Consumer Theory

Lecture 9: Calculation and Determinants of Demand Elasticity

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Recap: Lecture 8 ⏪

What we covered last time:

- **Individual demand**: derived from utility maximization as price changes
- **Market demand**: horizontal sum of all individual demands
- **Linear demand**: $P = b - mQ$ (inverse form)
- **Movements along** (price change) vs. **Shifts** (income, preferences, related goods)
- **Consumer surplus**: net benefit consumers get from market participation

The Key Question Today: How **sensitive** is quantity demanded to price changes? 🤔

Not all demand curves are created equal â€” some goods see huge changes in quantity when price moves; others barely budge!

Introduction to Elasticity

Why Elasticity Matters 📈

THE REVENUE PROBLEM

A hotel wants to increase revenue. Should it raise or lower prices? The answer depends on **how responsive** tourists are to price changes!

Two scenarios:

Scenario A: ✈️ Flights from Lisbon to Paris

Price increases 10% → Bookings drop 25%

👉 **Passengers are very sensitive** to price

Revenue **falls** when price rises!

Scenario B: 🔌 Hotel electricity

Price increases 10% → Usage drops 2%

👉 **Hotels barely respond** to price

Revenue **rises** when price rises!

Elasticity measures this **price sensitivity** precisely. Essential for pricing, taxation, and policy!

What Is Price Elasticity of Demand? 📏

PRICE ELASTICITY OF DEMAND (PED OR ε_d)

The **percentage change** in quantity demanded when price changes by **1%**, *ceteris paribus*.

$$\varepsilon_d = \frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q/Q}{\Delta P/P} = \frac{\Delta Q}{\Delta P}\frac{P}{Q}$$

Key properties:

- ε_d is almost always **negative** (law of demand: $\uparrow P \Rightarrow \downarrow Q$)
- We often report the **absolute value**: $|\varepsilon_d|$
- It's **unit-free**: same whether measuring in euros or dollars, trips or thousands of trips

Example: If $\varepsilon_d = -2$, then a **1% increase** in price causes a **2% decrease** in quantity demanded.

💡 Elasticity tells us the **proportional response**, not the absolute change!

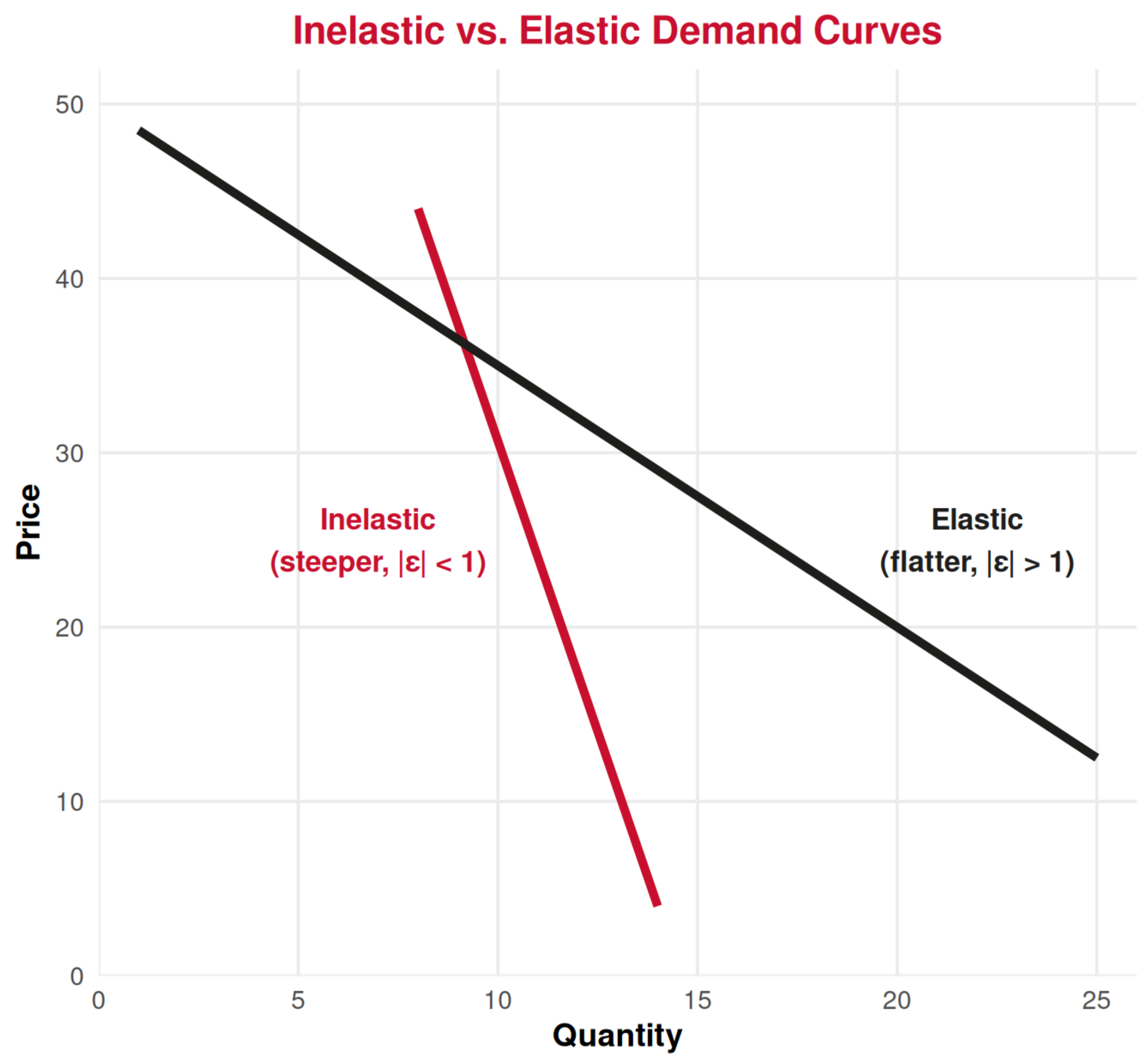
Elastic vs. Inelastic Demand ⚖️

We classify demand based on $|\varepsilon_d|$:

Category	Condition	Interpretation	Example
Perfectly Inelastic	$\|\varepsilon_d\| = 0$	Quantity doesn't change at all	Life-saving medicine
Inelastic	$0 < \|\varepsilon_d\| < 1$	Quantity changes *less* than price	Gasoline (short run)
Unit Elastic	$\|\varepsilon_d\| = 1$	Quantity changes *equally* with price	Some textbooks estimate for housing
Elastic	$\|\varepsilon_d\| > 1$	Quantity changes *more* than price	Restaurant meals, tourism
Perfectly Elastic	$\|\varepsilon_d\| = \infty$	Any price increase → demand drops to zero	Competitive market goods

Key insight: If $|\varepsilon_d| > 1$ (elastic), consumers are **very responsive**. If $|\varepsilon_d| < 1$ (inelastic), they are **not very responsive**.

Visualizing Elasticity 📉



Visual intuition:

Inelastic demand (red):

- **Steeper** curve
- Quantity barely responds to price
- $|\varepsilon_d| < 1$

Elastic demand (black):

- **Flatter** curve
- Quantity responds a lot to price
- $|\varepsilon_d| > 1$

⚠️ **Careful**: Steepness depends on units! Elasticity is the proper measure.

Calculating Elasticity

Two Methods of Calculation 🔢

Method 1: Point Elasticity (at a specific point on the demand curve)

$$\varepsilon_d = \frac{dQ}{dP} \cdot \frac{P}{Q}$$

- Use when you have a **demand function** and want elasticity at a particular (P, Q)
- For linear demand $Q = a - bP \Rightarrow \frac{dQ}{dP} = -b$

Method 2: Arc Elasticity (between two points)

$$\varepsilon_d = \frac{\Delta Q / Q_{avg}}{\Delta P / P_{avg}} = \frac{\frac{Q_2 - Q_1}{(Q_1 + Q_2)/2}}{\frac{P_2 - P_1}{(P_1 + P_2)/2}}$$

- Use when you observe **two discrete points** (e.g., before/after price change)
- Uses **midpoints** to avoid asymmetry issues (increasing 10% $\neq$ decreasing 10%)

Example 1: Point Elasticity ✏️

Demand for hotel rooms in Porto: $Q = 500 - 2P$ (rooms per night, P in €)

Question: What is the price elasticity of demand when $P = €100$?

Solution:

1. Find Q at $P = 100$: $Q = 500 - 2(100) = 300$ rooms
2. Calculate $\frac{dQ}{dP}$: For $Q = 500 - 2P$, we have $\frac{dQ}{dP} = -2$
3. Apply formula:

$$\varepsilon_d = \frac{dQ}{dP} \cdot \frac{P}{Q} = (-2) \cdot \frac{100}{300} = -\frac{200}{300} = -0.67$$

Interpretation: At $P = €100$, demand is **inelastic** ($|\varepsilon_d| = 0.67 < 1$). A 1% price increase causes only a 0.67% decrease in quantity demanded.

Example 2: Arc Elasticity ✏️

A museum in Sintra raises ticket prices from €10 to €12. Visitors fall from 1,000/day to 800/day.

Question: What is the arc elasticity of demand?

Solution:

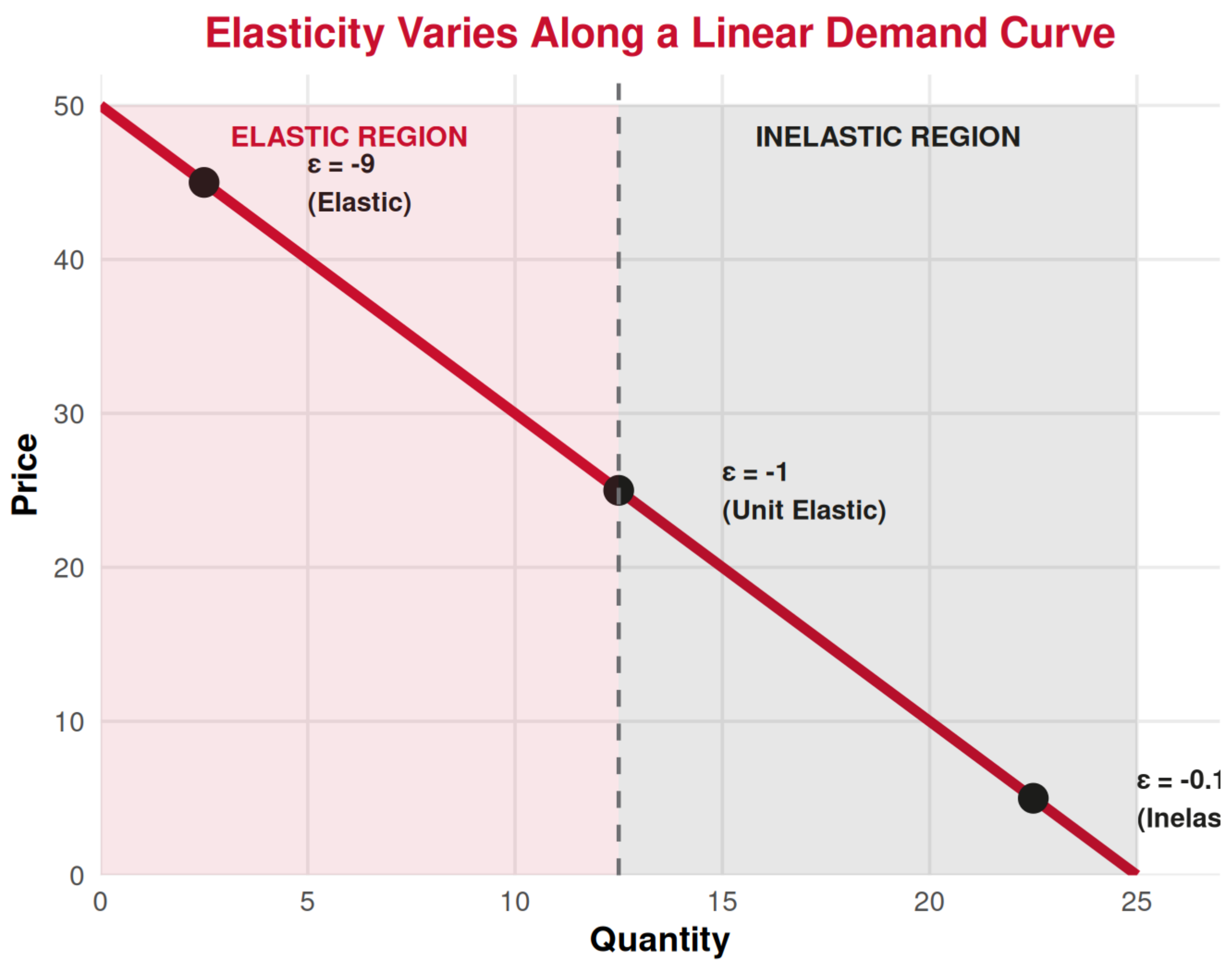
$$\varepsilon_d = \frac{Q_2 - Q_1}{(Q_1 + Q_2)/2} \div \frac{P_2 - P_1}{(P_1 + P_2)/2}$$

$$= \frac{800 - 1000}{(1000 + 800)/2} \div \frac{12 - 10}{(10 + 12)/2}$$

$$= \frac{-200}{900} \div \frac{2}{11} = -\frac{200}{900} \cdot \frac{11}{2} = -\frac{2200}{1800} \approx -1.22$$

Interpretation: Demand is **elastic** ($|\varepsilon_d| = 1.22 > 1$). A 1% price increase causes a 1.22% decrease in visitors.

Elasticity Along a Linear Demand Curve 📏



Key insight: For a **linear** demand curve, elasticity **varies** along the curve!

Why? $\varepsilon_d = \frac{dQ}{dP} \cdot \frac{P}{Q}$

- Slope $\frac{dQ}{dP}$ is constant
- But **ratio** $\frac{P}{Q}$ changes

Three regions:

- **Top** (high P, low Q): $|\varepsilon_d| > 1$ (elastic)
- **Middle** (midpoint): $|\varepsilon_d| = 1$ (unit elastic)
- **Bottom** (low P, high Q): $|\varepsilon_d| < 1$ (inelastic)

💡 The **same** demand curve is elastic at some prices, inelastic at others!

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Determinants of Elasticity

What Makes Demand More Elastic? 🤔

Five key determinants:

1. **Availability of substitutes** 🔄
   - More/closer substitutes → **more elastic**
   - Example: Brand-name hotels (many alternatives) vs. only hotel in remote village
2. **Share of budget** 💰
   - Larger expense → **more elastic**
   - Example: International vacation vs. coffee
3. **Necessity vs. luxury** 💎
   - Luxuries → **more elastic**; Necessities → **less elastic**
   - Example: Business travel (necessity) vs. leisure tourism (luxury)

4. **Time horizon** 🕐
   - Long run → **more elastic** (more time to adjust)
   - Example: After fuel price increase, tourists eventually switch to closer destinations
5. **Definition of the market** 🗺️
   - Narrowly defined → **more elastic**
   - Example: “TAP flights to Paris” (elastic) vs. “all flights to Paris” (less elastic) vs. “all air travel” (even less elastic)

Summary: Demand is more elastic when consumers have **options**, **time**, and the good is **less essential**.

Tourism Examples ✈️

Inelastic Tourism Demand:

1️⃣ **Business travel**: Fixed meetings, little flexibility → $|\varepsilon_d| \approx 0.3$

2️⃣ **Last-minute bookings**: Few alternatives, urgency → Low elasticity

3️⃣ **Travel to visit family**: Strong non-economic motivation

4️⃣ **Unique destinations** (e.g., Galápagos): No close substitutes

Elastic Tourism Demand:

1️⃣ **Leisure beach holidays**: Many substitutes (Algarve, Greece, Spain) → $|\varepsilon_d| \approx 2.5$

2️⃣ **Budget airlines**: Highly price-sensitive consumers

3️⃣ **Long-haul tourism**: Large budget share, can be postponed

4️⃣ **All-inclusive packages**: Many competing offers

💡 Tourism managers must understand *their* market's elasticity to price optimally!

Elasticity and Revenue

The Revenue Test 💰

TOTAL REVENUE AND ELASTICITY

Total Revenue: $TR = P \times Q$

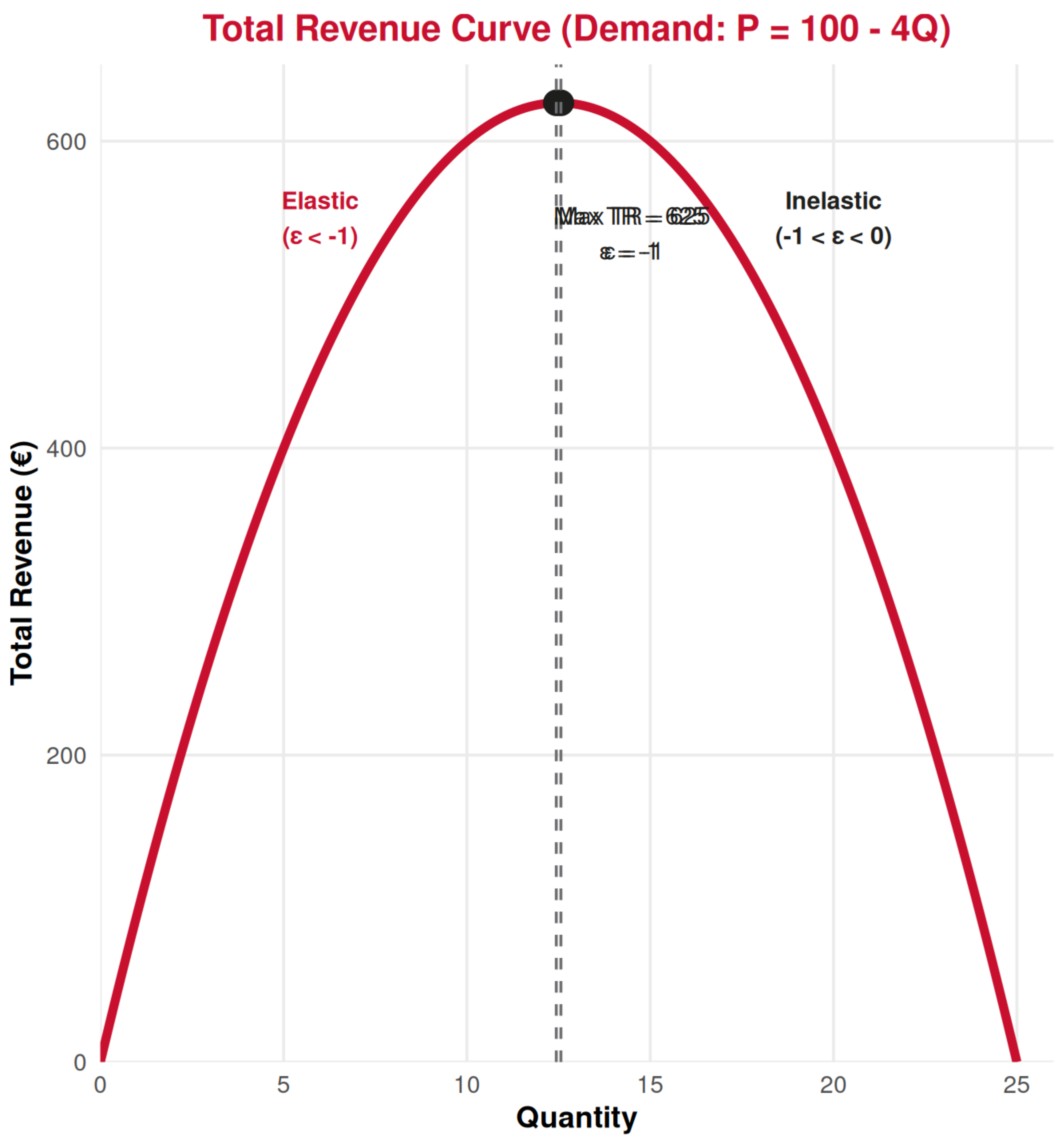
When price changes:

- If demand is **elastic** ($|\varepsilon_d| > 1$): Price and revenue move in **opposite** directions
- If demand is **inelastic** ($|\varepsilon_d| < 1$): Price and revenue move in the **same** direction
- If demand is **unit elastic** ($|\varepsilon_d| = 1$): Revenue is **maximized**

Why?

- **Elastic**: $\%\Delta Q > \%\Delta P$ → quantity effect dominates
- **Inelastic**: $\%\Delta Q < \%\Delta P$ → price effect dominates

Revenue and Elasticity: Graphical View 📈



Key observations:

- Revenue is **maximized** where $|\varepsilon_d| = 1$
- To the **left** (low Q, high P): elastic region, raising P **reduces** TR
- To the **right** (high Q, low P): inelastic region, raising P **increases** TR

Managerial insight:

If you're in the **elastic region**, **cut prices** to boost revenue!

If you're in the **inelastic region**, **raise prices** to boost revenue!

Example: Should the Museum Raise Prices? 🏛️

Gulbenkian Museum charges €8/ticket, sells 10,000 tickets/month. Managers estimate $\varepsilon_d = -1.5$.

Should they raise the price to €10?

Analysis:

- Current $TR = 8 \times 10{,}000 = €80{,}000$
- Demand is **elastic**: $|\varepsilon_d| = 1.5 > 1$
- **Revenue test**: Price and revenue move in **opposite directions**

If price rises, quantity will fall **more than proportionally** → revenue **decreases**.

Recommendation: **Do NOT raise the price**. Instead, consider **lowering** the price to increase revenue!

Verification: If $P = €10$, with $\varepsilon_d = -1.5$, a **25% price increase** causes approximately $-1.5 \times 25\% = -37.5\%$ decrease in quantity → $Q \approx 6{,}250$ → $TR = 10 \times 6{,}250 = €62{,}500 < €80{,}000$ ❌

Other Elasticities

Cross-Price Elasticity 🔄

CROSS-PRICE ELASTICITY OF DEMAND

Measures how quantity demanded of good i responds to a price change in good j:

$$\varepsilon_{ij} = \frac{\%\Delta Q_i}{\%\Delta P_j} = \frac{\Delta Q_i / Q_i}{\Delta P_j / P_j}$$

Interpretation:

- $\varepsilon_{ij} > 0$: Goods i and j are **substitutes** (e.g., flights vs. trains)
- $\varepsilon_{ij} < 0$: Goods i and j are **complements** (e.g., flights and hotels)
- $\varepsilon_{ij} = 0$: Goods are **independent** (e.g., milk and concert tickets)

Example: If $\varepsilon_{\text{hotels, flights}} = -0.4$, a **10% increase** in flight prices causes **4% decrease** in hotel demand.

Income Elasticity 💰

INCOME ELASTICITY OF DEMAND

Measures how quantity demanded responds to income changes:

$$\varepsilon_M = \frac{\%\Delta Q}{\%\Delta M} = \frac{\Delta Q/Q}{\Delta M/M}$$

Interpretation:

- $\varepsilon_M > 1$: **Luxury good** (tourism, fine dining)
- $0 < \varepsilon_M < 1$: **Normal good** (most goods)
- $\varepsilon_M < 0$: **Inferior good** (budget accommodations, bus travel)

Tourism application: International tourism has high income elasticity ($\varepsilon_M \approx 1.5 - 2.5$). During recessions, tourism demand falls sharply!

Summary of Elasticities 📋

Elasticity	Formula	Sign	Interpretation
Price Elasticity	$\frac{\%\Delta Q}{\%\Delta P}$	Usually **negative**	Responsiveness to own price
Cross-Price	$\frac{\%\Delta Q_i}{\%\Delta P_j}$	**Positive** (substitutes) / **Negative** (complements)	Relationship between goods
Income	$\frac{\%\Delta Q}{\%\Delta M}$	**Positive** (normal/ luxury) / **Negative** (inferior)	Responsiveness to income

💡 All measure **percentage changes** → comparable across markets and units!

Applications to Tourism

Dynamic Pricing in Tourism 🎫

How airlines and hotels use elasticity:

Segment 1: Business travelers

- Inelastic demand ($|\varepsilon_d| \approx 0.3$)
- Book last-minute
- Need flexibility
- **Strategy**: **High prices** (€300-500)

Segment 2: Leisure travelers

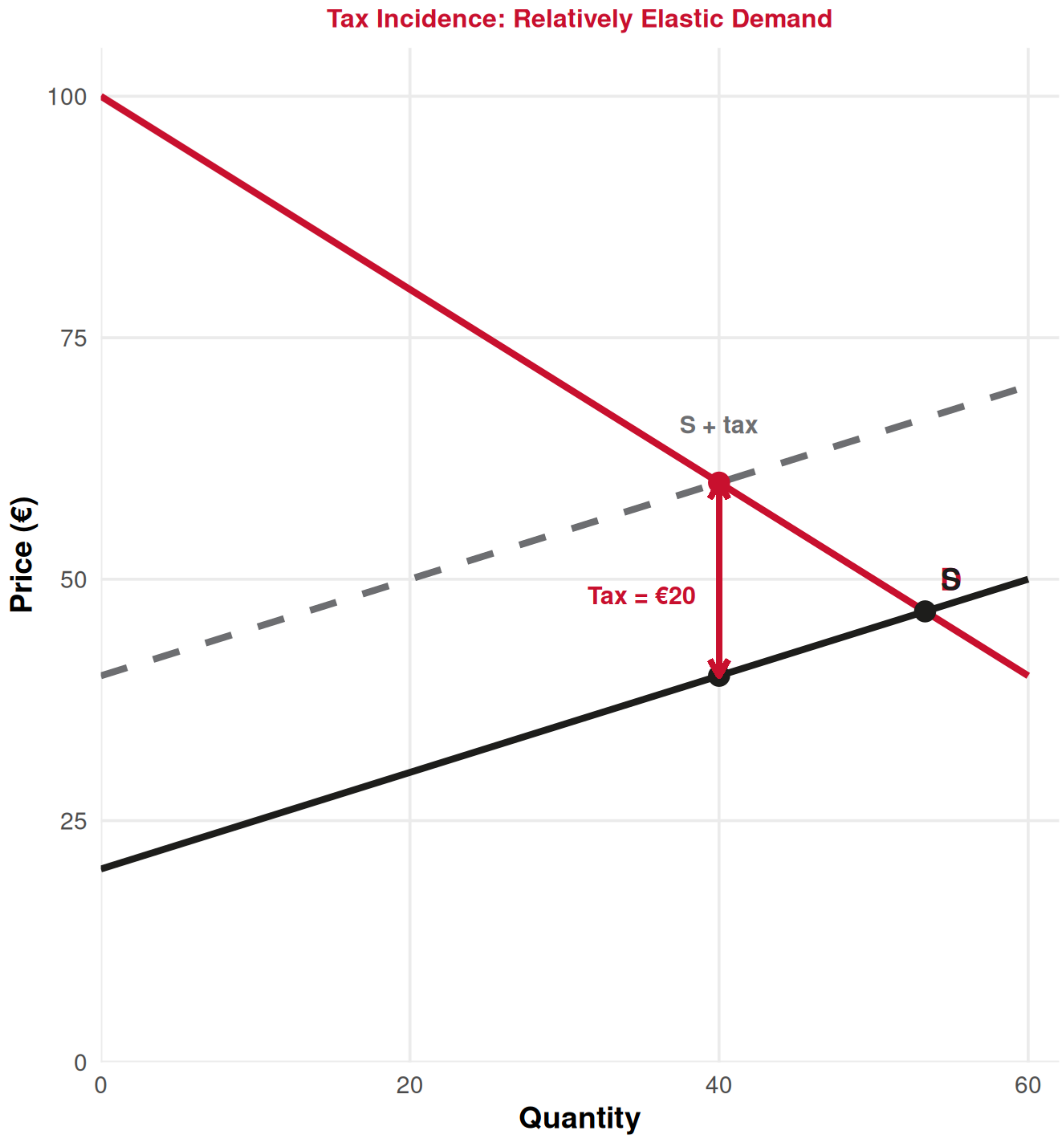
- Elastic demand ($|\varepsilon_d| \approx 2.5$)
- Book in advance
- Flexible dates
- **Strategy**: **Low prices** (€50-100)

Revenue maximization: Charge **different prices** to segments with **different elasticities**!

This is called **price discrimination**.

Taxation and Tourism 🏛️

Who really pays a tourism tax?



Tax incidence depends on **relative elasticities**!

Here, demand is relatively **elastic** (tourists can visit other destinations).

Result: A €20 tax causes:

- Price paid by tourists rises by **€13.33**
- Price received by hotels falls by **€6.66**
- **Sellers bear more** of the tax burden!

General rule: The side with **less elastic** response bears **more** of the tax.

👉 If tourists are price-sensitive (elastic demand), tourism businesses absorb most taxes!

Summary 📋

Today's Key Takeaways:

1. **Price elasticity of demand** (ε_d): percentage change in quantity per 1% price change
2. **Elastic** ($|\varepsilon_d| > 1$): quantity very responsive; **Inelastic** ($|\varepsilon_d| < 1$): not very responsive
3. **Calculation**: Point formula $\varepsilon_d = \frac{dQ}{dP} \cdot \frac{P}{Q}$ or arc formula (midpoint method)
4. **Determinants**: substitutes, budget share, necessity vs. luxury, time, market definition
5. **Revenue**: If elastic, price ↑ → revenue ↓; If inelastic, price ↑ → revenue ↑; Maximized at $|\varepsilon_d| = 1$
6. **Other elasticities**: cross-price (substitutes/complements), income (normal/inferior/luxury)
7. **Tourism applications**: dynamic pricing, tax incidence, demand forecasting

Connection: This builds on demand curves (L8) and leads into supply and market equilibrium (L11+).

Next (after Test 1): Producer theory🏭: costs, profits, and supply!

⚠️ **Test 1 is TOMORROW, March 13!** Covers Lectures 1–8 (Fundamentals + Consumer). Good luck! 🍀

Exercises

Practice Time! ✏️

Elasticity calculation and applications.

Exercise 1: Multiple Choice

Question: The demand for luxury cruises from Lisbon is estimated to have a price elasticity of $\varepsilon_d = -2.5$. If cruise operators raise prices by 8%, what happens to total revenue?

A. Total revenue increases
B. Total revenue decreases
C. Total revenue stays constant
D. Cannot determine without knowing the initial price

Answer: B

Demand is **elastic** ($|\varepsilon_d| = 2.5 > 1$), so price and revenue move in **opposite directions**. An 8% price increase causes approximately $-2.5 \times 8\% = -20\%$ decrease in quantity. Since quantity falls more than price rises, **total revenue decreases**.

This is the **revenue test**: elastic demand → price ↑ → revenue ↓.

Exercise 2: Multiple Choice

Question: Airbnb and traditional hotels are substitutes. If the cross-price elasticity between Airbnb and hotels is $\varepsilon_{AH} = 0.8$, and Airbnb prices increase by 10%, what happens to hotel demand?

A. Hotel demand increases by 8%
B. Hotel demand decreases by 8%
C. Hotel demand increases by 10%
D. Hotel demand increases by 1.25%

Answer: A

Cross-price elasticity formula: $\varepsilon_{AH} = \frac{\%\Delta Q_{\text{hotels}}}{\%\Delta P_{\text{Airbnb}}}$

$$0.8 = \frac{\%\Delta Q_{\text{hotels}}}{10\%} \Rightarrow \%\Delta Q_{\text{hotels}} = 0.8 \times 10\% = 8\%$$

Since $\varepsilon_{AH} > 0$ (substitutes), when Airbnb price rises, hotel demand **increases** by 8%.

Exercise 3: Open Question ✍️

The Algarve Tourism Authority is studying demand for beach resorts. They have the following demand function:

$$Q = 10{,}000 - 20P$$

where Q is the number of tourists per month and P is the average price per night (in €).

a) Calculate the price elasticity of demand when $P = €100$.

b) Is demand elastic or inelastic at this price? What does this mean for revenue if resorts raise prices?

c) At what price is demand unit elastic ($|\varepsilon_d| = 1$)? What is the quantity demanded and total revenue at this price?

d) The income elasticity for Algarve tourism is estimated at $\varepsilon_M = 1.8$. If European incomes rise by 5% next year, by what percentage will demand for Algarve tourism increase?

Exercise 3: Solution for Parts a & b

a) Price elasticity at P = €100:

Demand: $Q = 10{,}000 - 20P$

At $P = 100$: $Q = 10{,}000 - 20(100) = 8{,}000$ tourists

Calculate elasticity: $\varepsilon_d = \frac{dQ}{dP} \cdot \frac{P}{Q}$

$$\frac{dQ}{dP} = -20$$

$$\varepsilon_d = (-20) \cdot \frac{100}{8000} = -\frac{2000}{8000} = -0.25$$

b) Elasticity or inelastic?

$|\varepsilon_d| = 0.25 < 1$ → Demand is **inelastic** at $P = €100.$

Revenue implication: Since demand is inelastic, price and revenue move in the **same direction**. If resorts **raise prices**, total revenue will **increase**. Quantity falls, but by a smaller percentage than price rises, so $TR = P \times Q$ increases.

Exercise 3: Solution for Part c

c) Unit elastic demand ($|\varepsilon_d| = 1$):

For linear demand $Q = 10{,}000 - 20P$ (or inverse: $P = 500 - 0.05Q$), unit elasticity occurs at the **midpoint**.

Method 1 (midpoint of inverse demand):

Choke price (intercept): $P = 500$ when $Q = 0$

Maximum quantity: $Q = 10{,}000$ when $P = 0$

Midpoint: $P^* = \frac{500}{2} = €250, Q^* = \frac{10{,}000}{2} = 5{,}000$ tourists

Verify: $\varepsilon_d = (-20) \cdot \frac{250}{5000} = -\frac{5000}{5000} = -1$ ✓

Total Revenue at unit elasticity:

$$TR = P^* \times Q^* = 250 \times 5{,}000 = €1{,}250{,}000$$

This is the **maximum** possible revenue on this demand curve!

Exercise 3: Solution for Parts d

d) Income elasticity:

$\varepsilon_M = 1.8$ means a **1% increase in income** causes a **1.8% increase in demand**.

If incomes rise by **5%**:

$$\%\Delta Q = \varepsilon_M \times \%\Delta M = 1.8 \times 5\% = 9\%$$

Demand for Algarve tourism will **increase by 9%**.

💡 Since $\varepsilon_M > 1$, tourism is a **luxury good**, highly responsive to income changes!

Next Lecture

March 19, 2026: Market from a Cost Perspective: Geometry of Costs

We shift from **consumers** to **producers**!

⚠️ **Tomorrow, March 13**: Test 1 covering Fundamentals (L1–4) and Consumer (L5–8)

Study tips:

- Review budget constraints, preferences, MRS, utility maximization
- Practice demand curve derivation and consumer surplus
- Understand elasticity calculations and determinants

Thank You!

Questions? 🙋

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Test 1: Thursday, March 13, 2026

Next class (L10): Thursday, March 19, 2026