Searching for Subspace Trails and Truncated Differentials

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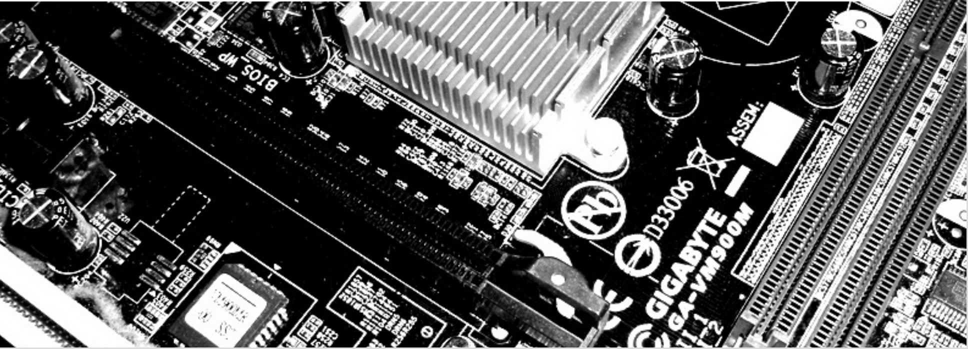


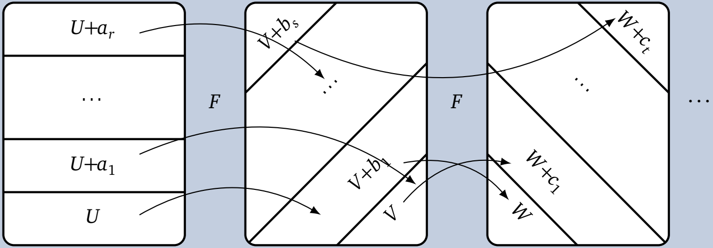
Differential Cryptanalysis

Structural Attacks

Subspace Trail Cryptanalysis

RUB

Main Idea



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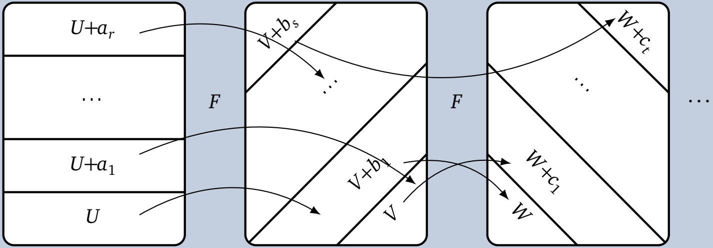

Subspace Trail Cryptanalysis [GRR16] (Last Year's FSE)

Let U, V be subspaces of $\mathbb{F}_2^n$, and $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$. We write $U \xrightarrow{F} V$, iff

$$\forall a \in U^{\perp} : \exists b \in V^{\perp} : F(U+a) \subseteq V+b$$

Outline

RUB

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Intuition

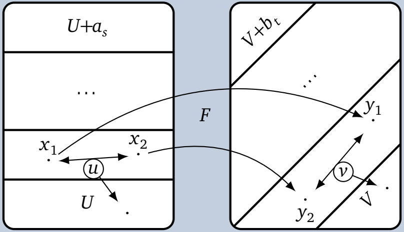
The Image of the Derivative is in the Subspace

RUB

Lemma

Let $U \xrightarrow{F} V$ *be a subspace trail. Then for all* x: $F(x) + F(x+u) \in V$.

Proof



Link to Truncated Differentials

Definition [Knu94; BLN14]

Let $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$. A *truncated differential* of probability one is a pair of affine subspaces $U+s$ and $V+t$ of $\mathbb{F}_2^n$, s. t.

$$\forall u \in U : \forall x \in \mathbb{F}_2^n : F(x) + F(x+u+s) \in V+t$$

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RUB

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- Direct consequence from above Lemma:

Link: Subspaces Trails are Truncated Differentials with probability one

Let $U \xrightarrow{F} V$ be a subspace trail. Then $U+0$ and $V+0$ are a truncated differential with probabiliy one.

Provable Resistant against Subspace Trails

How to search efficiently for Subspace Trails?

RUB

Security against Subspace Trails?

Given the round function $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$ of an SPN cipher, prove the resistance against subspace trail attacks!

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Main problem: Too many possible starting points.

Already for initially one-dimensional subspaces there are $2^n - 1$ possibilities.

Can't we just activate a single S-box and check to what this leads us?

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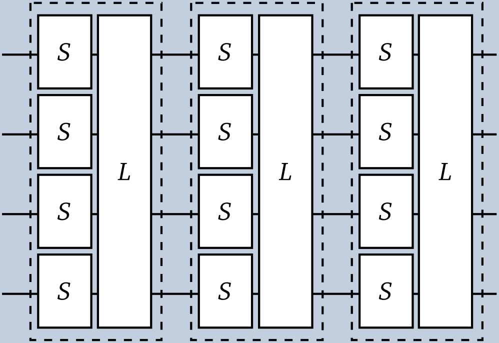
The short answer is:
No![1]

[1] The long answer is: Read our paper ☺

Approach to the Algorithm

SPN Structure



Easy parts

- Given a starting subspace, computing the trail is easy.
- The effect of the linear layer L to a subspace U is clear:

$$U \xrightarrow{L} L(U)$$

How to reduce the number of starting points?

Two possibilities, depending on the S-box S.

Possibility I

The short one

Observation

For an S-box S and $U \xrightarrow{S} V$, because of the above lemma, $\forall x \in \mathbb{F}_2^n$ and $\forall u \in U$:

$$S(x) + S(x+u) \in V$$

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By definition, $V^{\perp}$ is thus the set of zero-linear structures of S.

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Theorem

Let $F : \mathbb{F}_2^{kn} \to \mathbb{F}_2^{kn}$ be an S-box layer that applies k S-boxes with no non-trivial linear structures in parallel. Then every essential subspace trail $U \xrightarrow{F} V$ is of the form

$$U = V = U_1 \times \cdots \times U_k,$$

where $U_i \in \left\{\{0\}, \mathbb{F}_2^n\right\}$.

Possibility I

Algorithm

Algorithm

- Simply (de-)activate S-boxes
- Compute resulting subspace trail

Complexity (No. of starting Us)

For k S-boxes: 2^k (can be further decreased to k).

In particular, in this case, bounds from activating S-boxes are optimal.

This approach is independent of the S-box, i. e. any S-box without linear structures behaves the same with respect to subspace trails.

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The problem with S-boxes that have linear structures

Subspace trails through S-box layers with *one*-linear structures are not necessarily a direct product of subspaces (see e. g. Present).

Possibility II

The long one, but only the idea

Observation

If $U_1 \xrightarrow{F} U_2$ is a subspace, then for any $V_1 \subseteq U_1$ there exists a $V_2 \subseteq U_2$, s. t. $V_1 \xrightarrow{F} V_2$:

$$\begin{array}{ccc} U_1 & \xrightarrow{F} & U_2 \\ \cup\!\shortmid & & \cup\!\shortmid \\ V_1 & \xrightarrow{F} & V_2 \end{array}$$

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Complexity (Size of $\mathbb{W}$)

For an S-box layer $F : \mathbb{F}_2^{kn} \to \mathbb{F}_2^{kn}$ with k S-boxes, each n-bit: $|\mathbb{W}| = k \cdot (2^n - 1)$

Algorithm Idea

- Find a good set $\mathbb{W}$, s. t. for any possible subspace trail over the S-box layer $U \xrightarrow{F} V$, there is an element $W \in \mathbb{W}$ s. t. $\{W\} \subseteq V$.
- Compute the subspace trails for any starting point $W \in \mathbb{W}$.

Conclusion/Questions

Thank you for your attention!

Main Result

- Provable bound length of *every possible* subspace trail in SPN cipher

Open Problems

- Other structures then SPNs?
- Truncated Differentials?

Mainboard & Questionmark Images: flickr

References I

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[BLN14] C. Blondeau, G. Leander, and K. Nyberg. “Differential-Linear Cryptanalysis Revisited”. In: *FSE’14*. Vol. 8540. LNCS. Springer, 2014, pp. 411–430. doi: 10.1007/978-3-662-46706-0_21.

[GRR16] L. Grassi, C. Rechberger, and S. Rønjom. “Subspace Trail Cryptanalysis and its Applications to AES”. In: *IACR Trans. Symmetric Cryptol.* 2016.2 (2016), pp. 192–225. doi: 10.13154/tosc.v2016.i2.192-225.