Intro. to Monte Carlo Methods

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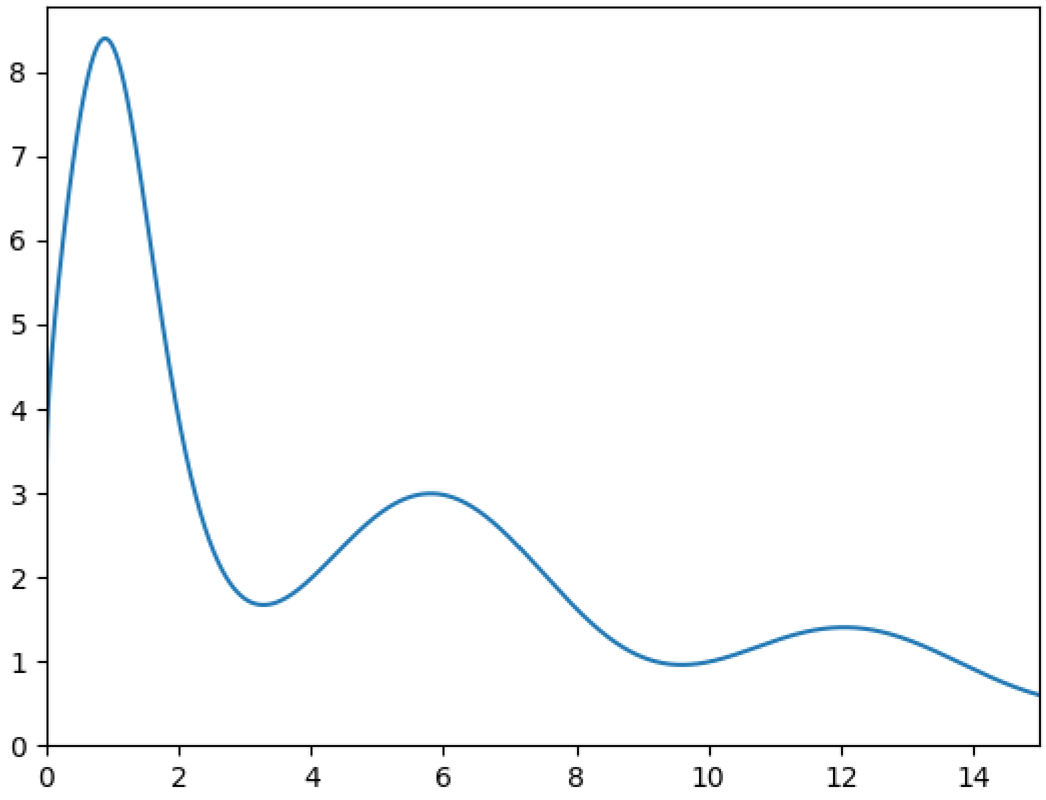
Basic Concepts

- Use of random samples to estimate quantitative variables;
- N independent iterations, in which each of them has no significance by itself;
- Highly dependent of the assumptions.

Basic Example

- Calculating the area below a function using Monte Carlo.

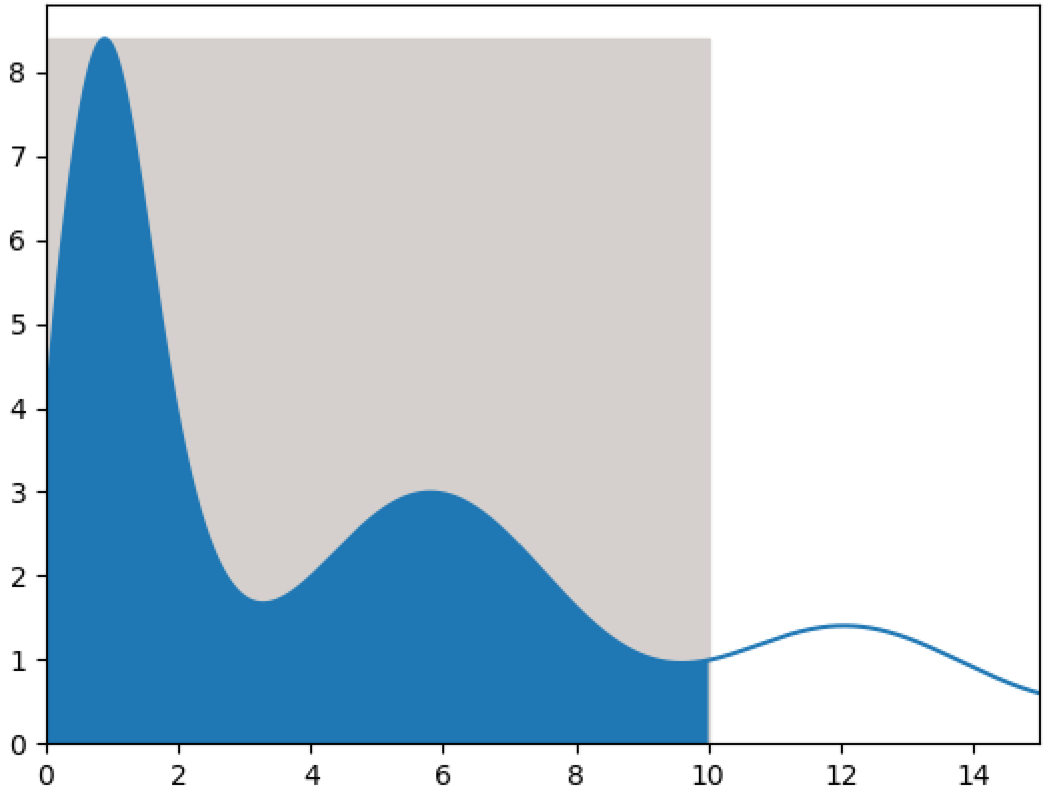


$$f(x) = -2e^{x} \cdot \frac{\cos(x) + \ln\left(x + 2^{\left(\sqrt{\frac{1}{x+1}+x}\right)}\right)}{\sqrt{x} + x^{2} - \pi^{x}}$$

$$A = \int_{0}^{10} f(x)\,\mathrm{dx}$$

Basic Example

- Define the area of interest, i.e. $x \in (0, 10)$

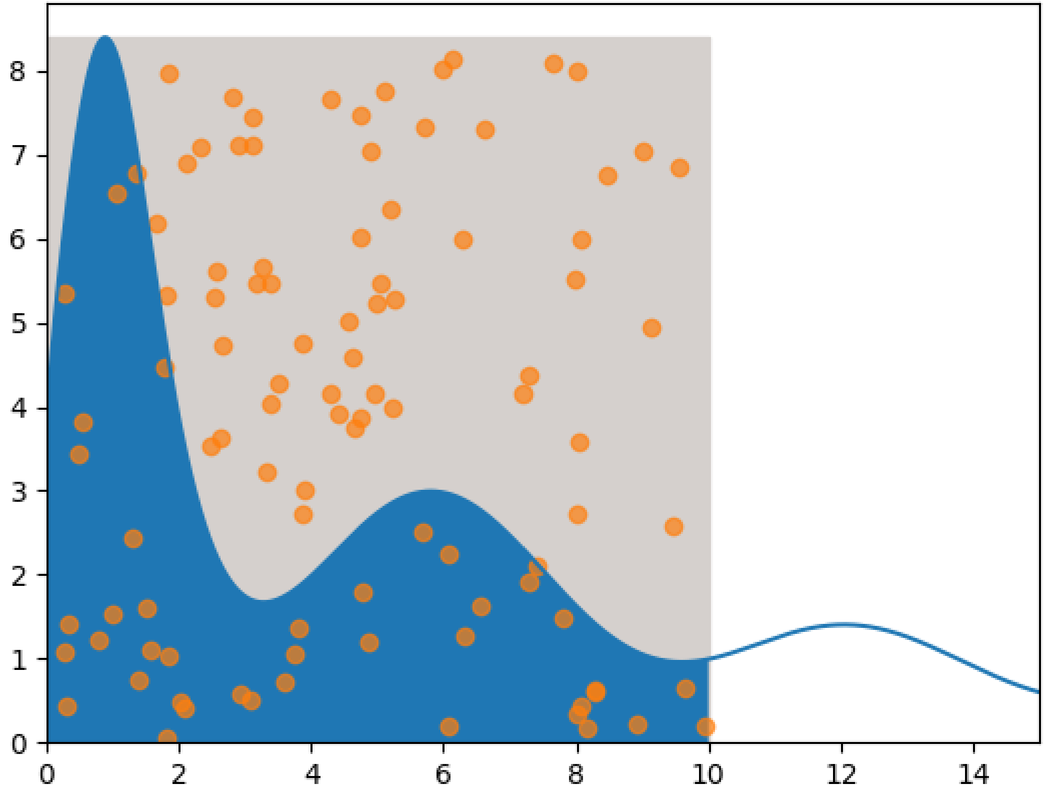


$$f(x) = -2e^{x} \cdot \frac{\cos(x) + \ln\left(x + 2^{\left(\sqrt{\frac{1}{x+1}+x}\right)}\right)}{\sqrt{x} + x^2 - \pi^x}$$

$$A = \int_0^{10} f(x)\,\mathrm{dx}$$

Basic Example

- Sample N random uniformly distributed (x, y) coordinates inside the known area of interest (grey).



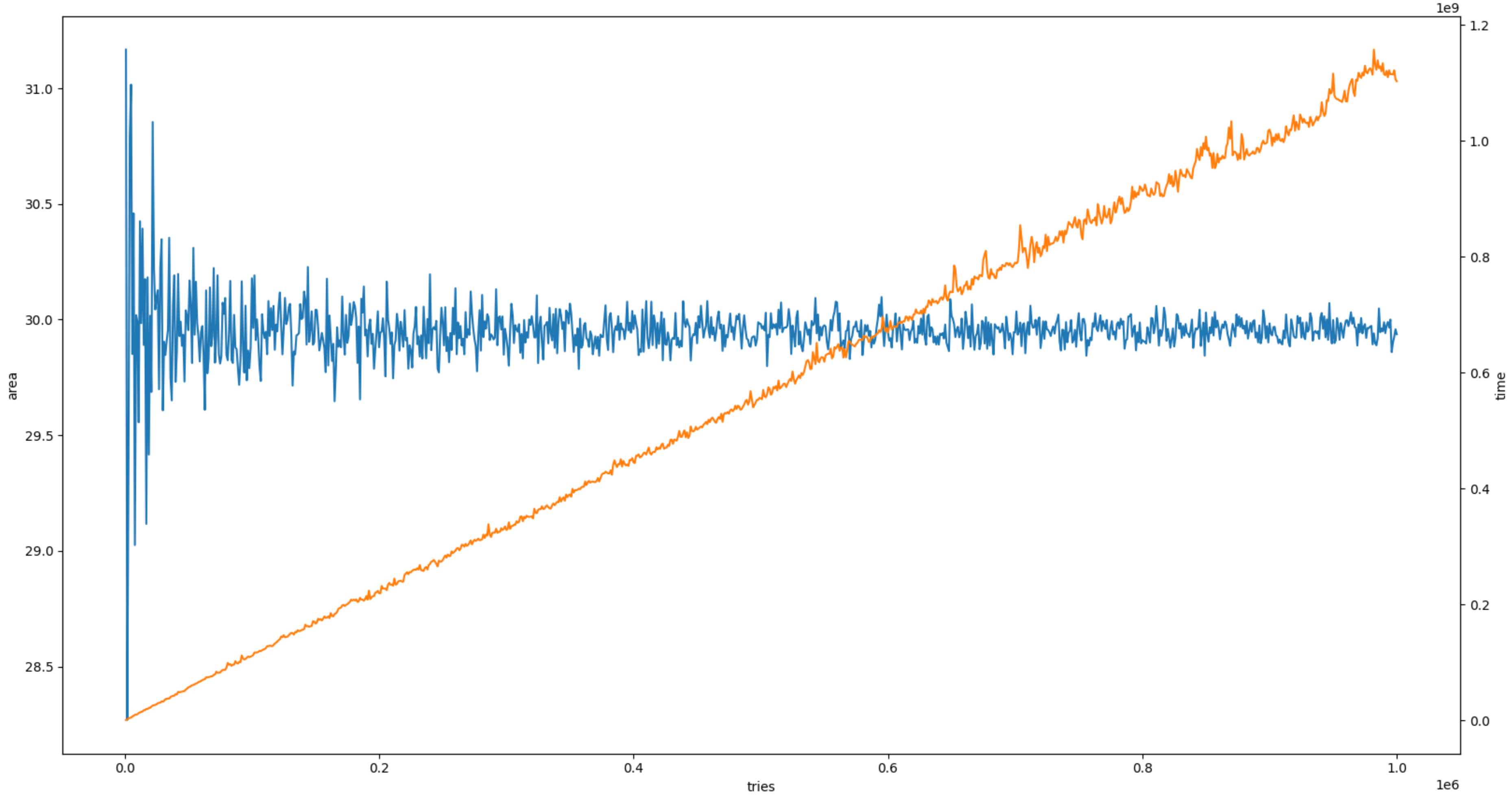
We can estimate the blue area as:

$$A \cong \frac{N_{below}}{N_{total}} * A_{grey}$$

Multiply the percentage of samples beneath the function by the total grey area.

Basic Example

- As we increase N_{total}, the estimated area converges and its standard deviation decreases, but the time taken to calculate the area increases.

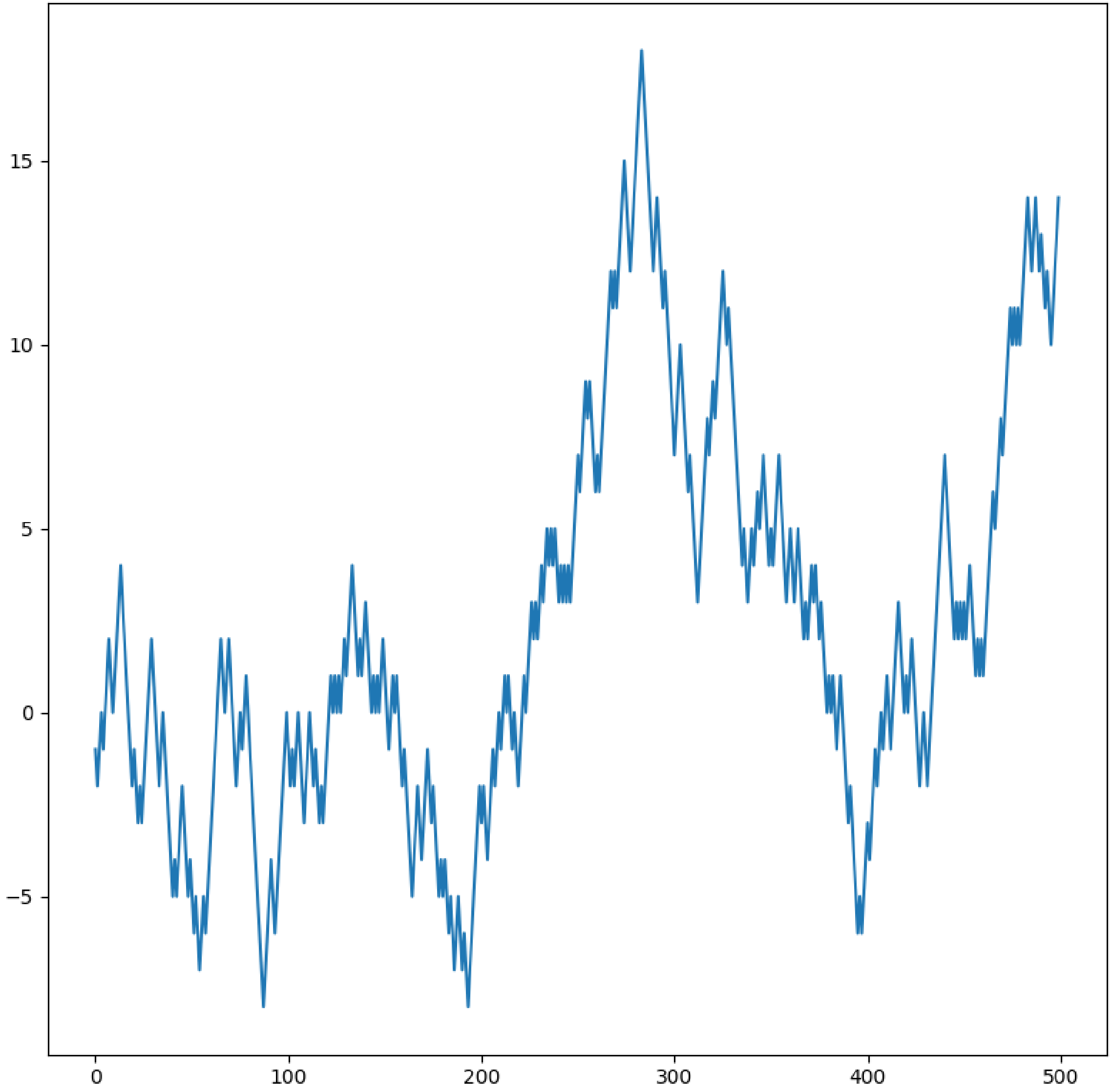


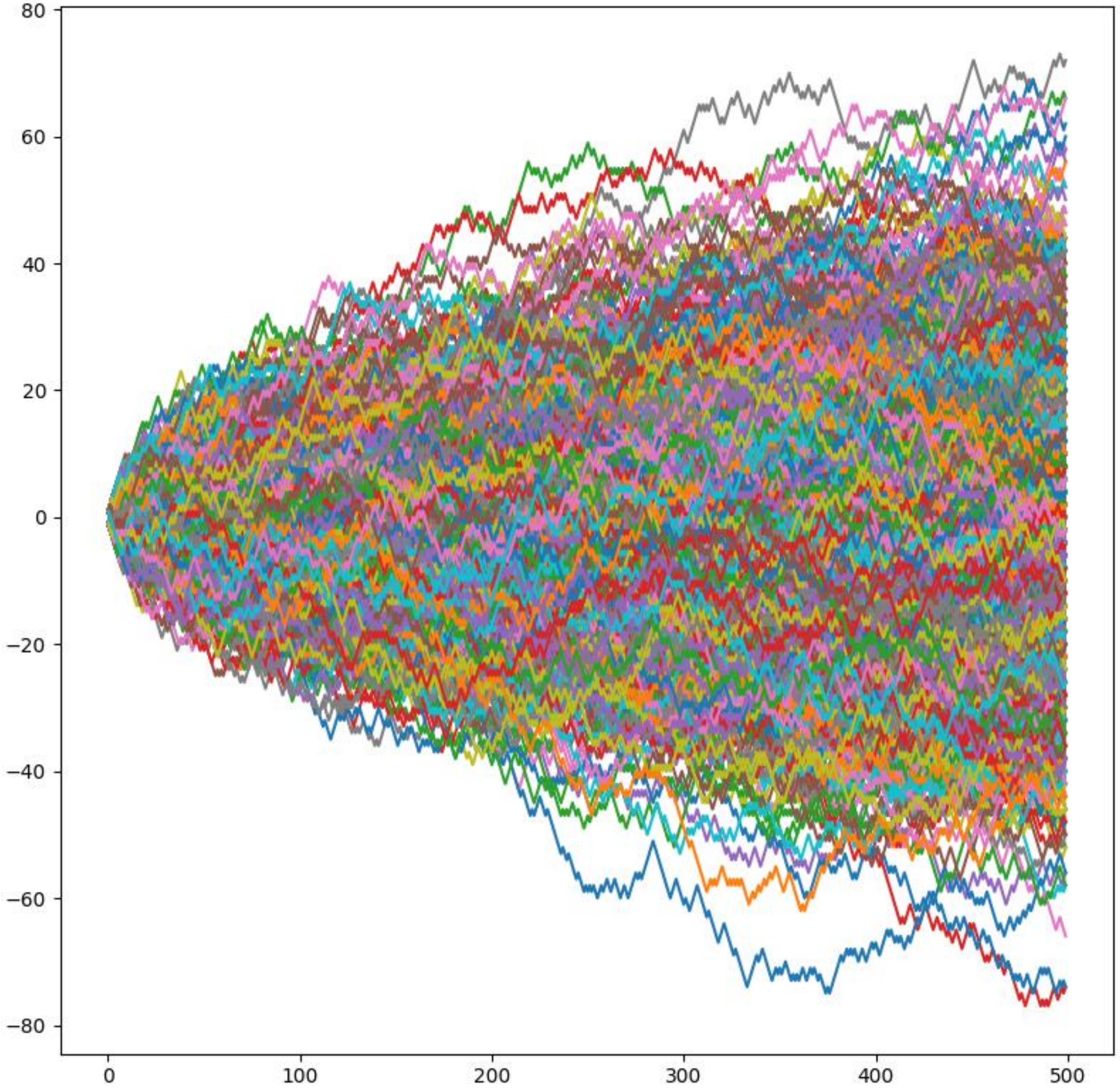
Modeling a Stochastic Behaviour

- Simulating a coin toss game where the player wins 1 point if he gets Heads and loses 1 point if he gets Tails.

$$p(H) = p(T) = 0.5$$
$$f(H) \rightarrow +1; f(T) \rightarrow -1$$
$$E\big(f(.)\big) \rightarrow 0$$

Modeling a Stochastic Behaviour





Modeling a Stock Price

- A Monte Carlo simulation can be used to simulate the price movements of financial assets;

- Such simulations have many applications in risk modeling and asset pricing, which will be shown in another moment.

- Since we haven't covered the basics of stochastic calculus, we won't look too deep in the formulas. Instead, we'll focus on modeling the price movement.

Modeling a Stock Price

$$\Delta S = rS\Delta t + \sigma S \epsilon \sqrt{\Delta t}$$

$$\frac{\Delta S}{S} = r\Delta t + \sigma \epsilon \sqrt{\Delta t}$$

Onde:

- S : stock price
- t : time
- r : risk-free rate for a Δt period
- σ : volatility (<u>not standard deviation</u>)
- ϵ : normally distributed random variable ($N(0,1)$)

Modeling a Stock Price