RUHR-UNIVERSITÄT BOCHUM

Cryptanalysis of Clyde and Shadow

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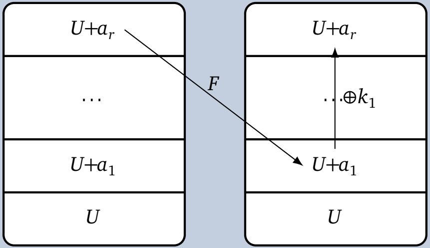
Overview

Section 1

Invariant Attacks – Round Constants

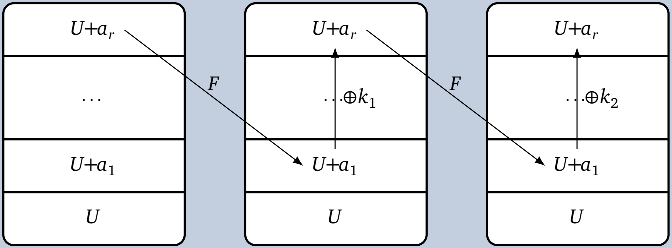
Invariant Attacks

Main Idea: Invariant Subspaces



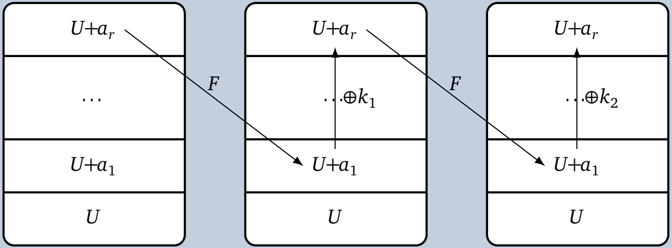
Invariant Attacks

Main Idea: Invariant Subspaces



Invariant Attacks

Main Idea: Invariant Subspaces

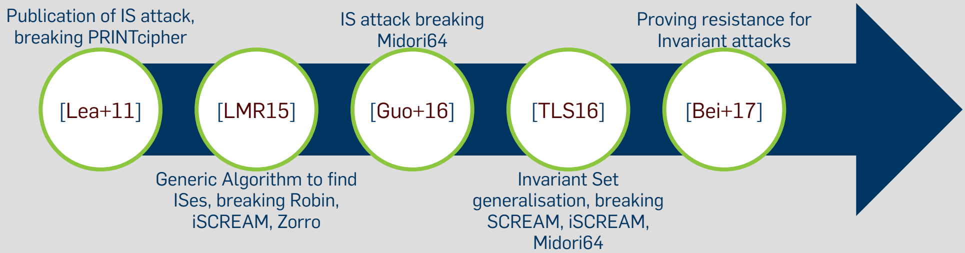


Invariant Subspace Attacks [Lea+11] (CRYPTO'11)

Let $U \subseteq \mathbb{F}_2^n$, $c, d \in U^\perp$, and $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$. Then U is an *invariant subspace* (IS) if and only if $F(U{+}c) = U{+}d$ and all round keys in $U{+}(c+d)$ are *weak keys*.

Invariant Attacks

A Short History



Invariant Attacks

Proving Resistance

Goal: Apply security argument from

C. Beierle, A. Canteaut, G. Leander, and Y. Rotella. "Proving Resistance Against Invariant Attacks: How to Choose the Round Constants". In: CRYPTO 2017, Part II. 2017. doi: 10.1007/978-3-319-63715-0_22. iacr: 2017/463.

What do we get from this?

- Non-existence of invariants for both parts of the ronud function (S-box and linear layer)

Issues

- Other partitionings of the round function might allow invariants (Christof B. found examples)
- Not clear how to prove the general absence of invariant attacks (best we can currently prove)
- All known attacks exploit exactly this structure (splitting in S-box and linear layer)

Invariant Attacks

Recap Security Argument (I)

Observation

- Invariants for the linear layer L and round key addition have to contain special linear structures.
- Denote by $c_1, \ldots, c_t$ the round constant differences for rounds with the same round key.
- Then the linear structures of any invariant have to contain $W_L(c_1, \ldots, c_t)$.

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Linear Structures

Let $f : \mathbb{F}_2^n \rightarrow \mathbb{F}_2$. Then its *linear structures* are

$$\mathrm{LS} := \{a \mid f(x) + f(x+a) \text{ is constant}\} .$$

The smallest L-invariant subspace

$W_L(c_1, \ldots, c_t)$ is the *smallest L-invariant subspace* of $\mathbb{F}_2^n$ containing all c_i

$$\Leftrightarrow \forall x \in W_L(c_1, \ldots, c_t) : L(x) \in W_L(c_1, \ldots, c_t)$$

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The simple case

If $W_L(c_1, \ldots, c_t) = \mathbb{F}_2^n$, only trivial invariants for L and key addition are possible (constant 0 and 1 function).

Invariant Attacks

Recap Security Argument (II)

Application to Clyde

- Find the important round constant differences:
  (the differences where the same tweakey is added)

Invariant Attacks

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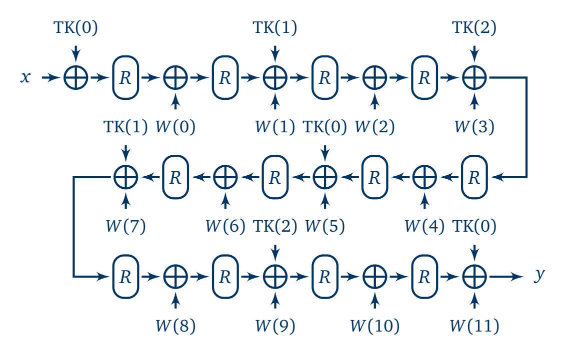

$$D = D_{\mathrm{TK}(0)} \cup D_{\mathrm{TK}(1)} \cup D_{\mathrm{TK}(2)} \cup D_0$$

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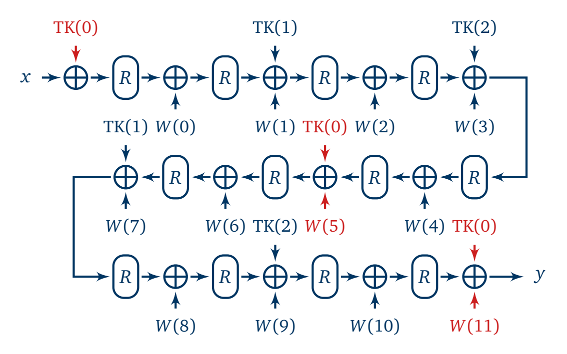

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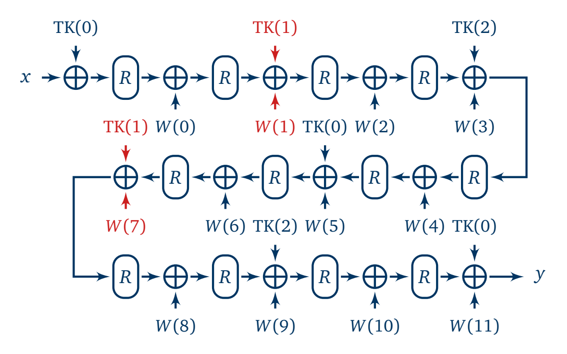

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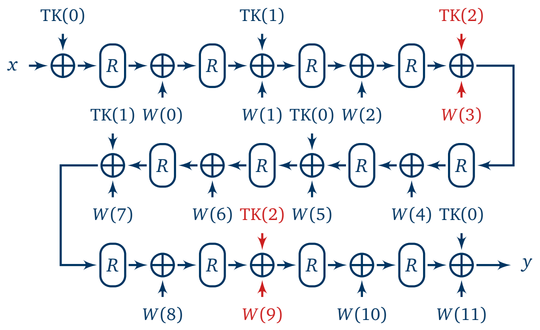

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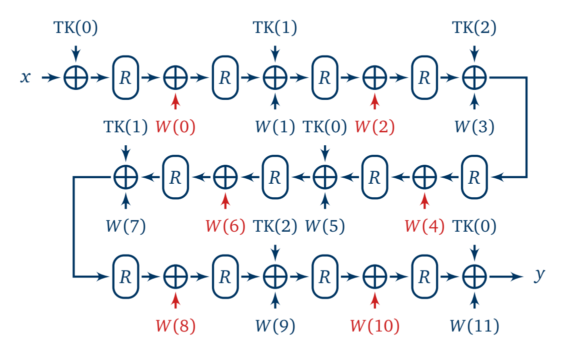

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$$D_{\mathrm{TK}(2)} = \{W(3) + W(9)\}$$

$$D_0 = \left\{a + b \;\middle|\; a, b \in D', a \neq b\right\}$$

$$D' = \{W(0), W(2), W(4), W(6), W(8), W(10)\}$$

Invariant Attacks

Application to Clyde

- Computing W_L is efficiently doable (takes $\approx$ 10 seconds on my laptop).
- For the round constants chosen for Clyde, $\dim W_L(D) = 128 = n$.

- Thus, we can apply:

Proposition 2 [Bei+17]

Suppose that the dimension of $W_L(D)$ is n. Then any invariant g is constant (and thus trivial).

- We conclude that we cannot find any non-trivial g for Clyde which is at the same time invariant for the S-box layer and for the linear layer.

Invariant Attacks

Improvable?

Bounding the dimension of W_L, [Bei+17, Theorem 1]

Given a linear layer L. Denote by Q_i its *invariant factors*. Then

$$\max_{c_1,\ldots,c_t \in \mathbb{F}_2^n} \dim W_L(c_1,\ldots,c_t) = \sum_{i=1}^{t} \deg Q_i \, .$$

Invariant Attacks

Improvable?

RUB

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Application to Clyde

- Compute invariant factors of linear layer:
- This gives a lower bound on the number of rounds:

Invariant Attacks

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Application to Clyde

- Compute invariant factors of linear layer: $4 \times (x^{32}+1)$
- This gives a lower bound on the number of rounds: 3 steps/6 rounds

Invariant Attacks

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Application to Clyde

- Compute invariant factors of linear layer: $4 \times (x^{32}+1)$
- This gives a lower bound on the number of rounds: 3 steps/6 rounds

- 3 stps/6 rnds: $\dim W_L(c_1,\ldots,c_4) = 96$
- 4 stps/8 rnds: $\dim W_L(c_1,\ldots,c_8) = 128$
- 5 stps/10 rnds: $\dim W_L(c_1,\ldots,c_{13}) = 128$
- 6 stps/12 rnds: $\dim W_L(c_1,\ldots,c_{20}) = 128$

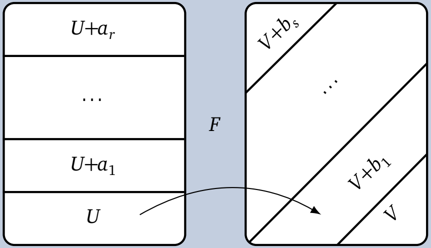
Section 2

Subspace Trails

Probability 1 Truncated Differentials

Subspace Trails

Main Idea: Subspace Trails



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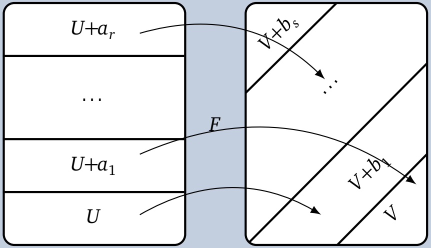

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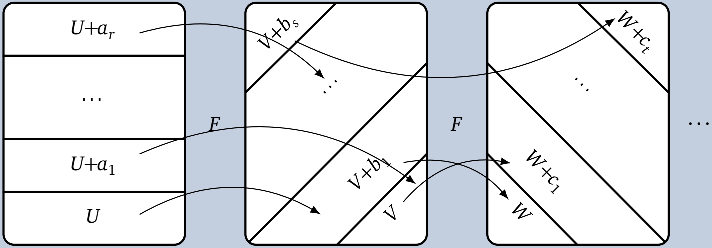

Subspace Trail Cryptanalysis [GRR16] (FSE'16)

Let $U_0, \ldots, U_r \subseteq \mathbb{F}_2^n$, and $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$. Then these form a *subspace trail* (ST), $U_0 \xrightarrow{F} \cdots \xrightarrow{F} U_r$, iff

$$\forall a \in U_i^{\perp} : \exists b \in U_{i+1}^{\perp} : \qquad F(U_i{+}a) \subseteq U_{i+1}{+}b$$

Computing Subspace Trails

Given a starting subspace U, we can efficiently compute the corresponding longest subspace trail.

Lemma

Let $U \xrightarrow{F} V$ be a ST. Then for all $u \in U$ and all x: $F(x) + F(x+u) \in V$.

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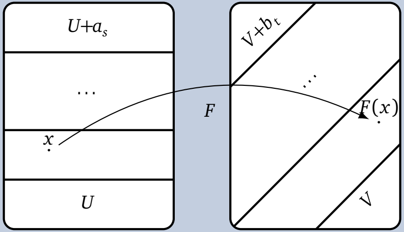

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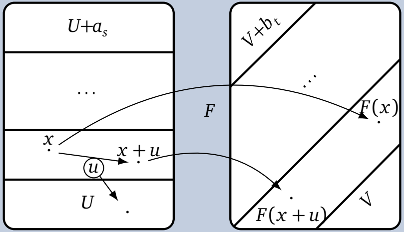

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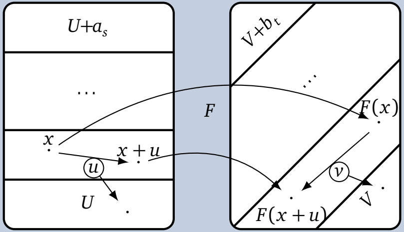

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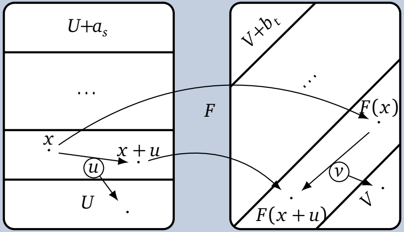


Computing the subspace trail

- To compute the next subspace, we have to compute the image of the derivatives.

Computing Subspace Trails

Algorithm

Compute Subspace Trails

Input: A nonlinear, bijective function $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$ and a subspace U.
Output: The longest ST starting in U over F.

function $\mathrm{Compute\ Trail}(F, U)$
    if $\dim(U) = n$ **then**
        return U
    $V \leftarrow \emptyset$
    for u_i basis vectors of U **do**
        for enough $x \in_R \mathbb{F}_2^n$ **do** ▷ e. g. $n + 20$ x's are enough
            $V \leftarrow V \cup \Delta_{u_i}(F)(x)$ ▷ $\Delta_a(F)(x) := F(x) + F(x + a)$
    $V \leftarrow \mathrm{span}(V)$
    return the subspace trail $U \to \mathrm{Compute\ Trail}(F, V)$

Subspace Trails

Proving Resistance

Goal: Apply security argument from

G. Leander, C. Tezcan, and F. Wiemer. "Searching for Subspace Trails and Truncated Differentials". In: ToSC 2018.1 (2018). doi: `10.13154/tosc.v2018.i1.74-100`.

What do we get from this?

- (Tight) upper bound on the length of any ST for an SPN construction

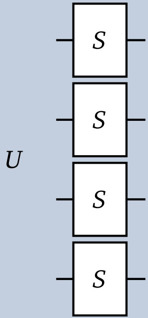
Why is the Compute Trail algorithm not enough?

- Exhaustively checking all possible starting points is to costly.

Subspace Trails

How to bound the length of any subspace trail

Observation



$$V \ni \begin{pmatrix} 0 \\ \alpha \\ 0 \\ 0 \end{pmatrix}$$

Subspace Trails

How to bound the length of any subspace trail

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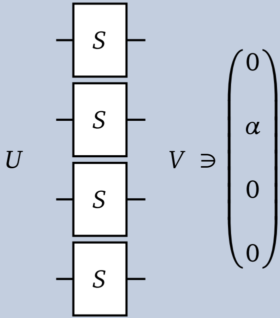


Algorithm Idea

Compute the subspace trails for any starting point $W_{i,\alpha} \in \mathcal{W}$, with

$$W_{i,\alpha} := (\underbrace{0,\ldots,0}_{i-1}, \alpha, 0, \ldots, 0)$$

Complexity (Size of $\mathcal{W}$)

For an S-box layer $S : \mathbb{F}_2^{kn} \to \mathbb{F}_2^{kn}$ with k S-boxes, each n-bit:

$$|\mathcal{W}| = k \cdot (2^n - 1)$$

Subspace Trails

Algorithm

RUB

Generic Subspace Trail Search

Input: A linear layer matrix $M : \mathbb{F}_2^{n \cdot k \times n \cdot k}$, and an S-box $S : \mathbb{F}_2^n \to \mathbb{F}_2^n$.
Output: A bound on the length of all STs over $F = M \circ S^k$.

function $\mathrm{Generic\ Subspace\ Trail\ Length}(M, S)$
  empty list L
  for possible initial subspaces represented by $W_{i,\alpha} \in \mathcal{W}$ **do** ▷ Overall $k \cdot (2^n - 1)$ iterations
    $L.\mathrm{append}(\mathrm{Compute\ Trail}(S^k \circ M, \{W_{i,\alpha}\}))$ ▷ S^k denotes the S-box layer
  return $\max \{\mathrm{len}(t) \mid t \in L\}$

Overall Complexity

Algorithm Complexity	Compute Trail	Generic Subspace Trail Length	Overall	Clyde	Shadow
	$\mathcal{O}(k^2 n^2)$	$\mathcal{O}(k 2^n)$	$\mathcal{O}(k^3 n^2 2^n)$	2^{23}	2^{29}

Subspace Trails

Results

Clyde

- Generic Subspace Trail Length Bound: 2 Rounds

Shadow

- Generic Subspace Trail Length Bound: 4 Rounds

Section 3

Division Property

Division Property

RUB

Main Idea: Division Property

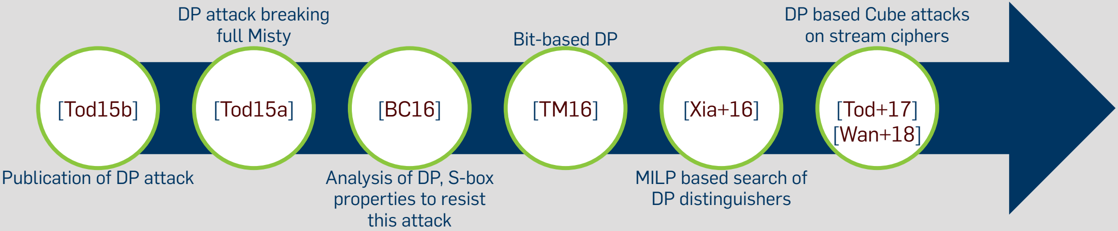
- Generalisation of Integral and Higher Order Differential attacks
- Captures properties of bits in a set
- For standard integral attacks: zero-sum, all or constant
- The Division Property allows to capture properties “in between” these (even if they do not have such a nice description as e. g. the zero-sum)

Division Property

???

Division Property

Related Work



Division Trails

Division Trail

???

Propagating Bit-Based Division Trails

copy : $x \mapsto (x, x)$

$$\mathcal{D}^1_x \xrightarrow{\text{copy}} \begin{cases} \mathcal{D}^1_{(0,0)} & \text{if } x = 0 \\ \mathcal{D}^1_{(0,1),(1,0)} & \text{if } x = 1 \end{cases}$$

xor : $(x, y) \mapsto x + y$

$$\mathcal{D}^{1,2}_{(k_0,k_1)} \xrightarrow{\text{xor}} \mathcal{D}^1_{k_0+k_1}$$

S-box $S : \mathbb{F}_2^n \to \mathbb{F}_2^n$:
see [Xia+16, Algorithm 2],
computes for all $u \in \mathbb{F}_2^n$

$$\mathcal{D}^{1,n}_u \xrightarrow{S} \mathcal{D}^{1,n}_V$$

s. t. $u \to v$ is a DT $\forall v \in V$.

Division Property

RUB

Goal: Apply security argument from

Z. Xiang, W. Zhang, Z. Bao, and D. Lin. "Applying MILP Method to Searching Integral Distinguishers Based on Division Property for 6 Lightweight Block Ciphers". In: ASIACRYPT 2016, Part I. 2016. doi: 10.1007/978-3-662-53887-6_24. iacr: 2016/857.

What do we get from this?

Number of rounds for which a division property/integral distinguisher exists.

Approach (similiar to Subspace Trails)

- Pick starting DPs in a way that covers all possibilities
- Model division trail propagations as MILP
- Find solutions for this over increasing number of rounds

Division Property

MILP model

Mixed Integer Linear Programs

Typical description of a MILP

Objective	max/min	$c^\top x$
linear inequalities	subject to	$Ax \leqslant b$

- A, b, c known coefficients
- x unknown variables

Division Property

MILP model

RUB

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Applying MILPs to find Division Properties

Goal: Model Division Property as a MILP

We need:

- Objective function
- Starting DP
- Propagation Rules
- Stopping Rule

Division Property

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Division Property

Modeling Propagation Rules: copy

Propagation Rule

$$\text{copy} : x \mapsto (x, x)$$

$$\mathcal{D}^1_x \xrightarrow{\text{copy}} \begin{cases} \mathcal{D}^1_{(0,0)} & \text{if } x = 0 \\ \mathcal{D}^1_{(0,1),(1,0)} & \text{if } x = 1 \end{cases}$$

Valid Transitions

1. $(0) \xrightarrow{\text{copy}} (0, 0)$
2. $(1) \xrightarrow{\text{copy}} (0, 1)$
3. $(1) \xrightarrow{\text{copy}} (1, 0)$

Division Property

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3. $(1) \xrightarrow{\text{copy}} (1, 0)$

MILP Model

- Given division trail $(x) \xrightarrow{\text{copy}} (y, z)$
- Propagation represented by the (in)equality

$$x - y - z = 0$$

$$x, y, z \in \{0, 1\}$$

Division Property

Modeling Propagation Rules: xor

Propagation Rule

$$\mathrm{xor} : (x, y) \mapsto x + y$$

$$\mathcal{D}^{1,2}_{(k_0,k_1)} \xrightarrow{\mathrm{xor}} \mathcal{D}^{1}_{k_0+k_1}$$

Valid Transitions

1. $(0, 0) \xrightarrow{\mathrm{xor}} (0)$
2. $(1, 0) \xrightarrow{\mathrm{xor}} (1)$
3. $(0, 1) \xrightarrow{\mathrm{xor}} (1)$

Division Property

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MILP Model

- Given division trail $(x, y) \xrightarrow{\mathrm{xor}} (z)$
- Propagation represented by the (in)equality:

$$x + y - z = 0$$
$$x, y, z \in \{0, 1\}$$

Division Property

Modeling Propagation Rules: S-box

RUB

Propagation Rule

S-box $S : \mathbb{F}_2^n \to \mathbb{F}_2^n$:
see [Xia+16, Algorithm 2],
computes for all $u \in \mathbb{F}_2^n$

$$\mathcal{D}_u^{1,n} \xrightarrow{S} \mathcal{D}_V^{1,n}$$

Valid Transitions

$$\begin{array}{ll} \boxed{1} \quad u \xrightarrow{S} v_1 & \\ \quad \ldots & \text{for } v_i \in V \\ \boxed{k} \quad u \xrightarrow{S} v_k & \end{array}$$

Division Property

Modeling Propagation Rules: S-box

Propagation Rule

S-box $S : \mathbb{F}_2^n \to \mathbb{F}_2^n$:
see [Xia+16, Algorithm 2],
computes for all $u \in \mathbb{F}_2^n$

$$\mathcal{D}_u^{1,n} \xrightarrow{S} \mathcal{D}_V^{1,n}$$

Valid Transitions

1. $u \xrightarrow{S} v_1$

$\dots$ for $v_i \in V$

k. $u \xrightarrow{S} v_k$

MILP Model

- Interpret set of all $(u, v) \in \mathbb{F}_2^{2n}$ as polyhedron
- Choose inequalities from its H-representation

Division Property

MILP model

Mixed Integer Linear Programs

Typical description of a MILP

Objective	max/min	$c^\top x$
linear inequalities	subject to	$Ax \leqslant b$

- A, b, c known coefficients
- x unknown variables

Applying MILPs to find Division Properties

Goal: Model Division Property as a MILP

We need:

- Objective function
- Starting DP
- Propagation Rules
- Stopping Rule

Division Property

MILP model

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Division Property

Start, Stop, Objective

Division Property

MILP model

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Applying MILPs to find Division Properties

Goal: Model Division Property as a MILP

We need:

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- Propagation Rules
- Stopping Rule

Using this, we can now model the DP search for Clyde

Conclusion

Conclusion

Thanks for your attention!

Future Work/Cryptanalysis

- Cryptagraph [HV18]
- Post cryptanalysis results on mailinglist?
- Eprint Write-Up?

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