RUHR-UNIVERSITÄT BOCHUM

Security Arguments and Tool-based Design of Block Ciphers

PhD Defense

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The setting

Block Ciphers and Security Notion

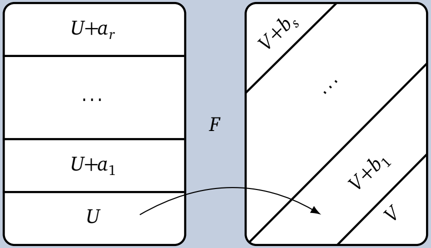
Block Ciphers

Security

Substitution Permutation Networks

Overview

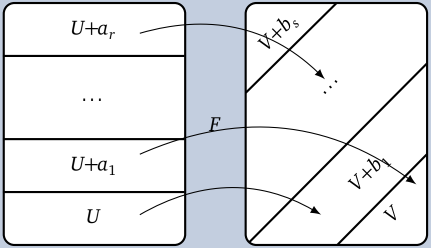
Main Idea of Subspace Trails



Subspace Trail Cryptanalysis

RUB

Main Idea of Subspace Trails



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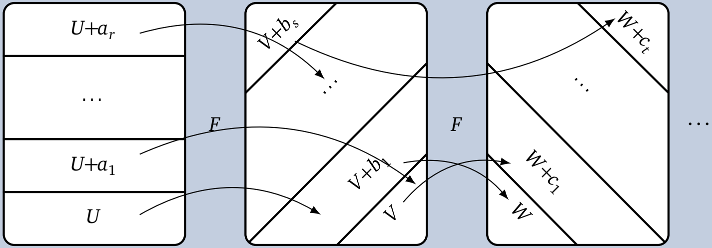

Subspace Trail Cryptanalysis [GRR16] (FSE'16)

Let $U_0, \dots, U_r \subseteq \mathbb{F}_2^n$, and $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$. Then these form a *subspace trail* (ST), $U_0 \xrightarrow{F} \cdots \xrightarrow{F} U_r$, iff

$$\forall a \in U_i^\perp : \exists b \in U_{i+1}^\perp : \qquad F(U_i + a) \subseteq U_{i+1} + b$$

Our Goal

RUB

Problem: Security against Subspace Trails

Given an SPN with round function F, consisting of

- k parallel applications of an S-box $S : \mathbb{F}_2^n \to \mathbb{F}_2^n$ and
- a linear layer $L : \mathbb{F}_2^{kn} \to \mathbb{F}_2^{kn}$.

Compute an upper bound on the length of any subspace trail through the cipher.

Subspace Propagation

Given a starting subspace U, we can efficiently compute the corresponding longest subspace trail.

Lemma

Let $U \xrightarrow{F} V$ be a ST. Then for all $u \in U$ and all x: $F(x) + F(x+u) \in V$.

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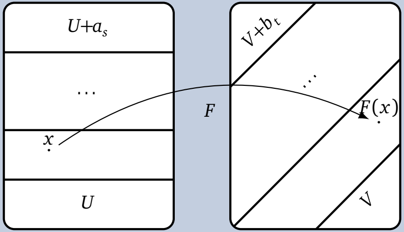

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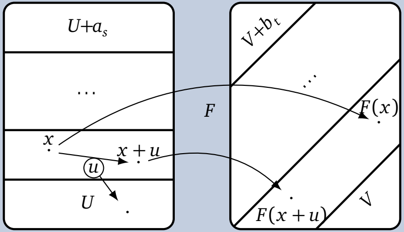

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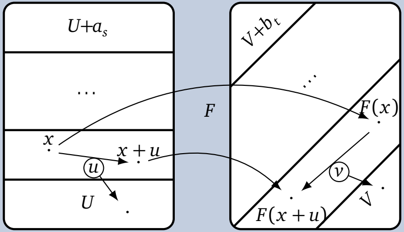

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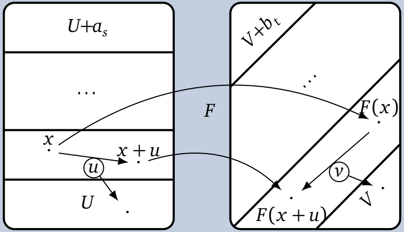

Computing the subspace trail

- To compute the next subspace, we have to compute the image of the derivatives.

Propagate a Basis

Actually it is enough to compute only the image of the derivatives in direction of U's basis vectors.

Lemma

Given $U \subseteq \mathbb{F}_2^n$ *with basis* $\{b_1, \dots, b_k\}$. *Then* $\operatorname{Span}\left\{\bigcup_{u \in U} \operatorname{Im} \Delta_u(F)\right\} = \operatorname{Span}\left\{\bigcup_{b_i} \operatorname{Im} \Delta_{b_i}(F)\right\}$.

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Proof: $\supseteq$ trivial, $\subseteq$ by induction over the dimension k of U

Let $u = \sum_{i=1}^{k} \lambda_i b_i$ and $v \in \operatorname{Im} \Delta_u(F)$, i. e. there exists an x s. t.

$$v = F(x) + F(x + \sum_{i=1}^{k} \lambda_i b_i) = F(y + \lambda_k b_k) + F(y + \sum_{i=1}^{k-1} \lambda_i b_i) = \lambda_k \Delta_{b_k}(F)(y) + \lambda' \Delta_{u'}(F)(y)\,.$$

Thus $v \in \operatorname{Span}\left\{\operatorname{Im} \Delta_{b_k}(F) \cup \operatorname{Im} \Delta_{u'}(F)\right\}$, where u' is contained in a $(k-1)$ dimensional subspace.

`ComputeTrail` Algorithm

Computation of Subspace Trails

Input: A nonlinear function $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$, a subspace U.
Output: A subspace trail $U \rightrightarrows^F \cdots \rightrightarrows^F V$.

1 **function** $\texttt{ComputeTrail}(F, U)$
2   **if** $\dim U = n$ **then return** U
3     $V \leftarrow \emptyset$
4   **for** u_i basis vectors of U **do**
5   **for** enough $x \in_{\mathrm{R}} \mathbb{F}_2^n$ **do**
6             $V \leftarrow V \cup \Delta_{u_i}(F)(x)$
7     $V \leftarrow \mathrm{Span}\{V\}$
8   **return** $U \rightrightarrows^F \texttt{ComputeTrail}(F, V)$

Correctness: previous two lemmata

Runtime:

- Line 4: max. n iterations
- Line 5: $n + c$ random vectors are enough
- Overall: $\mathcal{O}(n^2)$ evaluations of F

Remaining Problem: cyclic STs

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Give an upper bound on the length of any subspace trail.

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$\forall U \subseteq \mathbb{F}_2^m$ run $\texttt{ComputeTrail}(F, U)$

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$\forall u \subseteq \mathbb{F}_2^m \setminus \{0\}$ run $\texttt{ComputeTrail}(F, \text{Span}\,\{u\})$

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Still $2^m - 1$ starting subspaces.

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Often used heuristic

Activate single S-boxes only. That is, for a round function with k S-boxes which are n-bit wide, choose $U = \{0\}^i \times V \times \{0\}^{k-i-1}$, where $V \subseteq \mathbb{F}_2^n$.

Activating a single S-box only

Problem

Heuristic not valid in general when we want to prove a bound on the subspace trail length. In particular one can construct examples where the best subspace trail does activate more than one S-box in the beginning.

The good case

However, we will see next a sufficient condition for the case when the heuristic is valid.

The Connection to Linear Structures

Let us observe how a single S-box S behaves regarding subspace trails:

Given a subpsace trail $U \overset{S}{\rightrightarrows} V$, this implies

$$\Delta_u(S)(x) \in V \qquad \text{for all } x \in \mathbb{F}_2^n \text{ and } u \in U.$$

By definition of the dual space $V^\perp$:

$$\langle \alpha, \Delta_u(S)(x) \rangle = 0 \qquad \text{for all } \alpha \in V^\perp,$$

which are exactly the *linear structures* of S:

$$\mathrm{LS}(S) := \{(\alpha, u) \mid \langle \alpha, \Delta_u(S)(x) \rangle \text{ is constant for all } x\}$$

S-boxes without Linear Structures

This observation implies that S-boxes without linear structures (e. g. the AES S-box) exhibit only two important subspace trails:

$$\{0\} \rightrightarrows \{0\} \qquad \text{and} \qquad \mathbb{F}_2^n \rightrightarrows \mathbb{F}_2^n$$

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Theorem

Let F be an S-box layer of k parallel S-boxes $S : \mathbb{F}_2^n \to \mathbb{F}_2^n$. If S has no non-trivial linear structures, then for every subspace trail $U \rightrightarrows^F V$:

$$U = V = U_1 \times \cdots \times U_k \,,$$

with $U_i \in \left\{\{0\}, \mathbb{F}_2^n\right\}$.

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Proof

For all $\alpha = (\alpha_1, \ldots, \alpha_k) \in V^\perp$: $\langle \alpha, \Delta_u(F)(x) \rangle = \left\langle \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_k \end{pmatrix}, \begin{pmatrix} \Delta_{u_1}(S)(x_1) \\ \vdots \\ \Delta_{u_k}(S)(x_k) \end{pmatrix} \right\rangle = \sum_{i=1}^{k} \left\langle \alpha_i, \Delta_{u_i}(S)(x_i) \right\rangle = 0$

S-boxes without Linear Structures

The length ℓ of any subspace trail is upper bounded by

$$\ell = \max_{U \in \left\{\{0\}, \mathbb{F}_2^n\right\}^k} \left| \mathtt{ComputeTrail}(F, U) \right|,$$

which needs 2^k evaluations of the $\mathtt{ComputeTrail}$ algorithm.

S-boxes with Linear Structures

Compared to the no-linear-structures-case, $V^\perp$ can now contain much more elements, namely all combinations of linear structures, such that their corresponding constants sum to zero.

Instead, we can show that (for any not-trivially-insecure S-box) the subspace after the first S-box layer contains at least one element of a specific structure:

$$W_{i,\alpha} = \{0\}^{i-1} \times \{0, \alpha\} \times \{0\}^{k-i} \,.$$

The length ℓ of any subspace trail is then upper bounded by

$$\ell = \max_{W_{i,\alpha}} \left| \mathtt{ComputeTrail}(F', W_{i,\alpha}) \right| + 1 \,,$$

which needs $k \cdot 2^n$ evaluations of the `ComputeTrail` algorithm.

Note that F' first applies the linear layer, then the S-box layer (b/c of the skipped first S-box layer).

Conclusion

Thanks for your attention!

Applications of `ComputeTrail`

- Bound longest probability-one subspace trail
- Link to Truncated Differentials
- Finding key-recovery strategies