# A Vignette for the R Package panelTVP

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## Abstract

In this vignette, the R package panelTVP is introduced. The package allows for the estimation of flexible Bayesian regression models for panel data. The model is flexible in the sense that both regression effects and random intercepts are allowed to vary over time. The use of hierarchical shrinkage priors prevents overfitting and makes it possible to identify whether an effect is time-varying, time-invariant or zero. The response variable can either be Gaussian, binary or a (zero-inflated) count.

Keywords: Bayesian statistics, Panel data, Time-varying parameter models, Shrinkage priors, Pólya-Gamma data augmentation

#### 1. Introduction

Panel data are commonly encountered in econometrics and the social sciences. When analyzing data from longitudinal studies, standard estimation techniques such as OLS (*ordinary least squares*) are not appropriate as the repeated measurements of a subject are typically correlated. Therefore, models with random effects are used to account for the clustered data structure.

panelTVP makes it possible to estimate regression models with random effects and additionally allows the parameters to vary over time. To avoid overfitting, hierarchical shrinkage priors are considered. The method implemented in panelTVP was inspired by Bitto & Frühwirth-Schnatter (2019) who discussed the use of shrinkage priors in time-varying parameter models for the analysis of time series data. Their method has later been implemented in the R package shrinkTVP (Knaus, Bitto-Nemling, Cadonna, Frühwirth-Schnatter, 2021), which was partly used for creating the algorithm in panelTVP.

The package supports Gaussian response data as well as binary (Probit and Logit link) and count outcomes. For analyzing count data, either a standard Negative Binomial or a Zero-Inflated Negative Binomial (ZINB) model can be fitted. Compared to a Poisson model, the use of the Negative Binomial distribution has the advantage that overdispersion can be handled more naturally through the overdispersion parameter of the Negative Binomial distribution.

This vignette is structured as follows: Section 2 introduces the reader to the methodology. This includes the model, the prior distributions and basic information of the Markov Chain Monte Carlo (MCMC) sampler that is used in panelTVP. Details of the sampler are not discussed in this vignette. Section 3 provides a small case study for Gaussian response data, whereas Section 4 and 5 contain case studies for binary and count data, respectively. Section 6 presents a small simulation study, which illustrates how to simulate data using a function of the package. Moreover, the estimated effects are compared for two different prior specifications to demonstrate the usefulness of the package. Finally, Section 7 concludes this vignette and provides a brief summary of the content in the package.

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#### 2. Methodology

## 2.1. Shrinkage in time-varying parameter panel models

We consider a balanced panel  $y_{it}$  with t = 1, ..., T observations for i = 1, ..., n subjects, where  $y_{it}$  is either a Gaussian, binary or count response. The linear predictor of the model is given as

$$\eta_{it} = \mathbf{x}_{it}^{\mathsf{T}} \boldsymbol{\beta}_t + f_i \lambda_t, \tag{1}$$

where  $\mathbf{x}_{it}$  is a  $d \times 1$  vector of covariate values (including a 1 for the intercept as the first value),  $\boldsymbol{\beta}_t$  is a  $d \times 1$  vector of regression effects at time t,  $\lambda_t$  is a time-specific factor loading and  $f_i$  is a subject-specific factor score. The factor model with time-specific factor loadings in  $\eta_{it}$  can be interpreted as a model with a time-varying random intercept. For reasons discussed below, we refer to Equation (1) as the linear predictor of the model in centered representation.

The goal is now to identify whether an effect is time-varying, time-invariant or zero. For this, a random walk specification of the effects is considered, i.e.,

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\nu}_t^{\beta}, \qquad \boldsymbol{\nu}_t^{\beta} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}), \quad t = \{2, \dots, T\},$$
$$\lambda_t = \lambda_{t-1} + \boldsymbol{\nu}_t^{\lambda}, \qquad \boldsymbol{\nu}_t^{\lambda} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\psi}^2), \quad t = \{2, \dots, T\},$$

where  $\mathbf{Q} = \operatorname{diag}(\theta_1^2, \dots, \theta_d^2)$  and we assume that the process starts at T = 1 with starting distributions

$$\boldsymbol{\beta}_1 \sim \mathcal{N}(\boldsymbol{\beta}, c^{\beta} \mathbf{Q}),$$
  
 $\lambda_1 \sim \mathcal{N}(\lambda, c^{\lambda} \psi^2).$ 

Alternatively, it is also possible to let the processes start at T = 0. Simulations have shown that results are very similar when starting at T = 0 or T = 1.

In order to place prior distributions on all model parameters, we exploit the non-centered parameterization that was used for state space models (Frühwirth-Schnatter and Wagner, 2010), where the time-varying parameters are expressed as

$$\beta_t = \beta + \Theta \tilde{\beta}_t,$$
$$\lambda_t = \lambda + \psi \tilde{\lambda}_t,$$

where  $\Theta = \operatorname{diag}(\theta_1, \dots, \theta_d)$ .

In the non-centered representation the random processes are defined as

$$\tilde{\boldsymbol{\beta}}_{t} = \tilde{\boldsymbol{\beta}}_{t-1} + \tilde{\boldsymbol{\nu}}_{t}^{\beta}, \quad \tilde{\boldsymbol{\nu}}_{t}^{\beta} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$
$$\tilde{\lambda}_{t} = \tilde{\lambda}_{t-1} + \tilde{\boldsymbol{\nu}}_{t}^{\lambda}, \quad \tilde{\boldsymbol{\nu}}_{t}^{\lambda} \sim \mathcal{N}(0, 1)$$

with starting distributions  $\tilde{\boldsymbol{\beta}}_0 \sim \mathcal{N}(\mathbf{0}, c^{\beta}\mathbf{I})$  and  $\tilde{\lambda}_0 \sim \mathcal{N}(0, c^{\lambda})$  or  $\tilde{\boldsymbol{\beta}}_1 \sim \mathcal{N}(\mathbf{0}, c^{\beta}\mathbf{I})$  and  $\tilde{\lambda}_1 \sim \mathcal{N}(0, c^{\lambda})$  depending on the random walk specification. The linear predictor in non-centered representation is then given by

$$\eta_{it} = \mathbf{x}_{it}^{\mathsf{T}} \boldsymbol{\beta} + \mathbf{x}_{it}^{\mathsf{T}} \boldsymbol{\Theta} \tilde{\boldsymbol{\beta}}_t + f_i \lambda + f_i \psi \tilde{\lambda}_t. \tag{2}$$

A major advantage when using the non-centered model representation is that the parameters of interest  $\beta$ ,  $\Theta$ ,  $\lambda$  and  $\psi$  are all regression coefficients and, thus, well-known prior distributions from regression analysis can be placed on those coefficients. As in Bitto and Frühwirth-Schnatter (2019) and Pfeiler and Wagner (2024), we consider Normal-Gamma shrinkage priors

$$\theta_{j}|\xi_{j}^{2} \sim \mathcal{N}(0,\xi_{j}^{2}), \quad \xi_{j}^{2}|a^{\xi}, \kappa^{\xi} \sim \mathcal{G}\left(a^{\xi}, \frac{a^{\xi}\kappa^{\xi}}{2}\right), \ j = \{1, \dots, d\},$$

$$\beta_{j}|\tau_{j}^{2} \sim \mathcal{N}(0,\tau_{j}^{2}), \quad \tau_{j}^{2}|a^{\tau}, \kappa^{\tau} \sim \mathcal{G}\left(a^{\tau}, \frac{a^{\tau}\kappa^{\tau}}{2}\right), \ j = \{1, \dots, d\},$$

$$\psi|\zeta^{2} \sim \mathcal{N}(0,\zeta^{2}), \quad \zeta^{2}|a^{\zeta}, \kappa^{\zeta} \sim \mathcal{G}\left(a^{\zeta}, \frac{a^{\zeta}\kappa^{\zeta}}{2}\right),$$

$$\lambda|\phi^{2} \sim \mathcal{N}(0,\phi^{2}), \quad \phi^{2}|a^{\phi}, \kappa^{\phi} \sim \mathcal{G}\left(a^{\phi}, \frac{a^{\phi}\kappa^{\phi}}{2}\right),$$

where the scale parameters of the stochastic processes  $\theta_1, \dots, \theta_d, \psi$  have support on  $\mathbb{R}$  and are therefore defined as the positive and negative square root of the variance parameters  $\theta_1^2, \dots, \theta_d^2, \psi^2$ .

Hyperparameters of the model are either fixed or sampled using Metropolis-Hastings updates.

#### 2.2. Pólya-Gamma data augmentation

Bayesian inference for binary and count data is challenging as the posterior is in general not available in closed form and, thus, MCMC techniques are necessary. Within the machinery of MCMC, a Metropolis-Hastings (MH) algorithm based on an IWLS (iteratively weighted least squares) proposal distribution (Gamerman, 1997) is one possibility. However, updating the parameters using MH is potentially costly due to low acceptance rates in high-dimensional parameters settings (Fahrmeir, Kneib, Lang and Marx, 2021). In their seminal paper, Albert and Chib (1993) discuss a data augmentation scheme for Probit regression based on the latent utility representation of binary choice models. For Logit models, their sampler cannot be used due to the non-Gaussianity of errors in the latent utility representation of the Logit model. However, Polson, Scott and Windle (2013) introduced an efficient data augmentation scheme for posterior inference in models with binomial likelihoods. Their method can be used for Bayesian inference in Logit and Negative Binomial regression models and is briefly outlined in the following as it is also used in the panelTVP package.

The basic idea is that conditional on a Pólya-Gamma distributed random variable, Bayesian inference in the Logit or Negative Binomial model is based on a Gaussian posterior. A random variable  $\omega$  follows a Pólya-Gamma distribution with parameters b>0 and  $c\in\mathbb{R}$ , if

$$\omega \stackrel{D}{=} \frac{1}{2\pi^2} \sum_{k=1}^{\infty} \frac{g_k}{(k-1/2)^2 + c^2/(4\pi^2)},$$

where  $g_k \sim \mathcal{G}(b, 1)$  are independent Gamma random variables. Algorithms for sampling from the Pólya-Gamma distribution are implemented in the R-package BayesLogit (Polson, Scott and Windle, 2013).

Polson, Scott and Windle (2013) established the following integral identity

$$\frac{(\exp(\eta))^a}{(1+\exp(\eta))^b} = 2^{-b} \exp(\eta \delta) \int_0^\infty \exp\left(-\frac{\omega \eta^2}{2}\right) p(\omega) \ d\omega,\tag{3}$$

where  $\eta$  is a linear predictor,  $\delta = a - b/2$  and  $\omega \sim \mathcal{PG}(b,0)$  follows a Pólya-Gamma distribution.

The likelihood contribution in the Logit panel model in centered parameterization (see Equation (1)) is given by

$$p(\mathbf{y}_{it}|\mathbf{x}_{it},\boldsymbol{\beta}_t,f_i,\lambda_t,\omega_{it}) = \frac{(\exp(\eta_{it}))^{\mathbf{y}_{it}}}{1+\exp(\eta_{it})} \propto \exp\left(-\frac{\omega_{it}}{2}\left(\frac{\mathbf{y}_{it}-1/2}{\omega_{it}}-\eta_{it}\right)^2\right),$$

which follows from Equation (3) and by completing the square. Hence, conditional on the Pólya-Gamma variable  $\omega$ , the coefficients in the Logit model can be sampled from a Gaussian posterior. Moreover, the distribution of  $\omega$  - conditional on the parameters in the linear predictor  $\eta$  - is a Pólya-Gamma distribution as well, which makes Gibbs sampling feasible.

In order to perform inference in the Bayesian Negative Binomial model, we first consider a specific parameterization of the Negative Binomial distribution, where

$$p(\mathbf{y}_{it}|\mathbf{x}_{it},\boldsymbol{\beta}_t,f_i,\lambda_t) \propto \left(1 - \frac{\exp(\eta_{it})}{1 + \exp(\eta_{it})}\right)^r \left(\frac{\exp(\eta_{it})}{1 + \exp(\eta_{it})}\right)^{\mathbf{y}_{it}} = \frac{(\exp(\eta_{it}))^{\mathbf{y}_{it}}}{(1 + \exp(\eta_{it}))^{\mathbf{y}_{it}+r}}$$

Using this parameterization, the conditional moments are then given by

$$\mathbb{E}(\mathbf{y}_{it}|\mathbf{x}_{it},\boldsymbol{\beta}_t,f_i,\lambda_t) \equiv \mu_{it} = r \exp(\eta_{it}), \quad \mathbb{V}(\mathbf{y}_{it}|\mathbf{x}_{it},\boldsymbol{\beta}_t,f_i,\lambda_t) = r \exp(\eta_{it})(1 + \exp(\eta_{it})).$$

After some algebra and by using the integral identity from Equation (3), the likelihood contribution in the Negative Binomial panel data model can be expressed as

$$p(\mathbf{y}_{it}|\mathbf{x}_{it},\boldsymbol{\beta}_t,f_i,\lambda_t,\omega_{it}) \propto \frac{(\exp(\eta_{it}))^{\mathbf{y}_{it}}}{(1+\exp(\eta_{it}))^{\mathbf{y}_{it}+r}} \propto \exp\left(-\frac{\omega_{it}}{2}\left(\frac{\mathbf{y}_{it}-r}{2\omega_{it}}-\eta_{it}\right)^2\right).$$

Hence, inference in Bayesian Negative Binomial models is again based on a Gaussian posterior conditional on the Pólya-Gamma variables that are sampled in an additional data augmentation step (see Pillow and Scott (2012) for more details on Pólya-Gamma data augmentation in Negative Binomial models).

## 2.3. Zero-Inflated Negative Binomial model

For modelling both zero-inflated and overdispersed count data, we assume that there are two latent classes of zeros to account for the excess zeros: structural and at-risk zeros. A structural zero belongs to an observation that is not at risk of experiencing the event, whereas an at-risk zero belongs to an observation that is at-risk of experiencing the event but has for some reason a zero recorded. It is therefore assumed that the outcome in the Zero-Inflated Negative Binomial model  $y_{it}$  is a realization of a mixture distribution with a point mass at zero (for the structural zeros) and a standard Negative Binomial count model for observations that are at-risk, i.e., this includes both at-risk zeros and positive counts. The ZINB model can, thus, be stated as

$$y_{it}|r, \mu_{it}, w_{it} \sim (1 - \pi_{it}) \cdot \mathbb{I}_{(w_{it}=0)} + \pi_{it} \cdot \mathcal{NB}(\mu_{it}, r) \mathbb{I}_{(w_{it}=1)},$$

where  $w_{it}$  is a latent at-risk indicator such that with probability  $1 - \pi_{it}$ ,  $w_{it} = 0$  and with probability  $\pi_{it}$ ,  $w_{it} = 1$ , and  $\mu_{it}$  is the mean of the Negative Binomial distribution, as defined previously.

There are two separate linear predictors in the ZINB model. The first linear predictor  $\eta_{it}^{\text{logit}}$  is related to the zero-inflated part of the model through  $\pi_{it} = \frac{\exp(\eta_{it}^{\text{logit}})}{1+\exp(\eta_{it}^{\text{logit}})}$ , whereas the second linear predictor  $\eta_{it}^{\text{nb}}$  is the linear predictor of the Negative Binomial regression model. As the zero-inflation part is typically modelled with a Logit model, the Zero-Inflated Negative Binomial model contains both a Logit and a Negative Binomial regression model (for more details on Bayesian Zero-Inflated Negative Binomial regression see Neelon, 2019).

## 2.4. MCMC sampling schemes

In the following, the general sampling steps for the various regression models in panelTVP are briefly outlined. Details on the sampling steps are omitted in this vignette. For the sampling steps, we define  $\lambda = (\lambda_1, \dots, \lambda_T)^{\top}$ ,  $\mathbf{f} = (f_1, \dots, f_n)^{\top}$ ,  $\boldsymbol{\omega} = (\omega_{11}, \dots, \omega_{nT})^{\top}$  and  $\mathbf{X}_t$ , is an  $(n \times d)$  regressor matrix with rows  $\mathbf{X}_t^{\top}$ .

## 2.4.1. Gaussian model

The Gaussian panel data model in centered representation is given by

$$\mathbf{y}_{it} = \mathbf{x}_{it}^{\top} \boldsymbol{\beta}_t + f_i \lambda_t + \varepsilon_{it}, \quad \varepsilon_{it} \sim \mathcal{N}(0, \sigma^2),$$

where  $\sigma^2$  is the constant error variance. In the Gaussian model, posterior inference is easier than in the other models as a Pólya-Gamma data augmentation step is not required. However, the homoscedastic error variance must be sampled unlike in the other models. For this, we assume a conjugate Inverse-Gamma prior for  $\sigma^2$ , i.e.,

$$\sigma^2 | C_0 \sim \mathcal{G}^{-1}(c_0, C_0), \quad C_0 \sim \mathcal{G}(g_0, G_0),$$

where the parameters  $c_0$ ,  $g_0$  and  $G_0$  are set to fixed values.

Bayesian inference in the Gaussian panel data model then involves looping over the following steps:

- 1. Sample the regression effects  $\beta_1, \ldots, \beta_T$  conditional on the covariates  $\mathbf{X}_1, \ldots, \mathbf{X}_T$  the factors  $\mathbf{f}$ , the loadings  $\boldsymbol{\lambda}$  and the homoscedastic error variance  $\sigma^2$  from the posterior of a Normal regression model with response  $\mathbf{y}_{it} f_i \lambda_t$ .
- 2. Sample the factors  $\mathbf{f}$  conditional on the covariates  $\mathbf{X}_1, \dots, \mathbf{X}_T$ , the regression effects  $\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_T$ , the loadings  $\boldsymbol{\lambda}$  and the homoscedastic error variance  $\sigma^2$  from the posterior of a Normal regression model with response  $\mathbf{y}_{it} \mathbf{x}_{it}^{\top} \boldsymbol{\beta}_t$ .
- 3. Sample the loadings  $\lambda$  conditional on the covariates  $\mathbf{X}_1, \dots, \mathbf{X}_T$ , the regression effects  $\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_T$ , the factors  $\mathbf{f}$  and the homoscedastic error variance  $\sigma^2$  from the posterior of a Normal regression model with response  $\mathbf{y}_{it} \mathbf{x}_{it}^{\top} \boldsymbol{\beta}_t$ .
- 4. Sample the homoscedastic error variance  $\sigma^2$  conditional on the covariates  $\mathbf{X}_1, \dots, \mathbf{X}_T$ , the regression effects  $\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_T$ , the factors  $\mathbf{f}$  and the factor loadings  $\boldsymbol{\lambda}$  from an Inverse-Gamma posterior.

# 2.4.2. Probit model

The Probit model can be represented as a latent utility model with standard Normal noise, i.e.,

$$\tilde{\mathbf{y}}_{it} = \mathbf{x}_{it}^{\top} \boldsymbol{\beta}_t + f_i \lambda_t + \varepsilon_{it}, \quad \varepsilon_{it} \sim \mathcal{N}(0, 1),$$

where  $\tilde{y}_{it}$  is the latent utility that is linked to the observed binary response via a threshold mechanism: The observed response is 1 when  $\tilde{y}_{it} > 0$ , and it is 0 when  $\tilde{y}_{it} \leq 0$ . The latent utility  $\tilde{y}_{it}$  can easily be sampled from a truncated Normal distribution (Albert and Chib, 1993). Therefore, Bayesian inference in the Probit panel data model involves looping over the following steps:

- 1. Sample the latent utility  $\tilde{\mathbf{y}}_{it}$  either from a positively truncated Normal distribution, when  $\mathbf{y}_{it}=1$ , or from a negatively truncated Normal distribution, when  $\mathbf{y}_{it}=0$ .
- 2. Sample the regression effects  $\beta_1, \dots, \beta_T$  conditional on the covariates  $\mathbf{X}_1, \dots, \mathbf{X}_T$  the factors  $\mathbf{f}$ , and the loadings  $\lambda$  from the posterior of a Normal regression model with response  $\tilde{\mathbf{y}}_{it} f_i \lambda_t$ .
- 3. Sample the factors  $\mathbf{f}$  conditional on the covariates  $\mathbf{X}_1, \dots, \mathbf{X}_T$ , the regression effects  $\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_T$ , and the loadings  $\boldsymbol{\lambda}$  from the posterior of a Normal regression model with response  $\tilde{\mathbf{y}}_{it} \mathbf{x}_{it}^{\top} \boldsymbol{\beta}_t$ .
- 4. Sample the loadings  $\lambda$  conditional on the covariates  $\mathbf{X}_1, \dots, \mathbf{X}_T$ , the regression effects  $\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_T$ , and the factors  $\mathbf{f}$  from the posterior of a Normal regression model with response  $\tilde{\mathbf{y}}_{it} \mathbf{x}_{it}^{\top} \boldsymbol{\beta}_t$ .

# 2.4.3. Logit model

Bayesian inference in the Logit model requires a Pólya-Gamma data augmentation step. Hence, Gibbs sampling involves looping over the following steps:

- 1. Sample  $\omega_{it}|\mathbf{x}_{it}, \boldsymbol{\beta}_t, f_i, \lambda_t \sim \mathcal{PG}(1, \eta_{it})$  and define the new response as  $\mathbf{y}_{it}^* \equiv (\mathbf{y}_{it} 1/2)/\omega_{it} \ \forall i, t.$
- 2. Sample the regression effects  $\beta_1, \ldots, \beta_T$  conditional on the covariates  $\mathbf{X}_1, \ldots, \mathbf{X}_T$  the factors  $\mathbf{f}$ , the loadings  $\lambda$  and the weights  $\omega$  from the posterior of a Normal regression model with response  $y_{it}^* f_i \lambda_t$ .
- 3. Sample the factors  $\mathbf{f}$  conditional on the covariates  $\mathbf{X}_1, \dots, \mathbf{X}_T$ , the regression effects  $\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_T$ , the loadings  $\boldsymbol{\lambda}$  and the weights  $\boldsymbol{\omega}$  from the posterior of a Normal regression model with response  $\mathbf{y}_{it}^* \mathbf{x}_{it}^{\top} \boldsymbol{\beta}_t$ .
- 4. Sample the loadings  $\lambda$  conditional on the covariates  $\mathbf{X}_1, \dots, \mathbf{X}_T$ , the regression effects  $\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_T$ , the factors  $\mathbf{f}$  and the weights  $\boldsymbol{\omega}$  from the posterior of a Normal regression model with response  $\mathbf{y}_{it}^* \mathbf{x}_{it}^\top \boldsymbol{\beta}_t$ .

#### 2.4.4. Negative Binomial model

Bayesian inference in the Negative Binomial model also requires a Pólya-Gamma data augmentation step and an additional step to sample the dispersion parameter of the Negative Binomial distribution. The dispersion parameter could be sampled by Metropolis-Hastings, but we achieved better results with univariate Slice sampling. In this MCMC technique, the parameter of interest is drawn uniformly from a region under the graph of its posterior distribution. The Slice sampler, thus, requires less tuning than the Metropolis-Hastings sampler. Moreover, the random walk behavior of the chain can be suppressed by adding an overrelaxation step (see Neal (2003) for further details on Slice sampling). Posterior inference in the Negative Binomial model then involves looping over the following steps:

- 1. Sample  $\omega_{it}|\mathbf{x}_{it}, \boldsymbol{\beta}_t, f_i, \lambda_t, \mathbf{y}_{it}, r \sim \mathcal{PG}(\mathbf{y}_{it} + r, \eta_{it})$  and define the new response as  $\mathbf{y}_{it}^* \equiv (\mathbf{y}_{it} r)/(2\omega_{it}) \ \forall i, t.$
- 2. Sample the regression effects  $\beta_1, \ldots, \beta_T$  conditional on the covariates  $\mathbf{X}_1, \ldots, \mathbf{X}_T$  the factors  $\mathbf{f}$ , the loadings  $\boldsymbol{\lambda}$ , the weights  $\boldsymbol{\omega}$  and the dispersion parameter r from the posterior of a Normal regression model with response  $y_{it}^* f_i \lambda_t$ .
- 3. Sample the factors  $\mathbf{f}$  conditional on the covariates  $\mathbf{X}_1, \dots, \mathbf{X}_T$ , the regression effects  $\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_T$ , the loadings  $\boldsymbol{\lambda}$ , the weights  $\boldsymbol{\omega}$  and the dispersion parameter r from the posterior of a Normal regression model with response  $\mathbf{y}_{it}^* \mathbf{x}_{it}^{\top} \boldsymbol{\beta}_t$ .
- 4. Sample the loadings  $\lambda$  conditional on the covariates  $\mathbf{X}_1, \dots, \mathbf{X}_T$ , the regression effects  $\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_T$ , the factors  $\mathbf{f}$ , the weights  $\boldsymbol{\omega}$  and the dispersion parameter r from the posterior of a Normal regression model with response  $\mathbf{y}_{it}^* \mathbf{x}_{it}^{\mathsf{T}} \boldsymbol{\beta}_t$ .
- 5. Sample the dispersion parameter r conditional on the covariates  $\mathbf{X}_1, \dots, \mathbf{X}_T$ , the regression effects  $\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_T$ , the factors  $\mathbf{f}$ , the loadings  $\boldsymbol{\lambda}$  and the weights  $\boldsymbol{\omega}$  using univariate Slice sampling.

## 2.4.5. Zero-Inflated Negative Binomial model

As the Zero-Inflated Negative Binomial model consists of two sub-models, i.e., a Logit model for the zero-inflation component and a Negative Binomial model for the count component, the MCMC sampler is basically a combination of the two sampling schemes for Logit and Negative Binomial regression with an additional step for sampling the latent at-risk indicators. As a consequence, we have two separate linear predictors with time-varying regression and random effects, i.e.,

$$\begin{split} & \eta_{it}^{\text{logit}} = (\mathbf{x}_{it}^{\text{logit}})^{\top} \boldsymbol{\beta}_{t}^{\text{logit}} + f_{i}^{\text{logit}} \boldsymbol{\lambda}_{t}^{\text{logit}}, \\ & \eta_{it}^{\text{nb}} = (\mathbf{x}_{it}^{\text{nb}})^{\top} \boldsymbol{\beta}_{t}^{\text{nb}} + f_{i}^{\text{nb}} \boldsymbol{\lambda}_{t}^{\text{nb}}. \end{split}$$

The same set of covariates can be used for modelling both components of the Zero-Inflated Negative Binomial distribution, but it is not mandatory to include the same covariates in both predictors.

An MCMC scheme for sampling the parameters of a Bayesian Zero-Inflated Negative Binomial panel model then involves looping over the following steps:

- 1. Sample the latent at-risk indicators  $w_1, \ldots, w_n$  conditional on all model parameters.
- 2. Update the parameters in the predictor of the zero-inflation component  $\eta_{it}^{\text{logit}}$  by using the data augmentation sampler of the Logit model.
- 3. For all observations that are currently in the at-risk class, i.e. for which  $w_{it} = 1$ , update the parameters in the predictor of the count component  $\eta_{it}^{\text{nb}}$  and the dispersion parameter of the Negative Binomial distribution r by using the data augmentation sampler of the Negative Binomial model as well as the Slice sampler, respectively.

Note that the at-risk indicators are only updated for observations for which  $y_{it} = 0$ . For observations with a positive count,  $w_{it} = 1$  in all iterations of the MCMC sampler.

## 3. Basic structure of panelTVP

Time-varying parameter models for panel data of varying response distributions can be fitted by using the package's main function panelTVP::panelTVP(). The basic structure of the function is as follows:

```
panelTVP(formula = NULL,
         data = NULL,
         id = NULL,
         t = NULL,
         model = NULL,
         prior.reg,
         prior.var,
         prior.load,
         prior.reg_zinb.count,
         prior.load_zinb.count,
         prior.reg_zinb.inflation,
         prior.load zinb.inflation,
         mcmc.opt,
         settings.NegBin,
         HPD.coverage,
         R.WAIC,
         random.effects,
         progress.bar)
```

## 3.1. Essential arguments

The formula argument is used for specifying the model frame and works in the same way as the formula argument of lm() and glm() when fitting a model to either a Gaussian, binary or Negative Binomial response. For a Zero-Inflated Negative Binomial outcome, the formula was inspired by the function pscl::zeroinfl(), which is used for fitting frequentist Zero-Inflated Negative Binomial models to cross-sectional data.

The data argument expects a data.frame object that contains all the variables used in the formula argument.

The arguments id and t are each vectors that are typically variables in the data object. The former is a subject-indicator, whereas the latter is a time-indicator. It is mandatory to specify those parameters and covariates in the formula argument are not allowed to be named either id or t to avoid confusion.

The argument model defines the model that should be fitted to the data. This argument is a single character that is either "Gaussian" (for a Gaussian model), "Probit" (for a Probit model), "Logit" (for a Logit model), "NegBin" (for a Negative Binomial model) or "ZINB" (for a Zero-Inflated Negative Binomial model).

Those four arguments constitute the bare minimum for fitting a time-varying parameter model to balanced panel data with panelTVP(), e.g., a Logit model can be fitted by using

A Zero-Inflated Negative Binomial model with two linear predictors can be fitted by using

Note that when you specify model = "ZINB", then the formula argument consists of two parts: the count and the zero-inflation predictor. The two predictors are separated by the | sign. Moreover, it is possible to include the same set of covariates in both predictors. When all covariates in the data set should be included in both predictors, then you can write

```
panelTVP(formula = y ~ . | .,
    data = zinb.data,
    id = zinb.data$id,
    t = zinb.data$t,
    model = "ZINB")
```

This will also ignore the indicator variables id and t in your dataset, so you do not need to worry about excluding them.

## 3.2. Arguments for MCMC settings

The arguments mcmc.opt and settings.NegBin both deal with MCMC settings and are expected to be lists that contain additional parameters. Note that the elements in a list can easily be changed without the need to copy the entire list elements. To illustrate this, we consider the argument mcmc.opt that is used for setting the options of the MCMC sampler. Its default arguments are:

```
mcmc.opt(chain.length = 12000, burnin = 2000, thin = 10, asis = TRUE)
```

The argument chain.length is the total number of MCMC draws, while burnin defines the burn-in period (i.e., per default, the first 2000 draws are discarded) and thin defines the thinning factor (i.e., per default, every tenth draws is used). Finally, asis = TRUE means that the optional ASIS (ancillarity sufficiency interweaving strategy) step is used during MCMC sampling, which can increase sampling efficiency. Based on our simulations, we always recommend using ASIS as its computational burden is low and the gain in sampling efficiency can be tremendous. For more details on ASIS see, e.g., Yu and Meng (2011), Kastner and Frühwirth-Schnatter (2014) or Bitto and Frühwirth-Schnatter (2019).

Suppose that we want to change the burnin in period to a higher value - say 4000. Then we can modify the list argument mcmc.opt in the following way:

```
args <- rlang::fn_fmls(panelTVP) # extracts arguments of panelTVP
args.mcmc <- as.list(args$mcmc.opt) # selects mcmc.opt and converts it into a list
utils::modifyList(args.mcmc, list(burnin = 4000)) # this gives the altered list</pre>
```

```
## [[1]]
## list
##
## $chain.length
## [1] 12000
##
## $burnin
## [1] 4000
##
## $thin
## [1] 10
##
## $asis
## [1] TRUE
```

Alternatively, modification of elements in a list can also be done in a more condensed way:

```
## [[1]]
## list
##
## $chain.length
## [1] 12000
##
## $burnin
## [1] 4000
##
## $thin
## [1] 10
##
## $asis
## [1] TRUE
```

The argument settings.NegBin is a list that is only used when fitting a model to a count response variable. Its default structure is as follows:

The six arguments of settings.NegBin are used for Slice sampling of the dispersion parameter r of the Negative Binomial distribution. The arguments alpha.r and beta.r are shape and rate parameter of the Gamma prior on r, respectively. The argument expansion.steps defines the number of steps used in the stepping-out phase of the Slice sampler, while the width parameter defines the length of the slice interval. Those four arguments are required to perform standard Slice sampling. The standard Slice sampler can be extended to an overrelaxed Slice sampler, where the additional overrelaxation phase aims at increasing sampling efficiency by suppressing the random walk behavior of the Markov Chain. The accuracy of this procedure is governed by the argument accuracy.overrelax, which is the tuning parameter for mid-point refinement of the overrelaxed Slice sampler. If the accuracy limit is too small then the drawn value might not be in the Slice region, whereas high accuracy values will increase the computational burden of the sampler. Finally, the argument p.overrelax is the probability of performing an overrelaxed step. Hence, if p.overrelax = 0 then no overrelaxation will be done and instead standard Slice sampling will be used. When overrelaxation is desired, then we recommend relatively high values, e.g., p.overrelax = 0.95. For details on the Slice sampler see the seminal paper Neal (2003).

#### 3.3. Arguments for prior specification

#### 3.4. Additional arguments

There are four remaining arguments of the function panelTVP, which will be covered in this subsection. The first one is HPD.coverage, which is the coverage probability of Bayesian highest posterior density intervals for the parameters with a 95 % coverage as default.

The argument random.effects is a Boolean indicating whether you want to include a random effects / factor structure in your model to account for unobserved heterogeneity. Random effects are usually included in panel models and, thus, random.effects = TRUE is the default setting.

The argument R.WAIC relates to the computation of the Widely Applicable Information Criterion (WAIC), where the marginal WAIC is computed on the basis of R.WAIC replicated datasets, see, e.g., Quintero and Lesaffre (2018) for details on the replication method and a discussion on the conditional and marginal WAIC for longitudinal models.

Finally, the argument progress.bar is a Boolean that is set to FALSE as default and, thus, the progress of the sampler will not be printed out on the R-console. Change it to TRUE to keep track of the sampling progress.

## 3.5. Output

#### 4. Analyzing Gaussian response data

We now demonstrate the usage of the panelTVP package based on an application of a real-world dataset the Income dataset, which is contained in the package and a subset of the National Longitudinal Longitudinal Survey of Youth 1997. This survey started its first round in 1997, where most of the participants were children or young adults. As the goal of this illustration is to analyze income data, we do not focus on the first waves of the survey. Moreover, due to the availability of some variables we only focus on the last four waves of the study, i.e., survey rounds in the years 2015, 2017, 2019 and 2021.

In the following, the first ten observations are printed out for a quick overview.

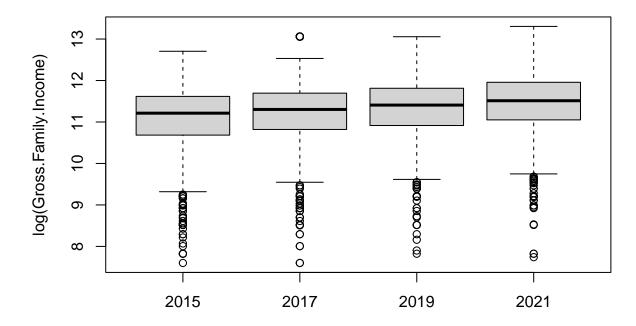
## head(Income, 10)

```
##
                                                                          Edu
         Sex
                                          Ethnicity Year
## 9
        male Mixed Race / Non-Black / Non-Hispanic 2015
                                                              college degree
## 11 female
                                           Hispanic 2015
                                                              college degree
## 12
                                           Hispanic 2015 high school degree
        male
## 22
        male
                                           Hispanic 2015 high school degree
## 27
        male
                                              Black 2015
                                                                   no degree
## 31
        male Mixed Race / Non-Black / Non-Hispanic 2015
                                                              college degree
## 33 female Mixed Race / Non-Black / Non-Hispanic 2015
                                                              college degree
## 37 female Mixed Race / Non-Black / Non-Hispanic 2015
                                                              college degree
## 41 female Mixed Race / Non-Black / Non-Hispanic 2015
                                                              college degree
## 48 female
                                              Black 2015
                                                              college degree
##
                   Hours.Worked Age t Gross.Family.Income log id Baseline.Age c
## 9
       worked at least 30 hours
                                  33 1
                                                      11.964001
                                                                                 3
## 11
       worked at least 30 hours
                                                                                 3
                                  33 1
                                                      10.491274 11
## 12
       worked at least 30 hours
                                  34 1
                                                      11.127263 12
                                                                                 4
       worked at least 30 hours
                                                      11.931636 22
                                                                                 3
                                  33 1
## 27 worked less than 30 hours
                                  34 1
                                                       8.987197 27
                                                                                 4
                                                      11.002100 31
                                                                                 3
       worked at least 30 hours
## 33 worked less than 30 hours
                                                      11.461632 33
                                                                                 4
                                                      12.180755 37
                                                                                 4
       worked at least 30 hours
       worked at least 30 hours
                                                      11.710374 41
                                                                                 2
                                  32 1
                                                      11.018629 48
                                                                                 2
## 48
       worked at least 30 hours
                                  32 1
```

The variable Edu is a factor with levels "no degree", "high school degree" and "college degree", which refers to the highest completed degree of the person. Hours.Worked is a factor with levels "worked less than 30 hours" and "worked at least 30 hours". The variable Baseline.Age\_c is the centered version of the baseline age of the person in the year 2015, i.e., the first study period considered in this specific subset of the survey. The variable t contains information the time point, whereas id is a subject indicator. Note that there are gaps in the ID enumeration, which is due to the fact that some subjects were removed for various reasons when the raw data were prepared for the purpose of creating this specific case study.

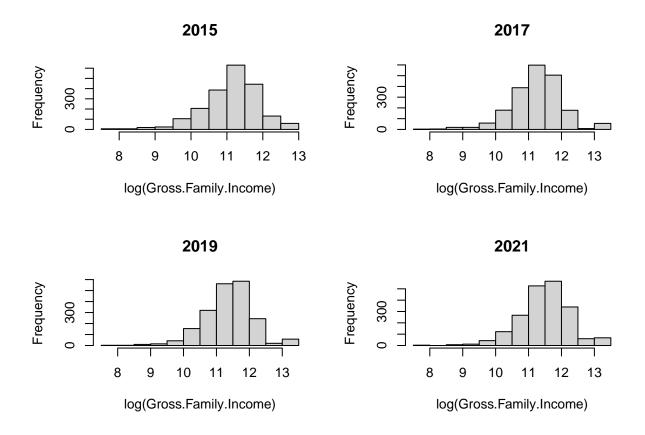
The response variable of interest is gross family income on the log-scale (Gross.Family.Income\_log). A quick descriptive analysis gives insight into the distribution of this variable as a function of time.

# Distribution of Income (log-scale) over Time



The boxplots suggest that the income distribution differs by time point and, thus, some covariates may differ in their effects as well.

The marginal distribution of the response by time point is given by



# layout(1)

Despite the slightly left-skewed marginal distribution of the response, we fit a Gaussian model with time-varying parameters by executing the following code:

Information on the duration of the MCMC sampler can be accessed via

# Gauss.vignette\$runtime

# ## [1] "Total Runtime for Bayesian Normal Model: 182.35 seconds"

The model comprises a lot of parameters, but the most important ones are the time-varying regression effects and factor loadings. The S3 functions summary() and plot() can be used to show those effects in a table and in a plot, respectively.

When summarizing the results in a table, the user can order the results either by covariate

```
##
## -----
## Posterior Summary of the Bayesian Normal Model with Time-Varying Coefficients:
## Warning in center_text(paste("Estimates for", nami[i])): At least one variable
## name is too long for fully displaying it.
Estimates for (Intercept)
##
##
    Lower (HPD) Posterior Mean Upper (HPD)
                      10.5389 0.0919
## t=1
     10.1770
             10.3579
               10.4612
## t=2
      10.2783
                        10.6299 0.0890
               10.6017
      10.4387
                       10.8027 0.0896
## t=3
## t=4
      10.5488
               10.7237
                       10.9099 0.0904
##
Estimates for Sexfemale
##
##
   Lower (HPD) Posterior Mean Upper (HPD)
## t=1 -0.1150 -0.0538 0.0082 0.0326
## t=2
      -0.1394
               -0.0778
                       -0.0233 0.0308
## t=3
               -0.0971
      -0.1548
                       -0.0341 0.0310
## t=4
      -0.1682
                -0.1047
                        -0.0468 0.0319
##
Estimates for Baseline.Age c
##
##
   Lower (HPD) Posterior Mean Upper (HPD)
## t=1
      -0.0033
                0.0120
                        0.0321 0.0107
      -0.0044
                0.0107
## t.=2
                        0.0305 0.0101
      -0.0051
                 0.0101
                         0.0297 0.0099
## t=3
## t=4
      -0.0076
                 0.0086
                        0.0294 0.0099
Estimates for EthnicityHispanic
##
   Lower (HPD) Posterior Mean Upper (HPD)
## t=1
      0.1923
                0.2753
                      0.3494 0.0408
       0.1968
                0.2743
                        0.3528 0.0405
## t=2
## t=3
       0.1901
                0.2729
                       0.3487 0.0406
       0.1913
                0.2711
## t=4
                        0.3521 0.0416
##
## -----
```

## Estimates for EthnicityMixed Race / Non-Black / No

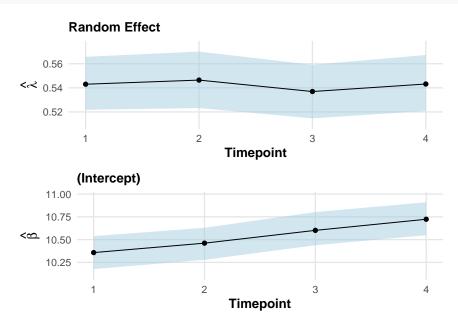
```
##
##
    Lower (HPD) Posterior Mean Upper (HPD)
## t=1
       0.3335
               0.4026
                          0.4691 0.0348
        0.3481
                  0.4082
                           0.4777 0.0344
## t=2
## t=3
        0.3377
                 0.4029
                           0.4686 0.0346
                  0.4087
## t=4
        0.3471
                           0.4802 0.0346
##
## -----
      Estimates for Eduhigh school degree
##
   Lower (HPD) Posterior Mean Upper (HPD)
## t=1
       0.0921
                0.2419
                        0.3833 0.0751
        0.0894
                  0.2379
                           0.3655 0.0735
## t=2
## t=3
       0.0912
                 0.2313
                           0.3757 0.0745
       0.0780
                 0.2262
## t=4
                           0.3757 0.0764
##
Estimates for Educollege degree
##
   Lower (HPD) Posterior Mean Upper (HPD)
##
## t=1 0.4449
               0.5980 0.7433 0.0769
                 0.6148
## t=2
       0.4693
                          0.7510 0.0744
## t=3
                 0.6224
       0.4776
                           0.7620 0.0753
                  0.6220
## t=4
        0.4788
                           0.7710 0.0762
## -----
## Estimates for Hours.Workedworked at least 30 hours
##
    Lower (HPD) Posterior Mean Upper (HPD)
## t=1
      0.0369
             0.1005 0.1721 0.0342
## t=2
        0.0394
                  0.1074
                           0.1704 0.0330
        0.0343
                  0.0996
                           0.1731 0.0348
## t=3
        0.0345
                  0.1089
                           0.1793 0.0359
## t=4
##
Estimates for Factor Loading
##
   Lower (HPD) Posterior Mean Upper (HPD)
## t=1
               0.5430 0.5658 0.0112
       0.5218
## t=2
        0.5232
                           0.5701 0.0120
                 0.5464
## t=3
       0.5147
                 0.5369
                          0.5591 0.0117
## t=4
        0.5208
                 0.5432
                           0.5672 0.0119
  or by time point
summary(Gauss.vignette, by = "timepoint")
```

```
## -----
## Posterior Summary of the Bayesian Normal Model with Time-Varying Coefficients:
## -----
           Estimates for Timepoint 1
##
##
                                           Lower (HPD) Posterior Mean
## (Intercept)
                                              10.1770 10.3579
## Sexfemale
                                              -0.1150
                                                           -0.0538
## Baseline.Age c
                                              -0.0033
                                                           0.0120
## EthnicityHispanic
                                               0.1923
                                                           0.2753
## EthnicityMixed Race / Non-Black / Non-Hispanic
                                               0.3335
                                                           0.4026
## Eduhigh school degree
                                                           0.2419
                                               0.0921
## Educollege degree
                                               0.4449
                                                            0.5980
## Hours.Workedworked at least 30 hours
                                               0.0369
                                                            0.1005
## Factor Loading
                                               0.5218
                                                            0.5430
##
                                           Upper (HPD)
                                                        SD
## (Intercept)
                                              10.5389 0.0919
## Sexfemale
                                               0.0082 0.0326
## Baseline.Age_c
                                               0.0321 0.0107
## EthnicityHispanic
                                               0.3494 0.0408
## EthnicityMixed Race / Non-Black / Non-Hispanic
                                               0.4691 0.0348
## Eduhigh school degree
                                               0.3833 0.0751
## Educollege degree
                                               0.7433 0.0769
                                               0.1721 0.0342
## Hours.Workedworked at least 30 hours
## Factor Loading
                                               0.5658 0.0112
## -----
            Estimates for Timepoint 2
##
##
                                           Lower (HPD) Posterior Mean
## (Intercept)
                                              10.2783 10.4612
## Sexfemale
                                              -0.1394
                                                           -0.0778
## Baseline.Age_c
                                              -0.0044
                                                           0.0107
## EthnicityHispanic
                                               0.1968
                                                            0.2743
## EthnicityMixed Race / Non-Black / Non-Hispanic
                                               0.3481
                                                            0.4082
## Eduhigh school degree
                                               0.0894
                                                            0.2379
## Educollege degree
                                               0.4693
                                                            0.6148
## Hours.Workedworked at least 30 hours
                                               0.0394
                                                            0.1074
## Factor Loading
                                               0.5232
                                                            0.5464
                                           Upper (HPD)
## (Intercept)
                                              10.6299 0.0890
## Sexfemale
                                              -0.0233 0.0308
## Baseline.Age_c
                                               0.0305 0.0101
## EthnicityHispanic
                                               0.3528 0.0405
## EthnicityMixed Race / Non-Black / Non-Hispanic
                                               0.4777 0.0344
                                               0.3655 0.0735
## Eduhigh school degree
## Educollege degree
                                               0.7510 0.0744
## Hours.Workedworked at least 30 hours
                                               0.1704 0.0330
                                               0.5701 0.0120
## Factor Loading
```

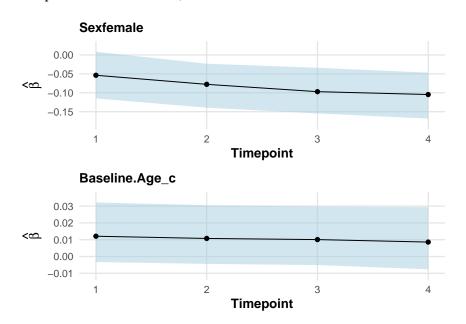
```
##
##
             Estimates for Timepoint 3
##
##
                                              Lower (HPD) Posterior Mean
## (Intercept)
                                                  10.4387 10.6017
## Sexfemale
                                                  -0.1548
                                                                -0.0971
## Baseline.Age_c
                                                  -0.0051
                                                                0.0101
## EthnicityHispanic
                                                   0.1901
                                                                0.2729
## EthnicityMixed Race / Non-Black / Non-Hispanic
                                                                0.4029
                                                   0.3377
## Eduhigh school degree
                                                   0.0912
                                                                0.2313
## Educollege degree
                                                   0.4776
                                                                0.6224
## Hours.Workedworked at least 30 hours
                                                   0.0343
                                                                 0.0996
## Factor Loading
                                                   0.5147
                                                                 0.5369
##
                                              Upper (HPD)
## (Intercept)
                                                  10.8027 0.0896
## Sexfemale
                                                  -0.0341 0.0310
## Baseline.Age_c
                                                   0.0297 0.0099
## EthnicityHispanic
                                                   0.3487 0.0406
## EthnicityMixed Race / Non-Black / Non-Hispanic
                                                   0.4686 0.0346
## Eduhigh school degree
                                                   0.3757 0.0745
## Educollege degree
                                                   0.7620 0.0753
## Hours.Workedworked at least 30 hours
                                                   0.1731 0.0348
## Factor Loading
                                                   0.5591 0.0117
##
Estimates for Timepoint 4
## -----
##
##
                                              Lower (HPD) Posterior Mean
## (Intercept)
                                                  10.5488
                                                                10.7237
## Sexfemale
                                                  -0.1682
                                                               -0.1047
## Baseline.Age_c
                                                  -0.0076
                                                                0.0086
## EthnicityHispanic
                                                   0.1913
                                                                0.2711
## EthnicityMixed Race / Non-Black / Non-Hispanic
                                                   0.3471
                                                                 0.4087
## Eduhigh school degree
                                                   0.0780
                                                                 0.2262
## Educollege degree
                                                   0.4788
                                                                 0.6220
## Hours.Workedworked at least 30 hours
                                                   0.0345
                                                                 0.1089
## Factor Loading
                                                   0.5208
                                                                 0.5432
##
                                                             SD
                                              Upper (HPD)
## (Intercept)
                                                  10.9099 0.0904
## Sexfemale
                                                  -0.0468 0.0319
## Baseline.Age_c
                                                   0.0294 0.0099
## EthnicityHispanic
                                                   0.3521 0.0416
## EthnicityMixed Race / Non-Black / Non-Hispanic
                                                  0.4802 0.0346
## Eduhigh school degree
                                                   0.3757 0.0764
## Educollege degree
                                                   0.7710 0.0762
## Hours.Workedworked at least 30 hours
                                                   0.1793 0.0359
## Factor Loading
                                                   0.5672 0.0119
```

Effect plots can be easily obtained via

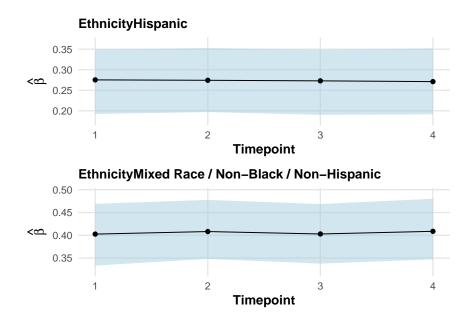
# plot(Gauss.vignette, nplots = 2)



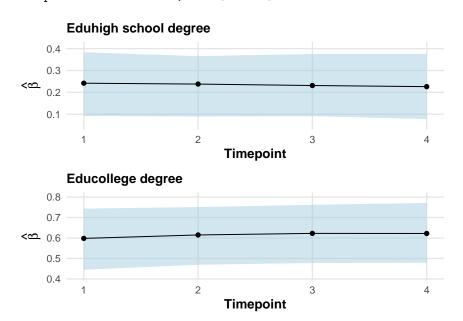
## The plots are out there, hit [Return] to see  $\dots$ 



## The plots are out there, hit [Return] to see  $\dots$ 

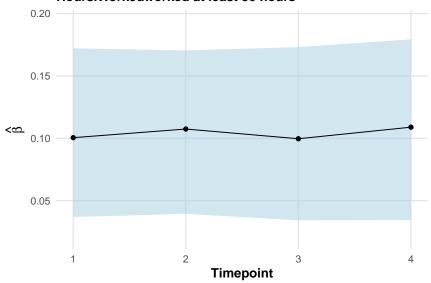


## The plots are out there, hit [Return] to see ...



## The plots are out there, hit [Return] to see  $\dots$ 

#### Hours.Workedworked at least 30 hours



The first effect plot is always the random effect or the factor loading, which is followed by the global intercept. Afterwards, the time-varying effects of covariates are given. Based on the results we observe an increase in the overall income level over time and a significant decrease for the women. Hence, the interviewed women have ceteris paribus a lower gross family income than men and this effect is more pronounced as they get older. The other effect estimates are shrunken to time-invariance. We see that people with a high school or college degree earn on average more money (or at least have a higher family income) than people with no completed degree. Furthermore, working at least 30 hours yields to an increase in the gross family income compared to people who work less than 30 hours.

Aside from the inspection of the time-varying parameters, Bayesian inference with MCMC requires proper evaluation of the generated Markov Chains. The remaining MCMC draws after applying burnin and thinning for all parameters can be accessed via

#### head(Gauss.vignette\$mcmc, 4)

```
##
        beta_t11 beta_t12 beta_t13 beta_t14
                                               beta_t21
                                                           beta_t22
                                                                      beta_t23
## [1,] 10.44387 10.54999 10.68535 10.75595 -0.11399866 -0.12826341 -0.1332617
## [2,] 10.36887 10.49241 10.66686 10.79378 -0.02807614 -0.07490815 -0.1002995
   [3,] 10.43375 10.52945 10.67197 10.79683 -0.01333708 -0.07377731 -0.1235973
##
   [4.]
       10.36396 10.47429 10.58853 10.70264 -0.06882912 -0.11018192 -0.1505653
##
           beta_t24
                         beta_t31
                                       beta_t32
                                                     beta_t33
                                                                   beta_t34
## [1,] -0.15070571
                     3.702441e-03
                                   0.0037157278
                                                 0.0038962246
                                                              0.0023979584
## [2,] -0.09927611
                    3.768037e-04
                                  0.0006411826 -0.0002872084 -0.0001843646
## [3,] -0.10039401 -4.095493e-05 -0.0001094372 -0.0001362720 -0.0002314573
## [4,] -0.13846180 2.076244e-02 0.0107683800
                                                 0.0100131834 0.0022898896
         beta_t41 beta_t42 beta_t43 beta_t44
##
                                                 beta t51 beta t52 beta t53
## [1,] 0.1919325 0.1919116 0.1918210 0.1920599 0.3706134 0.4097664 0.3443580
## [2,] 0.1868633 0.1875196 0.1874116 0.1862384 0.3765959 0.3757303 0.3747769
  [3,] 0.2265296 0.2264692 0.2270813 0.2264614 0.4408758 0.4405047 0.4403487
## [4,] 0.3051175 0.3041564 0.3047913 0.3050399 0.4242387 0.4773669 0.4232808
##
         beta_t54 beta_t61 beta_t62 beta_t63 beta_t64 beta_t71
## [1,] 0.3955467 0.2243026 0.2223148 0.2226232 0.2458123 0.5692582 0.5955511
## [2,] 0.3757758 0.2667827 0.2425325 0.2119134 0.1745347 0.6058749 0.6058278
## [3,] 0.4406143 0.1917788 0.1917759 0.1917791 0.1917729 0.4869194 0.5516268
```

```
## [4,] 0.4446442 0.2301241 0.2301311 0.2301256 0.2301233 0.5843568 0.5902506
##
         beta_t73 beta_t74
                              beta_t81 beta_t82
                                                   beta_t83
                                                              beta_t84
## [1,] 0.6142144 0.6411122 0.10302578 0.1055438 0.10702309 0.10845499 10.51355
## [2,] 0.6058872 0.6061404 0.12043767 0.1267505 0.12518459 0.13211203 10.49056
## [3,] 0.5633139 0.5696016 0.10480391 0.1143182 0.08238619 0.08364508 10.36190
## [4,] 0.5838170 0.5735417 0.07953443 0.0848327 0.14585243 0.15456917 10.59974
##
               beta2
                             beta3
                                       beta4
                                                 beta5
                                                           beta6
## [1,] -0.037905304
                     7.540648e-03 0.1919981 0.2145166 0.2201216 0.5741728
## [2,] -0.004879238
                     2.063433e-07 0.1864386 0.3781858 0.2670730 0.6058916
## [3,] -0.009461096
                    1.607629e-07 0.2262566 0.4410600 0.1917904 0.3527311
## [4,] -0.052205094 -1.959871e-07 0.3055418 0.4083764 0.2301097 0.5820129
##
             beta8
                       theta1
                                   theta2
                                                theta3
                                                              theta4
## [1,] 0.09958768 -0.1244162
                              0.05620747 0.0020570045 -0.0001079763 0.1029707474
                              0.01812659 0.0005113931 -0.0017106338 0.0007763536
## [2,] 0.13277618 -0.1279928
## [3,] 0.14199795 -0.1070822
                              0.03033023 0.0001314508 0.0005025183 0.0002887696
## [4,] 0.11492109 0.1158389 -0.04157812 0.0112413837 -0.0009651928 0.0619698782
##
               theta6
                             theta7
                                          theta8
                                                     tau21
                                                                  tau22
## [1,] -7.943579e-03 -0.0365450695 -0.001998671
                                                  60.71629 0.0053509274
## [2,] -4.010307e-02 0.0002463352 -0.012090209
                                                  61.58600 0.0115708661
                                    0.028257506 162.02628 0.0001783864
## [3,] -5.279756e-06 -0.1199411879
## [4,] 1.060278e-05 -0.0139054569 -0.044650712
                                                 58.78335 0.0116680637
##
               tau23
                          tau24
                                     tau25
                                                 tau26
                                                            tau27
## [1,] 7.020765e-05 0.01193802 65.4477443 187.7544372 0.16867667
                                                                   0.07579487
## [2,] 3.795668e-13 0.05206231 0.1475576
                                             0.2763538 1.50798234 16.27691262
                                             0.1963824 0.03801637
## [3,] 3.146930e-14 8.46000716 16.2338844
                                                                   0.15360257
## [4,] 5.258971e-14 3.58657134 13.2951325
                                             0.1410246 0.46093041
                                                                   1.12431768
##
                           xi22
                                                     xi24
                                                                  xi25
                                        xi23
## [1,] 0.171369511 0.007066971 5.773325e-04 4.294428e-08 2.054407e-03
## [2,] 0.302247597 0.002154247 1.131716e-07 3.568133e-06 6.115310e-07
## [3,] 0.007075938 0.039362529 1.691021e-06 3.387866e-05 3.890994e-04
## [4,] 0.007400079 0.012309663 1.377170e-03 1.664418e-05 2.102253e-03
                xi26
                             xi27
                                          xi28
                                                    a.tau kappa.tau
                                                                         a.xi
## [1,] 3.102158e-05 4.801811e-04 9.541107e-05 0.14985253 0.1297529 0.1329226
## [2,] 2.724440e-03 5.511761e-08 1.528679e-04 0.08159118 0.7632204 0.1215459
## [3,] 8.516701e-10 5.384080e-02 6.443829e-02 0.11601315 0.1173748 0.1349858
## [4,] 5.217748e-06 8.746377e-04 4.541086e-04 0.08097238 0.1109783 0.1495742
##
                     sigma2 lambda_t1 lambda_t2 lambda_t3 lambda_t4
         kappa.xi
## [1,]
        88.92907 0.1788861 0.5389640 0.5406923 0.5281776 0.5472028 0.5333181
## [2,]
        54.04817 0.1811933 0.5449713 0.5395289 0.5340582 0.5444099 0.5409153
        15.08552 0.1840817 0.5389138 0.5327645 0.5355356 0.5330704 0.5357220
## [4,] 449.32039 0.1846950 0.5345261 0.5465252 0.5414073 0.5362844 0.5375839
##
                                      zeta2 a.phi
                                                     kappa.phi a.zeta
                psi
                          phi2
                                                                        kappa.zeta
## [1,] -0.022275078 61.2992209 0.039290394
                                                  0.048936634
                                                                    1
                                                                        32.2354402
1 1296.1812925
                                                1
                                                  0.007379546
        0.007318261
                     0.1190598 0.009182661
                                                1 16.354238412
                                                                       106.1703867
                                                                    1
## [4,]
        0.005572515 9.0650754 1.031545035
                                                1
                                                  0.283040547
                                                                    1
                                                                         0.5351148
```

#### 5. Analyzing binary response data

In this section we focus on the analysis of binary outcomes using the Logit model. A corresponding Probit regression model can easily be set up by changing the model argument accordingly and, thus, will not be discussed here.

For demonstration, we will use the Marijuana dataset that is contained in the panelTVP package. This dataset is a pre-processed subset of the National Longitudinal Longitudinal Survey of Youth 1997 and contains data of the first seven waves of this study. The response variable is Used.Mari.Since.DLI and indicates if the person has consumed Marijuana since the last interview date. A one means that the person has consumed the drug. For the first wave the question was, if the person has ever consumed Marijuana in his/her life. The goal is to explain the probability of drug consumption as a function of covariates and to assess whether effects of covariates are subject to temporal change.

To get an overview of the data, the first 10 rows are given in the following:

#### head (Marijuana, 10)

```
##
       Bullied.Before.12
                              Sex
                                                                Ethnicity Year Age
## 1
                       no female Mixed Race / Non-Black / Non-Hispanic 1997
                                                                                 15
## 10
                      yes
                            male
                                                                 Hispanic 1997
                                                                                 14
## 22
                       no female
                                                                 Hispanic 1997
## 35
                            male
                                                                 Hispanic 1997
                                                                                 15
                       no
## 52
                       no female Mixed Race / Non-Black / Non-Hispanic 1997
                                                                                 16
                            male Mixed Race / Non-Black / Non-Hispanic 1997
## 57
                                                                                 15
## 64
                       nο
                            male Mixed Race / Non-Black / Non-Hispanic 1997
## 72
                       no female
                                                                 Hispanic 1997
                                                                                 15
## 84
                            male
                                                                 Hispanic 1997
                                                                                 1.5
                      ves
## 103
                       no female
                                                                 Hispanic 1997
                                                                                 15
##
       Residence t id Baseline.Age_c Used.Mari.Since.DLI
## 1
           Urban 1
                                     3
                     1
## 10
           Urban 1
                                     2
                                                           0
                     2
## 22
           Urban 1
                                     3
                                                           0
## 35
           Urban 1
                     5
                                     3
                                                           1
## 52
           Urban 1
                                     4
                     8
                                                           0
## 57
           Urban 1 9
                                     3
                                                           0
## 64
           Urban 1 10
                                     2
                                                           0
## 72
           Urban 1 11
                                     3
                                                           0
## 84
           Urban 1 12
                                     3
                                                           0
## 103
           Urban 1 15
                                     3
                                                           0
```

The variable Bullied.Before.12 indicates whether the person was a victim of bullying before the age of 12. The variable Residence gives information on the location where the person lives in and is either Urban or Rural. Finally, the variable Baseline.Age\_c is a centered version of the person's baseline age, i.e., the age in the first survey round in 1997.

A Bayesian Logit model with time-varying parameters and shrinkage priors on the effects can be estimated as follows:

The total runtime of the sampler was considerably longer than for the Gaussian model. This is due to the fact that the Logit model requires sampling of Pólya-Gamma weights in each step of the sampler.

## logit.vignette\$runtime

## t=3

## t=4

## t=5

## t=6

0.2114

0.0229

-0.1201

-0.1527

0.2905

0.1134

-0.0336

-0.0597

## ## [1] "Total Runtime for Bayesian Logit Model: 1092.35 seconds"

The estimated effects can again be summarized in a table

```
summary(logit.vignette)
## Posterior Summary of the Bayesian Logit Model with Time-Varying Coefficients:
## -----
## Warning in center_text(paste("Estimates for", nami[i])): At least one variable
## name is too long for fully displaying it.
##
           Estimates for (Intercept)
##
##
     Lower (HPD) Posterior Mean Upper (HPD)
## t=1
       -4.3070
                -3.9889
                          -3.6606 0.1754
                   -3.6023
        -3.9034
                            -3.2692 0.1588
## t=2
## t=3
        -3.9702
                   -3.6012
                            -3.2705 0.1781
## t=4
        -3.5495
                   -3.1704
                            -2.7928 0.1925
## t=5
        -3.4261
                   -3.0374
                            -2.6662 0.1973
## t=6
        -3.2125
                   -2.8364
                            -2.4198 0.2002
## t=7
        -3.0040
                   -2.6366
                            -2.2144 0.2011
## -----
##
           Estimates for Sexfemale
##
##
     Lower (HPD) Posterior Mean Upper (HPD)
## t=1
       -0.3470 -0.1531 -0.0108 0.0856
## t=2
        -0.3680
                   -0.1861
                            -0.0114 0.0889
## t=3
        -0.4666
                  -0.2656
                           -0.0822 0.0969
## t=4
        -0.6299
                   -0.4140
                            -0.1741 0.1181
## t=5
        -0.6300
                   -0.4102
                            -0.1639 0.1185
## t=6
        -0.8790
                   -0.6215
                            -0.3532 0.1358
## t=7
        -0.6910
                   -0.4846
                            -0.2689 0.1122
##
Estimates for Baseline. Age c
##
     Lower (HPD) Posterior Mean Upper (HPD)
##
## t=1
        0.5691
               0.6418
                             0.7138 0.0373
                   0.3676
## t=2
        0.2921
                             0.4352 0.0366
```

0.3697 0.0401

0.1946 0.0446

0.0598 0.0473

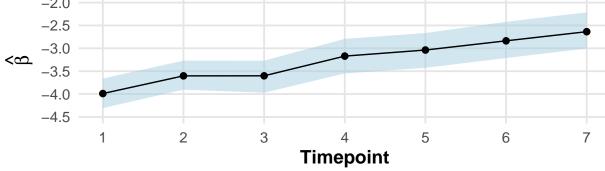
```
## t=7
      -0.2155 -0.1367 -0.0531 0.0418
##
Estimates for ResidenceUrban
   Lower (HPD) Posterior Mean Upper (HPD)
##
## t=1 0.0548 0.2184 0.4127 0.0958
                  0.2503
## t=2
        0.0924
                            0.4436 0.0874
## t=3
        0.0521
                  0.2380
                            0.4245 0.0976
## t=4
       0.1414
                  0.3405
                            0.5276 0.0988
## t=5
       0.1897
                  0.3898
                            0.5882 0.1035
## t=6
       0.2713
                   0.5116
                            0.7420 0.1228
## t=7
        0.2836
                   0.5654
                             0.8177 0.1363
##
        Estimates for Bullied.Before.12yes
##
    Lower (HPD) Posterior Mean Upper (HPD)
                          0.7252 0.0963
## t=1
        0.3574
                  0.5458
                   0.5426
                            0.7222 0.0948
        0.3636
## t.=2
## t=3
        0.3535
                   0.5418
                            0.7309 0.0980
## t=4
        0.3255
                   0.5312
                            0.7156 0.1011
## t=5
        0.3244
                  0.5251
                            0.7281 0.1033
## t=6
       0.3156
                  0.5179
                            0.7418 0.1083
## t=7
        0.3290
                   0.5255
                            0.7404 0.1046
## -----
        Estimates for EthnicityHispanic
##
    Lower (HPD) Posterior Mean Upper (HPD)
## t=1
      -0.0521 0.2033 0.4608 0.1315
## t=2
        0.1180
                   0.4162
                             0.7192 0.1537
                   0.3387
## t=3
        0.0009
                             0.6296 0.1617
## t=4
        -0.1021
                   0.2777
                             0.5809 0.1747
                   0.1289
## t=5
        -0.2538
                             0.4629 0.1880
## t=6
        -0.5365
                   -0.1828
                             0.1601 0.1822
## t=7
        -0.5699
                   -0.2354
                             0.1361 0.1767
##
## Estimates for EthnicityMixed Race / Non-Black / No
##
     Lower (HPD) Posterior Mean Upper (HPD)
##
## t=1
                          0.6420 0.1143
                   0.4186
        0.1947
## t=2
        0.5657
                   0.7858
                            1.0272 0.1207
## t=3
        0.7263
                  0.9996
                            1.2795 0.1400
## t=4
        0.8031
                   1.1098
                            1.3957 0.1527
## t=5
        0.8607
                   1.1682
                            1.4774 0.1673
## t=6
        0.6814
                            1.2902 0.1551
                   1.0126
```

```
0.6317
                            0.8932
## t=7
                                        1.1849 0.1481
##
##
              Estimates for Factor Loading
##
##
##
##
       Lower (HPD) Posterior Mean Upper (HPD)
                            1.2081
## t=1
            1.0805
                                        1.3500 0.0698
## t=2
            1.6222
                            1.7598
                                        1.9210 0.0796
## t=3
            2.1331
                            2.3278
                                        2.5366 0.1031
            2.5351
                            2.7809
## t=4
                                        3.0165 0.1204
## t=5
            2.8495
                            3.1207
                                        3.3941 0.1399
## t=6
            2.4912
                            2.7549
                                        2.9968 0.1269
## t=7
            2.0960
                            2.3056
                                        2.5120 0.1053
```

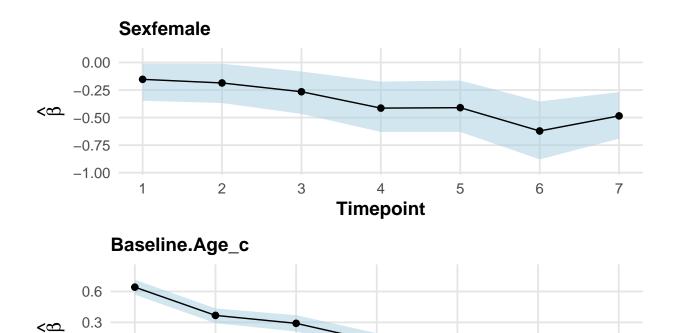
or displayed graphically

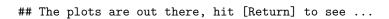
plot(logit.vignette, nplots = 2)

# **Random Effect** 3 << <sub>2</sub> 2 3 7 5 6 **Timepoint** (Intercept) -2.0-2.5 -3.0-3.5



## The plots are out there, hit [Return] to see  $\dots$ 

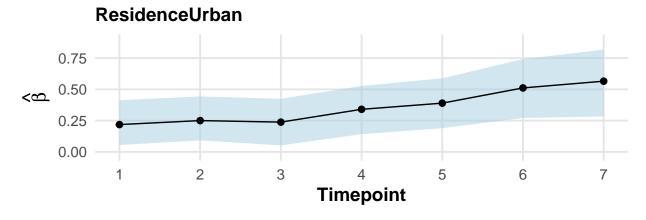


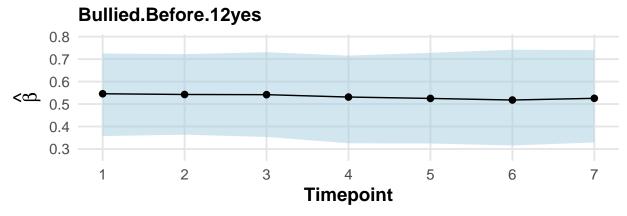


**Timepoint** 

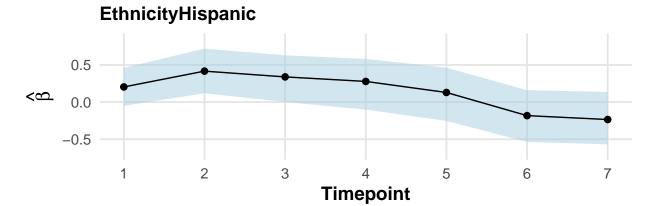
0.0

-0.3

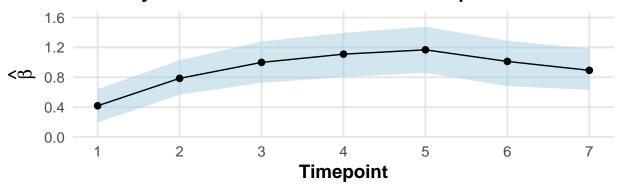




## The plots are out there, hit [Return] to see ...



# EthnicityMixed Race / Non-Black / Non-Hispanic



HIER NOCH INTERPRETATION DER EFFEKT HINGEBEN!!!!

- 6. Analyzing zero-inflated and overdispersed count response data
- 7. Analyzing simulated data with missing data