Monte Carlo Models in Finance

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Scaling Dispersion Metrics

$$Variance = \sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$$

$$Standard\ Dev. = \sigma = \sqrt{\sigma^2}$$

Scaling Variance

$$\frac{\sum_{i=1}^{N} 2 * (x_i - \mu)^2}{N} = 2 * \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N} = 2 * Variance$$

Which implies

$$Standard\ Dev. = \sqrt{2 * \sigma^2} = \sigma * \sqrt{2}$$

Wiener Processes

$$\Delta z = \epsilon\sqrt{\Delta t}\ |\epsilon \leftrightarrow N(0,1)$$

$$mean(\Delta z) = 0$$
$$std(\Delta z) = \sqrt{\Delta t}$$

Python 5

Generalized Wiener Processes

$$\Delta x = a\Delta t + b\Delta z$$

$$mean(\Delta z) = a\Delta t$$
$$std(\Delta z) = b\sqrt{\Delta t}$$

Python 6

Asset Price as a Generalized Wiener Process

$$\Delta S = \mu S \Delta t + \sigma S \epsilon \sqrt{\Delta t}$$
$$\frac{\Delta S}{S} = \mu \Delta t + \sigma \epsilon \sqrt{\Delta t}$$

$$mean\left(\frac{\Delta S}{S}\right) = \mu \Delta t$$
$$std\left(\frac{\Delta S}{S}\right) = \sigma \sqrt{\Delta t}$$

Python 7

Log Asset Price

$$\frac{d\ln(S)}{\ln(S)} = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma\epsilon\sqrt{dt}$$

$$mean\left(\frac{\Delta S}{S}\right) = e^{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t}$$

$$std(\Delta z) = e^{\sigma\sqrt{\Delta t}}$$

Python 8

Challenge 1

Make a Monte Carlo simulation from the historical returns of an asset. What distribution, mean and standard deviation are the returns going to have? How do they relate to the original asset's return?

Challenge 2

A Poisson process introduces discontinuous jumps in the model. Add these jumps as a function of the volatility to the model using the following property:

$$\Delta S_{Jump} = S * \sigma * \sqrt{\Delta t} * \overline{|k|} * dir \begin{cases} 1 \rightarrow 0.5 \\ -1 \rightarrow 0.5 \end{cases} * N \begin{cases} 1 \rightarrow p \\ 0 \rightarrow (1-p) \end{cases}$$

In which:

p: probability of a jump

$\bar{k}$: size of a jump in standard deviations

dir: np.random.choice([-1,1])

Logarithms

$$e^n = a \rightarrow log_e a = n$$

$$log_e a = \ln(a)$$

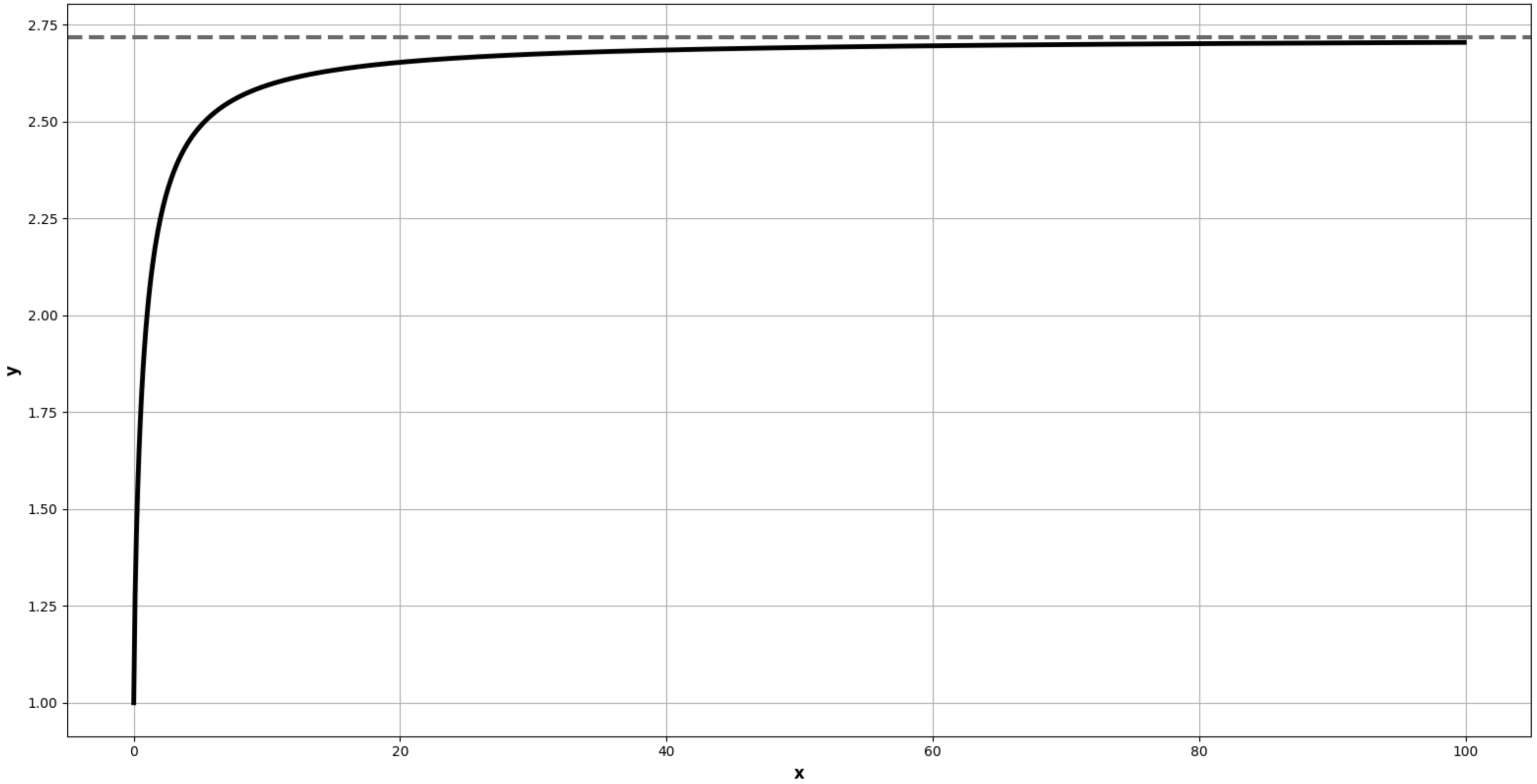
$$\ln(x * y) = \ln(x) + \ln(y)$$

$$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$$

$$\ln(x^y) = y * \ln(x)$$

Euler's Number

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$



Euler's Number

$$e^{r_c} = \left(1 + \frac{r_{exp}}{m}\right)^m \rightarrow e^{r_c t} = \left(1 + \frac{r_{exp}}{m}\right)^{mt}$$

$$e^{r_c t} \cong e^{rt} \cong e^{r_{exp} t}$$

Given that:

r: annualized interest rate

m: number of accruals per year

r_c: equivalent continuous interest rate

t: number of years

Euler's Number

$$e^{r_c} = \left(1 + \frac{r_{exp}}{m}\right)^m \rightarrow e^{r_c t} = \left(1 + \frac{r_{exp}}{m}\right)^{mt}$$

$$e^{r_c t} \cong e^{rt} \cong e^{r_{exp} t}$$

Example (fed funds rate):

r: 5.33%
m: 360
t: 1

$$Fator = \left(1 + \frac{5.33\%}{360} * 360 * 1\right) = 1.0533$$

$$\left(1 + \frac{5.19\%}{360}\right)^{360*1} = 1.0533 \therefore re_{xp} = 5.1932\%$$

$$e^{r_c} = 1.0533 \therefore r_c = 5.1928\%$$

Euler's Number

$$Factor = \left(1 + \frac{5.33\%}{360} * 360 * 1\right) = 1.0533$$

$$\left(1 + \frac{5.19\%}{360}\right)^{360*1} = 1.0533 \therefore re_{xp} = 5.1932\%$$

$$e^{r_c} = 1.0533 \therefore r_c = 5.1928\%$$

$$e^{r_c t} \cong e^{rt} \cong e^{r_{exp} t}$$

$$1.053300 \cong 1.054746 \cong 1.053304$$