Cryptanalysis of Clyde and Shadow

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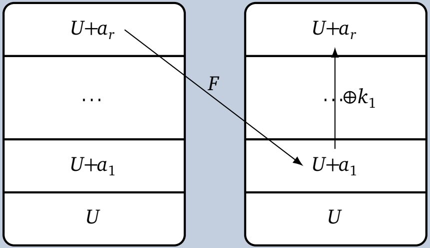
Overview

Invariant Attacks

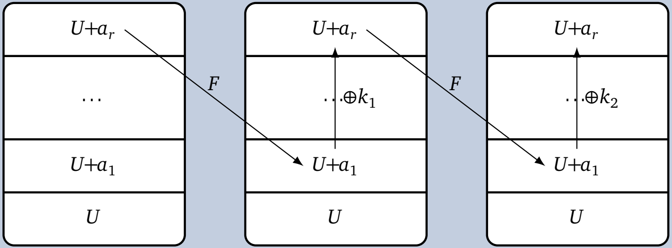
Invariant Attacks

Main Idea: Invariant Subspaces



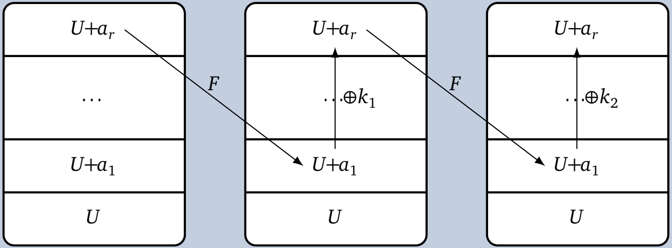
Invariant Attacks

Main Idea: Invariant Subspaces



Invariant Attacks

Main Idea: Invariant Subspaces

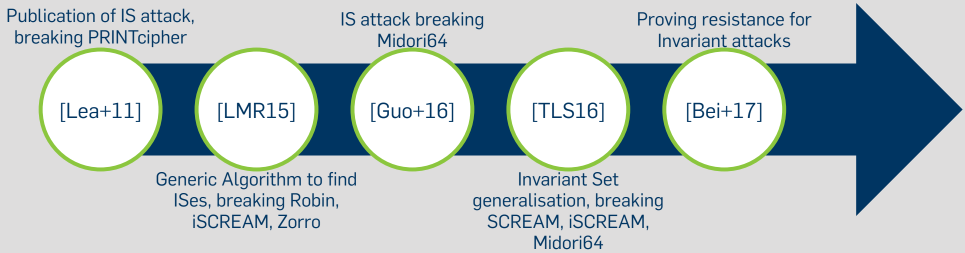


Invariant Subspace Attacks [Lea+11] (CRYPTO'11)

Let $U \subseteq \mathbb{F}_2^n$, $c, d \in U^\perp$, and $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$. Then U is an *invariant subspace* (IS) if and only if $F(U{+}c) = U{+}d$ and all round keys in $U{+}(c+d)$ are *weak keys*.

Invariant Attacks

A Short History



Invariant Attacks

Proving Resistance

RUB

Goal: Apply security argument from

C. Beierle, A. Canteaut, G. Leander, and Y. Rotella. "Proving Resistance Against Invariant Attacks: How to Choose the Round Constants". In: CRYPTO 2017, Part II. 2017. doi: 10.1007/978-3-319-63715-0_22. iacr: 2017/463.

What do we get from this?

- Non-existence of invariants for both parts of the ronud function (S-box and linear layer)

Issues

- Other partitionings of the round function might allow invariants (Christof B. found examples)
- Not clear how to prove the general absence of invariant attacks (best we can currently prove)
- All known attacks exploit exactly this structure (splitting in S-box and linear layer)

Invariant Attacks

Recap Security Argument (I)

Observation

- Invariants for the linear layer L and round key addition have to contain special linear structures.
- Denote by $c_1, \dots, c_t$ the round constant differences for rounds with the same round key.
- Then the linear structures of any invariant have to contain $W_L(c_1, \dots, c_t)$.

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Linear Structures

Let $f : \mathbb{F}_2^n \to \mathbb{F}_2$. Then its *linear structures* are

$$\mathrm{LS} := \{a \mid f(x) + f(x+a) \text{ is constant}\} .$$

The smallest L-invariant subspace

$W_L(c_1, \dots, c_t)$ is the *smallest L-invariant subspace* of $\mathbb{F}_2^n$ containing all c_i

$$\Leftrightarrow \forall x \in W_L(c_1, \dots, c_t) : L(x) \in W_L(c_1, \dots, c_t)$$

Invariant Attacks

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The simple case

If $W_L(c_1, \dots, c_t) = \mathbb{F}_2^n$, only trivial invariants for L and key addition are possible (constant 0 and 1 function).

Invariant Attacks

Recap Security Argument (II)

Application to Clyde

- Find the important round constant differences:
  (the differences where the same tweakey is added)

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Set of RC differences D below with $|D| = 20$

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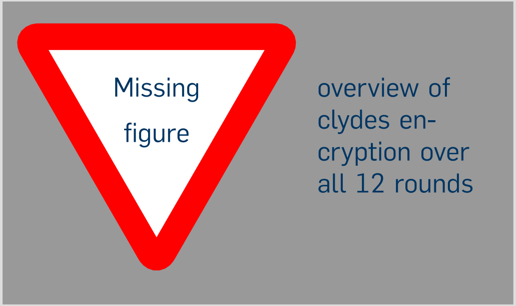

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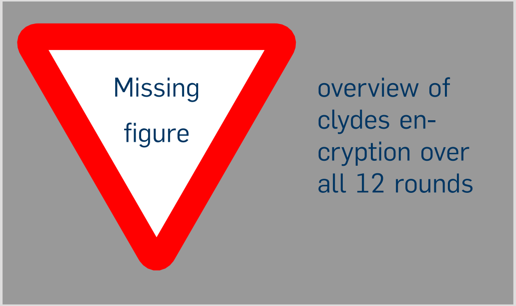

$$D = D_{\mathrm{TK}_0} \cup D_{\mathrm{TK}_1} \cup D_{\mathrm{TK}_2} \cup D_0$$

$$D_{\mathrm{TK}_0} = \{0 + W(5), 0 + W(11), W(5) + W(11)\}$$

$$D_{\mathrm{TK}_1} = \{W(1) + W(7)\}$$

$$D_{\mathrm{TK}_2} = \{W(3) + W(9)\}$$

$$D_0 = \left\{a + b \;\middle|\; a, b \in D', a \neq b\right\}$$

$$D' = \{W(0), W(2), W(4), W(6), W(8), W(10)\}$$

Invariant Attacks

Application to Clyde

- Computing W_L is efficiently doable (takes ≈ 10 seconds on my laptop).
- For the round constants chosen for Clyde, $\dim W_L(D) = 128 = n$.

- Thus, we can apply:

Proposition 2 [Bei+17]

Suppose that the dimension of $W_L(D)$ is n. Then any invariant g is constant (and thus trivial).

- We conclude that we cannot find any non-trivial g for Clyde which is at the same time invariant for the S-box layer and for the linear layer.

Invariant Attacks

Improvable?

Bounding the dimension of W_L, [Bei+17, Theorem 1]

Given a linear layer L. Denote by Q_i its *invariant factors*. Then

$$\max_{c_1,\ldots,c_t \in \mathbb{F}_2^n} \dim W_L(c_1,\ldots,c_t) = \sum_{i=1}^{t} \deg Q_i \, .$$

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- This gives a lower bound on the number of rounds:

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Application to Clyde

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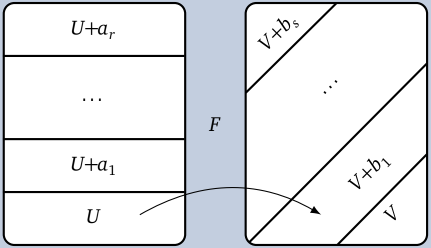
- 3 stps/6 rnds: $\dim W_L(c_1,\ldots,c_4) = 96$
- 4 stps/8 rnds: $\dim W_L(c_1,\ldots,c_8) = 128$
- 5 stps/10 rnds: $\dim W_L(c_1,\ldots,c_{13}) = 128$
- 6 stps/12 rnds: $\dim W_L(c_1,\ldots,c_{20}) = 128$

Subspace Trails

Probability 1 Truncated Differentials

Subspace Trails

Main Idea: Subspace Trails



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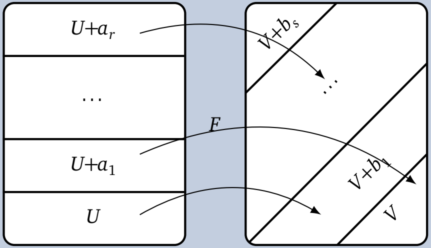

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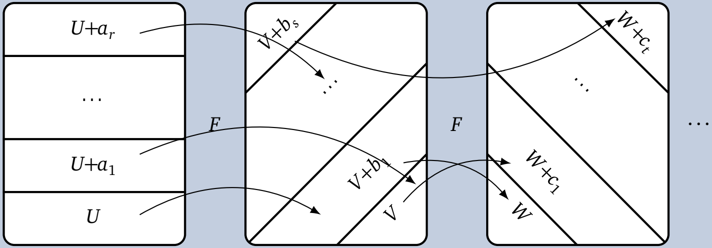

Subspace Trail Cryptanalysis [GRR16] (FSE'16)

Let $U_0, \dots, U_r \subseteq \mathbb{F}_2^n$, and $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$. Then these form a *subspace trail* (ST), $U_0 \xrightarrow{F} \cdots \xrightarrow{F} U_r$, iff

$$\forall a \in U_i^{\perp} : \exists b \in U_{i+1}^{\perp} : \qquad F(U_i + a) \subseteq U_{i+1} + b$$

Computing Subspace Trails

Given a starting subspace U, we can efficiently compute the corresponding longest subspace trail.

Lemma

Let $U \xrightarrow{F} V$ be a ST. Then for all $u \in U$ and all x: $F(x) + F(x+u) \in V$.

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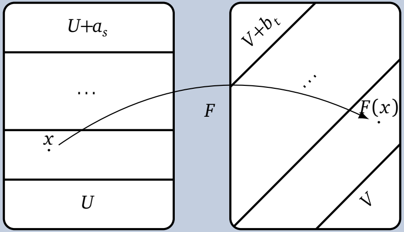

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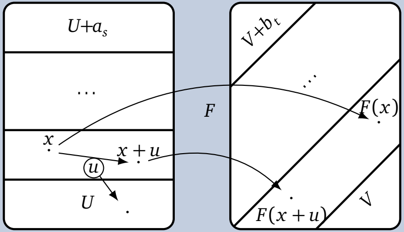

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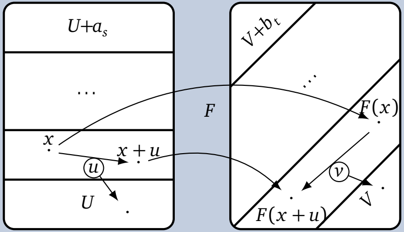

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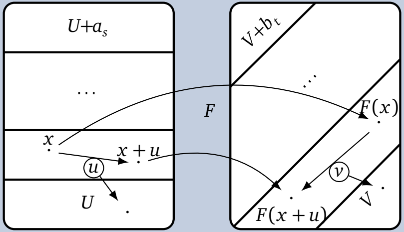


Computing the subspace trail

- To compute the next subspace, we have to compute the image of the derivatives.

Computing Subspace Trails

Algorithm

RUB

Compute Subspace Trails

Input: A nonlinear, bijective function $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$ and a subspace U.
Output: The longest ST starting in U over F.

function $\mathrm{Compute\ Trail}(F, U)$
  if $\dim(U) = n$ **then**
    return U
  $V \leftarrow \emptyset$
  for u_i basis vectors of U **do**
    for enough $x \in_R \mathbb{F}_2^n$ **do** ▷ e. g. $n + 20$ x's are enough
      $V \leftarrow V \cup \Delta_{u_i}(F)(x)$ ▷ $\Delta_a(F)(x) := F(x) + F(x + a)$
  $V \leftarrow \mathrm{span}(V)$
  return the subspace trail $U \to \mathrm{Compute\ Trail}(F, V)$

Subspace Trails

Proving Resistance

Goal: Apply security argument from

G. Leander, C. Tezcan, and F. Wiemer. "Searching for Subspace Trails and Truncated Differentials". In: ToSC 2018.1 (2018). doi: `10.13154/tosc.v2018.i1.74-100`.

What do we get from this?

- (Tight) upper bound on the length of any ST for an SPN construction

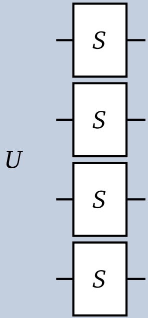
Why is the Compute Trail algorithm not enough?

- Exhaustively checking all possible starting points is to costly.

Subspace Trails

How to bound the length of any subspace trail

Observation

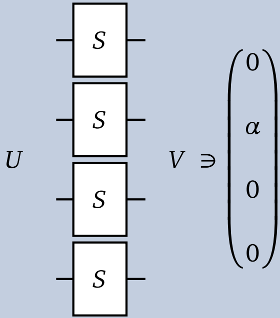


$$V \ni \begin{pmatrix} 0 \\ \alpha \\ 0 \\ 0 \end{pmatrix}$$

Subspace Trails

How to bound the length of any subspace trail

Observation



Algorithm Idea

Compute the subspace trails for any starting point $W_{i,\alpha} \in \mathcal{W}$, with

$$W_{i,\alpha} := (\underbrace{0,\dots,0}_{i-1}, \alpha, 0, \dots, 0)$$

Complexity (Size of $\mathcal{W}$)

For an S-box layer $S : \mathbb{F}_2^{kn} \to \mathbb{F}_2^{kn}$ with k S-boxes, each n-bit:

$$|\mathcal{W}| = k \cdot (2^n - 1)$$

Subspace Trails

Algorithm

Generic Subspace Trail Search

Input: A linear layer matrix $M : \mathbb{F}_2^{n \cdot k \times n \cdot k}$, and an S-box $S : \mathbb{F}_2^n \to \mathbb{F}_2^n$.
Output: A bound on the length of all STs over $F = M \circ S^k$.

function $\mathrm{Generic\ Subspace\ Trail\ Length}(M, S)$
  empty list L
  for possible initial subspaces represented by $W_{i,\alpha} \in \mathcal{W}$ **do** ▷ Overall $k \cdot (2^n - 1)$ iterations
    $L.\mathrm{append}(\mathrm{Compute\ Trail}(S^k \circ M, \{W_{i,\alpha}\}))$ ▷ S^k denotes the S-box layer
  return $\max \{\mathrm{len}(t) \mid t \in L\}$

Overall Complexity

Algorithm	Compute Trail	Generic Subspace Trail Length	Overall	Clyde	Shadow
Complexity	$\mathcal{O}(k^2 n^2)$	$\mathcal{O}(k 2^n)$	$\mathcal{O}(k^3 n^2 2^n)$	2^{23}	2^{29}

Division Property

Division Property

Goal: Apply security argument from

Z. Xiang, W. Zhang, Z. Bao, and D. Lin. "Applying MILP Method to Searching Integral Distinguishers Based on Division Property for 6 Lightweight Block Ciphers". In: ASIACRYPT 2016, Part I. 2016. doi: 10.1007/978-3-662-53887-6_24. iacr: 2016/857.

What do we get from this?

bla

Approach

Model division trail propagations as MILP, find solutions for this over increasing number of rounds.

Results

Results

Thanks for your attention!

Number of rounds

Technique	Clyde	Shadow
Invariants	6	—
Subspace Trails	2 (+1)	4 (+1)
Division Property	8	—

Future Work/Cryptanalysis

- Cryptagraph [HV18]
- Post cryptanalysis results on mailinglist?
- Eprint Write-Up?

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