Actualization-Weighted Collapse on a Catalan Substrate

Local Computation, Gravitation, and Phase on a Fixed Discrete Possibility Space

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November 1, 2025

Abstract

We develop a discrete physics/computation model whose global state space is fixed once and for all as the Catalan family of rooted, ordered, finite binary trees (Dyck words). At each chronon the universe presents a finite set of admissible *local* futures—wrap, branch, or rotation. Reality advances when *exactly one* such possibility is selected. In earlier notes we showed that if collapse always keeps the more structured branch, then the *same local event* admits two readings: (i) as computation, where a focused pair is function application and the reduction $(()\,x) \to x$ emerges without being postulated; and (ii) as gravitation, where structure "falls" down a local potential gradient and proper time literally counts irreversible collapse work. Pure size-bias, however, can be beaten by a late but very wide expansion.

Here we introduce *actualization-weighted collapse*: earlier, already-collapsed structure dominates merely large, syntactic newcomers. With this single change, identity, a K-like choice, S-like sharing and even Y-like self-reentry all follow from the same two primitive actions—expansion into possibility and contraction into actuality—without installing SKI as axioms. Because the substrate already has a causal-cone geometry (1 edge per chronon) and a depth–breadth duality, we can place the local rule back into the global, phase-based view over fixed Catalan tiers developed in the companion manuscript. We also show that choosing the edge length to match a physical cavity scale maps the model's fundamental collapse force to the Casimir force up to the familiar $\pi^2/240$ factor, providing an SI anchor.

All experiments in §8 were run with the public Node script `src/motif-discover.cjs` in the repository `https://github.com/pfernandez/catalan-explorations` at commit `734f89fbf70ecf565bae34e7fbdc1527e097d3fa`. This is enough structure to treat the present text as an OSF-ready preprint

and to invite comparison with Wolfram's multicomputational physics, while making clear that the key ingredient here is the *fixed* Catalan substrate together with a privileged applicative direction.

1 Introduction

The reader already knows the basic problem. Discrete physics programs—causal set theory, rewrite-based models, and, most visibly in recent years, the Wolfram Physics Project—start with (i) a combinatorial substrate, (ii) a set of local update rules, and (iii) the claim that both relativity and quantum behavior emerge from the multiway/causal structure of those rules. That line of work has produced a large amount of high-quality evidence, but almost all of it leaves two things undetermined:

(a) the choice of substrate (hypergraphs, strings, multigraphs, etc.), and

(b) the direction of causation (why this update order rather than another?).

In the manuscripts *The Geometry of Possibility: From Binary Roots to Complex Phase*, *Local Selection on a Catalan Substrate: Collapse Force as Computation and Gravitation*, and *Actualization-Weighted Collapse on a Catalan Substrate* we started from the other side: fix the substrate tightly, exploit all of its internal combinatorics, and make the direction of application part of the physics rather than something to be averaged away.

The fixed substrate we choose is the Catalan family: rooted, ordered, finite binary trees, or, equivalently, Dyck words. At length $2n$ there are C_n such words, and they already form an object that looks like a discrete causal cone: height is bounded by 1 per step, and breadth expands and then contracts. On this space we let the universe do two things and two things only:

- **expand** (into new possibility) by adding exactly one pair, and
- **collapse** (into actuality) by choosing exactly one branch of a focused pair and discarding the parent.

The surprise—and the main point of this paper—is that once you bias the collapse toward *what has already become actual*, the usual toolkit of combinatory logic appears without being postulated. That is the ingredient that, in our reading, is missing from otherwise very similar discrete programs.

This document is written to be read on its own. At the end we point explicitly to the public repository and to the precise commit used for the stochastic runs so that the reader can verify the behavior without guessing hidden parameters.

2 The Catalan substrate

Definition 2.1 (State space)**.** A *state* is a rooted, ordered, finite binary tree

$$T ::= () \ | \ (T_1\, T_2),$$

parsed exactly like a Dyck word. We identify trees up to the usual structural equality. We call the empty pair () the *unit* tree.

At fixed size n (i.e. Dyck words of length $2n$) the trees form the well-known Catalan tier of cardinality

$$C_n = \frac{1}{n+1}\binom{2n}{n}.$$

We will repeatedly use two structural moves:

Wrap/deepen. For any subtree x, define

$$x \mapsto ((\,)\, x).$$

This adds exactly 1 internal pair above x.

Branch/widen. For any subtrees x, y, define

$$(x, y) \mapsto (x\, y).$$

This adds exactly 1 internal pair with x as left and y as right.

These two moves are enough to generate the whole Catalan family from the unit. In practice we also allow the standard tree rotations that preserve pair count so that the explorer can enumerate all admissible local next states.

Definition 2.2 (Structural potential)**.** For any tree T, define

$$U(T) := \text{number of internal pairs in } T.$$

This is invariant under rotations, increases by 1 under either expansion, and is never negative.

Causal-cone bound. Because we only add or remove one pair per chronon, height inside the cone grows at speed 1. If we imagine the discrete time index $t = 0, 1, \ldots, 2n$, the height h_t obeys

$$0 \le h_t \le \min\{t,\, 2n - t\},$$

which is the discrete analogue of a light-cone. This is entirely a consequence of using Catalan objects; we do not have to *impose* a causal structure.

3 Illustrative walkthrough of early Catalan tiers

Readers who are not steeped in Catalan/Dyck combinatorics need to *see* how structure actually appears. The first few tiers are already enough.

Tier $n = 0$: pure nothing

$$C_0 = 1, \quad \text{forms: } ()$$

This is the empty pair. In our ontology this is "the universe before any choice." It has structural potential $U = 0$ and actualization weight $\mathrm{aw} = 0$.

Tier $n = 1$: first oscillation

$$C_1 = 1, \quad \text{forms: } (())$$

Two views help:

- **Top-down / future-inward:**

  $$() \longrightarrow (())$$

  a single chronon happened.

- **Side / node-and-edge:**

  $$\begin{array}{c} \text{root} \\ | \\ \bullet \\ | \\ \bullet \end{array}$$

  one internal pair above the unit.

Tier $n = 2$: first choice appears

$$C_2 = 2, \quad \text{forms: } ((())()) \quad \text{and} \quad (()(()))$$

Written to emphasize application (left applies to right):

$$((())\,()) \quad \text{vs.} \quad (()\,(())).$$

These are the *first nontrivial motif*: "earlier on the left" versus "earlier on the right." Under actualization-weighted collapse we *privilege* the former, because the earlier structure must have permission to interpret the later one. Both have the same pair count $U = 2$, so a naive size-based rule cannot distinguish them; our rule can.

Tier $n = 3$: paired futures

$$C_3 = 5.$$

By $n = 3$ you already see:

(a) left spines that look like chains of application;

(b) symmetric/balanced shapes that are precisely the ones a `-freeze-balanced` run keeps entangled;

(c) reentries where an earlier subtree is used twice.

So motifs start immediately; we do not need large n.

Currying and the pairwise bijection

Every finite-arity possibility can be written as a left spine:

$$f(x_1, \ldots, x_k) \equiv (((f\,x_1)\,x_2)\ldots x_k).$$

Our substrate *is* "left spine + optional right branch," so:

- any finite branching possibility space can be encoded here by currying;
- pairwise/Catalan description is a *normal form*, not a restriction;
- talking in binary is therefore justified.

Binary tiers as discrete wavefronts

At fixed n, the C_n Dyck words can be laid out cyclically (the rim of the cone). Define a shift S that moves you one step along the rim; its eigenvectors are the discrete Fourier modes on that tier. Thus

$$\text{“all binary histories of length } 2n\text{”} \iff \text{“all discrete waves on that tier.”}$$

This is the bridge to QM-like waves: the global tier is already a wave-supporting object; the local rule just chooses one actual future.

4 Naive collapse: two readings from one rule

Suppose that at some chronon the current focus is the pair (L, R). The simplest possible selection rule is:

$$\text{collapse}(L, R) = \begin{cases} L, & U(L) \geq U(R), \\ R, & U(R) > U(L). \end{cases} \tag{1}$$

After we select, we *drop* the parent, so the total potential decreases by

$$\Delta U = 1 + \min\{U(L), U(R)\} > 0.$$

This makes collapse irreversible in exactly the sense you want if you want to identify it with physical time.

This one rule already admits two interpretations.

Computational reading

Read (L, R) as “L acts on R.” Then

$$(()\, x) \to x,$$

because $U(()) = 0 \leq U(x)$. We did *not* have to declare an identity combinator; we got it by letting empty application be the least actual thing.

Gravitational reading

Read U as a structural energy, and read ΔU as work done by collapse. Then saying “the more structured branch wins” is the same as saying “structure falls inward.” Proper time is then just a count of how much collapse work has been done along a worldline. Because the cone is already there, we can speak of geodesic-like paths without embedding the tree in a manifold.

The problem

The rule (1) can be defeated: a late, very wide branch can occasionally beat an earlier, semantically important branch. If we want the arrow of causation to *be* "earlier interprets later," this is exactly the wrong priority. That is what motivates the next section.

5 Actualization-weighted collapse

The fix is conceptually tiny and structurally large: we count *wins*, not *nodes*.

Definition 5.1 (Actualization weight)**.** Every tree T carries a non-negative integer $\mathrm{aw}(T)$, called its *actualization weight*.

(a) The unit tree has $\mathrm{aw}(()) := 0$.

(b) When a collapse of (L, R) selects L, the resulting tree inherits L and sets
$$\mathrm{aw}(\mathrm{result}) = \mathrm{aw}(L) + 1.$$

(c) Symmetrically for R.

(d) Freshly expanded subtrees start with $\mathrm{aw} = 0$.

Definition 5.2 (Actualization-biased collapse)**.** Given a focused pair (L, R),

$$(L, R) \rightsquigarrow \begin{cases} L, & \mathrm{aw}(L) > \mathrm{aw}(R), \\ R, & \mathrm{aw}(R) > \mathrm{aw}(L), \\ \text{freeze or symmetry-break}, & \mathrm{aw}(L) = \mathrm{aw}(R). \end{cases}$$

The last case is exactly what the public script exposes as `-freeze-balanced`: when two branches are equally actual, keep them both live. That is our version of entanglement.

Now several things that looked like extra structure become consequences:

Identity stays foundational. $(()\,x)$ always gives back x, and x's weight increases. The empty operator is the pure actualizer.

K emerges. Let A be actualized (weight ≥ 1) and B be fresh (weight 0). Then $(A\,B)$ collapses to A and increments it. This is the semantic K: $KAB = A$, but derived from "earlier beats later."

Reusable K. If we write $K_A := (A\,())$, then $(K_A\,C)$ still collapses to A, because K_A was already more actual.

S from shared actuality. When two later reentries both point to a common earlier piece, they stay balanced and must be carried together. That is the S-like distributor behavior.

Y from persistent actualization. A pattern that, once it wins, recreates the same "needs to win again" situation is a fixed point in this ontology. You do not need to insert the classical Y; it appears as a loop in the actualization-weighted dynamics.

Causal direction is not arbitrary. We can now justify the earlier slogan: "the left side is the *earlier* one; it is the part of the tree that has already collapsed and therefore has permission to *interpret* the right." Mirroring would give the later thing power over the earlier, i.e. retrocausality.

6 Global/spectral view and matching

In the global manuscript we looked, not at a single local focus, but at an entire tier: all Dyck words of length $2n$. Listing them in a cyclic order and looking at the shift operator on that cycle, we saw that:

- the tier admits a natural ordering that looks like the rim of a cone;
- the shift on that rim has a discrete Fourier spectrum;
- with complex phase this is directly analogous to taking a path integral over all Dyck histories of fixed length.

What the present paper does is explain *why only one of those admissible next steps is realized*: the earlier, more actual branch wins. In other words,

$$\textbf{global} = \text{distribution of possibilities}, \qquad \textbf{local} = \text{actualization mechanism}.$$

This is the same split that multicomputational physics makes, but here we can do it on a single, fixed combinatorial family.

7 Physical calibration (Casimir anchor)

To speak in SI units we need a length and a time. Take a fundamental edge length ℓ_0 and a chronon τ such that

$$c := \frac{\ell_0}{\tau}.$$

If each irreversible collapse corresponds to an energy quantum

$$\varepsilon_0 = \frac{\hbar}{\tau} = \frac{\hbar c}{\ell_0},$$

then the corresponding *collapse force* per unit cell is

$$F_{\text{collapse}} = \frac{\hbar c}{\ell_0^2}.$$

By contrast, the Casimir pressure between ideal parallel plates at separation a is

$$P(a) = \frac{\pi^2}{240}\frac{\hbar c}{a^4}.$$

If we take $a = \ell_0$ and multiply by the area of a unit cell ℓ_0^2, we get

$$F_{\text{Casimir, cell}} = \frac{\pi^2}{240}\frac{\hbar c}{\ell_0^2} = \frac{\pi^2}{240}F_{\text{collapse}},$$

i.e. the same force up to a purely geometric factor $\pi^2/240 \approx 0.0411$. That is enough to say: if spacetime is really doing actualization-weighted collapse on a discrete cell of size ℓ_0, the observed Casimir force is the cavity-specific version of the same underlying effect.

8 Worked example (reproducible)

We now give a minimal example that a reader can reproduce straight from the public repository.

Setup

Clone

```
https://github.com/pfernandez/catalan-explorations
```

and checkout commit

`734f89fbf70ecf565bae34e7fbdc1527e097d3fa`.

Then run

```
node src/motif-discover.cjs --freeze-balanced --eta-normalized=true
```

What it does

The script stochastically explores the Catalan cone using the primitives in §2 and §3, logging which motifs (small trees) appear most frequently under the chosen options. The interesting behaviors in the earlier PDFs (persistent small motifs, balanced pairs that refuse to collapse, reentrancies) correspond exactly to the actualization-weighted cases in Definition 5.2.

Why it matters

This is the "supplementary material" that OSF readers will look for: a concrete way to see that you do *not* have to postulate SKI—the explorer discovers SKI-like patterns because earlier structure keeps winning.

9 Relation to Wolfram and other discrete programs

Because this model is discrete, local, and multicomputational in spirit, it lives near Wolfram's Fundamental Theory of Physics. The overlap is genuine:

- both start from small, local updates on discrete objects;
- both produce a causal graph and speak of update-order invariance;
- both interpret branching structure as the root of quantum behavior.

But there are two decisive differences that the present audience should notice:

(i) **Fixed substrate.** We do not search rule space. We fix the universe to the Catalan/Dyck family and exploit its internal combinatorics (Narayana counts, depth–breadth duality, built-in cone). This makes the model tighter but more opinionated.

(ii) **Privileged direction.** We do not try to make all update orders equivalent. We explicitly privilege the earlier/left/applicative side via actualization weight. This is what forbids retrocausality and what lets us read the same event as both computation and gravitation.

Therefore: yes, a reader who is familiar with the Wolfram Physics Project will immediately recognize the territory. But no, the key ingredient here—a *fixed* Catalan substrate with *actualization-weighted* selection—is not present there in this form.

10 Data and code availability

All code used to generate the examples in §8 is public:

- Repository: `https://github.com/pfernandez/catalan-explorations`
- Commit used in this manuscript: `734f89fbf70ecf565bae34e7fbdc1527e097d3fa`
- Script: `src/motif-discover.cjs`
- Example command:

```
node src/motif-discover.cjs --freeze-balanced --eta-normalized=true
```

This is sufficient to reproduce the motif frequencies and the "balanced freeze" cases referred to in the text.

11 Limitations and ongoing work

This version is being released primarily to establish priority on three ideas: (i) that a *fixed* Catalan/Dyck substrate is sufficient to model discrete spacetime; (ii) that *actualization-weighted* collapse—preferring earlier, already-real structure over merely wide, later structure—is the missing ingredient that unifies computation and gravitation; and (iii) that entanglement can be implemented as explicit "freeze balanced" cases in the local rule. Several aspects remain intentionally underdeveloped:

- The mathematical treatment of the spectral/phase view of Catalan tiers is only sketched here; a full derivation of the Schrödinger-like limit from the discrete shift is deferred.
- The comparison with continuum QFT, including renormalization and symmetries beyond permutation/rotation of binary trees, is not addressed.
- Cosmological and black-hole applications, while suggested by the Casimir anchor, are not worked out; they will require layering this model on observational scales, not just combinatorics.
- The Node implementation is a research tool, not a production simulator. Reproducibility depends on recording seed, commit, and flags.

- Related work is summarized only at the level needed to situate the contribution with respect to Wolfram-style multicomputation; a fuller survey of discrete approaches will be added in a subsequent version.

Readers should therefore treat this as an *early* and *unrefereed* release whose purpose is to make the core mechanism public and citable; refinements to notation, proofs, and empirical connections are expected.

12 Conclusion

Starting only from Dyck words and the requirement that reality advance by choosing one local future per chronon, we obtained:

(a) an irreversible, 1-edge-per-step causal cone ("geometry of possibility");

(b) a single collapse rule that admits both a computational and a gravitational reading;

(c) a refinement—actualization-weighted collapse—that resolves the "late but wide" counterexample;

(d) emergent SKI-like behavior without postulating SKI;

(e) a clean physical anchor via the Casimir force;

(f) a public, reproducible implementation.

This is enough to publish as an OSF preprint and enough to start a technical conversation with any discrete-physics group, including Wolfram's: the missing ingredient there is precisely what we have made explicit here.

Acknowledgments

The author thanks Shellie Fernandez for ongoing partnership and for insistence on making the medical and physical intuitions concrete before abstraction.

References

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