BISON
Instantiating the Withened Swap-Or-Not Construction

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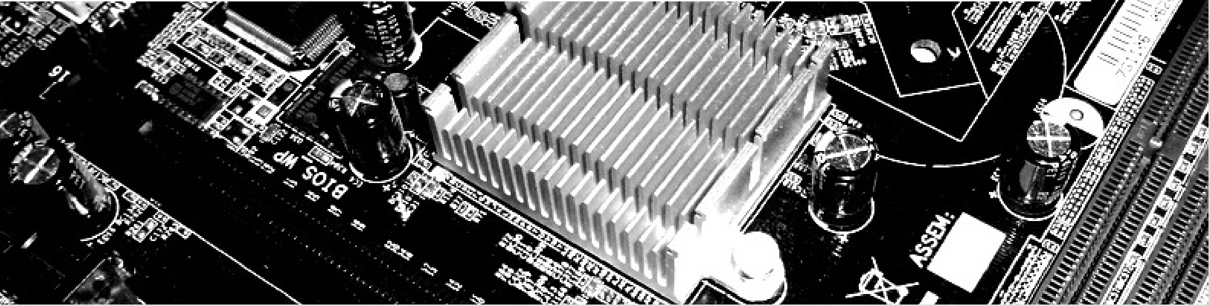
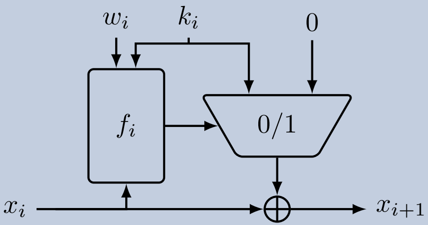


The WSN construction

Published by Tessaro [Tes15] at AsiaCrypt 2015.

Overview



Whitened Swap-Or-Not round function

$$x_i \mapsto x_i + f_{b(i)}(w_i + \max\{x_i, x_i + k_i\}) \cdot k_i$$

Security Proposition (informal)

The WSN construction with $\mathcal{O}(n)$ rounds is

$(2^{n-\mathcal{O}(\log n)}, 2^{n-\mathcal{O}(1)})$-secure.

(p,q)-secure: Attackers querying the encryption at most p and the underlying f_i's q times have only negl. advantage.

Generic Analysis

On the number of rounds

Observation

- The ciphertext is the plaintext plus a random subset of the round keys:

$$c = p + \sum_{i=1}^{r} \lambda_i k_i$$

- For pairs p_i, c_i: $\operatorname{span}\{p_i + c_i\} \subseteq \operatorname{span}\{k_j\}$.

Problematic because

- $\operatorname{span}\{k_j\} \subset \mathbb{F}_2^n$ reveals information on the round keys
- for $r < n$ there exists probability one linear hulls (exploitable: easy),
- for $r < 2n - 3$ there exists zero correlation linear hulls (exploitable: ???).

Rationale 1

Any instance must iterate at least n rounds; any set of n consecutive keys should be linear indp.

Generic Analysis

On the Boolean functions f_i

RUB

Observation

- If the f_i do not depend on a (linear combination of) bit(s), i. e.

$$f_i(x) = f_i(x + \delta)$$

this difference propagates through the whole encryption with non-negligible probability.

Why could this happen?

- For example, when the difference does not influence the lexicographic ordering of x and $x + k_i$.

Rationale 2

For any instance, the f_i should depend on all bits, and for any $\delta \in \mathbb{F}_2^n : \Pr[f_i(x) = f_i(x + \delta)] \approx \frac{1}{2}$.

Addressing Rationale 1

The Key Schedule

RUB

Rationale 1

Any instance must iterate at least n rounds; any set of n consecutive keys should be linear indp.

Design Decisions

- Choose number of rounds as $2 \cdot n$
- Round Keys derived from the state of LFSRs
- Add round constants c_i to w_i round keys

Implications

- Clocking an LFSR is cheap
- For an LFSR with feedback polynomial of degree n, every n consecutive states are linearly independent
- Round constants avoid structural weaknesses

Addressing Rationale 2

The Round Function

RUB

Rationale 2

For any instance, the f_i should depend on all bits, and for any $\delta \in \mathbb{F}_2^n :\ \Pr[f_i(x) = f_i(x+\delta)] \approx \frac{1}{2}$.

Design Decision

- Choose $f_i : \mathbb{F}_2^n \to \mathbb{F}_2$ to be bent
- Replace $x \mapsto \max\{x, x+k\}$ by

$$\Phi_k(x) : \mathbb{F}_2^n \to \mathbb{F}_2^{n-1}$$
$$\Phi_k(x) := (x + x[i(k)] \cdot k)[j]_{\substack{1 \leqslant j \leqslant n \\ j \neq i(k)}}$$

Implications

- With Φ_k we preserve the bent properties
- Bent functions only exists for even n
- Encryption now only possible for odd block lengths

BISON's round function

For round keys $k_i \in \mathbb{F}_2^n$ and $w_i \in \mathbb{F}_2^{n-1}$ the round function computes

$$R_{k_i,w_i}(x) := x + f_{b(i)}\big(w_i + \Phi_{k_i}(x)\big) \cdot k_i.$$

where

- Φ_{k_i} and $f_{b(i)}$ are defined as

$$\begin{aligned}\Phi_k(x) &: \mathbb{F}_2^n \to \mathbb{F}_2^{n-1} \\ \Phi_k(x) &:= (x + x[i(k)] \cdot k)[j]_{\substack{1 \leqslant j \leqslant n \\ j \neq i(k)}}\end{aligned}$$

$$\begin{aligned}f_{b(i)} &: \mathbb{F}_2^{\frac{n-1}{2}} \times \mathbb{F}_2^{\frac{n-1}{2}} \to \mathbb{F}_2 \\ f_{b(i)}(x, y) &:= \langle x, y \rangle + b(i),\end{aligned}$$

- and $b(i)$ is 0 if $i \leqslant \frac{r}{2}$ and 1 else.

BISON's key schedule

Given

- primitive $p_k, p_w \in \mathbb{F}_2[x]$ with $\deg(p_k) = n$ and $\deg(p_w) = n - 1$ and companion matrices C_k, C_w.
- master key $K = (k, w) \in \left(\mathbb{F}_2^n \times \mathbb{F}_2^{n-1}\right) \setminus \{0, 0\}$

The ith round keys are computed by

$$\begin{aligned} \mathrm{KS}_i : \mathbb{F}_2^n \times \mathbb{F}_2^{n-1} &\to \mathbb{F}_2^n \times \mathbb{F}_2^{n-1} \\ \mathrm{KS}_i(k, w) &:= (k_i, c_i + w_i) \end{aligned}$$

where

$$k_i = (C_k)^i k, \qquad c_i = (C_w)^{-i} e_1, \qquad w_i = (C_w)^i w.$$

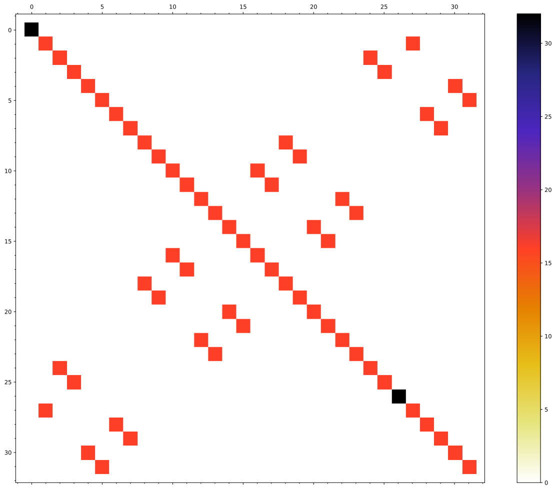
Differential Cryptanalysis

One round

Proposition

BISON's DDT consists of the entries

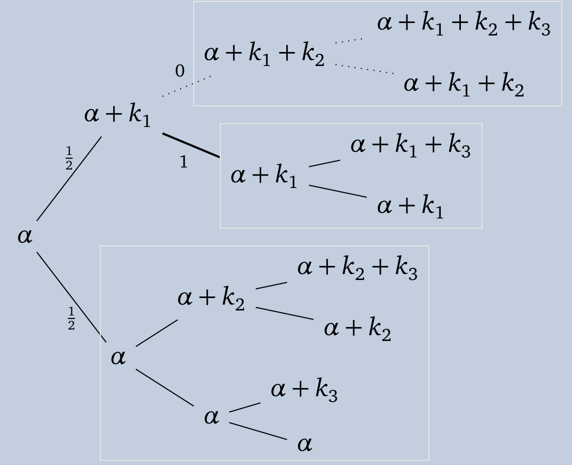
$$\mathrm{DDT}_R[\alpha, \beta] = \begin{cases} 2^n & \text{if } \alpha = \beta = k \text{ or } \alpha = \beta = 0 \\ 2^{n-1} & \text{else if } \beta \in \{\alpha, \alpha + k\} \\ 0 & \text{else} \end{cases} .$$



Differential Cryptanalysis

More rounds

Differences over $r = 3$ rounds



Probabilities of output differences

$$\Pr[\alpha \to \beta] = \begin{cases} 2^{-r} & \text{if } \beta \text{ in normal branch} \\ 2^{-r+1} & \text{if } \beta \text{ in collapsed branch} \\ 0 & \text{if } \beta \text{ in impossible branch} \end{cases}.$$

Collapsing

How many branches can collapse?

Once

For $r \leqslant n$ rounds and linearly indp. round keys this happens only once.

Further Cryptanalysis

RUB

- Linear Cryptanalysis
- Impossible Differentials
- Zero Correlation
- Invariant Attacks

Conclusion/Questions

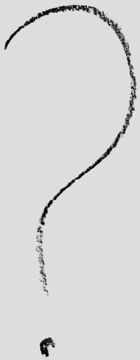
Thank you for your attention!

BISON

- A first instance of the WSN construction
- Good results for differential cryptanalysis

Open Problems

- Construction with similar good results for linear cryptanalysis
- Further analysis: division properties



References I

[Tes15] S. Tessaro. “Optimally Secure Block Ciphers from Ideal Primitives”. In: *ASIACRYPT'15*. Vol. 9453. LNCS. Springer, 2015, pp. 437–462. doi: 10.1007/978-3-662-48800-3_18.