RUHR-UNIVERSITÄT BOCHUM

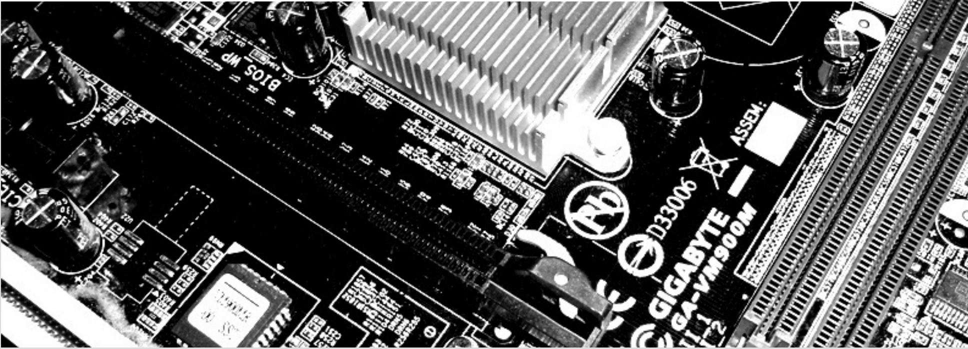
XOR Count

November 21st, 2017

FluxFingers

Workgroup Symmetric Cryptography
Ruhr University Bochum

Friedrich Wiemer



Overview

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Joint Work – Its not me alone

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RUHR UNIVERSITÄT BOCHUM RUB

Radboud University

Outline

1. Motivation
2. Preliminaries
3. State of the Art and Related Work
4. Future Work

What is the XOR count,
and why is it important?

Some facts

- Lightweight Block Ciphers
- Efficient Linear Layers
- MDS matrices are “optimal” (regarding security)[1]

[1] Are they?

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- MDS matrices are “optimal” (regarding security)[1]
- What is the lightest implementable MDS matrix?
- What about additional features (Involutory)?

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and why is it important?

Some facts

- Lightweight Block Ciphers
- Efficient Linear Layers
- MDS matrices are “optimal” (regarding security)[1]
- What is the lightest implementable MDS matrix?
- What about additional features (Involutory)?

The XOR count

- Metric for needed hardware resources
- Smaller is better

[1] Are they?

What is an MDS matrix?

Definition: MDS

A matrix M of dimension k over the field $\mathbb{F}$ is *maximum distance separable* (MDS), iff all possible submatrices of M are invertible (or nonsingular).

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Example

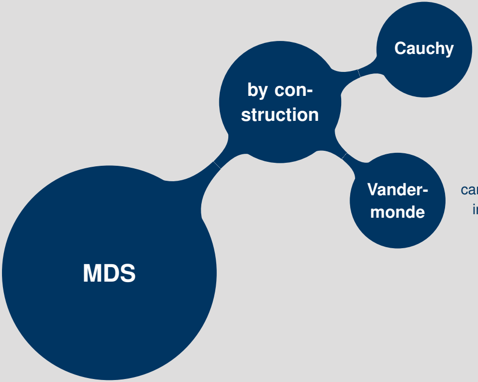
The AES MIXCOLUMN matrix is defined over $\mathbb{F}_{2^8} \cong \mathbb{F}[x]/\texttt{0x11b}$:

$$\begin{pmatrix} \texttt{0x02} & \texttt{0x03} & \texttt{0x01} & \texttt{0x01} \\ \texttt{0x01} & \texttt{0x02} & \texttt{0x03} & \texttt{0x01} \\ \texttt{0x01} & \texttt{0x01} & \texttt{0x02} & \texttt{0x03} \\ \texttt{0x03} & \texttt{0x01} & \texttt{0x01} & \texttt{0x02} \end{pmatrix} = \begin{pmatrix} x & x+1 & 1 & 1 \\ 1 & x & x+1 & 1 \\ 1 & 1 & x & x+1 \\ x+1 & 1 & 1 & x \end{pmatrix}$$

This is a (right) *circulant* matrix: $\mathrm{circ}(x, x+1, 1, 1)$.

What is an MDS matrix?

Constructions



can be made involutory

What is an MDS matrix?

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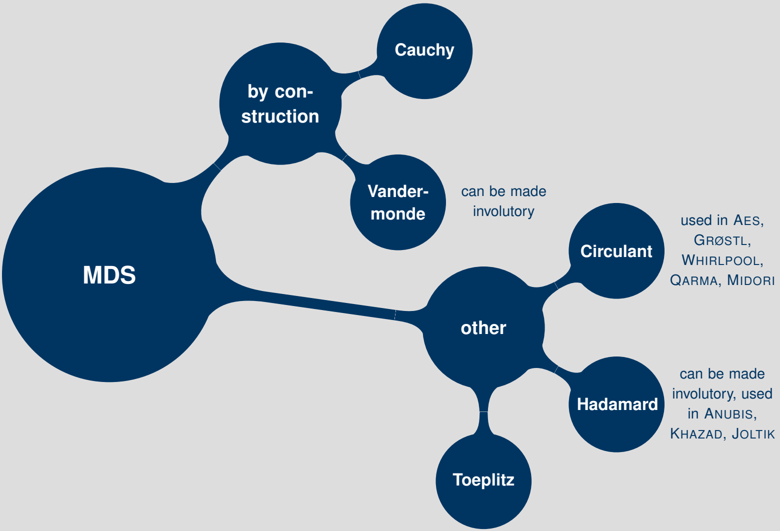

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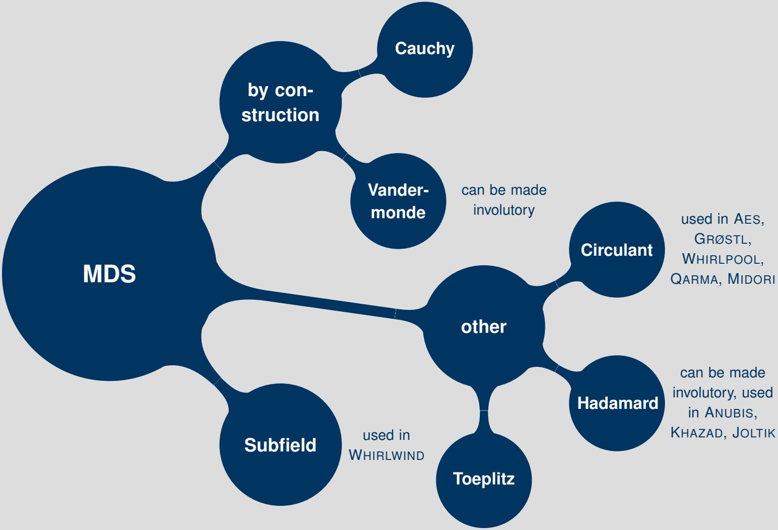

What is an MDS matrix?

Representations

How to implement this in hardware?

- This is about hardware implementations
- How do we implement a field multiplication in hardware?
- How do we implement a matrix multiplication in hardware?

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Example

$$\alpha \rightarrow \boxed{\cdot 1} \rightarrow \beta \qquad \alpha \rightarrow \boxed{\cdot x} \rightarrow \beta \qquad \alpha \rightarrow \boxed{\cdot (x+1)} \rightarrow \beta$$

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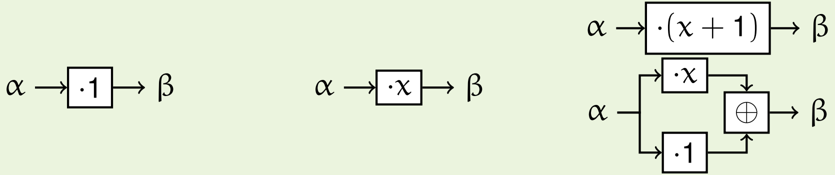
Example



Field Multiplication in Hardware

From $\mathbb{F}_2[x]/p(x)$ to $\mathbb{F}_2^n$

Implement $\alpha \longrightarrow \boxed{\cdot 1} \longrightarrow \beta$

OK, this one is easy ☺

Example in $\mathbb{F}_2[x]/0x13$:

Field Multiplication in Hardware

From $\mathbb{F}_2[x]/p(x)$ to $\mathbb{F}_2^n$

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Implement $\alpha \longrightarrow \boxed{\cdot 1} \longrightarrow \beta$

OK, this one is easy ☺

Example in $\mathbb{F}_2[x]/$0x13:

$$\begin{aligned}\alpha &= \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3\\ \beta &= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3\\ &= \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3\end{aligned}$$

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Example in $\mathbb{F}_2[x]/$0x13:

$$
\begin{aligned}
\alpha &= \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 \\
x^4 &\equiv x + 1 \bmod \text{0x13} \\
\beta &= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 \\
&= x \cdot (\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3) \\
&\equiv \alpha_3 + (\alpha_0 + \alpha_3) x + \alpha_1 x^2 + \alpha_2 x^3)
\end{aligned}
$$

Field Multiplication in Hardware

From $\mathbb{F}_2[x]/p(x)$ to $\mathbb{F}_2^n$

In matrix notation for $\mathbb{F}_2[x]/0x13$:

$$\beta = 1 \cdot \alpha \Leftrightarrow \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

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Companion Matrix

We call $M_{p(x)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ the *companion matrix* of the polynomial $p(x) = 0x13$. For any element $\gamma \in \mathbb{F}_2[x]/p(x)$, we denote by M_γ the matrix that implements the multiplication by this element in $\mathbb{F}_2^n$.

Counting XOR's

Example

We can rewrite the AES MIXCOLUMN matrix as:

$$\mathcal{M}_{\mathrm{AES}} = \mathrm{circ}(x, x+1, 1, 1) \cong \mathrm{circ}(M_x, M_{x+1}, M_1, M_1).$$

Starting in $(\mathbb{F}_2[x]/\mathtt{0x11b})^{4\times 4}$, we end up in $\left(\mathbb{F}_2{}^{8\times 8}\right)^{4\times 4} \cong \mathbb{F}_2^{32\times 32}$.

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A first XOR-count

To implement multiplication by γ, we need $\text{hw}(M_\gamma) - \dim(M_\gamma)$ many XOR's. Thus

$$\begin{aligned} \text{XOR-count}(\mathcal{M}_{\text{AES}}) &= 4 \cdot (\text{hw}(M_x) + \text{hw}(M_{x+1}) + 2 \cdot \text{hw}(M_1)) - 32 \\ &= 4 \cdot (11 + 19 + 2 \cdot 8) - 32 = 152. \end{aligned}$$

The General Linear Group

Generalise a bit

Instead of choosing elements from $\mathbb{F}_{2^n} \cong \mathbb{F}_2[x]/p(x)$ we can extend our possible choices for "multiplication matrices" by exploiting the following.

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Instead of choosing elements from $\mathbb{F}_{2^n} \cong \mathbb{F}_2[x]/p(x)$ we can extend our possible choices for "multiplication matrices" by exploiting the following.

Todo

Maybe remove this?

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The Stupidity of recent XOR Count Papers

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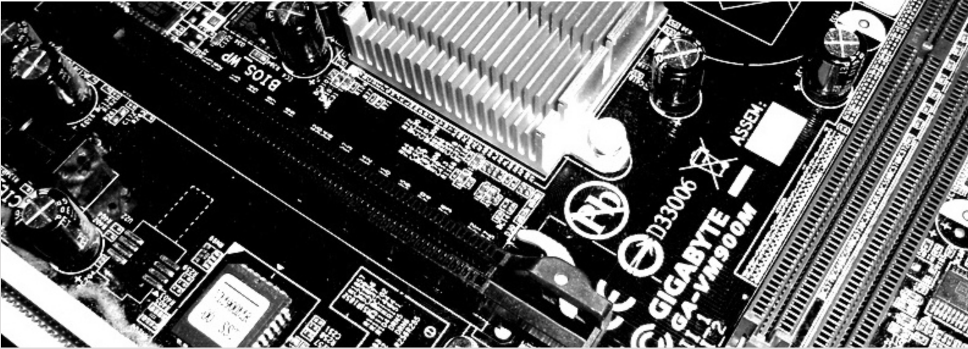

State of the Art

Before our Paper

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- You saw how to count XORs
- This count is split in the "overhead" and the XORs needed for the field multiplication
- Thus for AES we get $56 + 8 \cdot 3 \cdot 4 = 56 + 96 = 152$
- Finding a good matrix reduces now to find the cheapest elements for field multiplication
- There is a lot of work followng this line [BKL16; JPS17; LS16; LW16; LW17; Sim+15; SS16a; SS16b; SS17; ZWS17]

State of the Art

Best known Results

4×4 matrices over $\mathrm{GL}(8, \mathbb{F}_2)$		
Matrix	Naive	Literature
AES (Circulant)	152	**7+96**
[Sim+15] (Subfield)	136	40+96
[LS16] (Circulant)	128	32+96
[LW16]	106	10+96
[BKL16] (Circulant)	136	24+96
[SS16b] (Toeplitz)	123	27+96
[JPS17] (Subfield)	122	20+96

Related Work I

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Optimized Arithmetic for Reed-Solomon Encoders

Christof Paar*
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email: christof@ece.wpi.edu

1997 IEEE International Symposium on Information Theory, June 29 -- July 4, 1997, Ulm, Germany (extended version)

Abstract

Multiplication with constant elements from Galois fields of characteristic two is the major arithmetic operation in Reed-Solomon encoders. This contribution describes two optimization algorithms which yield low complexity constant multipliers for Ga-

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Related Work II

State of the Art

Best known Results (After our Paper)

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AES (Circulant)	152	**7+96**
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Best known Results (After our Paper)

4×4 matrices over $\mathrm{GL}(8, \mathbb{F}_2)$					
Matrix	Naive	Literature	PAAR1	PAAR2	BP
AES (Circulant)	152	**7+96**	108	108	97
[Sim+15] (Subfield)	136	40+96	100	98	100
[LS16] (Circulant)	128	32+96	116	116	112
[LW16]	106	10+96	102	102	102
[BKL16] (Circulant)	136	24+96	116	112	110
[SS16b] (Toeplitz)	123	27+96	110	108	107
[JPS17] (Subfield)	122	20+96	96	95	**86**

State of the Art

Finding better matrices?

Type	Previously Best Known		XOR count
$\mathrm{GL}(4, \mathbb{F}_2)^{4\times4}$	58	[JPS17; SS16b]	36
$\mathrm{GL}(8, \mathbb{F}_2)^{4\times4}$	106	[LW16]	72
$(\mathbb{F}_2[x]/\mathtt{0x13})^{8\times8}$	392	[Sim+15]	196
$\mathrm{GL}(8, \mathbb{F}_2)^{8\times8}$	640	[LS16]	392
$(\mathbb{F}_2[x]/\mathtt{0x13})^{4\times4\star}$	63	[JPS17]	42
$\mathrm{GL}(8, \mathbb{F}_2)^{4\times4}$	126	[JPS17]	84
$(\mathbb{F}_2[x]/\mathtt{0x13})^{8\times8}$	424	[Sim+15]	212
$\mathrm{GL}(8, \mathbb{F}_2)^{8\times8}$	663	[JPS17]	424

Future Work

Questions?

Thank you for your attention!

Mainboard & Questionmark Images: flickr

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