

# BISON Instantiating the Whitened Swap-Or-Not Construction

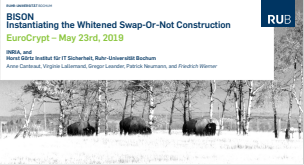
## EuroCrypt – May 23rd, 2019

INRIA, and  
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Anne Canteaut, Virginie Lallemand, Gregor Leander, Patrick Neumann, and *Friedrich Wiemer*



2019-05-15

### BISON Instantiating the Whitened Swap-Or-Not Construction



- Whitened Swap-Or-Not Construction developed by Hoang et al. and Tessaro
- Way of building block ciphers
- As this is one of the few talks here at EuroCrypt about block ciphers, we will start simple



## Two Parts

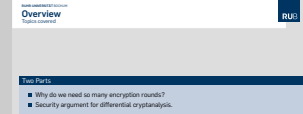
- Why do we need so many encryption rounds?
- Security argument for differential cryptanalysis.

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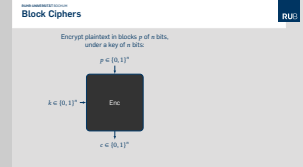
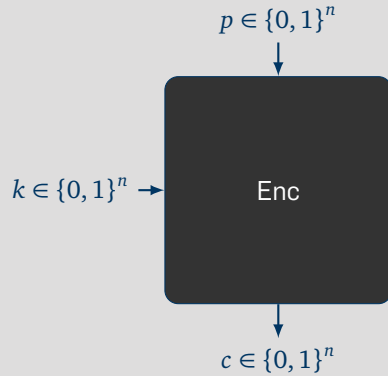
## BISON Instantiating the Whitened Swap-Or-Not Construction

## Overview

- Talk mainly about two parts of the paper:
  - Why do we need so many rounds  
(easy to understand argument)
  - Security against differential cryptanalysis  
(again relative simple argument that gives strong security here)

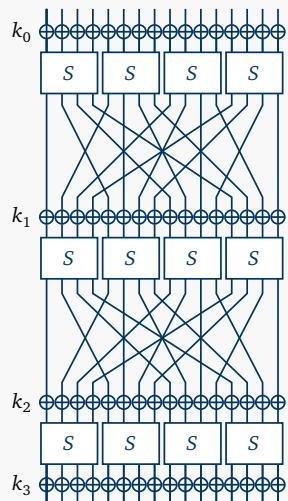
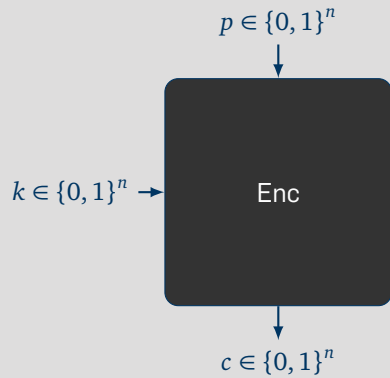


Encrypt plaintext in blocks  $p$  of  $n$  bits,  
under a key of  $n$  bits:



- Block ciphers encrypt *blocks* of  $n$ -bit inputs under an  $n$ -bit master key
- As a basic cryptographic primitive, we need special modes of operations, if the data to be encrypted is not of exactly  $n$ -bit length.
- This we do not consider here, instead we want to look at how to build this black box.
- Typicall approach is an SPN structure, where key-addition, S-box layer and a linear layer are iterated over several rounds.
- Relatively well understood
- Good security arguments against known attacks
- There are some problems: differentials and linear hull effects

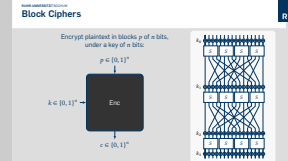
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— The WSN construction

— Block Ciphers

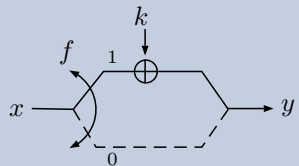


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# The WSN construction

Published by Tessaro at AsiaCrypt 2015 [ia.cr/2015/868].

## Overview round, iterated $r$ times



## Whitened Swap-Or-Not round function

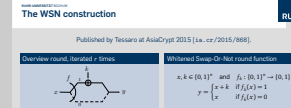
$$x, k \in \{0, 1\}^n \quad \text{and} \quad f_k : \{0, 1\}^n \rightarrow \{0, 1\}$$

$$y = \begin{cases} x + k & \text{if } f_k(x) = 1 \\ x & \text{if } f_k(x) = 0 \end{cases}$$

## BISON Instantiating the Whitened Swap-Or-Not Construction

└ The WSN construction

└ The WSN construction

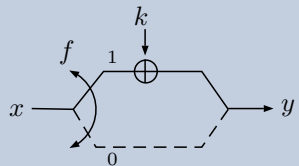


- Lets take a look at the WSN construction (simplified).
- Again, an iterated round function, where the input is fed into from the left.
- Next, a Boolean function decides if either the round key  $k$  is xored onto the input, or nothing happens.
- The result is the updated state, respective the output of the round.
- In other words,  $x$ , and  $k$  are both  $n$ -bit strings and  $f$  is an  $n$ -bit Boolean function.
- The round output  $y$  is either  $x + k$  if  $f_k(x) = 1$  or just  $x$  in the other case.
- So why is this nice?

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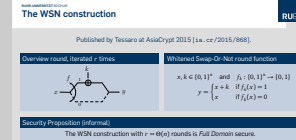
## Security Proposition (informal)

The WSN construction with  $r = \Theta(n)$  rounds is *Full Domain* secure.

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- So why is this nice?
- Tessaro was able to show that this construction, when iterated over  $\Theta(n)$  rounds, achieves *Full Domain* security (what ever that means).

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Construction  
└─ The WSN construction  
    └─ An Implementation

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An Implementation

RUB

- Sounds all very great.
- So from a practitioners point of view the natural next point is: lets implement it.

## Construction

- $f_k(x) := ?$
- Key schedule?
- $\Theta(n)$  rounds?

Theoretical vs. practical constructions



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BISON Instantiating the Whitened Swap-Or-Not Construction

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An Implementation

Construction

- $f_k(x) := ?$
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Theoretical vs. practical constructions



- Sounds all very great.
- So from a practitioners point of view the natural next point is: lets implement it.
- But ugh...
- How does this Boolean function  $f_k$  actually looks like?
- What about a key schedule? How do we derive the round keys?
- And how many are  $\Theta(n)$  rounds?
- So, from a theoretical point of view we have a nice construction.
- But from a practical point of view it is basically useless.
- OK, let us fix this.



Input

 $x$ 

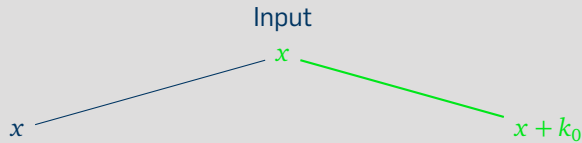
- We can observe an interesting first property, when looking at the encryption procedure round by round
- Starting with the plaintext  $x$ ...



# The WSN construction

Encryption

RUB



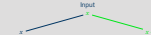
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Construction

└ Generic Analysis

└ The WSN construction

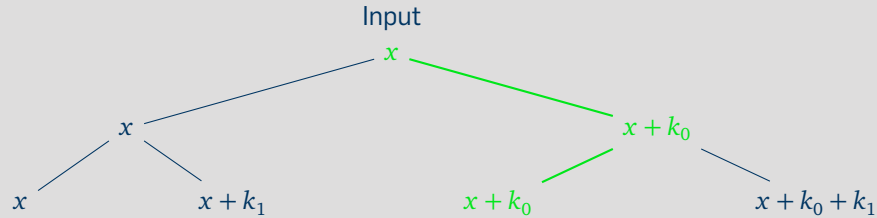


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- ...in each round, we either add the round key  $k_i$ , ...

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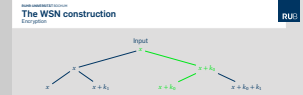


## BISON Instantiating the Whitened Swap-Or-Not Construction

Construction

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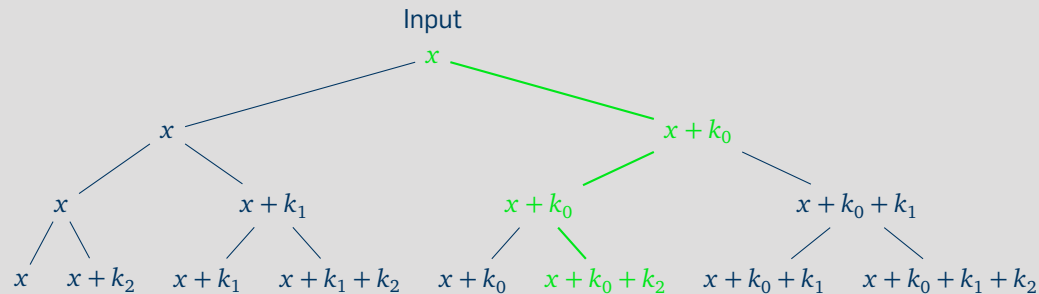
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- ...or not.

# The WSN construction

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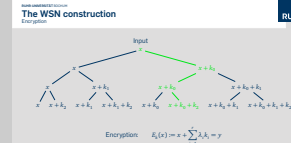


Encryption:  $E_k(x) := x + \sum_{i=1}^r \lambda_i k_i = y$

## BISON Instantiating the Whitened Swap-Or-Not Construction

Generic Analysis

The WSN construction



- We can observe an interesting first property, when looking at the encryption procedure round by round
- Starting with the plaintext  $x \dots$
- ...in each round, we either add the round key  $k_i$ , ...
- ...or not.
- Thus we end up with a binary tree of possible states.
- Furthermore, the encryption can also be written as the plaintext plus the sum of some round keys, chosen by the  $\lambda_i$ 's here.

## Observation

- The ciphertext is the plaintext plus a subset of the round keys:

$$y = x + \sum_{i=1}^r \lambda_i k_i$$

- For pairs  $x_i, y_i$ :  $\text{span} \{x_i + y_i\} \subseteq \text{span} \{k_j\}$ .

## BISON Instantiating the Whitened Swap-Or-Not

## Construction

## └ Generic Analysis

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- First observation:  $\text{span} \{x_i + y_i\} \subseteq \text{span} \{k_j\}$
- Leads to a simple distinguishing attack, *if number of rounds  $r < n$ .*

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Generic Analysis  
On the number of rounds

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Distinguishing Attack for  $r < n$  rounds

There is an  $u \in \mathbb{F}_2^n \setminus \{0\}$ , s. t.  $\langle u, x \rangle = \langle u, y \rangle$  holds always:

$$\begin{aligned} \langle u, y \rangle &= \left\langle u, x + \sum \lambda_i k_i \right\rangle \\ &= \langle u, x \rangle + \left\langle u, \sum \lambda_i k_i \right\rangle = \langle u, x \rangle + 0 \end{aligned}$$

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- It is easy to find a  $u$ , s. t.  $\langle u, y \rangle = \langle u, x \rangle = 0$  for all  $x, y = x, E(x)$ .
- Simply use the bilinearity of the scalar product.
- Then any  $u$  from the dual space spanned by the round keys fullfills the above equation.
- As long as there are less then  $n$  round keys, this dual space has dimension greater or equal then one.

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## Rationale 1

Any instance must iterate at least  $n$  rounds; any set of  $n$  consecutive keys should be linearly indp.

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- As long as there are less than  $n$  round keys, this dual space has dimension greater or equal then one.
- A first design rational is thus...

**Generic Analysis**  
On the number of rounds

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<p><b>Rationale 1</b></p> <p>Any instance must iterate at least <math>n</math> rounds; any set of <math>n</math> consecutive keys should be linearly indp.</p>	

A bit out of the clear blue sky, but:

## Rationale 2

For any instance,  $f_k$  has to depend on all bits, and for any  $\delta \in \mathbb{F}_2^n$ :  $\Pr[f_k(x) = f_k(x + \delta)] \approx \frac{1}{2}$ .

## BISON Instantiating the Whitened Swap-Or-Not

### Construction

#### Generic Analysis

#### Generic Analysis

- We also need this second rationale.
- Its not so easy explainable without going into more depth.
- So you have to believe me on this one.
- It basically says that for any input difference  $\delta \neq k$ :

$$\Pr[f_k(x) = f_k(x + \delta)] \approx \frac{1}{2}$$





# A genus of the WSN family: BISON

## Rationale 1

Any instance must iterate at least  $n$  rounds; any set of  $n$  consecutive keys should be linearly indep.

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## Generic properties of Bent whitened Swap Or Not (BISON)

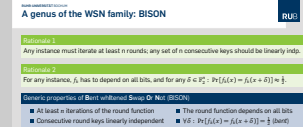
- At least  $n$  iterations of the round function
- The round function depends on all bits
- Consecutive round keys linearly independent
- $\forall \delta : \Pr[f_k(x) = f_k(x + \delta)] = \frac{1}{2}$  (*bent*)

## BISON Instantiating the Whitened Swap-Or-Not Construction

### Generic Analysis

### A genus of the WSN family: BISON

- A quick recap and implications for any WSN instance.
- Rationale 2 basically tells us, we have to use *bent* functions.
- Thats nice, as those functions are quite well understood and already well scrutinised.
- Also, this is the reason for the name: Bent Whitened Swap-Or-Not
- But, and thats not so nice...



# A genus of the WSN family: BISON

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## Generic properties of Bent whitened Swap Or Not (BISON)

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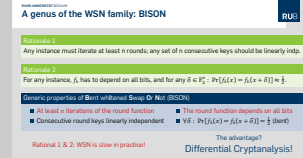
Rational 1 & 2: WSN is *slow* in practice!

The advantage?  
Differential Cryptanalysis!

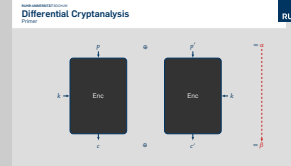
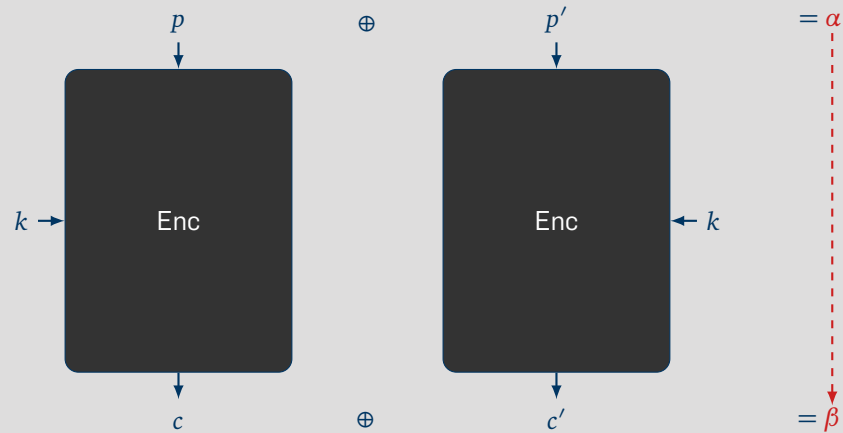
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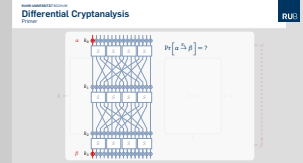
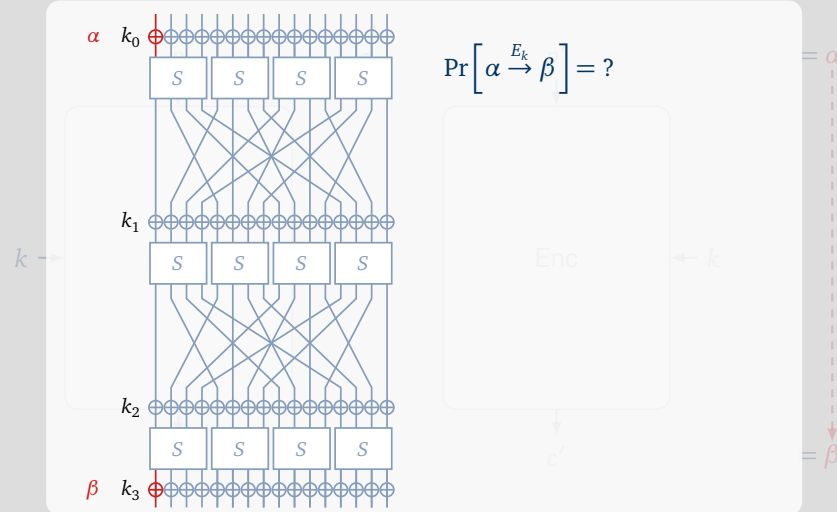
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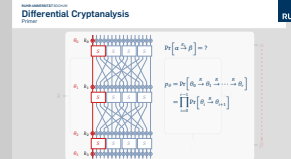
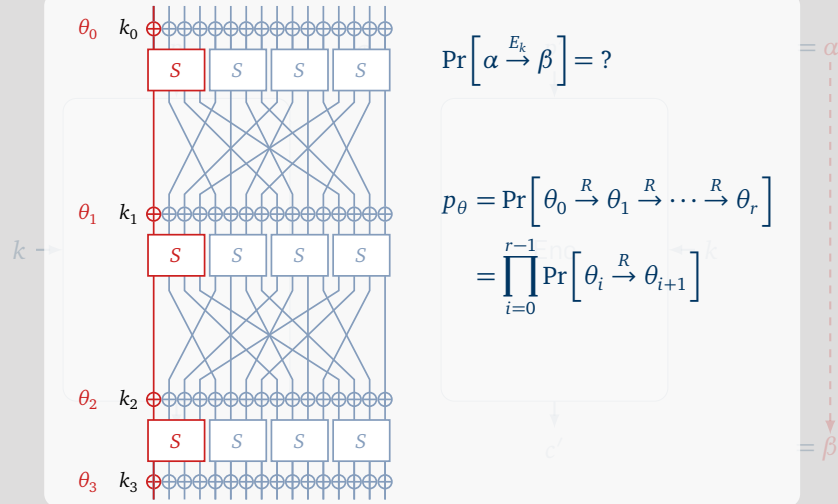
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- Thats nice, as those functions are quite well understood and already well scrutinised.
- Also, this is the reason for the name: Bent Whitened Swap-Or-Not
- But, and thats not so nice...
- $n$  iterations of a round function that depends on *all* bits will be slow
- Let me repeat this (Reviewer 2 argued that we should optimise more):  
No matter how good we will optimise this: *it will be slow*
- For example, assume you can do one round in one clock cycle, this is still an order of magnitude slower than AES.
- So, why should we care about any instance?
- All hope is not lost, let's have a look at differential cryptanalysis!



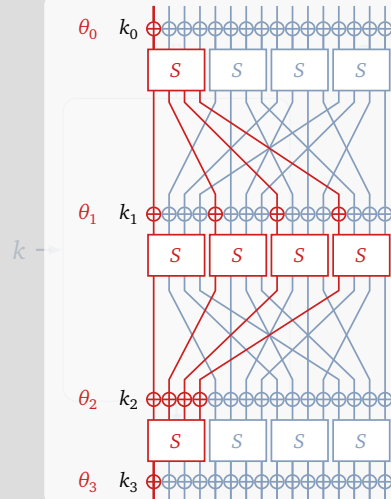
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- But, trails can go several alternative ways through non-linear parts, thus we have to cope with a branching effect...

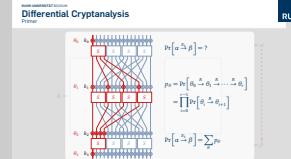


$$\Pr[\alpha \xrightarrow{E_k} \beta] = ?$$

$$p_\theta = \Pr[\theta_0 \xrightarrow{R} \theta_1 \xrightarrow{R} \dots \xrightarrow{R} \theta_r]$$

$$= \prod_{i=0}^{r-1} \Pr[\theta_i \xrightarrow{R} \theta_{i+1}]$$

$$\Pr[\alpha \xrightarrow{E_k} \beta] = \sum_{\theta} p_\theta$$



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- Now, computing the probability of one such trail is actually doable.
- But, trails can go several alternative ways through non-linear parts, thus we have to cope with a branching effect...
- And eventually, several of these trails cluster in a so called *differential*.
- While in this example it is still feasible, computing the *exact* probability in a real cipher is not.
- We thus have to restrain on bounding or approximating this probability.
- In the end, tight bounds for differentials over several rounds remain an open (but important!) problem.
- For BISON our aim is to give exactly this: a tight bound for any differential over several rounds.

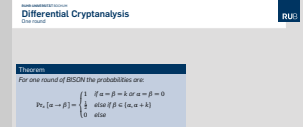
2019-05-15

## BISON Instantiating the Whitened Swap-Or-Not

## Construction

## └ Differential Analysis

## └ Differential Cryptanalysis



## Theorem

*For one round of BISON the probabilities are:*

$$\Pr_x[\alpha \rightarrow \beta] = \begin{cases} 1 & \text{if } \alpha = \beta = k \text{ or } \alpha = \beta = 0 \\ \frac{1}{2} & \text{else if } \beta \in \{\alpha, \alpha + k\} \\ 0 & \text{else} \end{cases}$$

- We start by understanding the differential one round behaviour.
- For the three possible cases, let us look at what differences are actually possible.

# Differential Cryptanalysis

One round

RUB

 BISON Instantiating the Whitened Swap-Or-Not Construction  
 └ Differential Analysis

2019-05-15

└ Differential Cryptanalysis

Differential Cryptanalysis	
<b>Theorem</b> For one round of BISON the probabilities are: $\Pr_x[\alpha \rightarrow \beta] = \begin{cases} 1 & \text{if } \alpha = \beta = k \text{ or } \alpha = \beta = 0 \\ \frac{1}{2} & \text{else if } \beta \in \{\alpha, \alpha + k\} \\ 0 & \text{else} \end{cases}$	<b>Possible differences</b> $\begin{aligned} x &+ f_k(x) \cdot k \\ \oplus x + \alpha &+ f_k(x + \alpha) \cdot k \\ = &\alpha + (f_k(x) + f_k(x + \alpha)) \cdot k \end{aligned}$
	<b>Properties of <math>f_k</math></b> $f_k(x) = f_k(x + k) \quad (1)$

## Theorem

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## Possible differences

$$\begin{aligned} x &+ f_k(x) \cdot k \\ \oplus x + \alpha &+ f_k(x + \alpha) \cdot k \\ = &\alpha + (f_k(x) + f_k(x + \alpha)) \cdot k \end{aligned}$$

## Properties of $f_k$

$$f_k(x) = f_k(x + k) \quad (1)$$

- We start by understanding the differential one round behaviour.
- For the three possible cases, let us look at what differences are actually possible.
- Remember that one round computes the output as  $x + f_k(x) \cdot k$ .
- With the input difference  $\alpha$  we get as possible output differences  $\beta \in \{0, \alpha, k, \alpha + k\}$ .
- For decryption we need that  $x$  and  $x + k$  are mapped to the same value by  $f_k$
- Thus,  $\beta = \alpha$  with probability one, if and only if  $\alpha = k$  or  $\alpha = 0$
- If  $\beta$  is not one of the above four values, such an input/output pair cannot occur, thus the probability is zero.



# Differential Cryptanalysis

One round

RUB

 BISON Instantiating the Whitened Swap-Or-Not Construction  
 └ Differential Analysis

└ Differential Cryptanalysis

2019-05-15

Differential Cryptanalysis	
<b>Theorem</b> For one round of BISON the probabilities are: $\Pr_x[\alpha \rightarrow \beta] = \begin{cases} 1 & \text{if } \alpha = \beta = k \text{ or } \alpha = \beta = 0 \\ \frac{1}{2} & \text{else if } \beta \in \{\alpha, \alpha + k\} \\ 0 & \text{else} \end{cases}$	<b>Possible differences</b> $\begin{aligned} x &+ f_k(x) && k \\ \oplus x + \alpha && + f_k(x + \alpha) &\cdot k \\ = && \alpha + (f_k(x) + f_k(x + \alpha)) &\cdot k \end{aligned}$
	<b>Properties of <math>f_k</math></b> $f_k(x) = f_k(x + k) \quad (1)$ $\Pr[f_k(x) = f_k(x + \alpha)] = \frac{1}{2} \quad (2)$

## Theorem

For one round of BISON the probabilities are:

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## Properties of $f_k$

$$f_k(x) = f_k(x + k) \quad (1)$$

$$\Pr[f_k(x) = f_k(x + \alpha)] = \frac{1}{2} \quad (2)$$

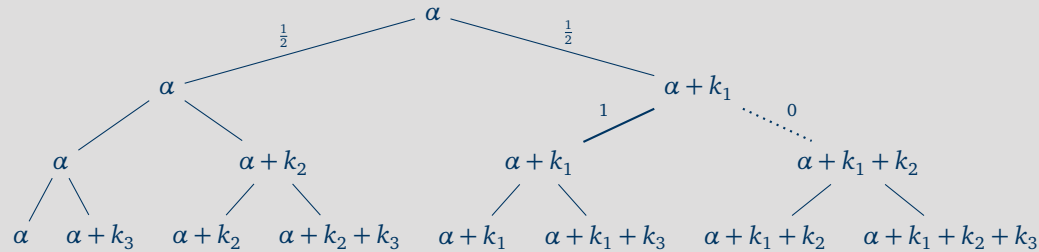
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- If  $\beta$  is not one of the above four values, such an input/output pair cannot occur, thus the probability is zero.
- For the last case, remember that for any input difference, we required that  $f_k$  collides with probability one half.

# Differential Cryptanalysis

More rounds

RUB

Example differences over  $r = 3$  rounds ( $\alpha = k_1 + k_2$ ):

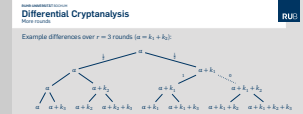


## BISON Instantiating the Whitened Swap-Or-Not Construction

Construction

└ Differential Analysis

└ Differential Cryptanalysis



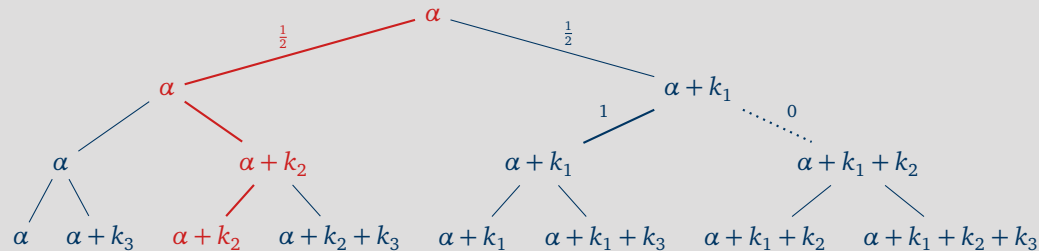
- When we look at more rounds, we can depict these cases again in this tree structure.
- Starting with the input difference  $\alpha$ , assume it is different from the first round key  $k_0$  and nonzero.
- After one round the two differences  $\alpha$  and  $\alpha + k_0$  occur with equal probability one half.
- As long as the intermediate difference is different from the next round key and nonzero, this argument iterates.
- At the point where the difference equals the next round key, these two choices collide into a deterministic propagation with probability one, where the round key is not added to the difference.

# Differential Cryptanalysis

More rounds

RUB

Example differences over  $r = 3$  rounds ( $\alpha = k_1 + k_2$ ):



## Theorem (Differentials in BISON)

Let  $\alpha, \beta \in \mathbb{F}_2^n$ . Then the probability for the differential  $\alpha \rightarrow \beta$  is  $\Pr[\alpha \rightarrow \beta] = \sum_{\theta} p_{\theta} = p_{(\alpha, \theta_1, \dots, \theta_{r-1}, \beta)}$ .

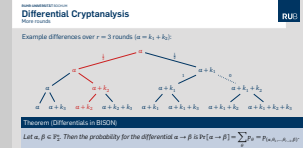
## BISON Instantiating the Whitened Swap-Or-Not Construction

Construction

Differential Analysis

Differential Cryptanalysis

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- After one round the two differences  $\alpha$  and  $\alpha + k_0$  occur with equal probability one half.
- As long as the intermediate difference is different from the next round key and nonzero, this argument iterates.
- At the point where the difference equals the next round key, these two choices collide into a deterministic propagation with probability one, where the round key is not added to the difference.
- Regarding differentials, the important observation is:
- For any input/output pair  $(\alpha, \beta)$ , there is only one path.
- In other words: no branching occurs and the differential consists of a single trail only.

## A concrete species



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## BISON Instantiating the Whitened Swap-Or-Not Construction

- └ The concrete Instance
  - └ BISON



- Up to now we do not have specified much more then the initial WSN construction had.
- For a concrete implementation, we still need to define
  - Number of rounds
  - Key Schedule
  - Boolean function  $f_k$
- So let us look at a concrete BISON species
- In particular, we discuss how to tackle Rationales 1 and 2.

# Addressing Rationale 1

The Key Schedule

RUB

## Rationale 1

Any instance must iterate at least  $n$  rounds; any set of  $n$  consecutive keys should be linearly indep.

### Design Decisions

- Choose number of rounds as  $3 \cdot n$
- Round keys derived from the state of LFSRs
- Add round constants  $c_i$  to  $w_i$  round keys

### Implications

- Clocking an LFSR is cheap
- For an LFSR with irreducible feedback polynomial of degree  $n$ , every  $n$  consecutive states are linearly independent
- Round constants avoid structural weaknesses

## BISON Instantiating the Whitened Swap-Or-Not Construction

└ The concrete Instance

└ Addressing Rationale 1

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Addressing Rationale 1 The Key Schedule	
<b>Rationale 1</b> Any instance must iterate at least $n$ rounds; any set of $n$ consecutive keys should be linearly indep.	
Design Decisions	Implications
<ul style="list-style-type: none"> <li>■ Choose number of rounds as <math>3 \cdot n</math></li> <li>■ Round keys derived from the state of LFSRs</li> <li>■ Add round constants <math>c_i</math> to <math>w_i</math> round keys</li> </ul>	<ul style="list-style-type: none"> <li>■ Clocking an LFSR is cheap</li> <li>■ For an LFSR with irreducible feedback polynomial of degree <math>n</math>, every <math>n</math> consecutive states are linearly independent</li> <li>■ Round constants avoid structural weaknesses</li> </ul>

- Due to analysis of the alg. deg., we chose  $3n$  rounds, see last slide.
- Deriving round keys from the states of LFSRs is efficiently implementable and fullfils our requirements for linear independent round keys.
- For those of you how attended Gregors talk at FSE:
  - While generating test vectors for the implementation we again noted some unwanted symmetries for encryptions with low hamming weight.
  - Thus we added round constants, to avoid these structural weaknesses.
  - (Sorry for fixing another cipher)

# Addressing Rationale 2

The Round Function

RUB

## Rationale 2

For any instance, the  $f_k$  should depend on all bits, and for any  $\delta \in \mathbb{F}_2^n$ :  $\Pr[f_k(x) = f_k(x + \delta)] \approx \frac{1}{2}$ .

### Design Decisions

- Choose  $f_k : \mathbb{F}_2^n \rightarrow \mathbb{F}_2$  s. t.

$$\delta \in \mathbb{F}_2^n : \Pr[f_k(x) = f_k(x + \delta)] = \frac{1}{2},$$

that is,  $f_k$  is a bent function.

- Choose the simplest bent function known:

$$f_k(x, y) := \langle x, y \rangle$$

### Implications

- Bent functions well studied
- Bent functions only exists for even  $n$
- Instance not possible for every block length  $n$

## BISON Instantiating the Whitened Swap-Or-Not Construction

└ The concrete Instance

└ Addressing Rationale 2

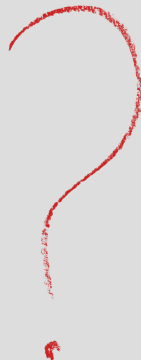
2019-05-15

Addressing Rationale 2	
<p><b>Rationale 2</b> For any instance, the <math>f_k</math> should depend on all bits, and for any <math>\delta \in \mathbb{F}_2^n</math>: <math>\Pr[f_k(x) = f_k(x + \delta)] \approx \frac{1}{2}</math>.</p>	
Design Decisions	Implications
<ul style="list-style-type: none"> <li>Choose <math>f_k : \mathbb{F}_2^n \rightarrow \mathbb{F}_2</math> s. t.  <math>\delta \in \mathbb{F}_2^n : \Pr[f_k(x) = f_k(x + \delta)] = \frac{1}{2}</math>,  that is, <math>f_k</math> is a bent function.</li> <li>Choose the simplest bent function known:  <math>f_k(x, y) := \langle x, y \rangle</math></li> </ul>	<ul style="list-style-type: none"> <li>Bent functions well studied</li> <li>Bent functions only exists for even <math>n</math></li> <li>Instance not possible for every block length <math>n</math></li> </ul>

- Just chose the simplest bent function, the scalar product.
- This is also efficiently implementable.
- But, another drawback:
- Bent functions exists only for even  $n$ .
- Thus BISON cannot be instantiated for every block length  $n$ .
- In particular, due to reasons not covered here, we can actually only instantiate it for *odd* block lengths.

## Construction

- $f_k(x) := ?$
- Key schedule?
- $\Theta(n)$  rounds?



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## BISON Instantiating the Whitened Swap-Or-Not

## Construction

└ The concrete Instance

└ An Implementation

- Coming back to our initial question.
- And basically only for the sake of completeness, as we already saw this is going to be slow.

## Construction

- $f_k(x) := ?$
- Key schedule?
- $\Theta(n)$  rounds?



## Construction

- $f_k(x, y) := \langle x, y \rangle$
- Key schedule: LFSRs.
- $\Theta(n) = 3n$  rounds.

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## BISON Instantiating the Whitened Swap-Or-Not

## Construction

└ The concrete Instance

└ An Implementation

## Construction

- $f_k(x, y) := \langle x, y \rangle$
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- We have specified everything, so let's benchmark against AES (what else).





Construction

- $f_k(x,y) := \langle x,y \rangle$
- Key schedule: LFSRs.
- $\Theta(n) = 3n$  rounds.

Cipher	Block size (bit)	Cycles/Byte mean
AES*	128	0.65
BISON†	129	3 064.08

\* AES-128 on Skylake Intel® Core i7-7800X @ 3.5GHz, see Daemen et al. [The design of Xoodoo and Xoofff, Table 5].  
† BISON on CoffeeLake Intel® Core i7-8700 @ 3.7 GHz.

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BISON Instantiating the Whitened Swap-Or-Not Construction

- └ The concrete Instance
- └ An Implementation

- Coming back to our initial question.
- And basically only for the sake of completeness, as we already saw this is going to be slow.
- We have specified everything, so let's benchmark against AES (what else).
- OK, told you so, BISON is like 4 700 times slower than AES.
- Or: more than three orders of magnitude.
- Optimising this will not help enough.

An Implementation

Construction

- $f_k(x,y) := \langle x,y \rangle$
- Key schedule: LFSRs.
- $\Theta(n) = 3n$  rounds.

Cipher	Block size (bit)	Cycles/Byte mean
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### Linear Cryptanalysis

For  $r \geq n$  rounds, the correlation of any non-trivial linear trail for BISON is upper bounded by  $2^{-\frac{n+1}{2}}$ .

### Zero Correlation

For  $r > 2n - 2$  rounds, BISON does not exhibit any zero correlation linear hulls.

### Algebraic Degree and Division Property

Algebraic degree grows *linearly*. Conservative estimate: for  $r \geq 3n$  rounds, no attack possible.

### Invariant Attacks

For  $r \geq n$  rounds, neither invariant subspaces nor nonlinear invariant attacks do exist for BISON.

### Impossible Differentials

For  $r > n$  rounds, there are no impossible differentials for BISON.

## BISON Instantiating the Whitened Swap-Or-Not Construction

### Further Analysis

### Further Cryptanalysis

- We did more cryptanalysis, but our results are more of the “classical” kind.
- For linear cryptanalysis, we bound the correlation of any non-trivial trail.
- Current known security arguments for resistance against invariant attacks apply.
- Zero correlation and impossible differentials do not exist for  $2n$  rounds or more.
- Best attacks seem to exploit the algebraic degree.
- We show that it grows only linearly – which is especially bad in comparison to SPN ciphers.
- The result on the algebraic degree also applies to NLFSRs or maximally unbalanced Feistel networks.
- Conservative estimation: might work for more than  $2n$  rounds, but not for  $3n$  or more.

Further Cryptanalysis	
<b>Linear Cryptanalysis</b> For $r \geq n$ rounds, the correlation of any non-trivial linear trail for BISON is upper bounded by $2^{-\frac{n+1}{2}}$ .	<b>Invariant Attacks</b> For $r \geq n$ rounds, neither invariant subspaces nor nonlinear invariant attacks do exist for BISON.
<b>Zero Correlation</b> For $r > 2n - 2$ rounds, BISON does not exhibit any zero correlation linear hulls.	<b>Impossible Differentials</b> For $r > n$ rounds, there are no impossible differentials for BISON.
<b>Algebraic Degree and Division Property</b> Algebraic degree grows linearly. Conservative estimate: for $r \geq 3n$ rounds, no attack possible.	

# Conclusion/Questions

Thank you for your attention!

RUB

## BISON

- A first instance of the WSN construction
- Good results for differential cryptanalysis

## Open Problems

- Construction for linear cryptanalysis?
- Similar args. for Unbalanced Feistel?

Thank you!

Questions?

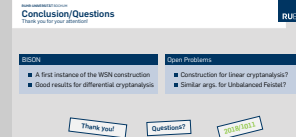
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BISON Instantiating the Whitened Swap-Or-Not Construction

└─ Further Analysis

└─ Conclusion/Questions



# Details

2019-05-15

BISON Instantiating the Whitened Swap-Or-Not  
Construction  
└─ Further Analysis

Details

## BISON's round function

For round keys  $k_i \in \mathbb{F}_2^n$  and  $w_i \in \mathbb{F}_2^{n-1}$  the round function computes

$$R_{k_i, w_i}(x) := x + f_{b(i)}(w_i + \Phi_{k_i}(x)) \cdot k_i.$$

where

- $\Phi_{k_i}$  and  $f_{b(i)}$  are defined as

$$\Phi_k(x) : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^{n-1}$$

$$\Phi_k(x) := (x + x[i(k)] \cdot k)[j]_{\substack{1 \leq j \leq n \\ j \neq i(k)}}$$

$$f_{b(i)} : \mathbb{F}_2^{\frac{n-1}{2}} \times \mathbb{F}_2^{\frac{n-1}{2}} \rightarrow \mathbb{F}_2$$

$$f_{b(i)}(x, y) := \langle x, y \rangle + b(i),$$

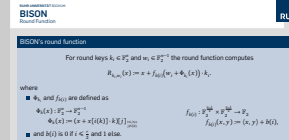
- and  $b(i)$  is 0 if  $i \leq \frac{r}{2}$  and 1 else.

## BISON Instantiating the Whitened Swap-Or-Not Construction

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Specification

BISON



## BISON's key schedule

Given

- primitive  $p_k, p_w \in \mathbb{F}_2[x]$  with degrees  $n, n-1$  and companion matrices  $C_k, C_w$ .
- master key  $K = (k, w) \in (\mathbb{F}_2^n \times \mathbb{F}_2^{n-1}) \setminus \{0, 0\}$

The  $i$ th round keys are computed by

$$\begin{aligned} \text{KS}_i &: \mathbb{F}_2^n \times \mathbb{F}_2^{n-1} \rightarrow \mathbb{F}_2^n \times \mathbb{F}_2^{n-1} \\ \text{KS}_i(k, w) &:= (k_i, c_i + w_i) \end{aligned}$$

where

$$k_i = (C_k)^i k, \quad c_i = (C_w)^{-i} e_1, \quad w_i = (C_w)^i w.$$

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- BISON Instantiating the Whitened Swap-Or-Not Construction
  - Specification
    - BISON

