

Attacks on Lattice Crypto

December 7th, 2016

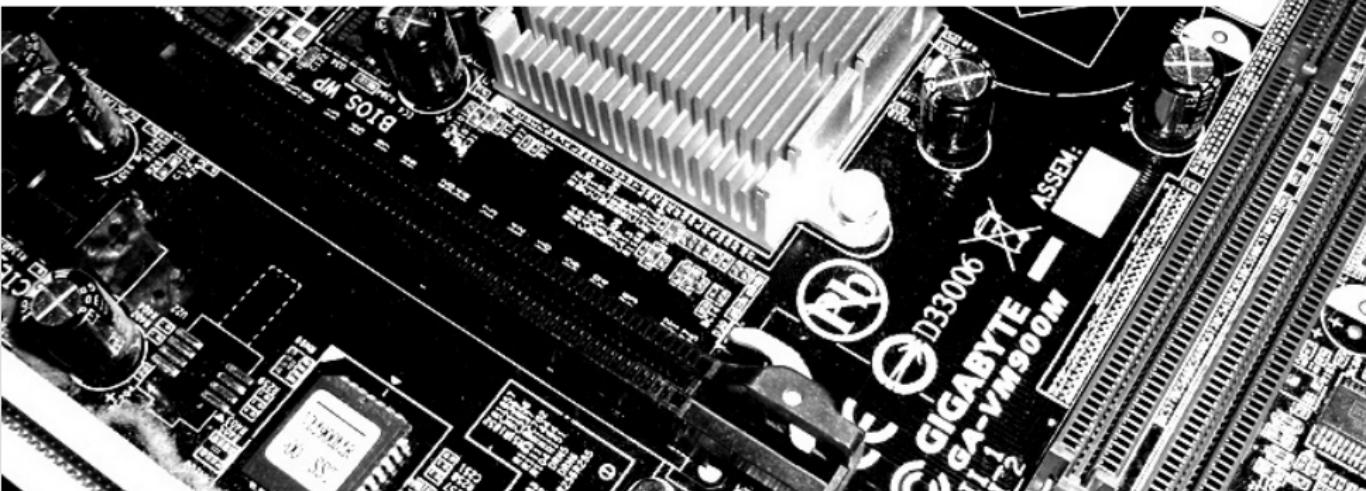
RUB

FluxFingers

Workgroup Symmetric Cryptography

Ruhr University Bochum

Friedrich Wiemer



Why is Lattice Based Crypto important?

Or interesting? Or...? Buzzword Bingo.

Some facts

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- You can build anything you want from it:
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- It allows to build even some of the most advanced cryptographic building blocks:
 - Fully Homomorphic Encryption (FHE), (The Holy Grail!)
 - Multi-linear Maps,
 - Identity-based Encryption (IBE),
 - ...

Why is Lattice Based Crypto important?

Is everything done?

Fully Homomorphic Encryption



GF(＼_(＼)＼_)

@hdevalence

Follow

Kirchner/Fouque: our attack lets us do FHE faster by just breaking the crypto & decrypting
eprint.iacr.org/2016/717.pdf

The parameters proposed for schemes using similar overstretched NTRU assumption, such as in homomorphic encryption [8, 31, 17, 18, 16, 12, 32, 20] or in private information retrieval [19], are also broken in practical time using LLL. For example, we recovered a decryption key of the FHE described in [17] in only 10 hours. For comparison, they evaluated AES in 29 h: that means that we can more efficiently than the FHE evalution, recover the secret, perform the AES evaluation, and then re-encrypt the result! A decryption key was recovered for [20] in 4 h. Other instanciations such as [11, 29] are harder, but within range of practical cryptanalysis, using BKZ with moderate block-size [13].

RETWEETS

33

LIKES

34



5:37 AM - 23 Jul 2016

The new cool kid in town.

What is this Hype?

- “Lattice based Crypto is one of the most promising PQC candidates blablabla” (almost every paper on lattices)

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- Google even implemented this in Chrome [Goob]
- So, research is really vibrant here

Everything was fine. And then Shor entered the stage...

A cryptographic thriller



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- ... and published an efficient CVP quantum algorithm [ES16]
- for one day the cryptographic community was shocked!



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- ... and published an efficient CVP quantum algorithm [ES16]
- for one day the cryptographic community was shocked!
- ... and then Regev saved us all by finding a flaw in the paper [Reg]
- but still, Google stopped its PQ key exchange experiment with New Hope [Gooa]



Enough motivation!

How does Lattice Crypto work?

How does Lattice Based Crypto work?

Wait! Lattice, wtf?

Definition:

A lattice L is an discrete, additive, abelian subgroup of \mathbb{R}^n .

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Definition:

Let $b_1, b_2, \dots, b_d \in \mathbb{R}^n$, $d \leq n$ linear independent. Then the set

$$L = \left\{ v \in \mathbb{R}^n \mid v = \sum_{i=1}^d a_i b_i, a_i \in \mathbb{Z} \right\}$$

is a lattice.

Hey! You promised, this will be easy!

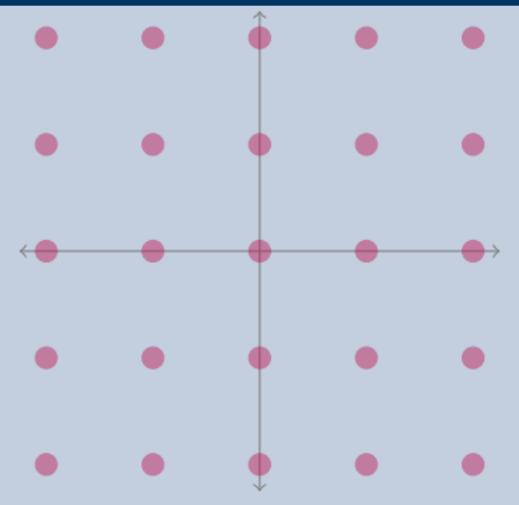
Lattice, dt.: Gitter



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OK, OK, we can say it easier: \mathbb{Z}^2 is a Lattice

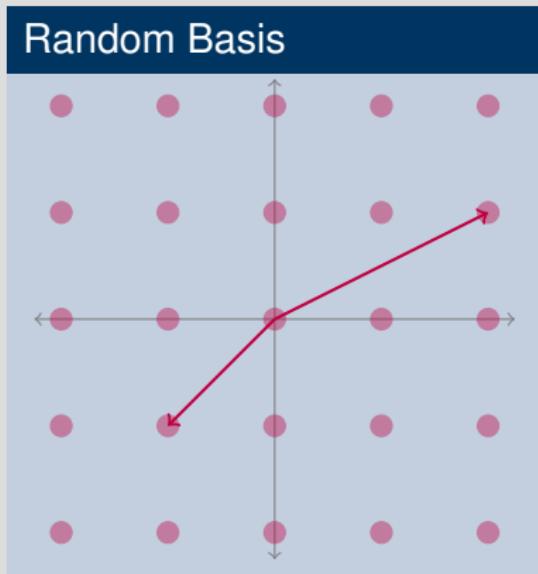
Example lattice



In general, basis reduction is a hard problem! The LLL and BKZ algorithm are available for this. NTL's implementation of BKZ has 2^{n^2} runtime.

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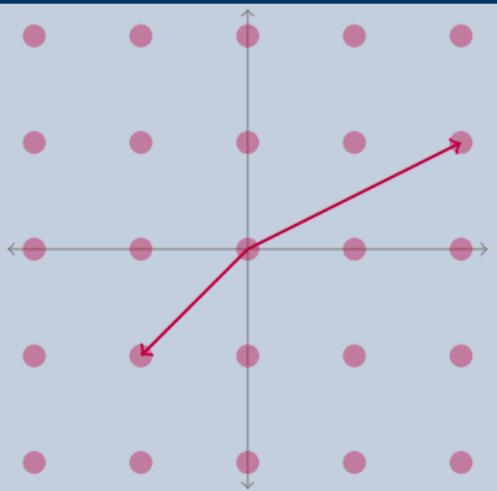


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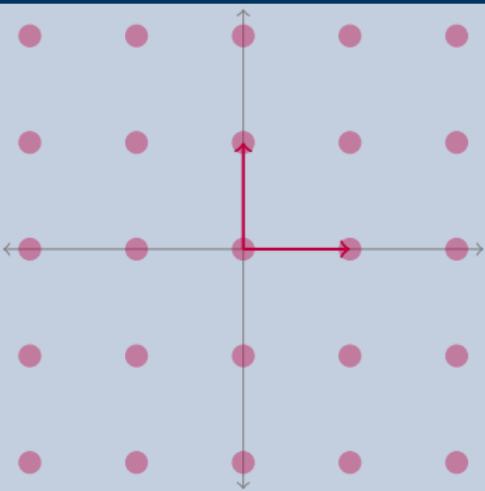
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Random Basis



Reduced Basis



In general, basis reduction is a hard problem! The LLL and BKZ algorithm are available for this. NTL's implementation of BKZ has 2^{n^2} runtime.

Hard Problems in Lattices...

...are what we need for crypto.

Shortest Vector Problem (SVP)

Given a lattice L , what is the shortest vector $v \in L \setminus \{0\}$?

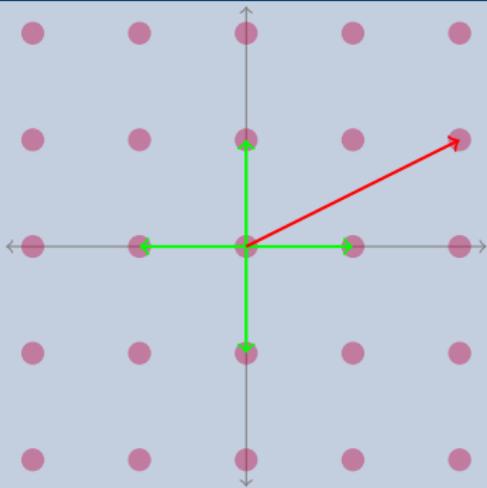
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Given a lattice L and a target $t \notin L$,
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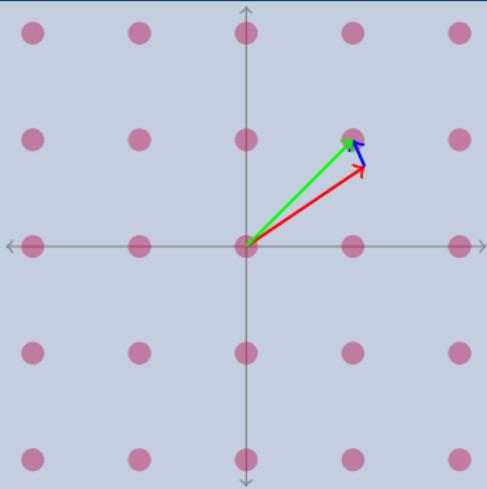
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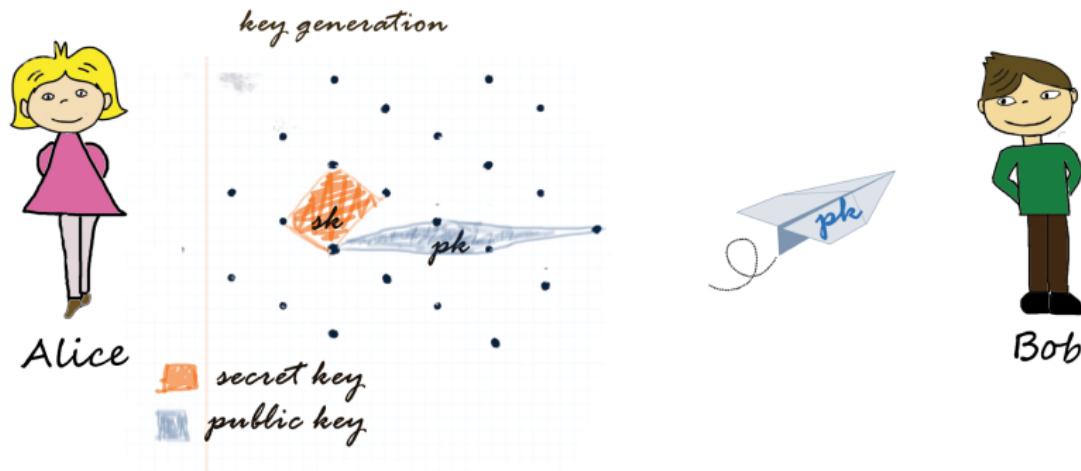
Example



Lattice Based Crypto

Learning With Errors – or: the equivalent to textbook RSA

Key Generation¹

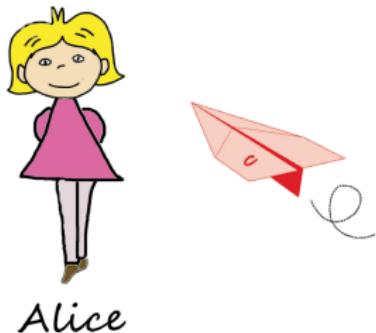


¹Thanks to Elena for the nice pictures.

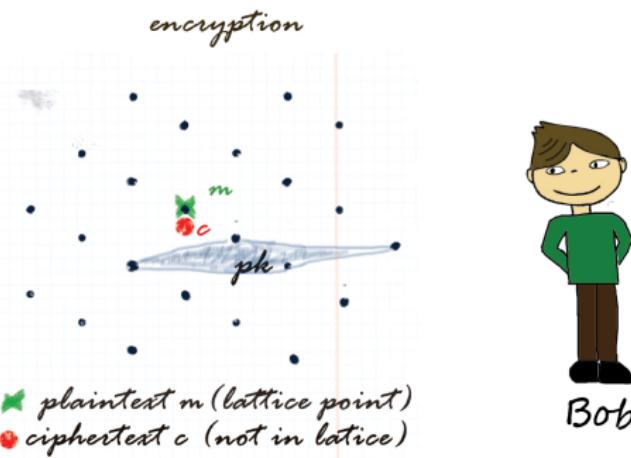
Lattice Based Crypto

Learning With Errors – or: the equivalent to textbook RSA

Encryption



Alice

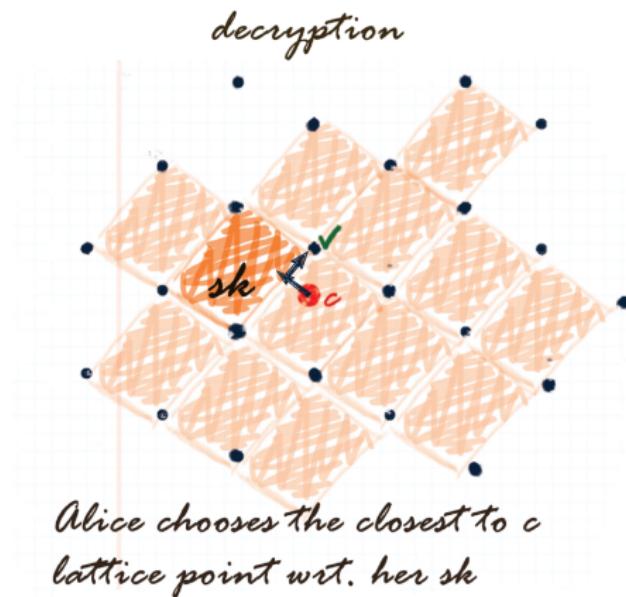


Bob

Decryption



Alice

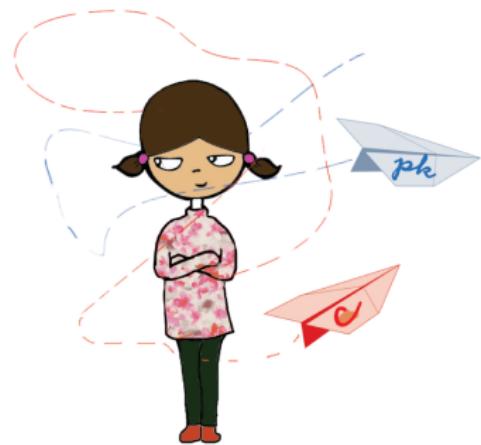


Attack Algorithm

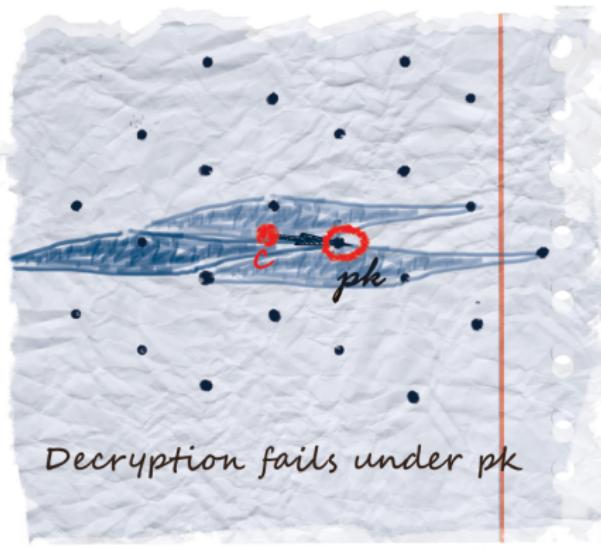
In practice most efficient strategy is Babai's Nearest Plane [Bab86], improved by Lindner and Peikert [LP11] and Gama *et al.* [GNR10].

Nearest Plane or BDD Enumeration

Attack



Eve

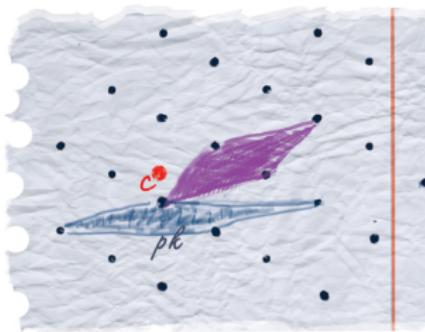


Step 1: Basis Reduction



Eve

step1: Find an approximation to sk

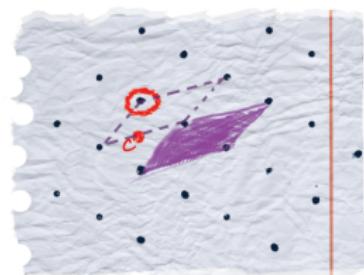
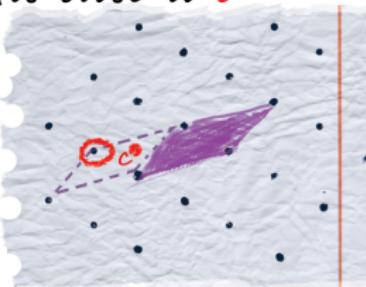
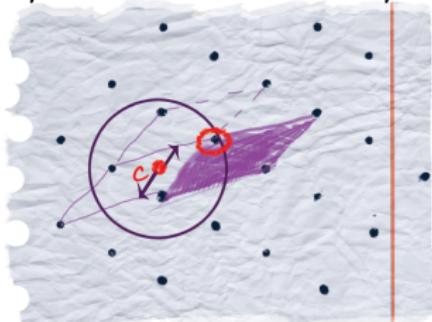


Nearest Plane

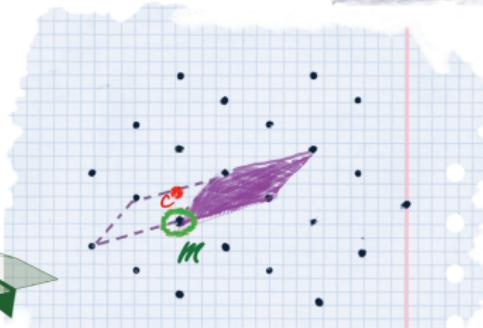
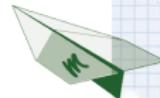
or BDD Enumeration

Step 2: Enumerate Nearest Planes

step 2: Enumerate all points close to c



Eve



Parallel Implementation of BDD enumeration for LWE

Finally, what we (joint work with Elena Kirshanova and Alex May) did:

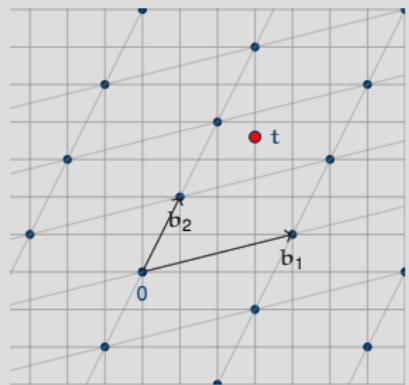
Research Project

- Goal: What is the *practical* runtime of BDD enumeration?
- Build a parallel implementation of NearestPlanes.
- Test this on some large scale parallel system.
- Hopefully break some real world parameters.

Parallelisation of Enumeration

Elena's explanation

Closest point search via depth-first tree-traversal:

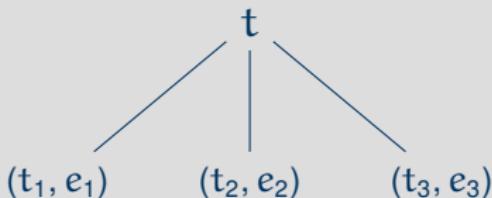
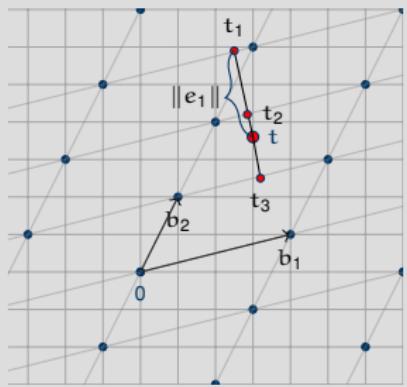


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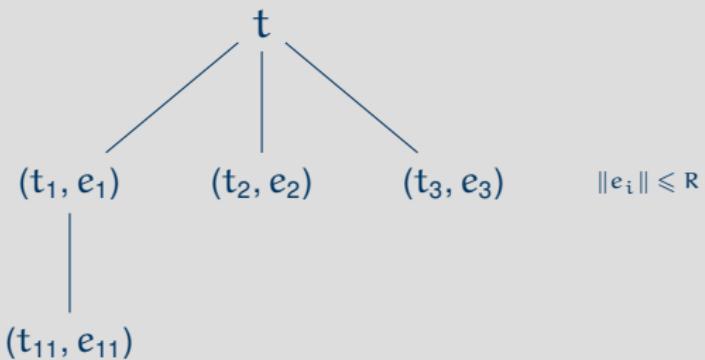
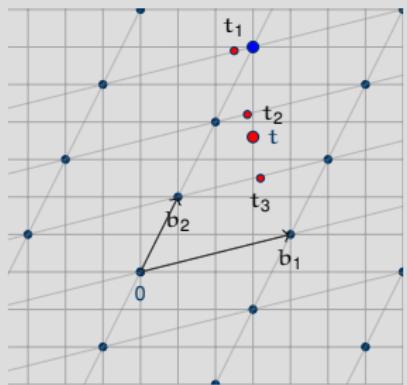
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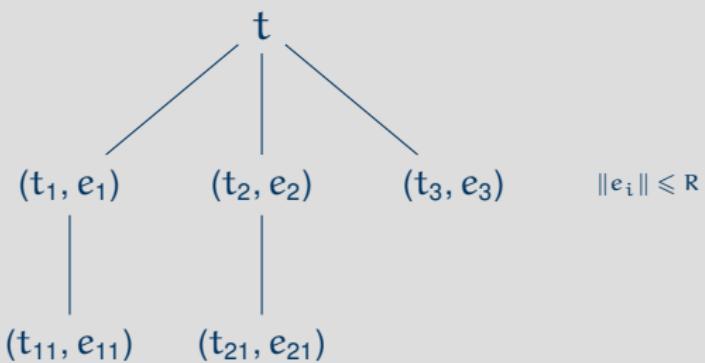
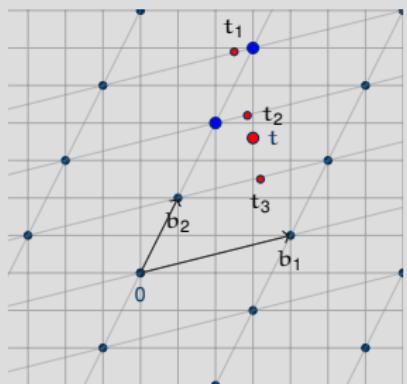
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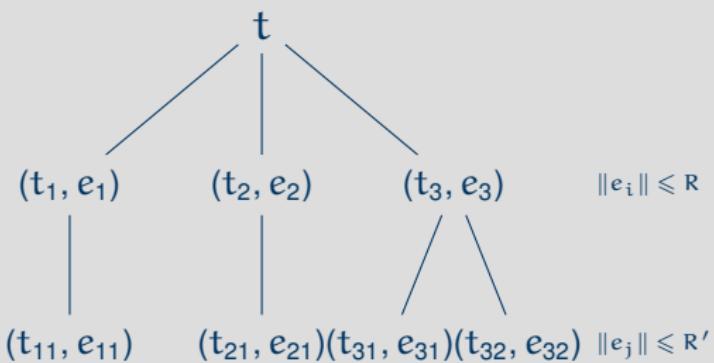
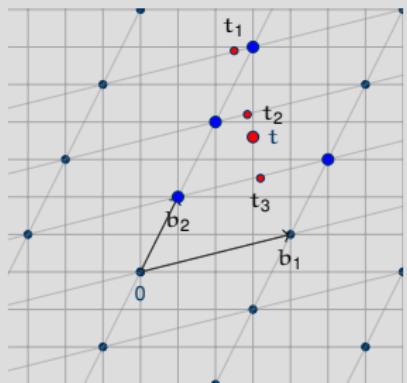
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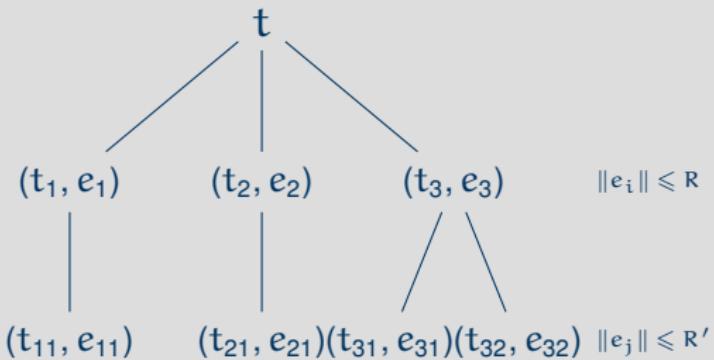
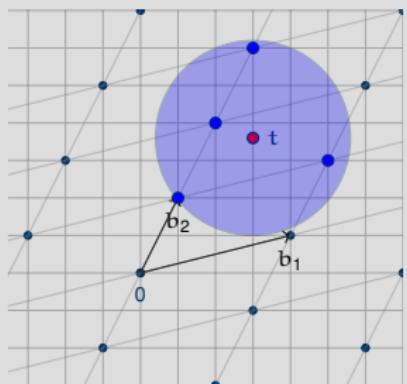
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Leaves to visit = $2^{n \log n}$ for n -dim BDD

Results

After more than one year of work, two submissions and something like over 9000 weeks of benchmarking

We ended up with:

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After more than one year of work, two submissions and something like over 9000 weeks of benchmarking

We ended up with:

- an open source implementation:
<https://github.com/pfasante/cvp-enum>
- an ACNS paper [KMW16] and a Best Student Paper Award ☺
- huge table of runtimes

Results: Numbers!

Standard LWE

LWE-parameters			BKZ-reduction	Enumeration	
n	q	$ e \leqslant$	T	# Threads	T
90	4093	10	11.3h	1	35h
90	4093	10	11.3h	10	3.6h
100	4093	10	7h	24	2.7h

To be compared with: ($n = 192$, $|e| < 18$, $q = 4093$) reaches 2^{87} -security level [LP11].

Results: Numbers!

LWE variant: Small secret

LWE-parameters			BKZ-reduction	Enumeration	
n	q	m	T	# Threads	T
140	16411	170	12h	1	16h
140	16411	170	12h	10	1.7h

To be compared with: ($n = 128$, $q = 16411$, $m = 2^{28}$, $T = 13h$) for combinatorial attack on LWE [KF15].

Results: Numbers!

LWE variant: Binary matrix

LWE-parameters			BKZ-reduction	Enumeration
n	q	m	T	T
256	500009	440	4.5h	2min

To be compared with: Estimation by Galbraith [Gal] roughly one day.

Questions?

Thank you for your attention!

Review

- Working as an engineer together with mathematicians can be fun
You can code, they... can do math
😊
- Even if you don't understand what you are implementing, you can get something working out of it
- Eventually you'll understand the math 🎓



Mainboard & Questionmark Images: flickr

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