RUHR-UNIVERSITÄT BOCHUM

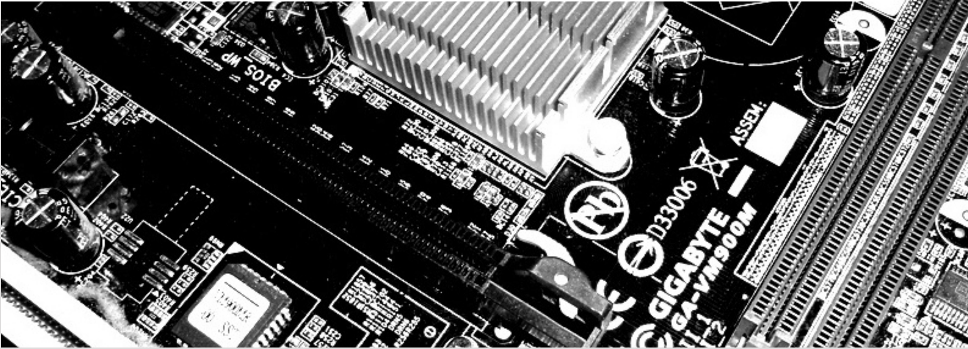
XOR Count

October XXth, 2017

FluxFingers

Workgroup Symmetric Cryptography
Ruhr University Bochum

Friedrich Wiemer



Overview

Joint Work – Its not me alone

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Outline

1. Motivation
2. Preliminaries
3. State of the Art and Related Work
4. Future Work

What is the XOR count,
and why is it important?

Some facts

- Lightweight Block Ciphers
- Efficient Linear Layers
- MDS matrices are “optimal” (regarding security)[1]

[1] Are they?

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- What is the lightest implementable MDS matrix?
- What about additional features (Involutory)?

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- What is the lightest implementable MDS matrix?
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The XOR count

- Metric for needed hardware resources
- Smaller is better

[1] Are they?

What is an MDS matrix?

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Definition: MDS

A matrix M of dimension k over the field $\mathbb{F}$ is *maximum distance separable* (MDS), iff all possible submatrices of M are invertible (or nonsingular).

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Example

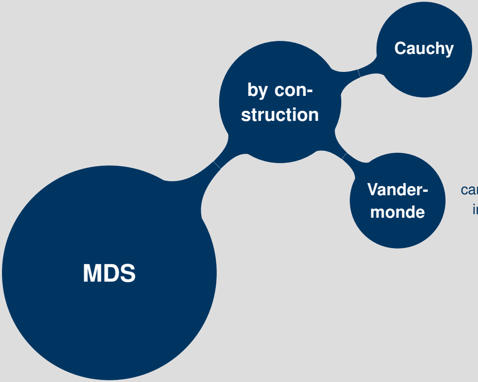
The AES MixColumn matrix is defined over $\mathbb{F}_{2^8} \cong \mathbb{F}[x]/\mathtt{0x11b}$:

$$\begin{pmatrix} \mathtt{0x02} & \mathtt{0x03} & \mathtt{0x01} & \mathtt{0x01} \\ \mathtt{0x01} & \mathtt{0x02} & \mathtt{0x03} & \mathtt{0x01} \\ \mathtt{0x01} & \mathtt{0x01} & \mathtt{0x02} & \mathtt{0x03} \\ \mathtt{0x03} & \mathtt{0x01} & \mathtt{0x01} & \mathtt{0x02} \end{pmatrix} = \begin{pmatrix} x & x+1 & 1 & 1 \\ 1 & x & x+1 & 1 \\ 1 & 1 & x & x+1 \\ x+1 & 1 & 1 & x \end{pmatrix}$$

This is a (right) *circulant* matrix: $\mathrm{circ}(x, x+1, 1, 1)$.

What is an MDS matrix?

Constructions



can be made involutory

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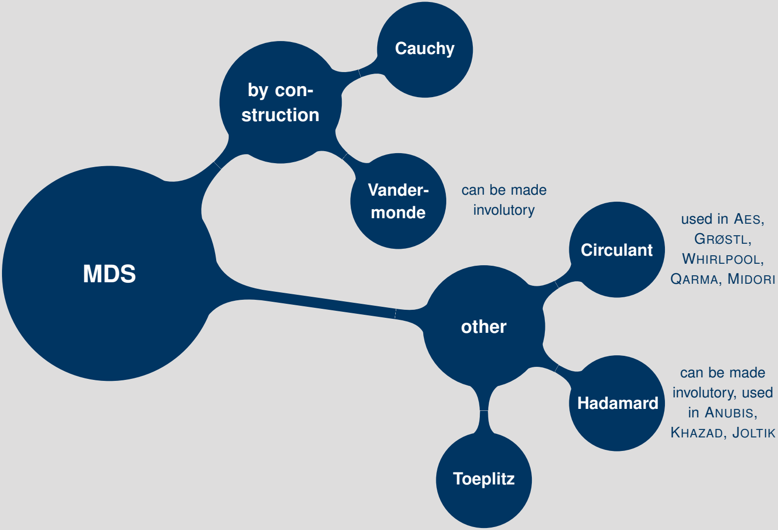

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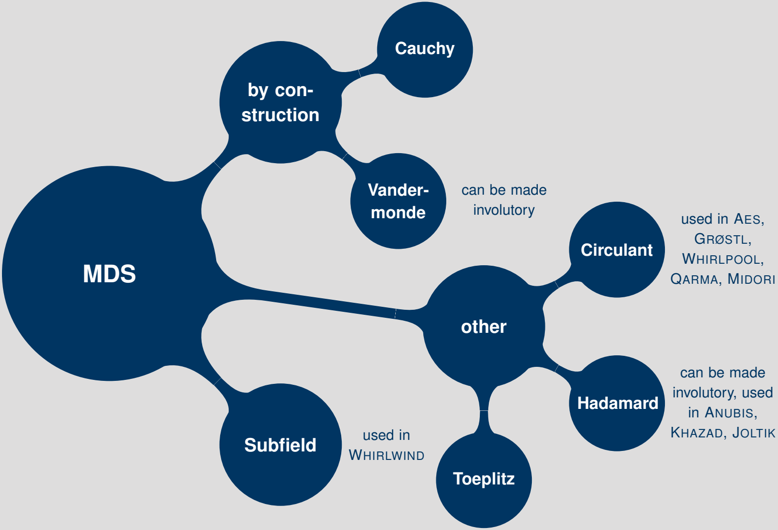

What is an MDS matrix?

Representations

How to implement this in hardware?

- This is about hardware implementations
- How do we implement a field multiplication in hardware?
- How do we implement a matrix multiplication in hardware?

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$$\alpha \longrightarrow \boxed{\cdot 1} \longrightarrow \beta \qquad \alpha \longrightarrow \boxed{\cdot x} \longrightarrow \beta \qquad \alpha \longrightarrow \boxed{\cdot (x+1)} \longrightarrow \beta$$

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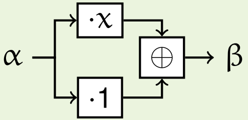

Field Multiplication in Hardware

From $\mathbb{F}_2[x]/p(x)$ to $\mathbb{F}_2^n$

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Implement $\alpha \longrightarrow \boxed{\cdot 1} \longrightarrow \beta$

OK, this one is easy ☺

Example in $\mathbb{F}_2[x]/0x13$:

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Example in $\mathbb{F}_2[x]/0x13$:

$$\begin{aligned}\alpha &= \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 \\ \beta &= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 \\ &= \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3\end{aligned}$$

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Example in $\mathbb{F}_2[x]/$0x13:

$$\begin{aligned} \alpha &= \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 \\ x^4 &\equiv x + 1 \bmod \text{0x13} \\ \beta &= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 \\ &= x \cdot (\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3) \\ &\equiv \alpha_3 + (\alpha_0 + \alpha_3)x + \alpha_1 x^2 + \alpha_2 x^3) \end{aligned}$$

Field Multiplication in Hardware

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In matrix notation for $\mathbb{F}_2[x]/0x13$:

$$\beta = 1 \cdot \alpha \Leftrightarrow \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

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Companion Matrix

We call $M_{p(x)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ the *companion matrix* of the polynomial $p(x) = 0x13$. For any element $\gamma \in \mathbb{F}_2[x]/p(x)$, we denote by M_γ the matrix that implements the multiplication by this element in $\mathbb{F}_2^n$.

Counting XOR's

Example

We can rewrite the AES MIXCOLUMN matrix as:

$$\mathcal{M}_{\text{AES}} = \text{circ}(x, x+1, 1, 1) \cong \text{circ}(M_x, M_{x+1}, M_1, M_1).$$

Starting in $(\mathbb{F}_2[x]/\text{0x11b})^{4\times 4}$, we end up in $\left({\mathbb{F}_2}^{8\times 8}\right)^{4\times 4} \cong \mathbb{F}_2^{32\times 32}$.

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A first XOR-count

To implement multiplication by γ, we need $\text{hw}(M_\gamma) - \dim(M_\gamma)$ many XOR's. Thus

$$\begin{aligned} \text{XOR-count}(\mathcal{M}_{\text{AES}}) &= 4 \cdot (\text{hw}(M_x) + \text{hw}(M_{x+1}) + 2 \cdot \text{hw}(M_1)) - 32 \\ &= 4 \cdot (11 + 19 + 2 \cdot 8) - 32 = 152. \end{aligned}$$

The General Linear Group

Generalise a bit

Instead of choosing elements from $\mathbb{F}_{2^n} \cong \mathbb{F}_2[x]/p(x)$ we can extend our possible choices for "multiplication matrices" by exploiting the following.

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Todo

Maybe remove this?

The Stupidity of recent XOR Count Papers

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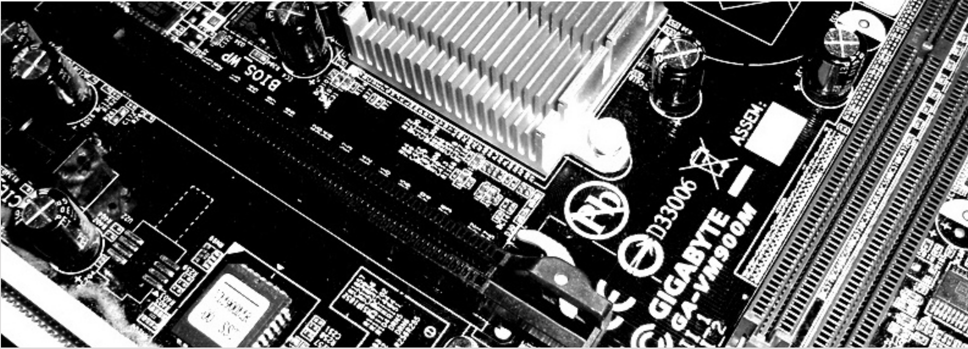

State of the Art

Before our Paper

Related Work I

Related Work II

State of the Art

After our Paper

Future Work

Questions?

Thank you for your attention!

Mainboard & Questionmark Images: flickr

References I