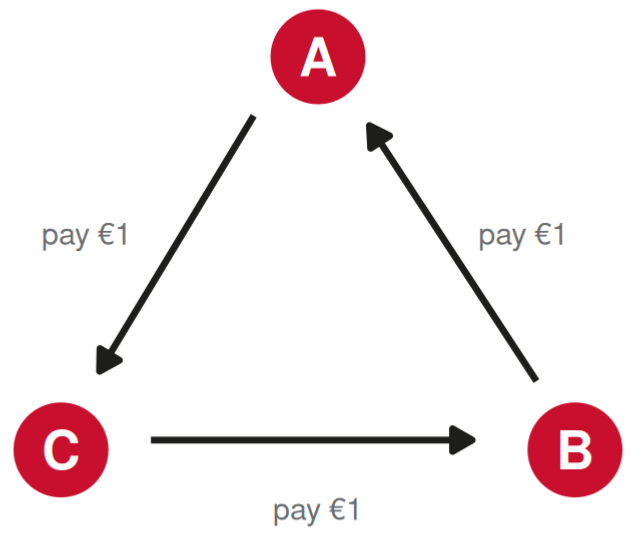
The "money pump" argument:

Imagine a tourist with intransitive preferences: $A \succ B \succ C \succ A$

1. Tourist holds bundle C
2. Offer to trade: $C \rightarrow B$ for a small fee (€1) — accepts! (prefers B)
3. Offer: $B \rightarrow A$ for €1 — accepts! (prefers A)
4. Offer: $A \rightarrow C$ for €1 — accepts! (prefers C)
5. Back to C… but **€3 poorer**! 💸

Repeat forever ➡️ lose all money

The Money Pump



Axiom 3: Monotonicity ("More is Better") ⬆️

MONOTONICITY

If bundle A has **at least as much** of every good as B, and **strictly more** of at least one good, then $A \succ B$.

In plain language: More of a good thing is always preferred, all else equal.

✅ **Example**:

$$(4 \text{ meals}, 3 \text{ museums}) \succ (3 \text{ meals}, 3 \text{ museums})$$

Same museums, one extra meal ➡️ strictly better

💡 **Implication**:

This rules out "bads" (things you'd rather have less of, like pollution)

For this course, we assume all goods are **desirable**.

Indifference Curves

What Is an Indifference Curve? 〰️

INDIFFERENCE CURVE

The set of all bundles that give the consumer the **same level of satisfaction**.

Along an indifference curve: $A \sim B$ for all points A, B on the curve.

Tourism intuition:

A tourist might be equally happy with:

- 5 beach days + 1 cultural tour
- 3 beach days + 3 cultural tours
- 1 beach day + 6 cultural tours

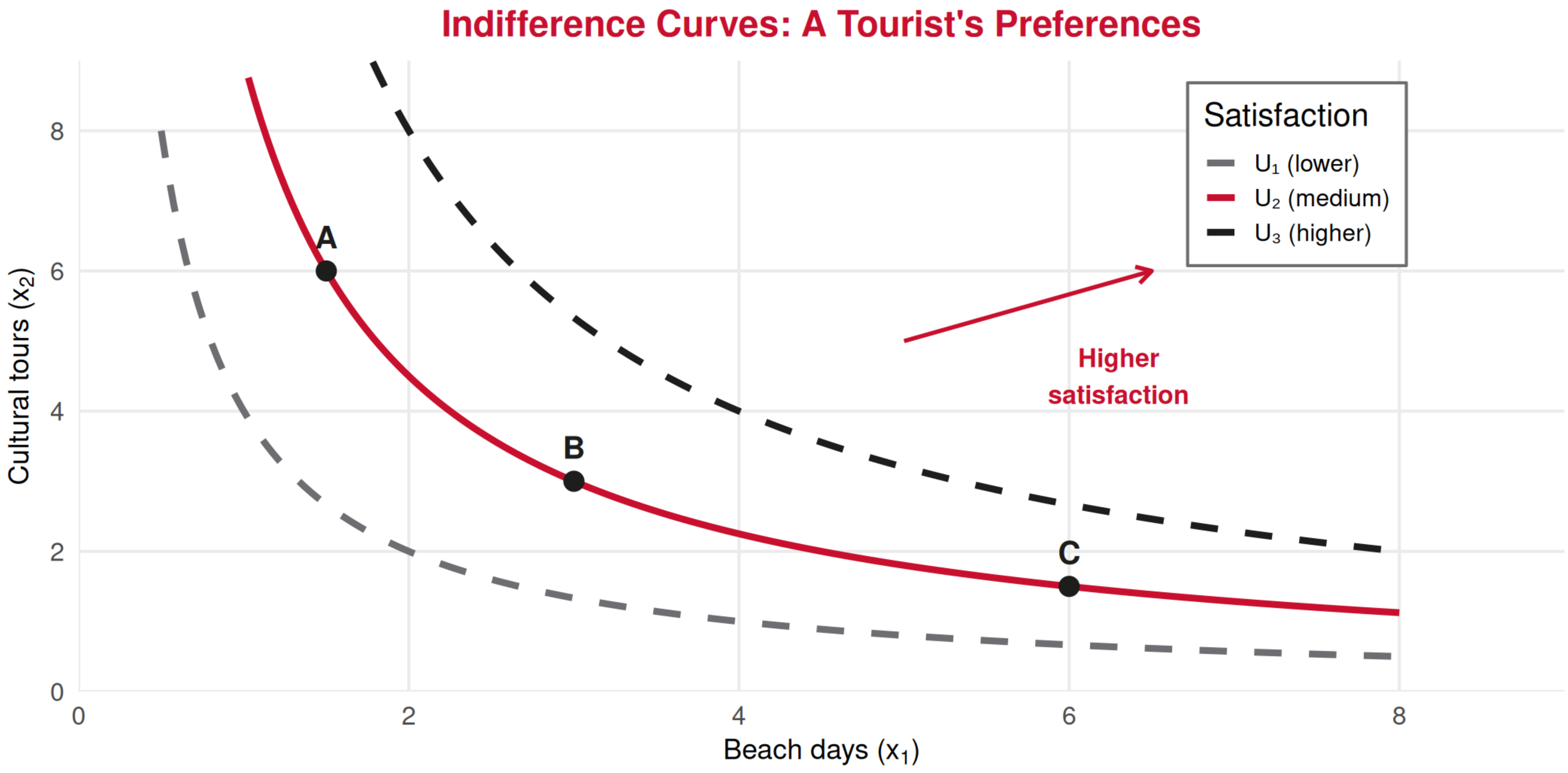
All on the **same** indifference curve!

👉 The consumer is **indifferent** between any two points on the same curve.

Moving to a **higher** curve = **better** (by monotonicity).

Moving to a **lower** curve = **worse**.

Drawing Indifference Curves 🎨



Points A, B, C are on the **same** curve (U_2): the tourist is **indifferent** among them.

Properties of Indifference Curves

The axioms imply **four key properties**:

1️⃣ **Higher curves are preferred**

(monotonicity: more is better)

2️⃣ **Downward sloping**

(to stay indifferent, getting more of one good requires giving up some of the other)

3️⃣ **Cannot cross**

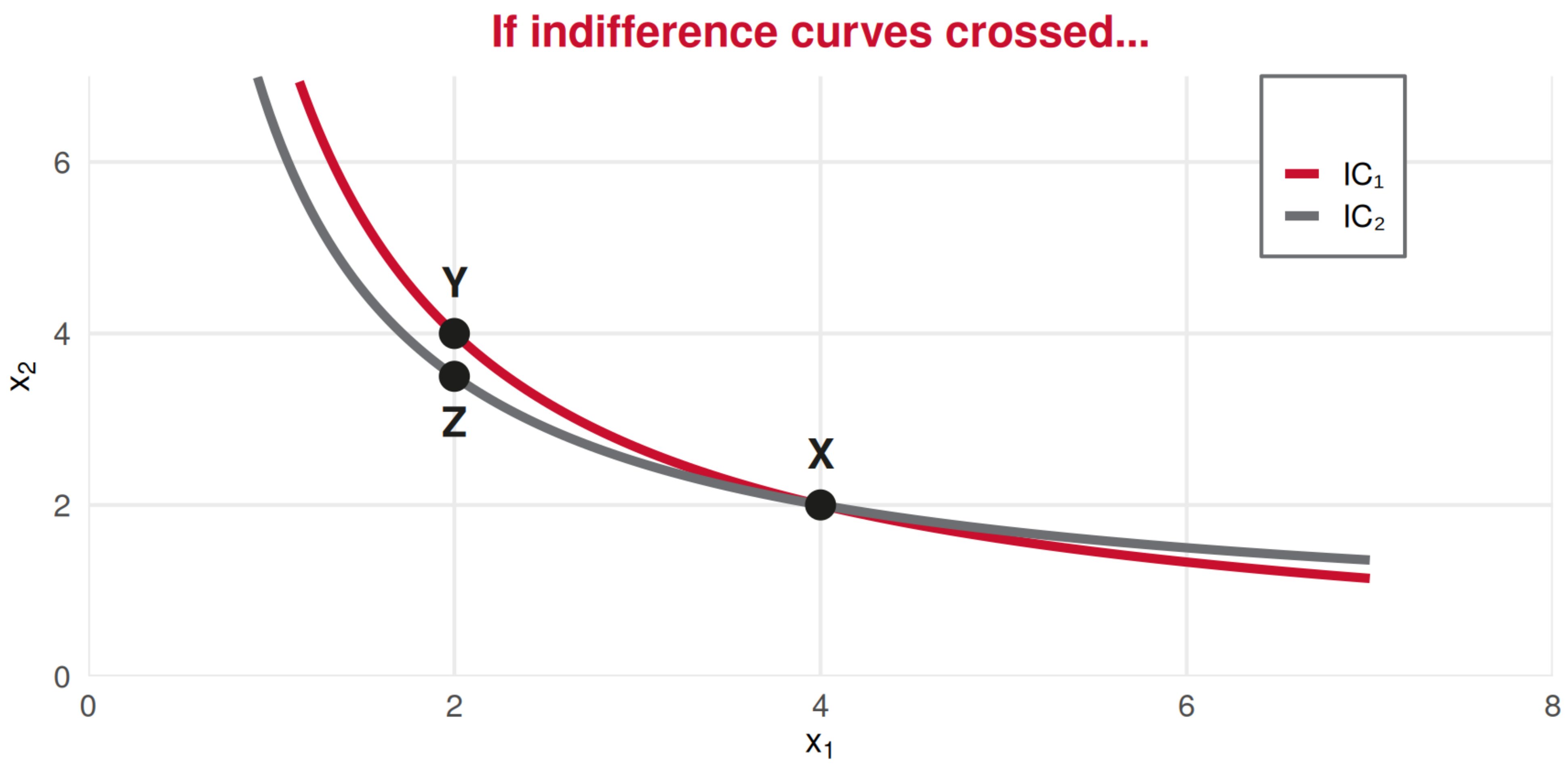
(would violate transitivity — we'll prove this next!)

4️⃣ **Exactly one curve through every point**

(completeness: every bundle belongs to some indifference level)

Why Can’t Indifference Curves Cross? ❌

Proof by contradiction (using the axioms):



- X and Y are on IC_1 ➡️ $X \sim Y$
- X and Z are on IC_2 ➡️ $X \sim Z$
- By **transitivity**: $Y \sim Z$
- But $Y = (2, 4)$ has more of x_2 than $Z = (2, 3.5)$ (same x_1) ➡️ **monotonicity** says $Y \succ Z$ 💥 **Contradiction!**

Axiom 4: Convexity

CONVEXITY

Averages (mixtures) of bundles are **at least as good** as extremes.

If $A \sim B$, then any weighted average $\lambda A + (1-\lambda)B$ is weakly preferred ($\succsim A$), where $0 < \lambda < 1$.

Intuition: People prefer **balanced** consumption to extremes.

A tourist equally happy with:

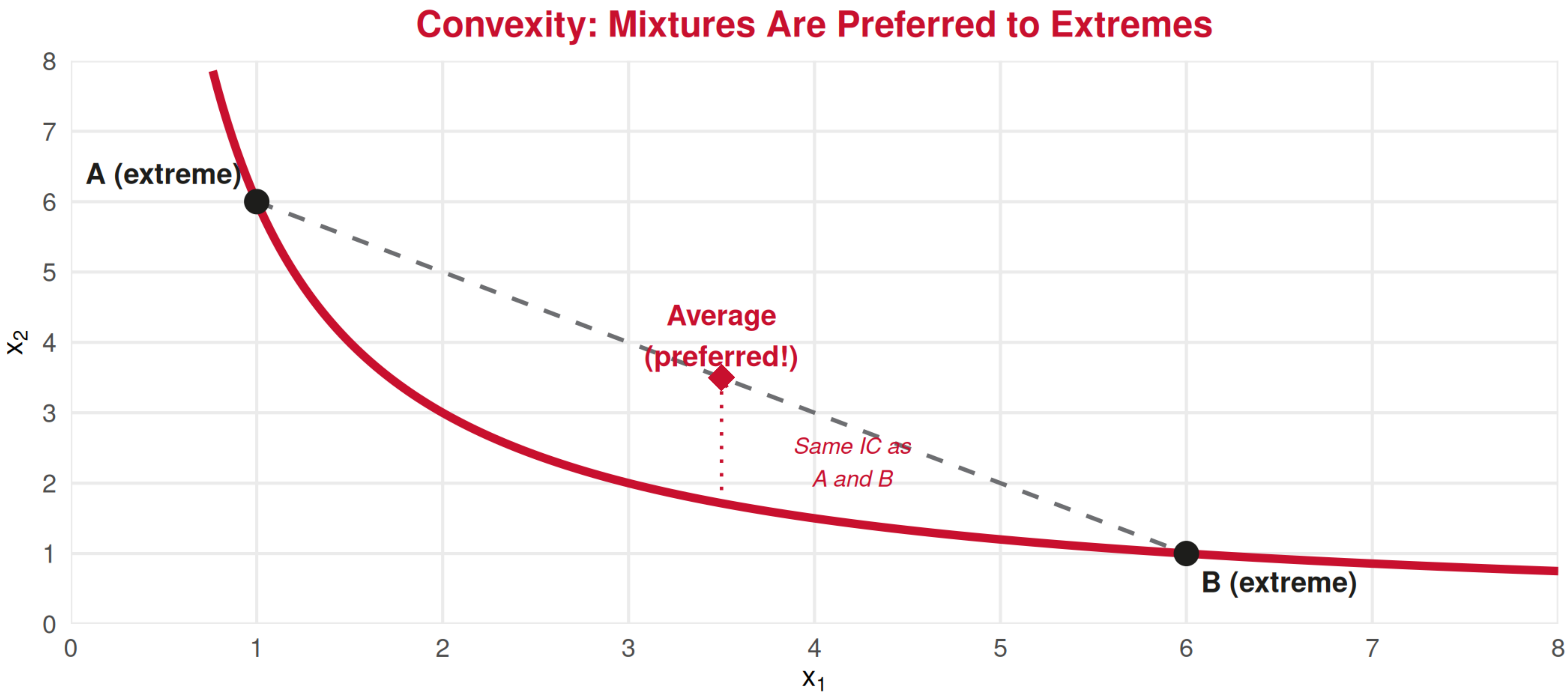
- 7 beach days + 0 museums
- 0 beach days + 7 museums

Would **prefer** 3.5 beach + 3.5 museums!

Graphically: Indifference curves are **bowed toward the origin** (convex shape).

👉 This will give us a **diminishing** Marginal Rate of Substitution (Lecture 7)

Convexity: Graphical Intuition 📉



The average of A and B lies **above** the indifference curve ➡️ it is on a **higher** curve ➡️ preferred!

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Summary of the Four Axioms

Axiom	Statement	Intuition
Completeness	Can always compare any two bundles	No indecision paralysis
Transitivity	If $A \succsim B$ and $B \succsim C$, then $A \succsim C$	No preference cycles
Monotonicity	More of a good is better	Goods are desirable
Convexity	Mixtures preferred to extremes	Variety is valued

👉 If these hold, preferences can be represented by a **utility function** $U(x_1, x_2)$ — that's Lecture 7!

👉 The utility function assigns a **number** to each bundle so that higher numbers = preferred bundles.

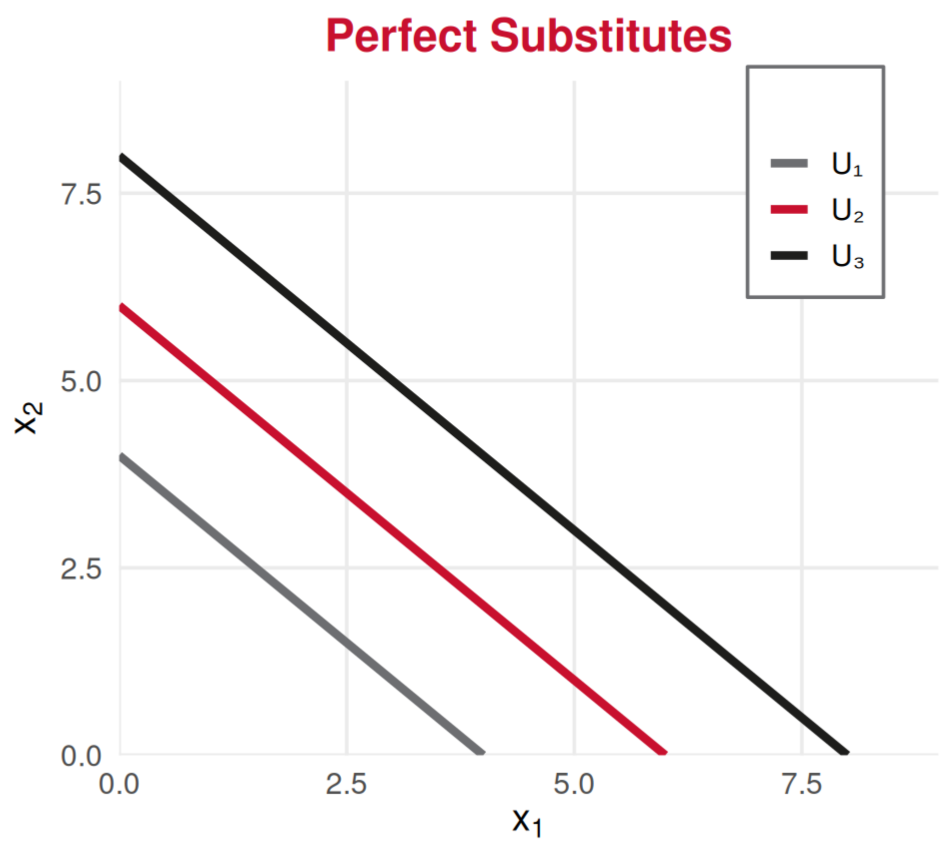
Special Cases

Special Case 1: Perfect Substitutes ↔

Definition: Consumer is willing to trade goods at a **constant rate**.

Indifference curves are **straight lines**.

Tourism example: A tourist is indifferent between a €10 lunch voucher and €10 cash — they substitute perfectly at a 1:1 rate.



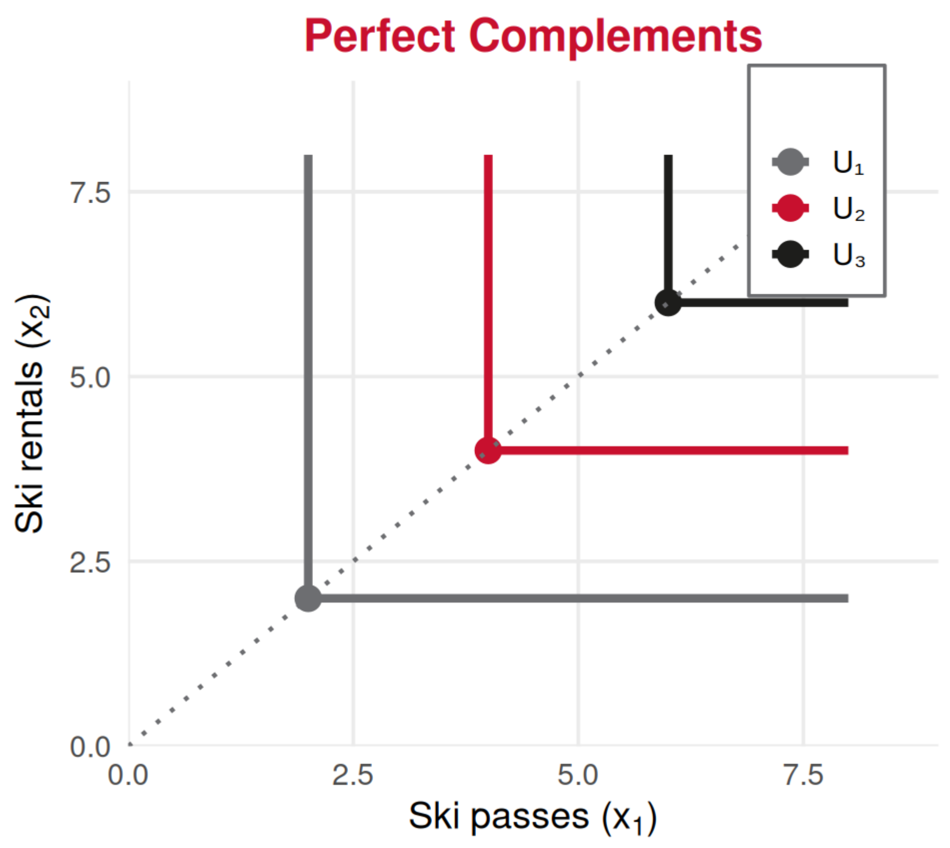
💡 **No** convexity here — the consumer doesn't care about "balance."

Special Case 2: Perfect Complements 🧩

Definition: Goods are always consumed in **fixed proportions**.

Indifference curves are **L-shaped** (right angles).

Tourism example: A ski pass and ski rental — having 3 ski passes and 1 ski rental is no better than having 1 of each (you need both to ski!).



💡 More of one good **without** the other gives **no** extra satisfaction.

The “Normal” Case 👍

Most goods lie **between** the two extremes:

Type	IC Shape	Substitutability	Example
Perfect Substitutes	Straight line	Complete	€10 cash vs €10 voucher
Normal goods	**Smooth convex curve**	**Partial**	**Beach days vs cultural tours**
Perfect Complements	L-shape	None	Ski pass + ski rental

👉 For the rest of this course, unless stated otherwise, we assume the **normal** case: smooth, convex, downward-sloping indifference curves.

This is where **convexity** applies — consumers value variety and are willing to trade goods, but at a **changing** rate. That changing rate will be the **MRS** in Lecture 7!

Tourism Application

Tourism: Revealed Preferences ✈️

How do we know tourist preferences in practice?

We can't read minds — but we can observe **choices**!

Revealed preference logic:

If a tourist *could afford* both packages A and B, but *chose* A, then:

$$A \succsim B$$

Their choice **reveals** their preference!

Tourism industry applications:

- 💻 Booking data reveals preferences for destinations, hotel stars, trip lengths
- 📊 A/B testing on travel websites reveals what tourists click on
- ✈️ Airline loyalty programs track revealed choices over time

💡 This is why tourism companies collect **so much data** — they're mapping your indifference curves!

Summary: Today’s Key Takeaways

Today’s Lecture:

1. **Preferences** rank bundles — independently of prices or income
2. **Four axioms** make preferences “rational” and modelable:
    - **Completeness**: always able to compare
    - **Transitivity**: no preference cycles
    - **Monotonicity**: more is better
    - **Convexity**: mixtures preferred to extremes
3. **Indifference curves**: connect bundles giving equal satisfaction
4. IC properties: downward-sloping, don’t cross, higher = better, convex
5. **Special cases**: perfect substitutes (lines) and perfect complements (L-shapes)

Connection to Lecture 5: Budget constraint shows what’s *feasible*; preferences show what’s *desirable*.

Next (Lecture 7): The **Marginal Rate of Substitution** (the slope of the IC) and **utility functions** — how we turn preferences into math and find the **optimal choice**!

Exercises

Application Time! ✏️

Preferences, axioms, and indifference curves.

Exercise 1: Multiple Choice

Question: A tourist says: “I prefer an Algarve beach holiday to a Douro wine tour, I prefer a Douro wine tour to a Lisbon city break, and I prefer a Lisbon city break to an Algarve beach holiday.” This violates:

A. Completeness
B. Transitivity
C. Monotonicity
D. Convexity

Answer: B

This is a **preference cycle**: Algarve $\succ$ Douro $\succ$ Lisbon $\succ$ Algarve. Transitivity requires that if Algarve $\succ$ Douro and Douro $\succ$ Lisbon, then Algarve $\succ$ Lisbon — but the tourist says the opposite. This violates transitivity and makes the tourist vulnerable to a “money pump.”

Exercise 2: Multiple Choice

Question: Indifference curves that are **L-shaped** represent goods that are:

A. Perfect substitutes
B. Normal goods with convex preferences
C. “Bads” (undesirable)
D. Perfect complements

Answer: D

L-shaped indifference curves indicate **perfect complements** — goods consumed in fixed proportions. Extra units of one good without the other provide no additional satisfaction (e.g., a left shoe without a right shoe, or a flight ticket without a hotel booking in a package deal).

Exercise 3: Open Question

Scenario: Consider a tourist choosing between two types of vacation days: 🌞 **Beach days** (x_1) and 🏛️ **City sightseeing days** (x_2). Assume preferences satisfy all four axioms.

Questions:

a. The tourist is indifferent between bundle $A = (2, 6)$ and bundle $B = (6, 2)$. Draw an indifference curve through both points (sketch it with the typical convex shape). Label it U_1.

b. Consider bundle $D = (4, 4)$. Using the axiom of convexity, explain why D is preferred to A (or B). Show D on your graph — is it above, on, or below U_1?

c. Now consider bundle $E = (1, 5)$. Can you definitively say whether E is better or worse than A without more information? Why or why not?

d. The tourist says: "I prefer bundle $F = (3, 3)$ to A, but I also prefer A to $G = (5, 5)$." Does this violate any axiom? Which one and why?

e. Suppose these are **perfect substitutes** (1 beach day = 1 sightseeing day). Redraw the indifference curve through A and B. What is different about its shape?

Exercise 3: Solution — Parts a & b

a) The indifference curve through $A = (2, 6)$ and $B = (6, 2)$ is a smooth, downward-sloping, **convex** curve (bowed toward the origin). All points along this curve give the same satisfaction level U_1.

b) Bundle $D = (4, 4)$ is the **average** of A and B:

$$D = \frac{1}{2}A + \frac{1}{2}B = \left(\frac{2+6}{2}, \frac{6+2}{2}\right) = (4, 4)$$

By the **convexity** axiom: if $A \sim B$, then any mixture is weakly preferred.

So $D \succsim A \sim B$.

Since D is on the straight line connecting A and B, and the indifference curve is convex (bowed below this line), D lies **above** U_1 ➡️ D is on a **higher** indifference curve ➡️ $D \succ A$.

Exercise 3: Solution — Parts c, d, e

c) Bundle $E = (1, 5)$ vs $A = (2, 6)$: E has **less** of x_1 (1 < 2) and **less** of x_2 (5 < 6).

By **monotonicity**: $A \succ E$. Yes, we **can** say E is worse — A dominates E in both goods.

d) The tourist says $F = (3, 3) \succ A$ and $A \succ G = (5, 5)$.

This violates **monotonicity**: $G = (5, 5)$ has strictly more of **both** goods than $F = (3, 3)$, so monotonicity requires $G \succ F$. Combined with $F \succ A \succ G$, by transitivity we'd need $F \succ G$. But G dominates F — **contradiction!**

The statement violates monotonicity (and by extension creates a transitivity conflict with monotonicity).

e) If perfect substitutes at 1:1, the indifference curve through $A = (2, 6)$ and $B = (6, 2)$ is a **straight line** with slope -1: $x_1 + x_2 = 8$. All bundles summing to 8 give equal satisfaction. The curve is linear, not convex — **no preference for variety**.

Next Lecture

February 26, 2026: Marginal Rate of Substitution, Utility Functions & Utility Maximization

We now know what's feasible (budget) and what's desirable (preferences).

Next: **How to find the best affordable bundle!** 🎯

Thank You!

Questions? 🙋

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Next class: Thursday, February 26, 2026