Consumer Theory

Lecture 5: Budget Set and Budget Constraint

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Recap: Fundamentals Block ⏪

What we covered in Lectures 1–4:

- **Scarcity** forces choices ➡️ trade-offs everywhere
- **Economic Systems**: Market, Centralized, Mixed
- **Opportunity Cost** = accounting cost + surplus of the best alternative
- **PPF**: Visualizes society's production trade-offs and efficiency

Today: We zoom in from society to the **individual consumer**

👉 How does a single person decide what to buy?

Welcome to Consumer Theory 🛒

The Big Question: Given limited income and market prices, what can a consumer afford?

Lectures 5–9 Roadmap:

5. 👉 **Budget Set & Constraint** (today)
6. Preferences & Rationality Axioms
7. MRS, Utility & Maximization
8. Individual & Market Demand
9. Demand Elasticity

Why it matters for tourism ✈️

Tourists are consumers! Understanding budget constraints explains:

- Why some choose hostels, others choose resorts
- How exchange rates affect travel decisions

The Consumer's Problem

Starting Point: What Can You Afford? 💸

Every consumer faces **three constraints**:

💰 **Income** (M)

The total money available to spend

🏷️ **Price of Good 1** (p_1)

How much each unit of good 1 costs

🏷️ **Price of Good 2** (p_2)

How much each unit of good 2 costs

THE CONSUMER'S PROBLEM

Given income M and prices p_1, p_2, what combinations of goods 1 and 2 can the consumer purchase?

A Tourism Example 🌴

Scenario: A tourist arrives in Lisbon with a daily budget of **€100**.

Two "goods" to spend on:

- 🍴 **Meals** at restaurants: €20 each
- 🎟️ **Museum tickets**: €10 each

Question: What combinations of meals and museum visits can this tourist afford?

Meals (x_1)	Museums (x_2)	Total Spent
0	10	€100
1	8	€100
2	6	€100
3	4	€100
5	0	€100

The Budget Constraint Equation

The consumer spends **all** income on two goods:

$$\underbrace{p_1 \cdot x_1 + p_2 \cdot x_2}_{Expenditure} = \underbrace{M}_{Budget}$$

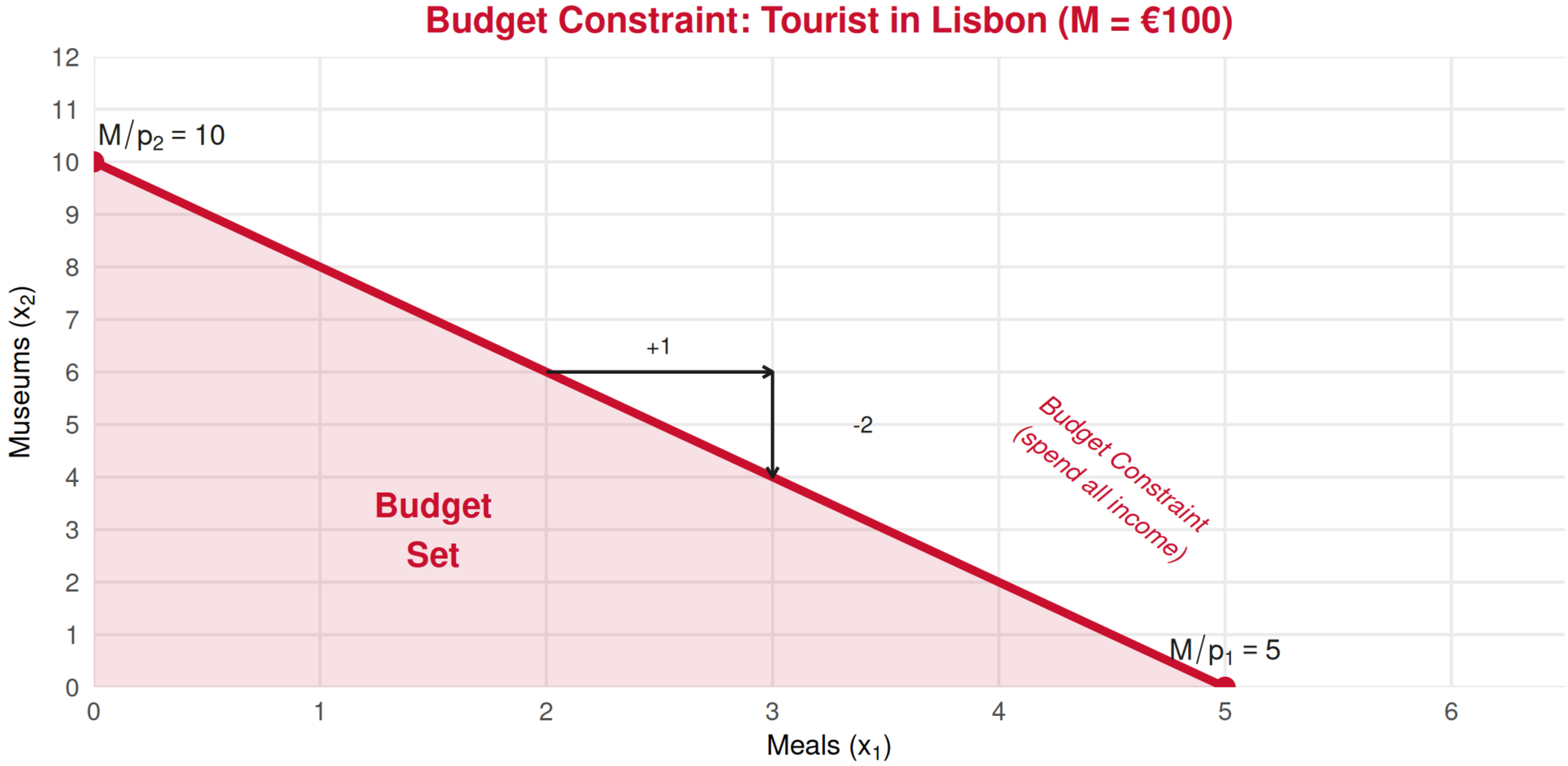
From our example: €20 $\cdot x_1$ + €10 $\cdot x_2$ = €100

Solving for x_2 (to graph it):

$$x_2 = \frac{M}{p_2} - \frac{p_1}{p_2} \cdot x_1$$

$$x_2 = \frac{100}{10} - \frac{20}{10} \cdot x_1 = 10 - 2x_1$$

Graphing the Budget Constraint



Key Elements of the Budget Constraint

Intercepts (maximum of each good):

- **Vertical** ($x_1 = 0$): $\frac{M}{p_2} = \frac{100}{10} = 10$ museums
- **Horizontal** ($x_2 = 0$): $\frac{M}{p_1} = \frac{100}{20} = 5$ meals

Slope of the budget line:

$$\text{Slope} = -\frac{p_1}{p_2} = -\frac{20}{10} = -2$$

👉 For every **1 extra meal**, the tourist gives up **2 museum visits**

This is the **economic rate of substitution** set by the market!
Also known as the **opportunity cost** of an extra meal!

Budget Set vs Budget Line 🗺️

BUDGET LINE

vs **Budget Set**

- **BUDGET LINE**

  All bundles where the consumer spends *exactly* all income: $p_1 x_1 + p_2 x_2 = M$

- **BUDGET SET**

  All *affordable* bundles (spend all or less): $p_1 x_1 + p_2 x_2 \leq M$

The budget set is the **shaded area** including the line itself.

Bundles **above** the budget line are **unaffordable** ❌

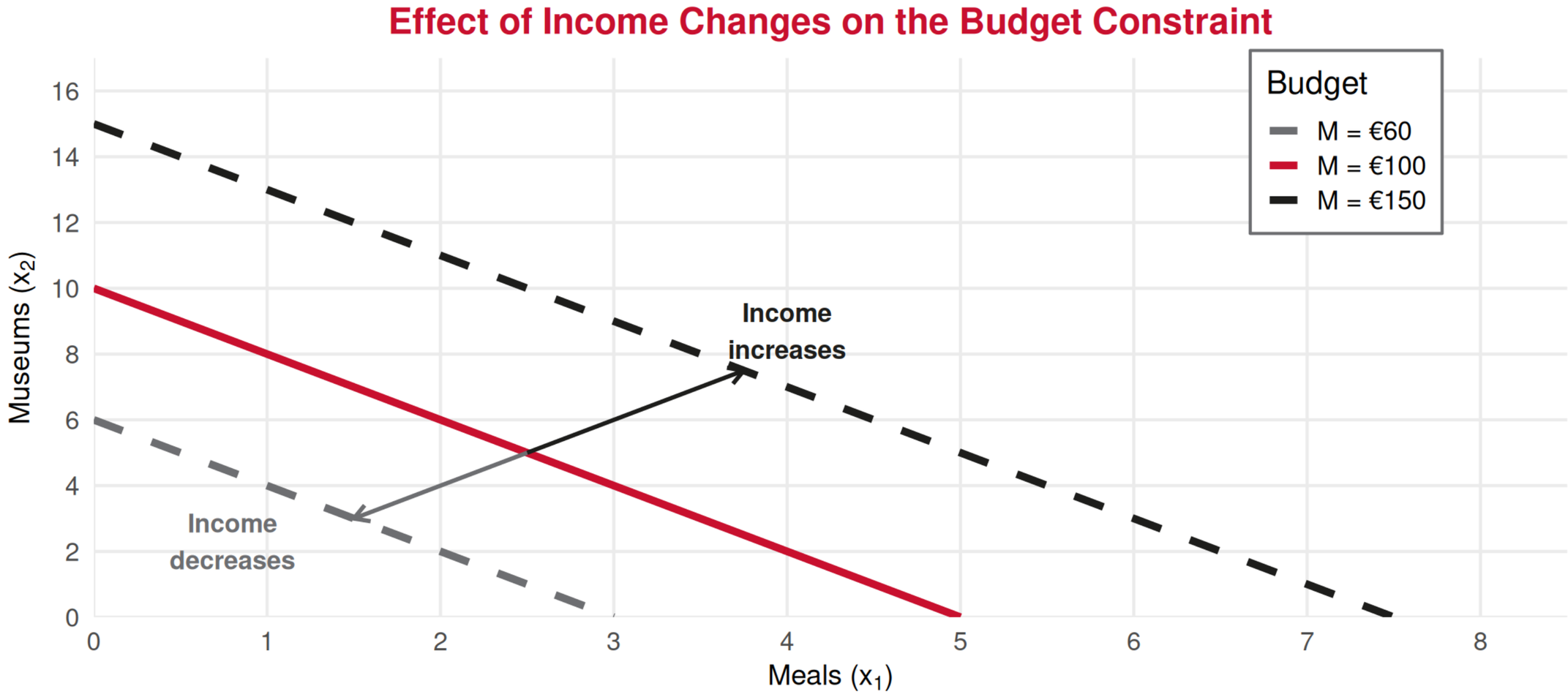
Bundles **on** the budget line: spend all income ✅

Bundles **inside** the budget set: affordable with money left over ✅

What Shifts the Budget Constraint?

Change in Income 📈

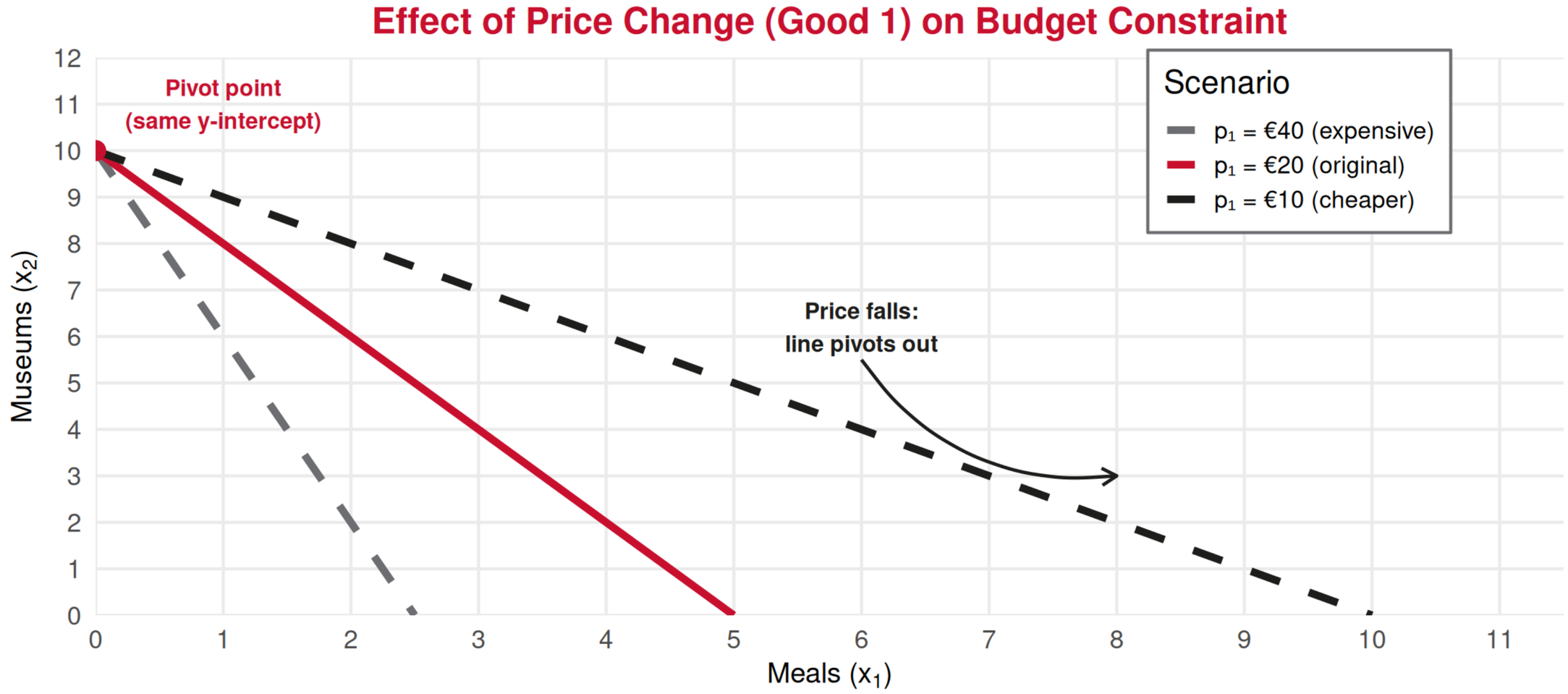
What happens if our tourist's budget increases from €100 to €150?



👉 Income change: **parallel shift** (slope unchanged at $-p_1/p_2$)

Change in Price of Good 1 🏷️

What if meal prices drop from €20 to €10?



👉 Price change of **one** good: **pivot** around the other intercept (slope changes!)

Summary: What Shifts What? 📝

Change	Effect on Budget Line	Slope	Intercepts
⬆️ Income (M)	Parallel shift **outward**	Same	Both increase
⬇️ Income (M)	Parallel shift **inward**	Same	Both decrease
⬇️ Price p_1	Pivot **outward** on x_1 axis	Flatter	x_1-intercept increases
⬆️ Price p_1	Pivot **inward** on x_1 axis	Steeper	x_1-intercept decreases
⬇️ Price p_2	Pivot **outward** on x_2 axis	Steeper	x_2-intercept increases
⬆️ Price p_2	Pivot **inward** on x_2 axis	Flatter	x_2-intercept decreases

💡 **Key insight**: The slope $-p_1/p_2$ is the **relative price** — what the *market* says one good costs in terms of the other.

The Slope as Opportunity Cost ⚖️

The slope of the budget constraint has a direct economic interpretation:

$$\text{Slope} = -\frac{p_1}{p_2}$$

This tells us the **market's exchange rate** between the two goods.

Our example: $-20/10 = -2$

For 1 extra meal, you **must** give up 2 museum visits.

This is **not** a preference — it is a constraint imposed by prices!

👉 Compare with the **PPF slope** from Lecture 4

- PPF slope: society's opportunity cost (technology)
- Budget line slope: individual's opportunity cost (prices)

Both represent trade-offs, at different scales!

Tourism Applications

Application: Exchange Rates & Tourist Budgets 💶

How exchange rates shift a tourist's budget constraint

A British tourist visits Portugal with **£500** to spend on:

- 🏨 **Accommodation**: €80/night
- 🍴 **Dining**: €20/meal

Scenario A: £1 = €1.15

Budget in €: £500 × 1.15 = **€575**

- Max nights: 575/80 ≈ **7.2**
- Max meals: 575/20 ≈ **28.8**

Scenario B: £1 = €1.30

Budget in €: £500 × 1.30 = **€650**

- Max nights: 650/80 ≈ **8.1**
- Max meals: 650/20 ≈ **32.5**

👉 A stronger pound = **parallel outward shift** of the budget constraint in euro terms. The tourist can afford more of *everything*!

Numerical Example: Step by Step 🧮

Problem: A tourist has **€200** to spend. Surfing lessons cost **€40** each. Fado show tickets cost **€25** each.

Step 1: Write the budget constraint

$$40x_1 + 25x_2 = 200$$

Step 2: Find intercepts

- If $x_1 = 0$: $x_2 = 200/25 = 8$ fado shows
- If $x_2 = 0$: $x_1 = 200/40 = 5$ surf lessons

Step 3: Find the slope

$$\text{Slope} = -\frac{p_1}{p_2} = -\frac{40}{25} = -1.6$$

👉 1 extra surf lesson costs 1.6 fado shows

Step 4: Check an interior bundle — (2 surf, 4 fado): $40(2) + 25(4) = 80 + 100 = 180 \leq 200$ ✅ (inside budget set, €20 unspent)

General Formulas: Cheat Sheet 📋

BUDGET CONSTRAINT FORMULAS

Equation: $p_1 x_1 + p_2 x_2 = M$

Solved for x_2: $x_2 = \frac{M}{p_2} - \frac{p_1}{p_2} x_1$

Vertical intercept ($x_1 = 0$): $\frac{M}{p_2}$

Horizontal intercept ($x_2 = 0$): $\frac{M}{p_1}$

Slope: $-\frac{p_1}{p_2}$ (the relative price of good 1 in terms of good 2)

Summary: Today's Key Takeaways

Today's Lecture Integration:

1. **Income** (M) and **prices** (p_1, p_2) define the consumer's constraint
2. **Budget line**: $p_1 x_1 + p_2 x_2 = M$ — all bundles spending exactly all income
3. **Budget set**: $p_1 x_1 + p_2 x_2 \leq M$ — all affordable bundles
4. **Slope** = $-p_1 / p_2$ = market rate of exchange between goods
5. Income changes ➡️ **parallel shift**
6. Price changes ➡️ **pivot** (rotation)

Connection to previous lectures: The budget line is the *individual-level analog* of the PPF — both show feasible combinations and trade-offs.

Next: Lecture 6 — Consumer **Preferences** and axioms of rationality. We'll ask: *among all affordable bundles, which one does the consumer actually want?*

Exercises

Application Time! ✏️

Budget constraint calculations and graphical analysis.

Exercise 1: Multiple Choice

Question: A consumer has income $M = 120$, with $p_1 = 15$ and $p_2 = 10$. The slope of the budget constraint is:

A. $-10/15$
B. $-15/10$
C. $-120/15$
D. $-120/10$

Answer: B

The slope of the budget line is always $-p_1/p_2 = -15/10 = -1.5$. This means for each additional unit of good 1, the consumer must give up 1.5 units of good 2. Note: income affects the *position* of the line, not its slope.

Exercise 2: Multiple Choice

Question: If the price of good 2 **doubles** while income and p_1 stay the same, the budget line:

A. Shifts outward in parallel
B. Pivots inward around the x_1-intercept
C. Pivots outward around the x_2-intercept
D. Pivots inward around the x_2-intercept

Answer: B

When p_2 doubles: the x_2-intercept (M/p_2) **halves** (moves down), while the x_1-intercept (M/p_1) **stays the same**. So the line **pivots inward around the x_1-intercept**. The consumer can buy less of good 2 but the same maximum of good 1. The slope $-p_1/p_2$ becomes smaller in absolute value (the line becomes flatter).

Exercise 3: Open Question

Scenario: A Portuguese tourism student plans a weekend trip. She has a budget of **€150**. She wants to split spending between:

- 🏄 **Surf lessons**: €30 each (x_1)
- 🍷 **Wine tasting tours**: €25 each (x_2)

Questions:

a. Write the budget constraint equation and solve for x_2.

b. Calculate and interpret the slope. What is the opportunity cost of one surf lesson in terms of wine tastings?

c. Find both intercepts. Draw the budget constraint with x_1 on the horizontal axis.

d. Is the bundle (2 surf lessons, 3 wine tastings) affordable? Is it on the budget line?

e. Suppose the student receives a €50 gift card (total budget now €200). Draw the new budget line on the same graph. What changed and what stayed the same?

f. Now instead of the gift card, suppose surf lesson prices drop to €25 (budget stays at €150). Draw this new line. How does it differ from part (e)?

Exercise 3: Solution — Parts a, b, c

a) Budget constraint: $30x_1 + 25x_2 = 150$

Solving for x_2: $x_2 = \frac{150}{25} - \frac{30}{25}x_1 = 6 - 1.2x_1$

b) Slope $= -p_1/p_2 = -30/25 = -1.2$

👉 Each surf lesson costs 1.2 wine tastings. The opportunity cost of 1 surf lesson is 1.2 wine tours foregone.

c) Intercepts:

- $x_1 = 0 \Rightarrow x_2 = 150/25 = 6$ wine tastings (vertical intercept)
- $x_2 = 0 \Rightarrow x_1 = 150/30 = 5$ surf lessons (horizontal intercept)

Graph: straight line from $(0, 6)$ to $(5, 0)$.

Exercise 3: Solution — Parts d, e, f

d) Bundle (2, 3): $30(2) + 25(3) = 60 + 75 = 135 \leq 150$ ✅

Affordable? Yes! **On the budget line?** No — she spends only €135, leaving €15 unspent. The bundle is **inside** the budget set.

e) New budget €200: $30x_1 + 25x_2 = 200 \Rightarrow x_2 = 8 - 1.2x_1$

- New intercepts: $(0, 8)$ and $(6.67, 0)$
- **Parallel shift outward** — slope unchanged at -1.2

f) Price drop $p_1 = 25, M = 150$: $25x_1 + 25x_2 = 150 \Rightarrow x_2 = 6 - x_1$

- New intercepts: $(0, 6)$ and $(6, 0)$
- **Pivot outward** around vertical intercept — slope changes to -1

Key difference: (e) is a parallel shift (more of both goods equally); (f) is a pivot (relatively more surf lessons become affordable, wine tasting maximum unchanged).

Next Lecture

February 20, 2026: Consumer Preferences & Axioms of Rationality

We answered: *What can the consumer afford?*

Next, we ask: *What does the consumer **want**?* 🤔

Thank You!

Questions? 🙋

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Next class: Tomorrow, Friday, February 20, 2026