1 High Energy Nuclear Physics

> *Three quarks for Muster Mark!*
> *Sure he has not got much of a bark*
> *And sure any he has it's all beside the mark.*
>
> — **James Joyce**
> (Finnegans Wake)

According to cosmological theories, in its early stages the Universe was extremely hot and dense. In the first few microseconds, the energy densisty was so high that hadrons could not be formed and their foundamental constituents were in a deconfined state. When the energy density has decreased enough, a phase transition led to the formation of the ordinary matter.

High Energy Nuclear Physics (HENP) investigates the hot and dense nuclear matter and the properties of its phase ransition into ordinary matter through the study of ultra-relativistic heavy-ion collisions. The aim is to improve our understandig of the behaviour of the matter in extereme conditions and of the Universe at the beginning of its life.

1.1 QCD: the theory of the Strong Interaction

In 1964 M. Gell-Mann and G. Zweig proposed independently a model that could explain the existence of the great variety of hadrons discovered in the 1950s and 1960s [1–3]. This model, known as the Static Quark Model, was based on the assumption that hadrons are not fundamental particles, but they are composed states of elementary constituents called quarks. In this way it was possible to explain the large number of particles observed and their properties, which showed some sort of pattern, in term of constituents properties. Furthermore, thanks to the Static Quark Model, it was possible predict new hadrons (e.g. Ω^-) and to explain why certain particles don't exist (e.g. baryons with $S = +1$). However, this model could not deal with many questions: why there is no evidence of free quarks? What hold quarks together in a hadron? Why the Δ^{++} baryon exists despite is forbidden by Pauli's Principle? In order to answer these questions it was necessary to introduce a new quantum number the colour [4]. The introduction of the colour led to the formulation of a quantum field theory for the Strong Interaction, inspired by the Quantum Electrodynamics (QED), the Quantum Chromodynamics (QCD).

The QCD is a non-Abelian quantum gauge theory, based on the invariance under local $\mathrm{SU}(3)_c$ group transformations. The choice of this particular symmetry group is due to the hypothesis that the colour comes in three different states: red, blue and green. The local invariance under $\mathrm{SU}(3)_c$ implies the existence of 8 massless gauge bosons mediators of the colour interaction, called gluons [5]. Therefore the QCD Lagrangian can be written as:

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i(i(\gamma^\mu D_\mu)_{ij} - m\delta_{ij})\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a \tag{1.1}$$

where the first term is related to quarks fields while the second is releted to gluons fields. In the first term $\psi_i(x)$ represents the quarks fields expressed in the fundamental representation of $\mathrm{SU}(3)_c$, while D_μ is the covariant derivative, defined as:

$$D_\mu = \partial_\mu - ig_s A^a_\mu \lambda_a. \tag{1.2}$$

In this derivarives shows up the coupling constant for the Strong Interaction g_s, the Gell-Mann matrices λ_α which gives a representation of the generators of $\mathrm{SU}(3)_c$ symmetry group, and the gluon field $A(x)$. This first term of the QCD Lagrangian represents the quarks-gluons interaction via a QED-like vertex.

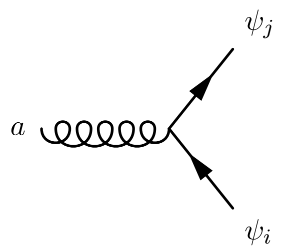

Fig. 1.1: Feynman diagram for the gluon-quarks interaction.

The second term of the Lagrangian $G^a_{\mu\nu}$ represents the gauge invariant gluon field strength tensor, and can be written as:

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu. \tag{1.3}$$

In this tensor $g_s f^{abc} A^b_\mu A^c_\nu$ is the non-Abelian part of the theory, which implies the self-interactions among gluons. This interactions have resulted from the fact that gluons carry a colour and an anti-colour charge, so they can interact among themselves. Therefore in addition to the QED-like vertex, in the QCD, 3 gluons and 4 gluons vertex are allowed at the tree level. The existence of the gluons vertex allows to have gluons loops.

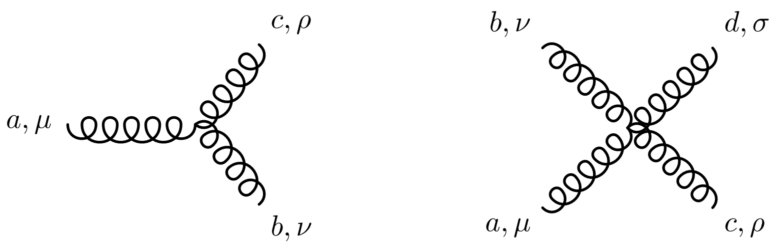

Fig. 1.2: Feynman diagrams for the gluon-gluon interactions at the tree level.

In the renormalization process of the theory, the non-Abelian nature of the QCD brings to the so-called *anti-screening* in colour interaction. Adding loop corrections to the gluons propagator, gluons loops contribute to the sum with opposite sign respect to the quarks loops. Therefore, in addition to the QED-like *screening* effect, there is an *anti-screening* effect due to gluons loops. As a result, the QCD shows up its specific features, *asymptotic freedom* and *confinement*.

Setting $\alpha_s = g_s^2/4\pi$ strong coupling constant can be written as [6]:

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)(33 - 2n_f)\ln(Q^2/\mu^2)} \tag{1.4}$$

where n_f is the number of quark families and μ is the renormalization scale of the theory. For high transferred momenta α_s goes to zero and the QCD becomes a free theory and this regime is called *asymptotic freedom*. At low Q^2 the Strong copuling diverges, forcing quarks to be strongly bound in hadrons: the so-called *confinement* regime. This behaviour of the QCD copuling constant has been confirmed by experimental results over the years as shown in Figure 1.3. The equation 1.4 can be rewritten fixing the energy scale:

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\ln(Q^2/\Lambda_{\text{QCD}})} \tag{1.5}$$

where Λ_{QCD} is the renormalization scale of QCD (typically ≈ 200 MeV).

In QCD the perturbative approach used to calculate the elements of the scattering matrix (pQCD), is possible only for high Q^2 processes ($Q^2 \gg \mu^2$, thus $\alpha_s \ll 1$). As already mentioned, in low transferred momentum processes α_s diverges. Therefore, is impossible to express the elements of the scattering matrix in terms of power series expansion of the Strong copuling constant. In this regime it is still possible to evaluate the Green's functions of the QCD Lagrangian on a space-time lattice with spacing a, as proposed in 1974 by Wilson [7]. With this method, called lattice QCD

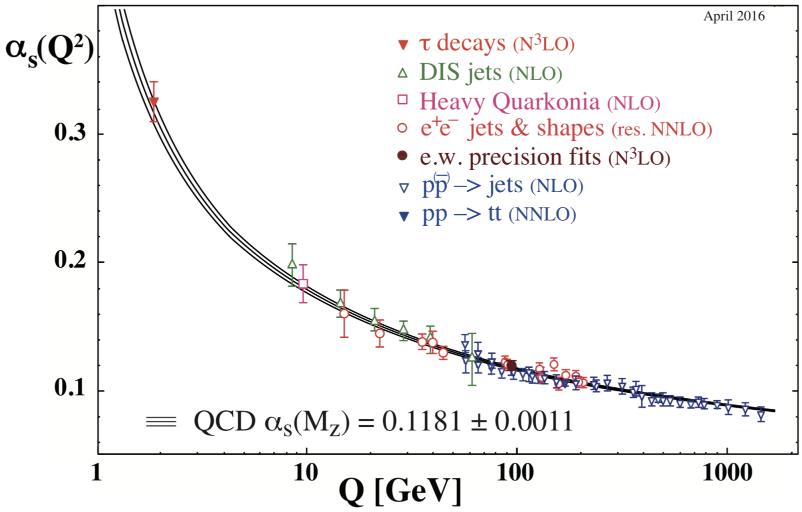

Fig. 1.3: Summary of measurements of α_s as a function of the energy scale Q [6].

(LQCD) is possible to extrapolate to the continuum ($a \to 0$) and get results to be compared with the experiments.

1.2 States of hadronic matter

One of the interesting consequences of the running copuling constant is the possibility of having different states of the hadronic matter. The state is essentially determined by the mean transferred momentum in the interactions which define the value of α_s. A system with low mean transferred momentum it is in the *confinement* regime, therefore quarks and gluons are required to be confined in hadrons. Otherwise in high mean transferred momentum systems, the *asymptotic freedom* regime allows the formation of a plasma where quarks and gluons are essentially free. This state of matter is called Quark Gluon Plasma (QGP) and is supposed that the universe was in this state in the first microseconds after the Big Bang. One of the main goals of the HENP is the study of the phase transitions between the different states of the hadronic matter.

Considering a system with finite dimensions, composed by hadronic matter, can be useful to describe it using thermodinamical variables like temperature (T) and chemical potential (μ). In this specific framework the chemical potential is interpreted as the energy required to create a baryonic state and it is called baryon chemical potential (μ_B). Figure 1.4 shows the phase diagram of the QCD matter predicted by the theory and the values of temperature T and baryon chemical potential (μ_B) which are accessible experimentally in high energy heavy ion collisions.

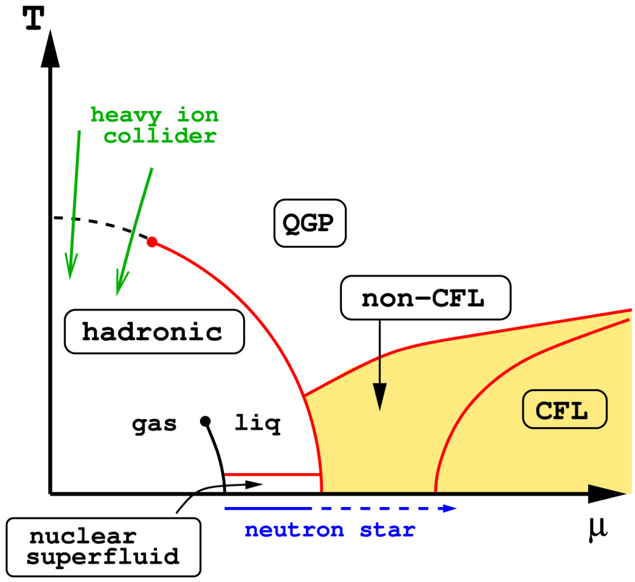

Fig. 1.4: Schematic representation of the nuclear matter phase diagram from [8]. QGP refers to the Quark Gluon Plasma state, CFL (Colour-Flavour Locked) corresponds to the colour superconducting phase that is present in systems with high baryon chemical potential. The green arrows represent the phase space investigated by collider experiments at the Relativistic Heavy Ion Collider (RHIC) and at the LHC.

The origin of the diagram (T$= \mu_B = 0$ GeV) corresponds to the QCD vacuum. At T$= 0$ GeV, μ_B is the energy required to create a baryonic state, therefore ordinary matter (proton, neutrons and nuclei) sits around 1 GeV on the μ_B axis. Along the μ_B axis lies a phase transition to a state, the Color Superconducting Phase, that has been hypothesised to be present in matter at high density, e.g. in the core of neutron stars [9]. Along the T axis, where $\mu_B = 0$, there is a phase transition when T $\gg \Lambda_{\mathrm{QCD}}$. At this temperatures the average momentum exchange between quarks and gluons is so high that they reach the *asymptotic freedom*, hence they are no longer confined in colour singlets states. In these conditions they constitute a plasma of free coloured partons, similar to the primordial universe: the aforementioned QGP.

The order of the phase transition is determined by the behaviour of the derivatives of the free energy of the system with respect to time. It basically describes how fast the free energy varies in a neighbourhood of the transition temperature. A first order transitions takes place when a latent heat is present, leading to a discontinuous free energy first derivatives and variation of entropy. If no latent heat is involved in the process, occours a second order transition. The free energy first derivative is continuous, while derivatives of higher than first order of the free energy are discontinuous. When the transition occours with a continuous behaviour of the free energy and its derivatives, it is called a *crossover* transition. In $\mu_B = 0$ conditions, the transition from hadron gas to the QGP takes place when T ≈ 150 MeV with a *crossover* transition.

1.3 Heavy Ion Collisions

The QCD phase diagram is derived by theories and models, but their predictions are difficult to test. For the T ≈ 0 GeV and high μ_B region, important suggestions can come from astronomic observations of neutron stars. For high T regions, instead, the only known way to cross the phase boundary between ordinary hadronic matter and QGP is by colliding ultrarelativistic heavy ions in the laboratory.

The journey of the High Energy Nuclear Physics started in the '70 at the Lawrence Berkeley National Laboratory where the first experiments on heavy ions collisions (HIC) was performed at modestly relativistic conditions (≈ 2 GeV/nucleon). In 1986, two HIC experiments started simultaneously at the Super Proton Syncrotron (SPS) at CERN and at the Alternate Gradient Syncrotron (AGS) at Brookhaven, colliding O ions at fixed target at higher energies. Nowdays the two main hadron colliders active with an HIC program and dedicated experiments are the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory (BNL) and the Large Hadron Collider (LHC) at CERN.

1.3.1 The "Little Bang" at the LHC

Atomic nuclei are composite systems of nucleons with finite dimensions. When they collide at ultrarelativistic energies the problem of the description of the collision, that can be very complex, arises. The Glauber Model [10] is a semi-classical model describing nucleus–nucleus interaction in terms of nucleon–nucleon (NN) interactions. The Glauber Model is based on the assumption of the *optical limit*:

- nucleons are point like and independent inside the nuclei;
- only hadronic interactions are considered: protons and neutrons cannot be distinguished;
- the collision does not deflect colliding nucleons: they travel in a straight line;
- the cross section for an elementary nucleon-nucleon interaction is constant during the whole process.

With this assumption the Glauber Model allows a quantitative calculation of the interaction probability, the number of elementary NN collisions (N_{coll}), the number of the partecipants nucleons (N_{part}) and extension of the overlap region. These quantities are expressed in terms of the impact parameter $\vec{b}$, which characterizes the collisions geometry. A direct experimental measurement of the impact parameter is precluded and the same goes for N_{coll} and N_{part}. However, the Glauber Model enables to correlate this variables with measurable quantities such as the total number of particles produced in the collision.

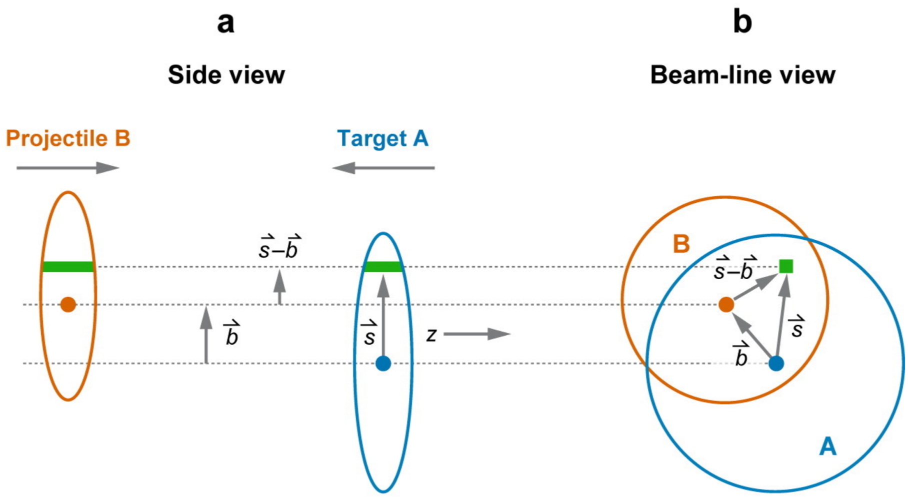

Fig. 1.5: Sketch of the longitudinal and transverse view of an heavy ion collision taken from [10]. In the side view, the colliding nuclei are squeezed to represent the Lorentz boost contraction due to their momentum.

Following the notation introduced in Figure 1.5 the nuclear overlap function for two colliding nuclei (A and B) can be written as:

$$T_{AB}(\vec{b}) = \int T_A(\vec{s})\, T_B(\vec{s}-\vec{b})\, d^2 s \tag{1.6}$$

and represents the probability of finding a nucleon in both the colliding nuclei inside the overlap region in the transverse plane. $T_A(\vec{s})$ and $T_B(\vec{s}-\vec{b})$ are the *thickness functions* the A and B nuclei. They represent the probability of finding a nucleon in the unit transverse area located at $\vec{s}$ given the probability per unit volume $\rho(\vec{s}, z)$:

$$T(\vec{s}) = \int \rho(\vec{s}, z)\, dz. \tag{1.7}$$

Bibliography

1. Gell-Mann, M. A schematic model of baryons and mesons. *Physics Letters* **8,** 214–215 (1964) (cit. on p. 1).

2. Zweig, G. An SU(3) model for strong interaction symmetry and its breaking. *Developments In The Quark Theory Of Hadrons* **1** (1964) (cit. on p. 1).

3. Zweig, G. An SU(3) model for strong interaction symmetry and its breaking II. *Developments In The Quark Theory Of Hadrons* **1** (1964) (cit. on p. 1).

4. Fritzsch, H. & Gell-Mann, M. Current Algebra: Quarks and What Else? *Proceedings of the XVI International Conference on High Energy Physics* **2,** 135 (1972) (cit. on p. 2).

5. Fritzsch, H. & ans Heinrich Leutwyler, M. G.-M. Advantages of the color octet gluon picture. *Physics Letters B* **47,** 365–368 (1973) (cit. on p. 2).

6. Tanabashi, M., Hagiwara, K., Hikasa, K., *et al.* Review of Particle Physics. *Physical Review D* **98,** 030001 (3 2018) (cit. on pp. 3, 4).

7. Wilson, K. G. Confinement of quarks. *Physical Review D* **10,** 2445–2459 (8 1974) (cit. on p. 3).

8. Braun-Munzinger, P., Koch, V., Schäfer, T. & Stachel, J. Properties of hot and dense matter from relativistic heavy ion collisions. *Physics Reports* **621.** Memorial Volume in Honor of Gerald E. Brown, 76–126 (2016) (cit. on p. 5).

9. Alford, M. G., Schmitt, A., Rajagopal, K. & Schäfer, T. Color superconductivity in dense quark matter. *Reviews of Modern Physics* **80,** 1455–1515 (4 2008) (cit. on p. 5).

10. Miller, M. L., Reygers, K., Sanders, S. J. & Steinberg, P. Glauber modeling in high energy nuclear collisions. *Ann. Rev. Nucl. Part. Sci.* **57,** 205–243 (2007) (cit. on pp. 6, 7).