RUHR-UNIVERSITÄT BOCHUM

Security Arguments and Tool-based Design of Block Ciphers

PhD Defense

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Topics of the Thesis

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Based on mainly four papers:

Security Arguments

- A. Canteaut, L. Kölsch, and F. Wiemer. *Observations on the DLCT and Absolute Indicators*. 2019. iacr: 2019/848
- A. Canteaut, V. Lallemand, G. Leander, P. Neumann, and F. Wiemer. "Bison – Instantiating the Whitened Swap-Or-Not Construction". In: *EUROCRYPT 2019, Part III*. 2019, pp. 585–616. iacr: 2018/1011

Tool-based Design

- T. Kranz, G. Leander, K. Stoffelen, and F. Wiemer. "Shorter Linear Straight-Line Programs for MDS Matrices". In: *IACR Trans. Symm. Cryptol.* 2017.4 (2017), pp. 188–211. iacr: 2017/1151
- G. Leander, C. Tezcan, and F. Wiemer. "Searching for Subspace Trails and Truncated Differentials". In: *IACR Trans. Symm. Cryptol.* 2018.1 (2018), pp. 74–100

The General Setting

Block Ciphers and Security Notion

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Def.: Block Cipher

- $E : \mathcal{K} \times \mathcal{M} \to \mathcal{C}$ family of permutations, with $\mathcal{K}$ the key space, $\mathcal{M}$ the message space, and $\mathcal{C}$ the cipher space
- In practice: $\mathcal{K} = \mathbb{F}_2^m$ and $\mathcal{M} = \mathcal{C} = \mathbb{F}_2^n$. $E : \mathbb{F}_2^m \times \mathbb{F}_2^n \to \mathbb{F}_2^n$ is the *encryption*, $D = E^{-1}$ the *decryption*, m the *key length* and n the *block lengh*.

Further $E_k = E(k, \cdot)$ and $\mathrm{Perm}_n = \left\{ f : \mathbb{F}_2^n \to \mathbb{F}_2^n \,\middle|\, f \text{ is permutation} \right\}$ the set of n-bit permutations.

Def.: Security

A block cipher E is (q, t, ε)-*secure*, if the (CPA)-advantage of every (q, t)-adversary is bound by ε:

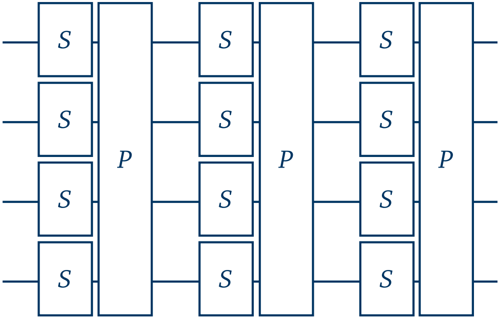
$$\mathrm{Adv}_E^{\text{PRP-CPA}}(\mathcal{A}_{q,t}) := \left| \Pr_{k \in_R \mathbb{F}_2^n}[\mathcal{A}_{q,t}^{F_k} = 1] - \Pr_{f \in_R \mathrm{Perm}_n}[\mathcal{A}_{q,t}^{f} = 1] \right| \leqslant \varepsilon \,.$$

In practice: security of a block cipher always security against known attacks.

Substitution Permutation Networks

The most common design structure for block ciphers

Exemplary SPN Structure

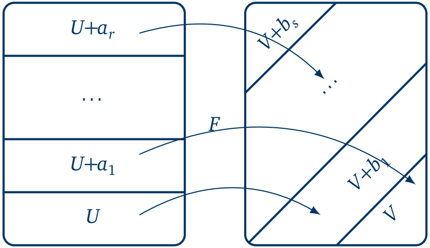


- S-box $S : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$ provides *confusion* and non-linearity on small blocks (typically $3 \leqslant n \leqslant 8$)
- Linear layer $P : \mathbb{F}_2^{kn} \rightarrow \mathbb{F}_2^{kn}$ provides *diffusion* and spreads the S-box influence over the whole state
- Key-alternating: the round keys are added in between the rounds

Overview

Subspace Trail Cryptanalysis

Idea: Subspace Trails



Def.: Subspace Trails [GRR16] (FSE'16)

Let $U_0, \ldots, U_r \subseteq \mathbb{F}_2^n$, and $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$. If

$$\forall a \in U_i^\perp : \exists b \in U_{i+1}^\perp : \quad F(U_i+a) \subseteq U_{i+1}+b \,,$$

these subspaces form a *subspace trail* (ST).

Notation: $U_0 \rightrightarrows^F \cdots \rightrightarrows^F U_r$.

The Problem/Our Goal

Find a solution to

Problem: Security against Subspace Trails

Given an SPN with round function F, consisting of

- k parallel applications of an S-box $S : \mathbb{F}_2^n \to \mathbb{F}_2^n$ and
- a linear layer $L : \mathbb{F}_2^{kn} \to \mathbb{F}_2^{kn}$.

Compute an upper bound on the length of any subspace trail through the cipher.

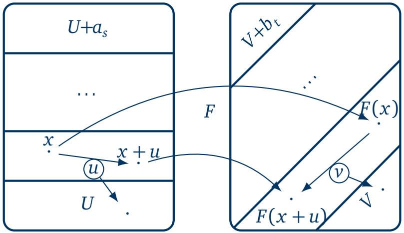
Subspace Propagation

Given a starting subspace U, we can efficiently compute the corresponding longest subspace trail.

Lemma

Let $U \xrightarrow{F} V$ *be a ST. Then for all* $u \in U$ *and all* x: $F(x) + F(x+u) \in V$.

Proof



Computing the subspace trail

- To compute the next subspace, we have to compute the image of the derivatives.

Propagate a Basis

Actually it is enough to compute only the image of the derivatives in direction of U's basis vectors.

Lemma

Given $U \subseteq \mathbb{F}_2^n$ *with basis* $\{b_1, \ldots, b_k\}$. *Then* $\mathrm{Span}\left\{\bigcup_{u \in U} \mathrm{Im}\, \Delta_u(F)\right\} = \mathrm{Span}\left\{\bigcup_{b_i} \mathrm{Im}\, \Delta_{b_i}(F)\right\}$.

Proof: $\supseteq$ ***trivial,*** $\subseteq$ ***by induction over the dimension*** k ***of*** U

Let $u = \sum_{i=1}^{k} \lambda_i b_i$ and $v \in \mathrm{Im}\, \Delta_u(F)$, i.e. there exists an x s.t.

$$v = F(x) + F(x + \sum_{i=1}^{k} \lambda_i b_i) = F(y + \lambda_k b_k) + F(y + \sum_{i=1}^{k-1} \lambda_i b_i) = \lambda_k \Delta_{b_k}(F)(y) + \lambda' \Delta_{u'}(F)(y)\,.$$

Thus $v \in \mathrm{Span}\left\{\mathrm{Im}\, \Delta_{b_k}(F) \cup \mathrm{Im}\, \Delta_{u'}(F)\right\}$, where u' is contained in a $(k-1)$ dimensional subspace.

`ComputeTrail` Algorithm

Computation of Subspace Trails

Input: A nonlinear function $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$, a subspace U.
Output: A subspace trail $U \rightrightarrows^F \cdots \rightrightarrows^F V$.

1 **function** $\texttt{ComputeTrail}(F, U)$
2   **if** $\dim U = n$ **then return** U
3     $V \leftarrow \emptyset$
4   **for** u_i basis vectors of U **do**
5   **for** enough $x \in_{\mathrm{R}} \mathbb{F}_2^n$ **do**
6             $V \leftarrow V \cup \Delta_{u_i}(F)(x)$
7     $V \leftarrow \mathrm{Span}\,\{V\}$
8   **return** $U \rightrightarrows^F \texttt{ComputeTrail}(F, V)$

Remaining Problem: cyclic STs

Correctness: previous two lemmata

Runtime:

- Line 4: max. n iterations
- Line 5: $n + c$ random vectors are enough
- Recursions: can stop after $r < n$ rounds
- Overall: $\mathcal{O}(n^3)$ evaluations of F

How to Bound the Length of Subspace Trails

Goal

Give an upper bound on the length of any subspace trail.

Naïve Approach I

$\forall U \subseteq \mathbb{F}_2^m$ run $\texttt{ComputeTrail}(F, U)$

Naïve Approach II

$\forall u \subseteq \mathbb{F}_2^m \setminus \{0\}$ run $\texttt{ComputeTrail}(F, \mathrm{Span}\,\{u\})$

Problem

Exponentially many starting subspaces.

Problem

Still $2^m - 1$ starting subspaces.

Often used heuristic

Activate single S-boxes only. That is, for a round function with k S-boxes which are n-bit wide, choose $U = \{0\}^i \times V \times \{0\}^{k-i-1}$, where $V \subseteq \mathbb{F}_2^n$.

Activating a single S-box only

Problem

Heuristic not valid in general when we want to prove a bound on the subspace trail length. In particular one can construct examples where the best subspace trail does activate more than one S-box in the beginning.

The good case

However, we will see next a sufficient condition for the case when the heuristic is valid.

The Connection to Linear Structures

Let us observe how a single S-box S behaves regarding subspace trails:

Given a subpsace trail $U \overset{S}{\rightrightarrows} V$, this implies

$$\Delta_u(S)(x) \in V \qquad \text{for all } x \in \mathbb{F}_2^n \text{ and } u \in U.$$

By definition of the dual space $V^\perp$:

$$\langle \alpha, \Delta_u(S)(x) \rangle = 0 \qquad \text{for all } \alpha \in V^\perp,$$

which are exactly the *linear structures* of S:

$$\mathrm{LS}(S) := \{(\alpha, u) \mid \langle \alpha, \Delta_u(S)(x) \rangle \text{ is constant for all } x\}$$

S-boxes without Linear Structures

This observation implies that S-boxes without linear structures (e. g. the AES S-box) exhibit only two important subspace trails:

$$\{0\} \rightrightarrows \{0\} \qquad \text{and} \qquad \mathbb{F}_2^n \rightrightarrows \mathbb{F}_2^n$$

We can further show that subspace trails over an S-box layer without linear structures are direct products of the above two subspace trails.

Theorem

Let F be an S-box layer of k parallel S-boxes $S : \mathbb{F}_2^n \to \mathbb{F}_2^n$. If S has no non-trivial linear structures, then for every subspace trail $U \rightrightarrows^F V$:

$$U = V = U_1 \times \cdots \times U_k \, ,$$

with $U_i \in \left\{\{0\}, \mathbb{F}_2^n\right\}$.

Proof

For all $\alpha = (\alpha_1, \ldots, \alpha_k) \in V^\perp$: $\langle \alpha, \Delta_u(F)(x) \rangle = \left\langle \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_k \end{pmatrix}, \begin{pmatrix} \Delta_{u_1}(S)(x_1) \\ \vdots \\ \Delta_{u_k}(S)(x_k) \end{pmatrix} \right\rangle = \sum_{i=1}^{k} \left\langle \alpha_i, \Delta_{u_i}(S)(x_i) \right\rangle = 0$

S-boxes without Linear Structures

Resistance of SPN against Subspace Trails, without linear structures

The length ℓ of any subspace trail is upper bounded by

$$\ell \leqslant \max_{U \in \left\{\{0\}, \mathbb{F}_2^n\right\}^k} \left|\mathtt{ComputeTrail}(F, U)\right|,$$

which needs 2^k evaluations of the `ComputeTrail` algorithm.

S-boxes with Linear Structures

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Compared to the no-linear-structures-case, $V^\perp$ can now contain much more elements, namely all combinations of linear structures, such that their corresponding constants sum to zero.

Instead, we can show that (for any not-trivially-insecure S-box) the subspace after the first S-box layer contains at least one element of a specific structure:

$$W_{i,\alpha} = \{0\}^{i-1} \times \{0, \alpha\} \times \{0\}^{k-i} \; .$$

Resistance of SPN against Subspace Trails, with linear structures

The length ℓ of any subspace trail is upper bounded by

$$\ell \leqslant \max_{W_{i,\alpha}} \left| \texttt{ComputeTrail}(F', W_{i,\alpha}) \right| + 1 \; ,$$

which needs $k \cdot 2^n$ evaluations of the `ComputeTrail` algorithm.

Note: F' first applies the linear layer, then the S-box layer (b/c of the skipped first S-box layer).

Conclusion

Thanks for your attention!

Applications of `ComputeTrail`

- Bound longest probability-one subspace trail
- Link to Truncated Differentials
- Finding key-recovery strategies