

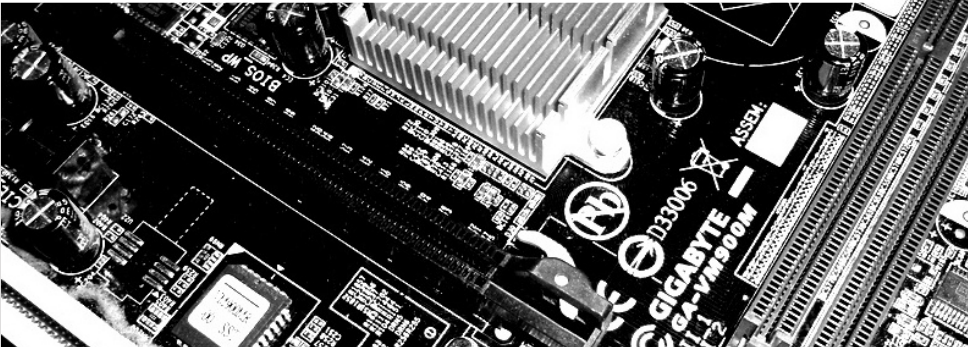
XOR Count

November 21st, 2017

FluxFingers

Workgroup Symmetric Cryptography
Ruhr University Bochum

Friedrich Wiemer



Joint Work – Its not me alone [Kra+17]¹

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Outline

- 1 Motivation
- 2 Preliminaries
- 3 State of the Art and Related Work
- 4 Future Work

¹available on eprint: <https://eprint.iacr.org/2017/1151>

What is the XOR count, and why is it important?

Some facts

- Lightweight Block Ciphers
- Efficient Linear Layers
- MDS matrices are “optimal” (regarding security)²

²Are they?

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- What is the lightest implementable MDS matrix?
- What about additional features (Involutory)?

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- What is the lightest implementable MDS matrix?
- What about additional features (Involutory)?

The XOR count

- Metric for needed hardware resources
- Smaller is better

²Are they?

What is an MDS matrix?

Definition: MDS

A matrix M of dimension k over the field \mathbb{F} is *maximum distance separable* (MDS), iff all possible submatrices of M are invertible (or nonsingular).

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Example

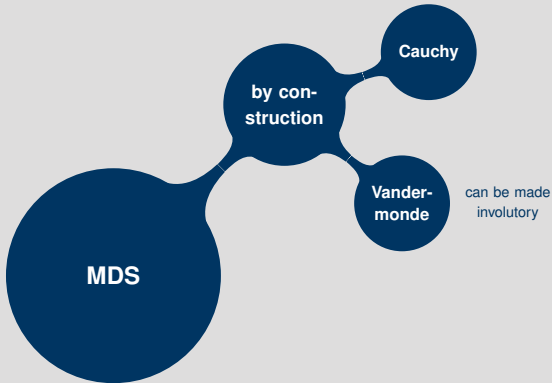
The AES MIXCOLUMN matrix is defined over $\mathbb{F}_{2^8} \cong \mathbb{F}[x]/0x11b$:

$$\begin{pmatrix} 0x02 & 0x03 & 0x01 & 0x01 \\ 0x01 & 0x02 & 0x03 & 0x01 \\ 0x01 & 0x01 & 0x02 & 0x03 \\ 0x03 & 0x01 & 0x01 & 0x02 \end{pmatrix} = \begin{pmatrix} x & x+1 & 1 & 1 \\ 1 & x & x+1 & 1 \\ 1 & 1 & x & x+1 \\ x+1 & 1 & 1 & x \end{pmatrix}$$

This is a (right) *circulant* matrix: $\text{circ}(x, x+1, 1, 1)$.

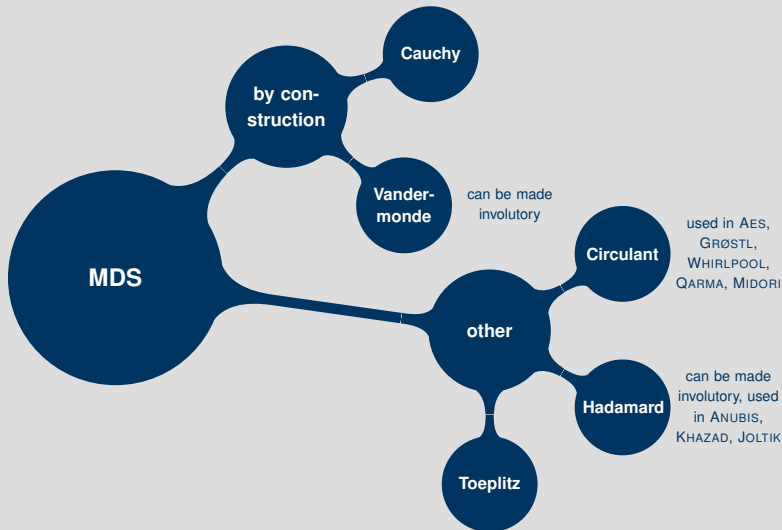
What is an MDS matrix?

Constructions



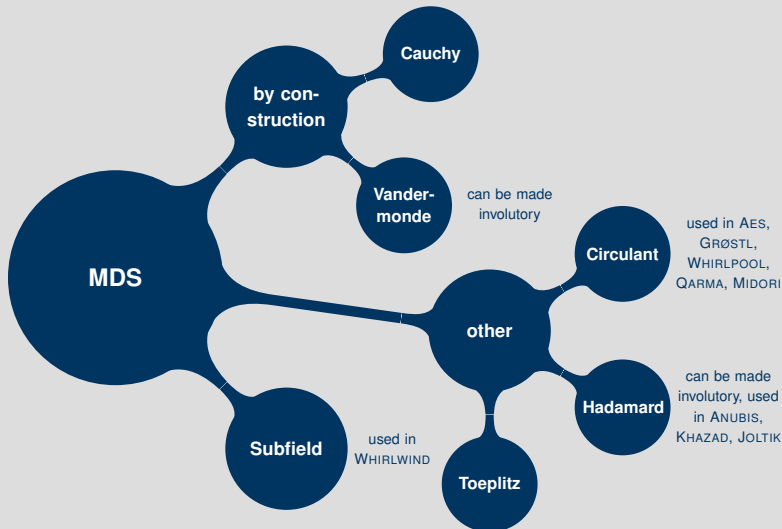
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What is an MDS matrix?

Representations

How to implement this in hardware?

- This is about hardware implementations
- How do we implement a field multiplication in hardware?
- How do we implement a matrix multiplication in hardware?

What is an MDS matrix?

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Example

$$\alpha \rightarrow \boxed{\cdot 1} \rightarrow \beta$$

$$\alpha \rightarrow \boxed{\cdot x} \rightarrow \beta$$

$$\alpha \rightarrow \boxed{\cdot (x + 1)} \rightarrow \beta$$

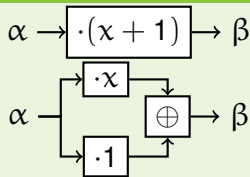
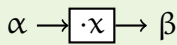
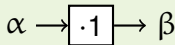
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Example



Field Multiplication in Hardware

From $\mathbb{F}_2[x]/p(x)$ to \mathbb{F}_2^n

Implement $\alpha \rightarrow \boxed{\cdot 1} \rightarrow \beta$

OK, this one is easy 😊

Example in $\mathbb{F}_2[x]/0x13$:

Field Multiplication in Hardware

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Example in $\mathbb{F}_2[x]/0x13$:

$$\alpha = \alpha_0 + \alpha_1x + \alpha_2x^2 + \alpha_3x^3$$

$$\beta = \beta_0 + \beta_1x + \beta_2x^2 + \beta_3x^3$$

$$= \alpha_0 + \alpha_1x + \alpha_2x^2 + \alpha_3x^3$$

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Example in $\mathbb{F}_2[x]/0x13$:

$$\alpha = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3$$

$$x^4 \equiv x + 1 \pmod{0x13}$$

$$\beta = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$$

$$= x \cdot (\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3)$$

$$\equiv \alpha_3 + (\alpha_0 + \alpha_3)x + \alpha_1 x^2 + \alpha_2 x^3$$

Field Multiplication in Hardware

From $\mathbb{F}_2[x]/p(x)$ to \mathbb{F}_2^n

In matrix notation for $\mathbb{F}_2[x]/0x13$:

$$\beta = 1 \cdot \alpha \Leftrightarrow \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

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Companion Matrix

We call $M_{p(x)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ the *companion matrix* of the polynomial $p(x) = 0x13$. For any element $\gamma \in \mathbb{F}_2[x]/p(x)$, we denote by M_γ the matrix that implements the multiplication by this element in \mathbb{F}_2^n .

Example

We can rewrite the AES MIXCOLUMN matrix as:

$$\mathcal{M}_{\text{AES}} = \text{circ}(x, x + 1, 1, 1) \cong \text{circ}(M_x, M_{x+1}, M_1, M_1).$$

Starting in $(\mathbb{F}_2[x]/0x11b)^{4 \times 4}$, we end up in $(\mathbb{F}_2^{8 \times 8})^{4 \times 4} \cong \mathbb{F}_2^{32 \times 32}$.

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A first XOR-count

To implement multiplication by γ , we need $\text{hw}(M_\gamma) - \dim(M_\gamma)$ many XOR's. Thus

$$\begin{aligned} \text{XOR-count}(\mathcal{M}_{\text{AES}}) &= 4 \cdot (\text{hw}(M_x) + \text{hw}(M_{x+1}) + 2 \cdot \text{hw}(M_1)) - 32 \\ &= 4 \cdot (11 + 19 + 2 \cdot 8) - 32 = 152. \end{aligned}$$

The General Linear Group

Generalise a bit

Instead of choosing elements from $\mathbb{F}_{2^n} \cong \mathbb{F}_2[x]/p(x)$ we can extend our possible choices for “multiplication matrices” by exploiting the following.

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Todo

Maybe remove this?

The Stupidity of recent XOR Count Papers

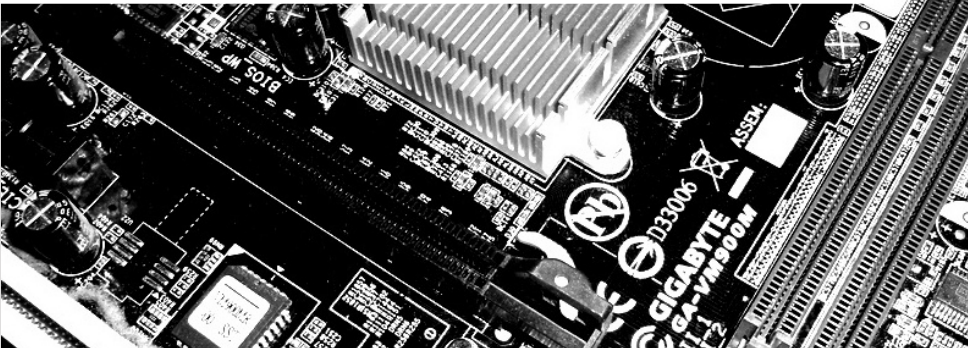
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- You saw how to count XORs
- This count is split in the “overhead” and the XORs needed for the field multiplication
- Thus for AES we get $56 + 8 \cdot 3 \cdot 4 = 56 + 96 = 152$
- Finding a good matrix reduces now to find the cheapest elements for field multiplication
- There is a lot of work following this line [BKL16; JPS17; LS16; LW16; LW17; Sim+15; SS16a; SS16b; SS17; ZWS17]

 4×4 matrices over $GL(8, \mathbb{F}_2)$

Matrix	Naive	Literature
AES (Circulant)	152	7+96
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[BKL16] (Circulant)	136	24+96
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[JPS17] (Subfield)	122	20+96

Optimized Arithmetic for Reed-Solomon Encoders

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1997 IEEE International Symposium on Information Theory, June 29 -- July 4, 1997,
Ulm, Germany (extended version)

Abstract

Multiplication with constant elements from Galois fields of characteristic two is the major arithmetic operation in Reed-Solomon encoders. This contribution describes two optimization algorithms which yield low complexity constant multipliers for Ga-

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Journal of
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Logic Minimization Techniques with Applications to Cryptology*

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Online publication 3 May 2012

State of the Art

Best known Results (After our Paper)

4×4 matrices over $GL(8, \mathbb{F}_2)$

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AES (Circulant)	152	7+96
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			PAAR1	PAAR2	BP
AES (Circulant)	152	7+96	108	108	97
[Sim+15] (Subfield)	136	40+96	100	98	100
[LS16] (Circulant)	128	32+96	116	116	112
[LW16]	106	10+96	102	102	102
[BKL16] (Circulant)	136	24+96	116	112	110
[SS16b] (Toeplitz)	123	27+96	110	108	107
[JPS17] (Subfield)	122	20+96	96	95	86

State of the Art

Finding better matrices?

Type	Previously Best Known	XOR count
$GL(4, \mathbb{F}_2)^{4 \times 4}$	58 [JPS17; SS16b]	36
$GL(8, \mathbb{F}_2)^{4 \times 4}$	106 [LW16]	72
$(\mathbb{F}_2[x]/0_{x^4}13)^{8 \times 8}$	392 [Sim+15]	196
$GL(8, \mathbb{F}_2)^{8 \times 8}$	640 [LS16]	392
$(\mathbb{F}_2[x]/0_{x^4}13)^{4 \times 4*}$	63 [JPS17]	42
$GL(8, \mathbb{F}_2)^{4 \times 4}$	126 [JPS17]	84
$(\mathbb{F}_2[x]/0_{x^4}13)^{8 \times 8}$	424 [Sim+15]	212
$GL(8, \mathbb{F}_2)^{8 \times 8}$	663 [JPS17]	424

Future Work; Questions?

Thank you for your attention!

Do your work!

Apply global optimization techniques that are known for years!

(But thanks for the easy paper
😊)



Mainboard & Questionmark Images: flickr

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