Actualization-Weighted Collapse on a Catalan Substrate

Emergent Identity, Selection, and Self-Reentry without Extra Primitives

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Abstract

In earlier notes (*The Geometry of Possibility* and *Local Selection on a Catalan Substrate*) we treated (i) the Catalan/Dyck hierarchy as a discrete possibility space and (ii) local, size-biased collapse as a physical/computational actualization event. In this paper we sharpen the model so that *all* observed combinatory behavior—in particular identity, constant/selection, and a distributor-like self-reentry—emerges from *one* principle: **contraction always prefers what has already been actualized**. Concretely, we replace naive "keep the syntactically bigger branch" with an *actualization-weighted* collapse: the branch that is *earlier* (has already won collapses) dominates the branch that is merely *large* (recent, uncollapsed expansion). With this change, the elementary rule

$$(()\,x)\ \to\ x$$

is enough to produce a K-like "keep the earlier thing" behavior *without* installing ad hoc special cases. Moreover, because expansion is understood as reentry of the same global structure, an S-like motif appears as the local situation in which the already-actualized part simultaneously holds two future reentries. Entanglement then becomes fundamental: it is just "two branches sharing an earlier, already-actualized piece." Finally, a Y-like fixed-point structure is seen as a shape that, when collapsed, recreates the condition for further collapse, so that the same two primitive actions—expansion and actualization—suffice for recursion.

1 Overview

The core claim is that we do *not* need to install SKI-style rules as external axioms. Once we (1) restrict ourselves to Catalan/Dyck trees, (2) accept only two primitive actions—*expansion* into possibility space and *contraction* back into the already-actual part—and (3) define contraction to prefer *actuality* rather than *syntactic size*, the familiar combinatory behaviors follow.

Two clarifications frame the rest:

(i) **Actuality vs. syntax.** A freshly expanded subtree can be arbitrarily large, but until it has *won* a collapse it is not authoritative. Conversely, a very small but earlier subtree can dominate later, larger expansions.

(ii) **Causal application.** We interpret a binary node (L, R) as "L acts on R" or "the already-actual part (L) incorporates the new contribution (R)." Mirroring would invert causality, which we will discuss briefly later.

Under this reading, the constant/selector behaviour that in ordinary SKI is written

$$K\,A\,B \to A$$

is reinterpreted here as

$$(\text{earlier } A,\ \text{later } B) \to A.$$

No extra primitive was added; we merely made the collapse metric faithful to the ontology already implicit in the previous papers.

2 Trees from Catalan words

We keep the same data model as before. Let

$$\mathcal{T} ::= () \ \mid\ (t_1 t_2), \quad t_1, t_2 \in \mathcal{T}$$

be binary application trees with a distinguished leaf (). Trees are obtained from *primitive* Dyck words by the right-associating parser we used in the Node script; this ensures a *left* application spine, i.e. curried form:

$$(((f\ a)\ b)\ c)$$

is represented as a left-branching chain.

Serialization is again

$$\mathrm{ser}(()) = (), \qquad \mathrm{ser}((t_1 t_2)) = (\mathrm{ser}(t_1)\,\mathrm{ser}(t_2)).$$

3 Actualization weight

3.1 Motivation

If collapse picks "the bigger side" where "bigger" means "more nodes," then a late, very wide expansion can wrongly beat an early, already-resolved piece: $(A\ B) \to A$ fails when A is small (e.g. identity) and B is a large, recent branch. To avoid introducing arbitrary special rewrite rules, we change *what* is being compared.

3.2 Definition

Definition 1 (Actualization weight)**.** *To each tree $t \in \mathcal{T}$ we associate a nonnegative integer* $\mathrm{aw}(t)$ *called its* actualization weight*, defined inductively:*

(a) $\mathrm{aw}(()) = 0$ *for the leaf.*

(b) If $t = (L, R)$ and the local contraction selects L, then $\mathrm{aw}(L)$ *is incremented to* $\mathrm{aw}(L) + 1$ *and the result of the contraction inherits this value.*

(c) If the contraction selects R, we analogously increment $\mathrm{aw}(R)$*.*

(d) A newly expanded subtree starts with $\mathrm{aw} = 0$*.*

Thus aw does *not* count nodes; it counts *wins*. It is a measure of "how much of reality has already flowed through here."

3.3 Actualization-biased collapse

Definition 2 (Actualization-biased collapse)**.** *Given $t = (L, R)$ with weights* $\mathrm{aw}(L)$ *and* $\mathrm{aw}(R)$*, the local collapse is*

$$(L, R) \rightsquigarrow \begin{cases} L, & \mathrm{aw}(L) > \mathrm{aw}(R), \\ R, & \mathrm{aw}(R) > \mathrm{aw}(L), \\ \textit{freeze}(L, R) \ \textit{or symmetry-breaking}, & \mathrm{aw}(L) = \mathrm{aw}(R). \end{cases}$$

The `--freeze-balanced` option in the script corresponds to the third case: if both sides are equally actual, we *do not* break the symmetry. This will be the basis of the entanglement discussion later.

4 Base rule: Identity

We keep the same η-like simplification

$$(()\,x) \;\rightsquigarrow\; x. \tag{1}$$

This is justified as follows: the left branch is the *pure* actualizer (the trivial application), the right branch is the fresh content, so the content is what remains. We regard the result x as having $\mathrm{aw}(x) := \mathrm{aw}(x) + 1$ because an actualization has just occurred.

Call

$$\mathsf{I} := (()()).$$

Then (1) simply says: $\mathsf{I}\,x \rightsquigarrow x$, and the resulting x is now *more actual* than any freshly expanded neighbour.

5 Emergence of K from actualization

We now revisit the example that was problematic under node-counting.

5.1 Setup

Let A be any already-actualized tree, i.e. $\mathrm{aw}(A) = a \geq 1$. Let B be a freshly expanded possibility, i.e. $\mathrm{aw}(B) = 0$. Form the application

$$T := (A\ B).$$

5.2 Collapse

Apply the actualization-biased rule:

$$\mathrm{aw}(A) = a \geq 1, \qquad \mathrm{aw}(B) = 0 \quad \Rightarrow \quad (AB) \rightsquigarrow A,$$

and we increment $\mathrm{aw}(A)$ to $a + 1$.

So we have shown:

$$(AB) \rightsquigarrow A \qquad \text{whenever } \mathrm{aw}(A) > \mathrm{aw}(B).$$

5.3 K-shape from Causal Immutability

Now consider building a *reusable* K-like shape by capturing a value once and then feeding it further arguments. Concretely, let

$$\mathsf{K}_A := (A\ ()),$$

where A is already-actualized and () is a fresh placeholder with weight 0. By the rule above,

$$\mathsf{K}_A \rightsquigarrow A,$$

and A's weight increments. If we now apply this to an additional argument C,

$$(\mathsf{K}_A\ C) \equiv ((A())C),$$

the left child is *even more* actual (it has now won at least twice), and the right child is new; hence it collapses again to A. Thus we obtain the characteristic behaviour

$$(\mathsf{K}_A\ C) \rightsquigarrow A$$

without installing a "K-rule" by hand. We only compared actualization weights.

5.4 Why the counterexample disappears

The refinement above resolves a gap in our previous description of applicative collapse, illustrated by the following counterexample: let $A = \mathsf{I}$ (syntactically tiny), let B be a huge tree; under node-counting, $(AB) \to B$. Under actualization-counting, if I has ever been used, $\mathrm{aw}(\mathsf{I}) \geq 1$ and $\mathrm{aw}(B) = 0$, so $(\mathsf{I}B) \rightsquigarrow \mathsf{I}$. So the model now does what the ontology says it should: *earlier reality beats later possibility.*

6 Emergence of S from reentry and sharing

With K explained, we can turn to the distributor-like pattern.

6.1 Expansion is reentry

In the global picture (Geometry of Possibility) expansion does not create a *different* universe or introduce new material; it unfolds more of the *same* Catalan structure, from which dimensionality emerges. We began with nothing (), and that nothing is conserved. Locally, this means: when a node expands, what appears on the right can be regarded as another reentry of the same total structure. In a fully entangled view, the entire tree is "re-entering itself" at each node.

6.2 Two futures for one past

Now suppose we have a node whose left child has high actualization weight—it is a piece of history that is already fixed—and that node branches into *two* such reentries. Schematically:

$$\text{actualized core} \rightsquigarrow (\text{core}, X) \quad \text{and also} \quad (\text{core}, Y).$$

If X and Y are both fresh ($\text{aw} = 0$) but the core is old ($\text{aw} \gg 0$), then both branches are *kept* in the sense that neither X nor Y is strong enough to overwrite the core. What we have created is: *two distinct spatial-looking branches that share one earlier piece.* That is exactly the structural analogue of S "feeding the argument to two consumers," except that we have not cloned X; we have *shared* the earlier part.

6.3 Balanced freeze = entanglement

The `--freeze-balanced` option in the script captures this: if two subtrees have the *same* actualization weight, we do not collapse either; we preserve the shared, unresolved configuration. In the present interpretation this is simply:

Definition 3 (Fundamental entanglement)**.** *Two branches are entangled iff they both reference a subtree whose actualization weight is equal on both sides and therefore is not yet collapsed.*

This is more primitive than duplication; spacetime separation is emergent from which branch collapses later.

6.4 S as a discovered motif

In the stochastic runs, the high-frequency shapes that look like

$$(((()())(()()))) \quad \text{and} \quad ((((()())())()))$$

are precisely those where an identity-like core is used on the left and combined with another, comparably actualized piece. These are the trees for which the local rule *cannot* immediately delete the right, and so they survive as motifs. In other words: the process has *discovered* an S-like situation: one actualized piece, multiple still-live reentries.

We therefore do not need to write

$$\mathsf{S}\ f\ g\ x \to f\ x\ (g\ x)$$

as a primitive. We can simply say: *the local collapse dynamics admits nodes where one earlier part fans into multiple later parts, and when those later parts remain equally unactualized, the structure stays shared.* That *is* the distributor, in the language of this model.

7 Y as persistent actualization

Once we accept that an S-like sharing shape can stay live, we can describe a Y-like self-feeding knot very simply:

- start from an already-actualized core C,
- reenter it to produce a fresh copy of (a view on) C,
- arrange so that the presence of the fresh copy is itself a reason to reenter again.

This gives a tree that, when collapsed once, produces a configuration of *the same form* but at a later stage of actualization. In ordinary SKI, this is the fixed-point combinator. Here it is a shape whose actualization weight never ceases to increase because it always recreates an unresolved piece on the right.

This is the point where the paper naturally turns back to physics: *not all histories terminate*; some are locally self-sourcing collapses.

8 Causal orientation

The question has arisen whether the choice "left applies to right" could be mirrored, perhaps allowing for a Copenhagen-like interpretation of collapse. With actualization weight the answer is clearer: the left side is the earlier one; it is the part of the tree that has already collapsed and therefore has permission to interpret the right. Mirroring would give the later thing power over the earlier, i.e. retrocausality. Since the physical reading in Local Selection was that collapse = gravitation-like, we keep the causal direction and, with it, the left-spine, curried representation. But this is ultimately an interpretation we choose.

9 Relation to the earlier notes

- **Geometry of Possibility** gave the possibility space. Here we clarified that local reentry can be read as "the same total tree appearing again," which is what makes sharing/entanglement natural.
- **Local Selection** gave the collapse-as-computation rule. Here we clarified that the collapse must compare *actualization*, not *syntax*, if we want selection (K-like) to be emergent rather than installed.
- **This paper** shows that, under that clarification, identity, selection, and S-like branching all follow from the same two primitive actions.

10 Conclusion

By replacing naive node-counting with actualization-weight, we made the original ontology consistent: reality prefers the earlier, not the wider. Once that is in place,

1. the identity rule $(()x) \to x$ is truly foundational;
2. K-like behaviour arises whenever an earlier piece meets a later one;
3. S-like behaviour arises whenever an earlier piece meets *two* equally unactualized reentries, i.e. in the entangled, frozen case;
4. Y-like behaviour is the limiting case where collapse recreates its own precondition.

All of this uses only the two actions we allowed ourselves at the beginning: expansion into possibility and contraction into actuality.

References

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