Local Selection on a Catalan Substrate: Collapse Force as Computation and Gravitation

Abstract

We describe a discrete spacetime model whose underlying substrate is the Catalan family of balanced binary structures. Each update of the universe generates a finite set of admissible local futures (all rotation- and depth/breadth-preserving rewrites); reality advances when exactly one of these futures is selected. We introduce a single local rule, *collapse*, which selects the more structured side of a focused pair according to a structural potential $U(T)$ equal to the number of internal pairs in the tree. This rule admits two compatible readings: (i) as *computation*, where collapse is function application and $(()\,x) \to x$ emerges without axiom; and (ii) as *gravitation*, where structure falls down a local potential gradient and the collapse work is $\Delta U = 1 + \min\{U(L), U(R)\}$. Imposing a locality constraint of one edge per update induces a Lorentz-like kinematics with invariant interval $s^2 = \Delta t^2 - \Delta x^2$, in which proper time is literally the amount of irreversible collapse work actually performed. Finally, by choosing the edge length equal to a physical cavity scale, the model can be calibrated to SI units using the Casimir force, yielding a concrete force scale $F_0 = \hbar c/\ell_0^2$. This paper is intended to be read alongside a companion paper that develops the global, spectral view over the full Catalan lattice; here we provide the local, energetic mechanism that explains how one history becomes actual.

1 Introduction

A standard way to get physics-like behavior from discrete objects is to start large and global: enumerate all histories, impose symmetries, then take a continuum limit. A different route, and the one we take here, is to start small and local: specify a substrate, specify what counts as a local future, and specify how one future is chosen. The thesis of this paper is that, on a Catalan substrate, a *single* local selection rule suffices to support two seemingly different interpretations:

- a **computational** interpretation, in which selection is application (a β-like reduction step);
- a **gravitational** interpretation, in which selection is falling down a structural potential.

Both readings act on the same object (a focused pair in a binary tree), both use the same local data (the size of the two subtrees), and both produce the same next state. In other words, the duality is not enforced from the outside; it is already present in the combinatorics of the substrate.

This paper provides the local dynamics that complements a companion manuscript that works at the global level of "all admissible Catalan histories". There, one views an entire Catalan layer as a space of possible histories and studies phase-like operations on that space. Here, we explain why *one* history is actually realized.

2 The Catalan substrate

We take as primitive objects rooted, ordered, finite binary trees. Equivalently, these are Dyck words or fully parenthesized expressions; their number of size n is the nth Catalan number. We will write a binary node as $\langle L, R\rangle$ and the empty tree as $()$.

Two constructive moves are sufficient to build the whole space:

1. **Wrap** (or deepen): $x \mapsto (()\, x)$; this introduces one new pair above x.
2. **Branch** (or widen): $x, y \mapsto \langle x, y\rangle$; this introduces one new pair with two subtrees.

In addition, we allow **local rotations** (or local re-associations) of adjacent subtrees; these do not change the number of internal pairs and simply enumerate the "set of possible next steps" from a given configuration. Intuitively, wrap and branch expand possibility; rotations explore possibility.

Example: first Catalan tiers and bijection

For orientation, the first few Catalan classes are:

- $C_0 = 1$: `()`
- $C_1 = 1$: `(())`
- $C_2 = 2$: `(()(())), ((()()))`
- $C_3 = 5$: (five binary trees / Dyck words of size 3)

Each Dyck word corresponds to exactly one binary tree (its parse tree), and each binary tree prints to exactly one Dyck word. This canonical bijection is the reason we are free to move between the Dyck-word presentation (used in the companion "Geometry of Possibility" paper) and the pairwise presentation used here. Taking the *pair* as the fundamental decomposition is therefore not an extra assumption: (i) Church already showed that ordered pairs are sufficient to encode all finite data; and (ii) the Catalan family is precisely the class of all well-formed binary nestings. We work with pairs because the local rule we will introduce—collapse at a binary node—is most naturally stated on trees.

3 Structural potential

To compare two subtrees structurally, we assign to every tree T a *structural potential*

$$U(T) := \text{number of internal pairs in } T. \tag{1}$$

This is the simplest nontrivial potential one can define on Catalan objects: it is invariant under local rotations, increases by 1 under wrap or branch, and is always nonnegative.

Given a focused pair $\langle L, R\rangle$, we will want to compare $U(L)$ and $U(R)$ and to define a "force" from their difference:

$$F_{\text{app}}(\langle L, R\rangle) := \big|\, U(L) - U(R)\, \big|. \tag{2}$$

We call F_{app} the *applicative force*. It is large when the two sides are strongly unbalanced and zero when they are equal.

4 Collapse as computation and gravitation

Consider now a single local update. Let the focus be on a pair $\langle L, R\rangle$. We define the *collapse* of this pair by

$$\mathrm{collapse}(\langle L, R\rangle) = \begin{cases} L, & \text{if } U(L) \geq U(R), \\ R, & \text{if } U(R) > U(L). \end{cases} \tag{3}$$

That is, we *keep the more structured side.* Since the parent pair itself disappears, the total potential drops by

$$\Delta U = \big(1 + U(L) + U(R)\big) - \max\{U(L), U(R)\} = 1 + \min\{U(L), U(R)\}. \tag{4}$$

This is always at least 1, so collapse is irreversible: it always performs nonzero structural work.

4.1 Computational reading

In a computational reading, a focused pair is an application site: the left branch is the operator context; the right branch is the operand. A neutral operator is simply the empty tree on the left:

$$(()\, x) = \langle (), x\rangle.$$

Since $U(()) = 0 \leq U(x)$, the rule (3) keeps x and we obtain

$$(()\, x) \;\longrightarrow\; x. \tag{5}$$

Thus the identity combinator *emerges* from the collapse rule; it does not need to be postulated. More complex operator trees can, by rearrangement, present a pair in which the desired branch is heavier at the moment of collapse, yielding selector-like (K) and distributor-like (S) behaviors. In this sense, collapse is the local, structural analogue of β-reduction: among many admissible rewrites, pick the one that actually advances evaluation.

4.2 Gravitational reading

In a gravitational reading, U is an energy or mass functional on structure. At $\langle L, R\rangle$ we have two "masses" $U(L)$ and $U(R)$; the rule (3) says that the *lighter* one is absorbed into the *heavier* one, and the heavier branch defines the local geometry thereafter. The drop (4) is exactly the work done by this local fall. With this interpretation, the force (2) is the structural analogue of a force as a gradient:

$$F_{\text{coll}} := F_{\text{app}} = \big|U(L) - U(R)\big|. \tag{6}$$

The greater the imbalance, the more decisive the collapse. Computation and gravitation are therefore two readings of the *same* local event.

5 Locality and induced kinematics

The collapse rule tells us what happens at a focused pair, but to recover spacetime-like behavior we must also bound how fast information can move. We impose a single **locality constraint**:

> In one update cycle, the focus may move by at most one edge in the tree.

An update cycle (a *chronon*) consists of zero or more rotations (which do not change U) followed by one collapse (3). The constraint means that in one chronon we can only affect nodes that are at graph distance 1 from the current focus.

This immediately defines a maximal speed

$$c := \frac{1 \text{ edge}}{1 \text{ chronon}}. \tag{7}$$

We may take $c = 1$ as a natural unit.

Consider two collapse events on the same evolving tree, separated by Δt chronons, during which the focus has drifted by Δx edges. By locality, $|\Delta x| \leq \Delta t$. Define the discrete interval

$$s^2 := \Delta t^2 - \Delta x^2. \tag{8}$$

This is the 1+1–dimensional Minkowski form with $c = 1$. Events with $|\Delta x| = \Delta t$ lie on the "light cone": all of the update budget was spent on motion, none on collapse. Events with $|\Delta x| < \Delta t$ are timelike: some budget was spent on actual structural work.

This suggests defining a *proper time* accumulated by a process:

$$\Delta\tau := \sqrt{\Delta t^2 - \Delta x^2} = \frac{\Delta t}{\sqrt{1 - v^2}}, \qquad v = \frac{\Delta x}{\Delta t}. \tag{9}$$

Because collapse is the only operation that reduces U, proper time literally measures how much computation/gravitational work has been done along a worldline. Processes that move quickly (large $|v|$) perform fewer collapses per chronon; processes at rest ($v = 0$) collapse every chronon and so "age" fastest. Thus the same locality that makes the substrate look like spacetime also bounds the rate of computation.

6 Physical calibration

So far all quantities have been in model units: edges, chronons, and pair-counts. We can map these to SI units by choosing a physical length scale for an edge.

Let ℓ_0 be the length in meters corresponding to one edge, and τ the time in seconds for one chronon. We set

$$c = \frac{\ell_0}{\tau} = 2.99792458 \times 10^8 \text{ m/s}. \tag{10}$$

Let ε_0 be the energy released by a unit collapse in joules. To match Planck's constant we take

$$\varepsilon_0 = \frac{\hbar}{\tau} = \frac{\hbar c}{\ell_0}. \tag{11}$$

The corresponding *fundamental collapse force* of the model is then

$$F_{\text{collapse}} := \frac{\varepsilon_0}{\ell_0} = \frac{\hbar c}{\ell_0^2}. \tag{12}$$

This is, by definition, the force associated with a unit structural collapse at the fundamental length ℓ_0. It is a property of the model, not of any particular experimental geometry.

To connect to experiment, we can anchor ℓ_0 to the Casimir effect between ideal plates at separation a. The Casimir pressure is

$$P(a) = \frac{\pi^2}{240} \frac{\hbar c}{a^4}. \tag{13}$$

If we identify $a = \ell_0$ and take a unit cell of area ℓ_0^2, the force on that cell is

$$F_{\text{Casimir, cell}} = P(a)\,\ell_0^2 = \frac{\pi^2}{240}\frac{\hbar c}{\ell_0^2} = \frac{\pi^2}{240}F_{\text{collapse}}. \tag{14}$$

Thus, up to the dimensionless factor $\pi^2/240 \approx 0.0411$ (reflecting the specific boundary conditions of the parallel-plate configuration), the model's unit collapse force coincides with the Casimir force at that scale. Choosing ℓ_0 (e.g. 100 nm) therefore fixes all remaining units.

7 Relation to the companion paper

The companion paper takes the *global* view: for a fixed Catalan level n, consider the entire set of admissible histories and act on it with a rotation/shift operator, possibly with phases. That view is spectral and Hilbert-like: all possibilities are considered at once.

The present paper provides the *local* view: from those many admissible next steps, a single next step is selected by a structural rule that can be read as either computation or gravitation. The two papers are consistent if we regard the global one as describing the *possibility* distribution and the local one as describing the *actualization* mechanism.

8 Conclusion

We have shown that a very small set of assumptions:

1. immutable binary trees as states (Catalan substrate);
2. a structural potential $U = \#\text{pairs}$;
3. a local collapse rule that keeps the more structured branch;
4. a locality bound of one edge per update;

already yields

- a computation-like dynamics in which identity and, by routing, richer combinators emerge;
- a gravity-like dynamics in which structure falls down a local potential gradient;
- a Lorentz-like kinematics in which proper time counts actual irreversible work;
- and a feasible route to SI calibration through the Casimir effect.

In this sense, "collapse" is a unifying operation: the same local event can be read as evaluation, as falling mass, or—with an added probabilistic/phase law over the admissible local moves—as a quantum-style selection. Follow-up work can focus on cataloguing the recurrent motifs (emergent S, K, I) and on introducing correlated collapses to model entanglement.