Projeto 1 – MMF 1, IMPA

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Preços de Opções Europeias

Parâmetros:

$S_0 = 9 \rightarrow$ Preço da ação

$K = 10 \rightarrow$ Strike

$N = 3\ anos \rightarrow$ Tempo para vencimento

$T = 15 \rightarrow$ Número de passos

$r = 6\%\ a.a. \rightarrow$ Risk free rate

$\sigma = 30\%\ a.a. \rightarrow$ Volatilidade

$$u = e^{\sigma\sqrt{\Delta t}+(r-0.5\sigma^2)\Delta t} \rightarrow e^{0.3\sqrt{\frac{3}{15}}+(0.06-0.5*0.3^2)\frac{3}{15}}$$

$$= e^{0.1342+0.003} = e^{0.1372} = 1.147$$

$$d = e^{-\sigma\sqrt{\Delta t}+(r-0.5\sigma^2)\Delta t} \rightarrow e^{0.3\sqrt{\frac{3}{15}}+(0.06-0.5*0.3^2)\frac{3}{15}}$$

$$= e^{-0.1342+0.003} = e^{-0.1312} = 0.877$$

$$\tilde{p} = \frac{e^{r*\Delta t} - d}{u - d} = \frac{e^{0.012} - 0.877}{1.147 - 0.877} = 0.5001007441$$

$$\tilde{q} = 1 - \tilde{p} = 0.4998992558$$

Preços de Opções Europeias

Iniciamos uma lista com $T = 15$ Heads e salvamos, substituímos o último H por T até que ajam apenas Ts, assim teremos um caminho que levará a cada preço final. Por exemplo, $T = 4$:

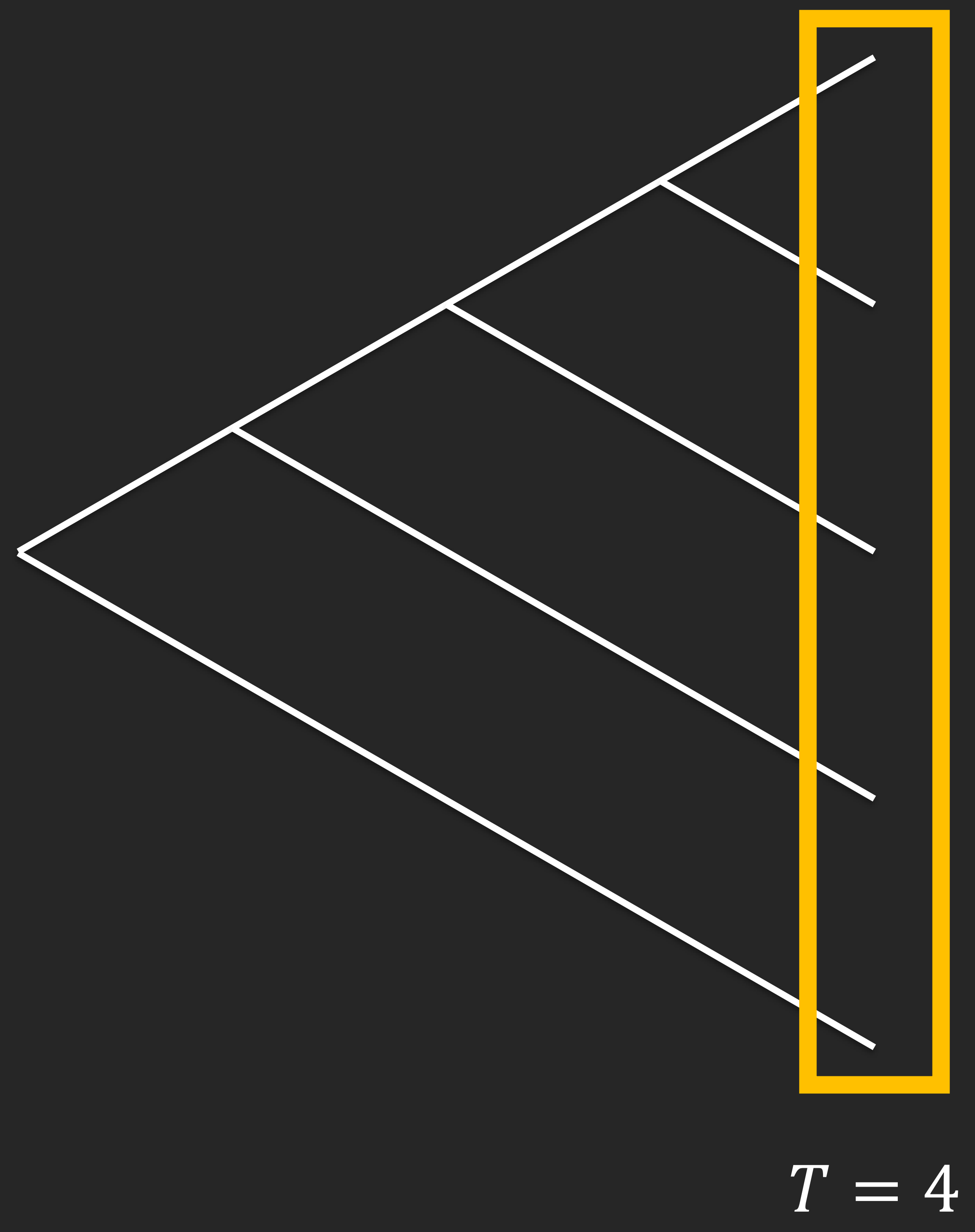
['H', 'H', 'H', 'H']

['H', 'H', 'H', 'T']

['H', 'H', 'T', 'T']

['H', 'T', 'T', 'T']

['T', 'T', 'T', 'T']



Preços de Opções Europeias

Com esse método, conseguimos calcular todos os preços finais possíveis para o ativo subjacente. Usamos eles para calcular o valor das opções no exercício usando uma função payoff.

	('H', 'H', 'H', 'H')	('H', 'H', 'H', 'T')	('H', 'H', 'T', 'T')	('H', 'T', 'T', 'T')	('T', 'T', 'T', 'T')
0	9.0000	9.0000	9.0000	9.0000	9.0000
1	11.8022	11.8022	11.8022	11.8022	7.0193
2	15.4768	15.4768	15.4768	9.2048	5.4745
3	20.2955	20.2955	12.0707	7.1790	4.2697
4	26.6145	15.8289	9.4143	5.5991	3.3301

	('H', 'H', 'H', 'H')	('H', 'H', 'H', 'T')	('H', 'H', 'T', 'T')	('H', 'T', 'T', 'T')	('T', 'T', 'T', 'T')
4	16.6145	5.8289	0	0	0

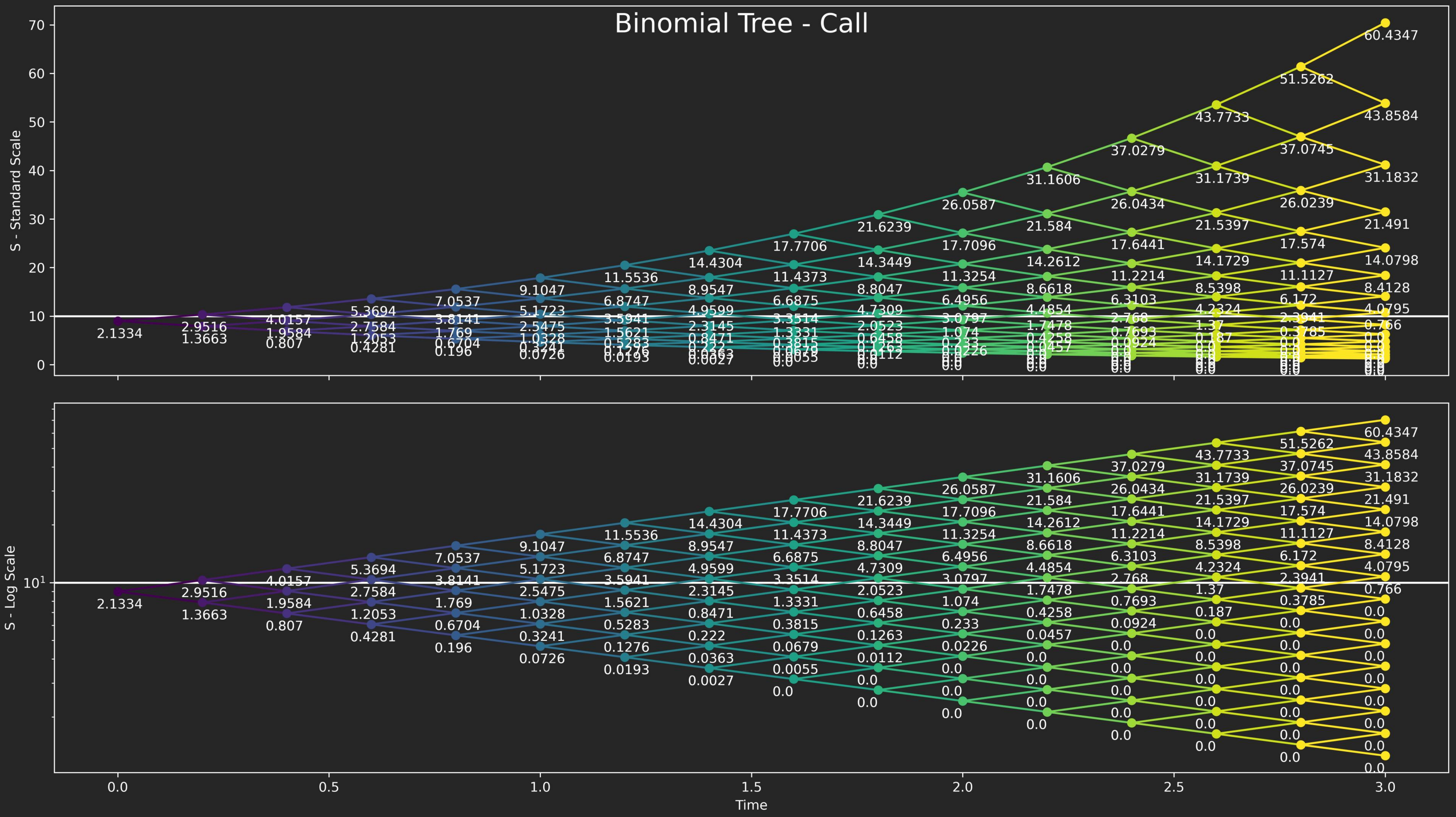
Preços de Opções Europeias

2 a 2 selecionamos os valores da opção e calculamos o seu valor no período anterior

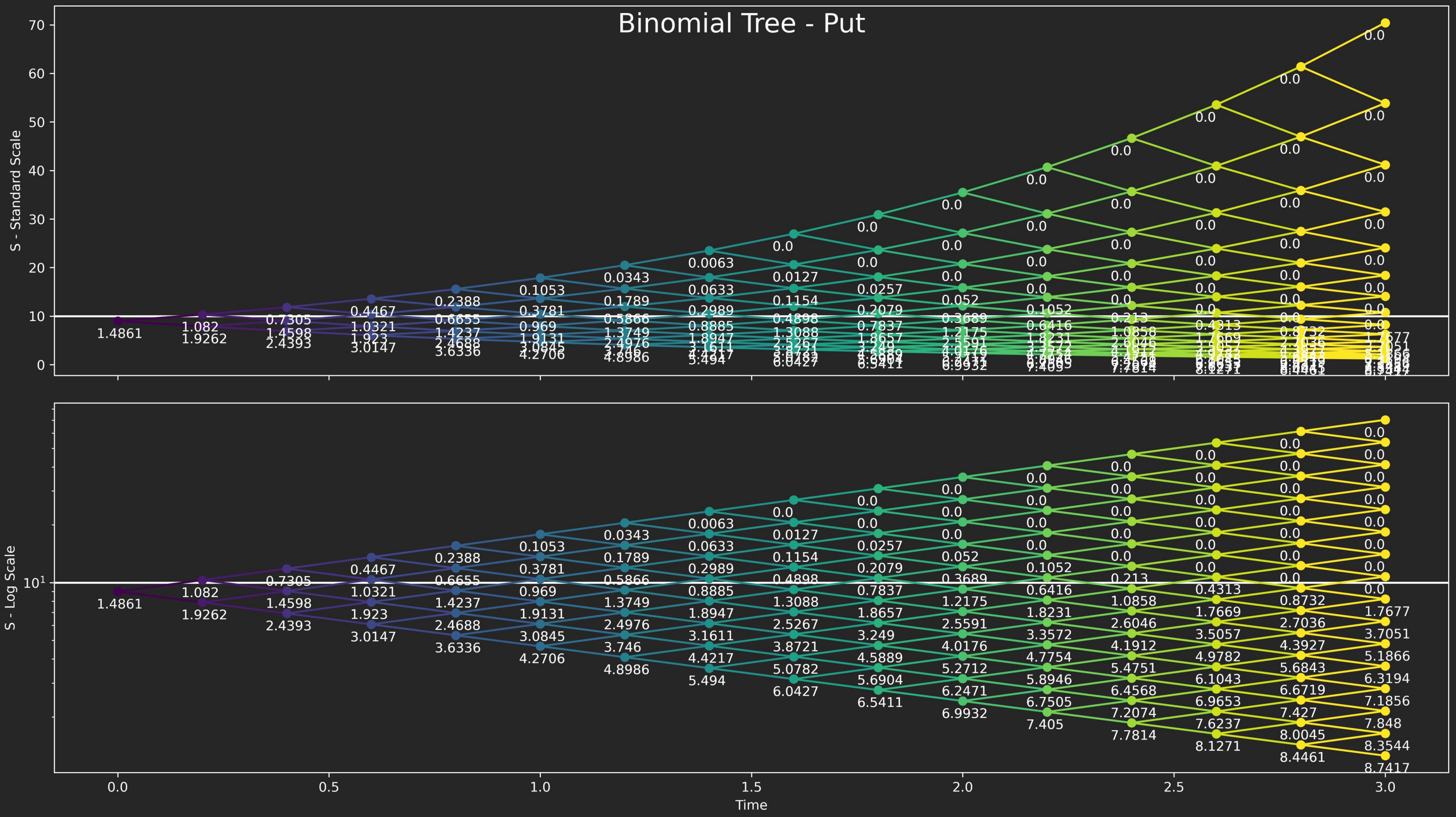
	('H', 'H', 'H', 'H')	('H', 'H', 'H', 'T')	('H', 'H', 'T', 'T')	('H', 'T', 'T', 'T')	('T', 'T', 'T', 'T')
4	16.6145	5.8289	0	0	0
3		10.7355			

$$V_n = e^{-r*\Delta t}(\tilde{p} * V_{n+1}(H) + \tilde{q} * V_{n+1}(T))$$

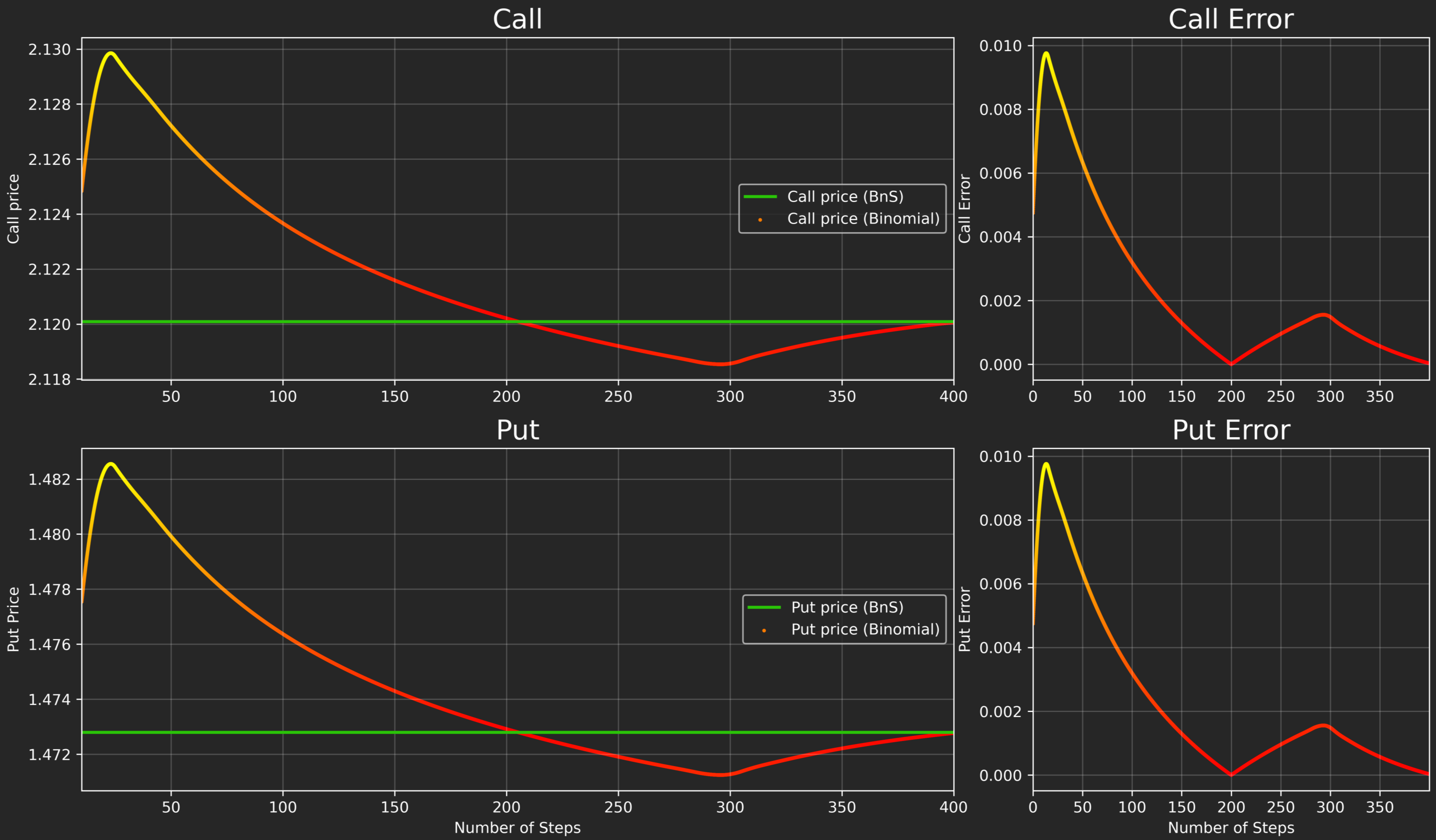
	('H', 'H', 'H', 'H')	('H', 'H', 'H', 'T')	('H', 'H', 'T', 'T')	('H', 'T', 'T', 'T')	('T', 'T', 'T', 'T')
4	16.6145	5.8289	0	0	0
3		10.7355	2.7903	0	0
2			6.4709	1.3357	0
1				3.7352	0.6394
0					2.0932



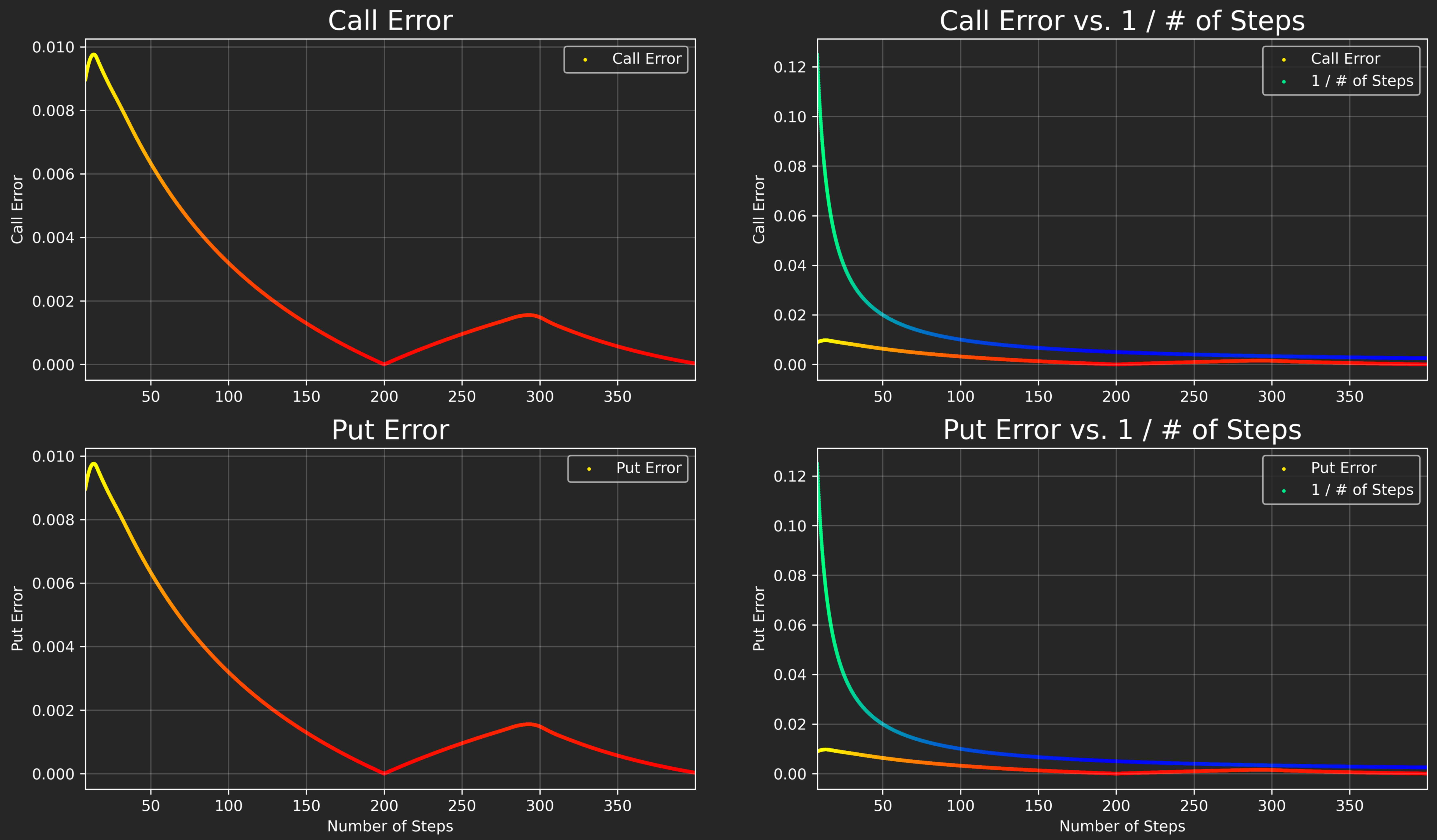
Binomial Tree - Call
S - Standard Scale
S - Log Scale
Time
0 10 20 30 40 50 60 70
10^1
0.0 0.5 1.0 1.5 2.0 2.5 3.0
2.1334
2.9516 1.3663
4.0157 1.9584 0.807
5.3694 2.7584 1.2053 0.4281
7.0537 3.8141 1.769 0.6704 0.196
9.1047 5.1723 2.5475 1.0328 0.3241 0.0726
11.5536 6.8747 3.5941 1.5621 0.5283 0.1276 0.0193
14.4304 8.9547 4.9599 2.3145 0.8471 0.222 0.0363 0.0027
17.7706 11.4373 6.6875 3.3514 1.3331 0.3815 0.0679 0.0055 0.0
21.6239 14.3449 8.8047 4.7309 2.0523 0.6458 0.1263 0.0112 0.0 0.0
26.0587 17.7096 11.3254 6.4956 3.0797 1.074 0.233 0.0226 0.0 0.0 0.0
31.1606 21.584 14.2612 8.6618 4.4854 1.7478 0.4258 0.0457 0.0 0.0 0.0 0.0
37.0279 26.0434 17.6441 11.2214 6.3103 2.768 0.7693 0.0924 0.0 0.0 0.0 0.0 0.0
43.7733 31.1739 21.5397 14.1729 8.5398 4.2324 1.37 0.187 0.0 0.0 0.0 0.0 0.0 0.0
51.5262 37.0745 26.0239 17.574 11.1127 6.172 2.3941 0.3785 0.0 0.0 0.0 0.0 0.0 0.0 0.0
60.4347 43.8584 31.1832 21.491 14.0798 8.4128 4.0795 0.766 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0


Binomial Tree - Put
S - Standard Scale
S - Log Scale
Time
0 10 20 30 40 50 60 70
10^1
0.0 0.5 1.0 1.5 2.0 2.5 3.0
1.4861
1.082 1.9262
0.7305 1.4598 2.4393
0.4467 1.0321 1.923 3.0147
0.2388 0.6655 1.4237 2.4688 3.6336
0.1053 0.3781 0.969 1.9131 3.0845 4.2706
0.0343 0.1789 0.5866 1.3749 2.4976 3.746 4.8986
0.0063 0.0633 0.2989 0.8885 1.8947 3.1611 4.4217 5.494
0.0 0.0127 0.1154 0.4898 1.3088 2.5267 3.8721 5.0782 6.0427
0.0 0.0 0.0257 0.2079 0.7837 1.8657 3.249 4.5889 5.6904 6.5411
0.0 0.0 0.0 0.052 0.3689 1.2175 2.5591 4.0176 5.2712 6.2471 6.9932
0.0 0.0 0.0 0.0 0.1052 0.6416 1.8231 3.3572 4.7754 5.8946 6.7505 7.405
0.0 0.0 0.0 0.0 0.0 0.213 1.0858 2.6046 4.1912 5.4751 6.4568 7.2074 7.7814
0.0 0.0 0.0 0.0 0.0 0.0 0.4313 1.7669 3.5057 4.9782 6.1043 6.9653 7.6237 8.1271
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.8732 2.7036 4.3927 5.6843 6.6719 7.427 8.0045 8.4461
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 1.7677 3.7051 5.1866 6.3194 7.1856 7.848 8.3544 8.7417

	400 steps	Black and Scholes	difference
call	2.1200648	2.1200938	-0.0000291
put	1.4727669	1.4727959	-0.0000291



Call
Call Error
Put
Put Error
Call price
Call Error
Put Price
Put Error
Call price (BnS)
Call price (Binomial)
Put price (BnS)
Put price (Binomial)
Number of Steps
Number of Steps



Call Error
Call Error
0.010
0.008
0.006
0.004
0.002
0.000
50
100
150
200
250
300
350
Call Error vs. 1 / # of Steps
Call Error
1 / # of Steps
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Put Error
Put Error
Put Error vs. 1 / # of Steps
1 / # of Steps
Number of Steps
Number of Steps

A. Referências

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Hull – Options, Futures e other Derivatives (2017)

Griffiths, Hill and Judge - Learning and Practicing Econometrics (1993)

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