Signal Conditioning
&
Filtering

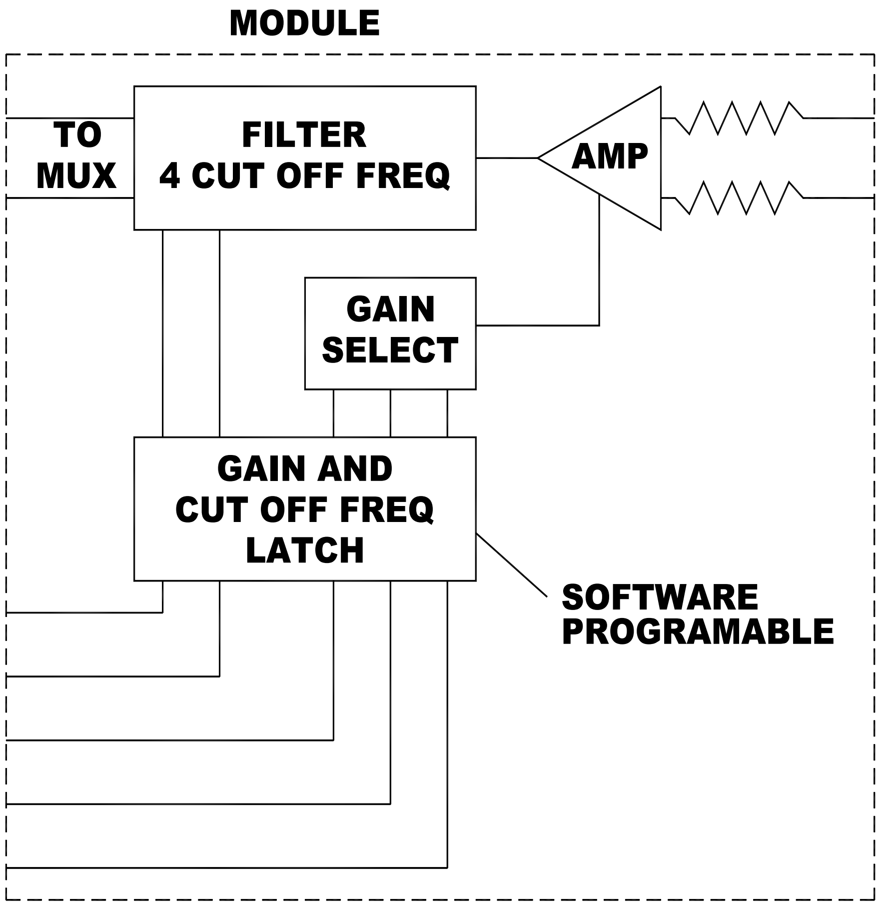
Signal Conditioning

- Signal conditioning is used to provide a match between the information (the sensor) and the rest of the data acquisition system.
  - Selection of proper signal conditioning is critical in order to ensure the measurement does not get contaminated with other undesired information (usually noise).
  - Signal conditioning that contains proper filtering can have the effect of removing noise or additional signal components from the part of the signal that is of real interest.
  - Signal conditioning also provides excitation for sensor and amplification and filtering needed to normalize the signal for the rest of the data acquisition systems (Typically 0 to 5 Volts).

Signal Conditioning

- Examples of signal conditioning include:
  - Analog Signal transformation such as converting a change in current into a voltage
  - Amplification , attenuation, and/or shifting the analog signal to make it compatible with the rest of the data acquisition system.
  - Converting of an analog signal to a digital format compatible with the rest of the data acquisition system.
  - Converting digital information from one format to another format that is better for processing the information

ANALOG DATA FILTER MODULE

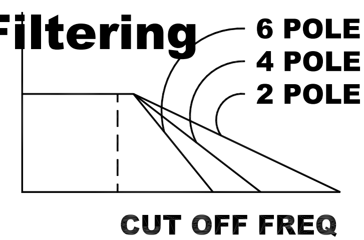


TRANSDUCER INPUT

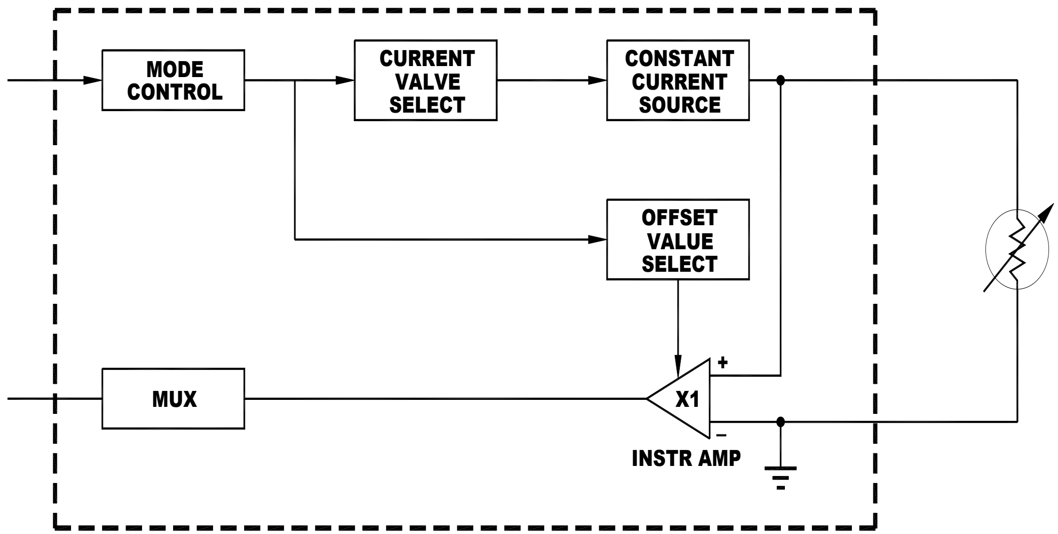
Normally Multi-Channel Unit

Programmable Primary and Secondary Gain

Filtering



RESISTIVE TEMPERATURE SENSOR SIGNAL CONDITIONER



Normally Multi Channel

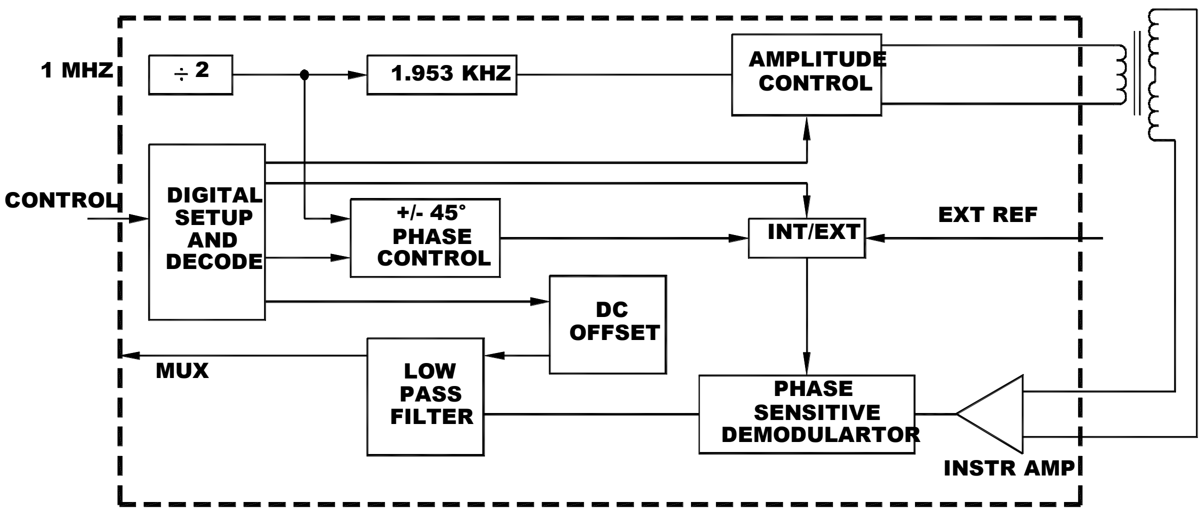
Programmable Constant Current

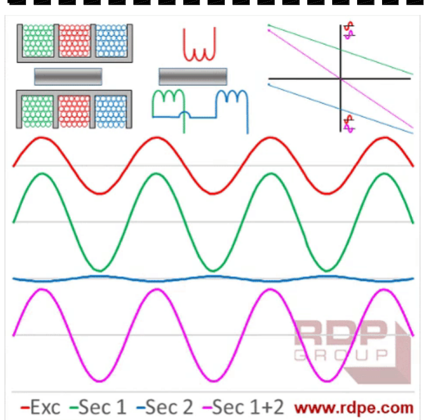
Programmable Offset Voltage

Range 50-1500 OHMS

PHASE SENSITIVE DEMODULATOR

LINEAR VARIABLE DIFFERENTIAL TRANSFORMER (LVDT)





Multi Channel

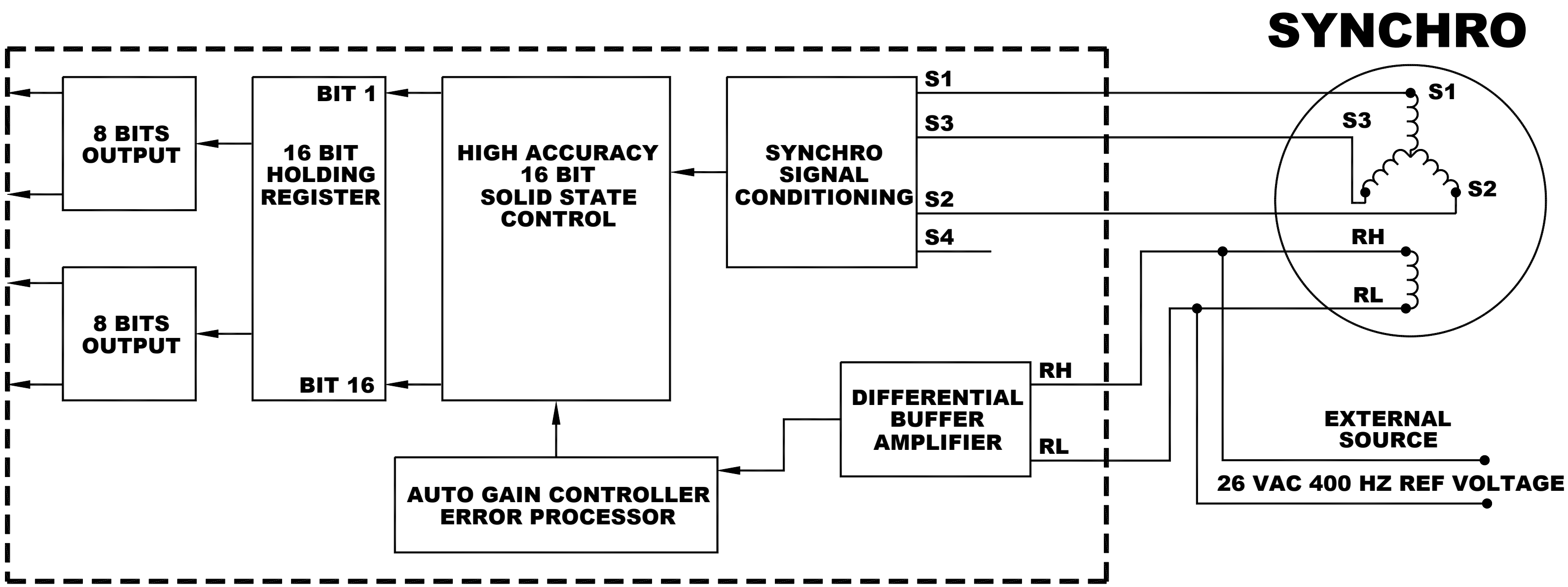
Programmable Amplitude Control

2.5, 5.0, 7.5 OR 10 VRMS

Phase Control

± 45.^{0} STEP 1.406^0

SYNCHRO/RESOLVER CONVERTER



Normally Multi Channel Unit

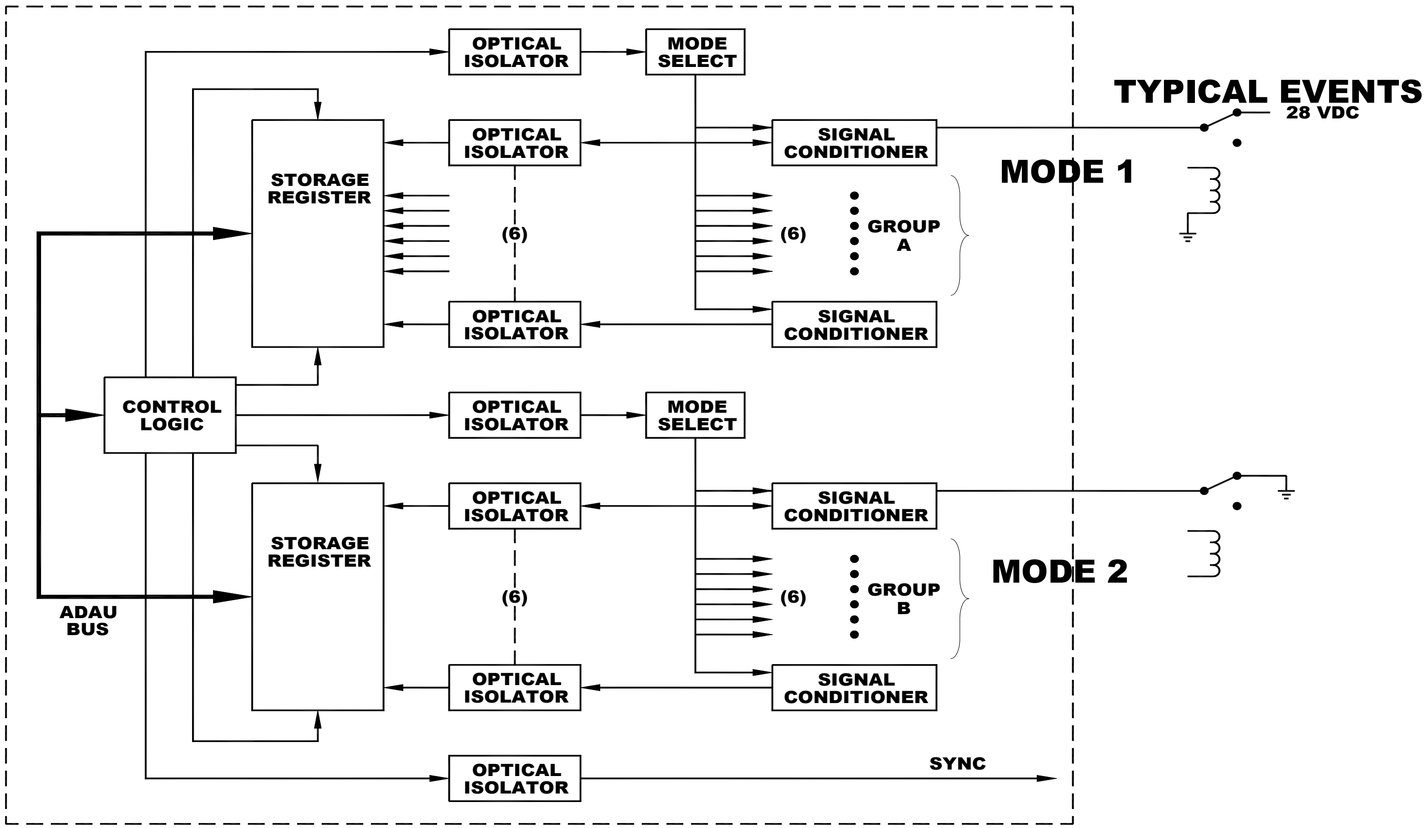
11.8 OR 90 V RMS Reference

TRACKING RATE

3600 ^{0}/SEC (14 BIT MODE)

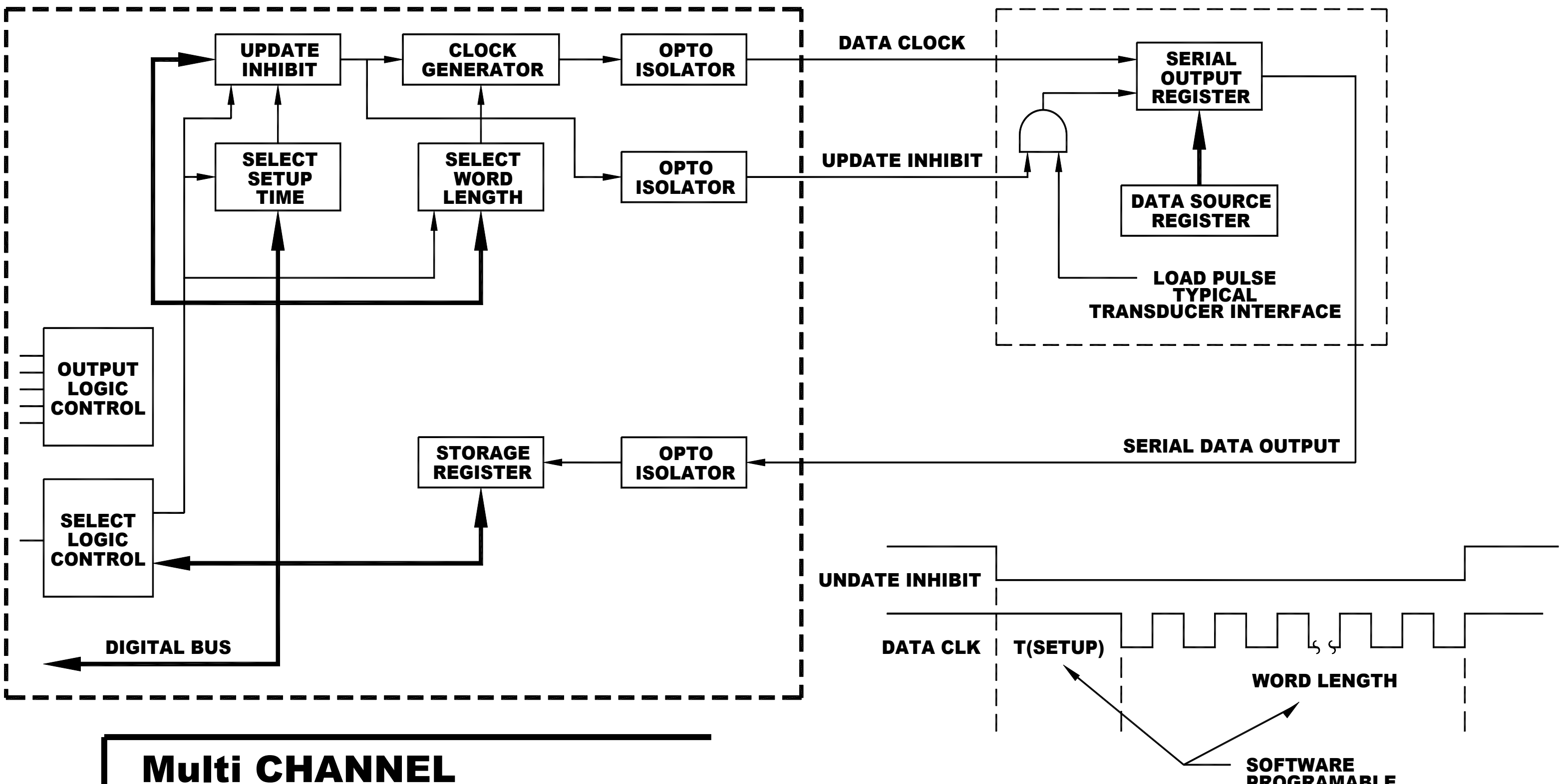
900 ^{0}/SEC (15 BIT MODE)

PARALLEL DIGITAL



Usually designed to support different signal levels and states by channel

SERIAL DIGITAL CONDITIONER



Multi CHANNEL

4 TO 30 BIT WORD

Signal Conditioning

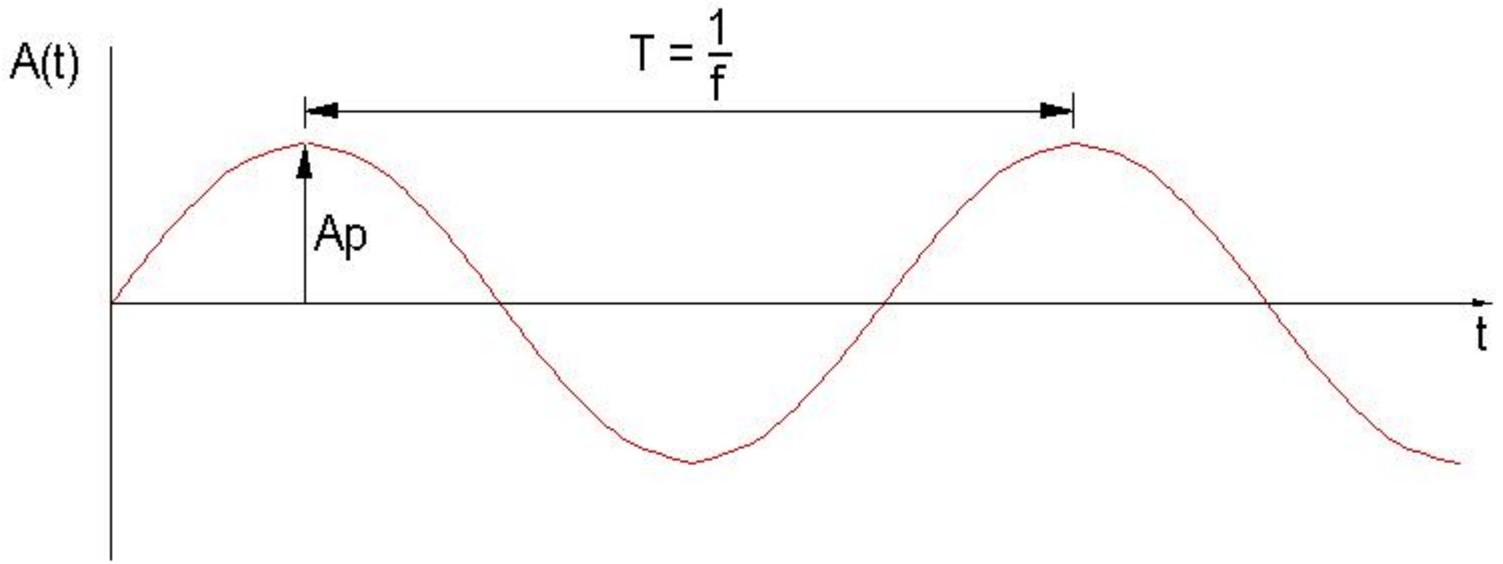
Frequencies and Signals

Some Concepts to Understand First: The Frequency Domain

- We are comfortable with measurements in the time domain. This is what we see on an oscilloscope or on a strip chart.

- Since filters are described in terms of frequency, they are represented in the frequency domain (as displayed on a spectrum analyzer). This makes a filter’s characteristics easier to see.

A Sine Wave in the Time Domain



- In the time-domain, the horizontal (x) axis is time, and the vertical (y) axis is the amplitude of the signal. The sine wave has a period T, that is measured between two like points on the sine wave. The frequency (f = 1/T) of the sine wave is the inverse of the period.

Mathematical Conversion to the Frequency Domain: The Fourier Transform

- To mathematically convert functions from the time domain to the frequency domain, the Fourier Transform is used.
- If f(*t*) is a function with respect to time *t*, then the Fourier Transform of f(*t*) is

$$F(f) = \int_{-\infty}^{+\infty} f(t) e^{-j2\pi ft} dt$$

f(*t*) is a function of time

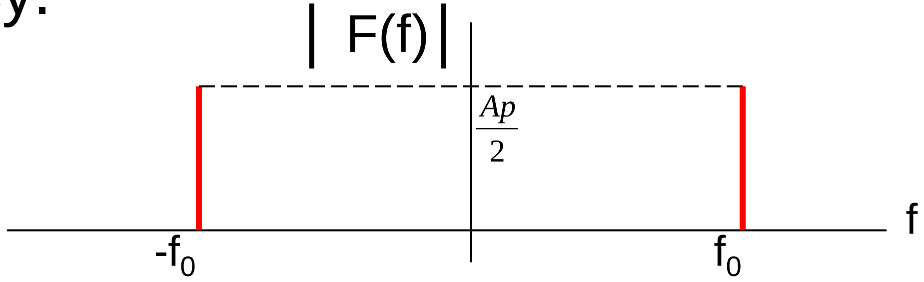
F(*f*) is a function of frequency

Sine Wave in the Frequency Domain

- A sine wave of amplitude A_p and frequency f_o is represented mathematically as $f(t) = A_p \sin(2\pi f_o t)$,
- doing all the math, its Fourier Transform would be:

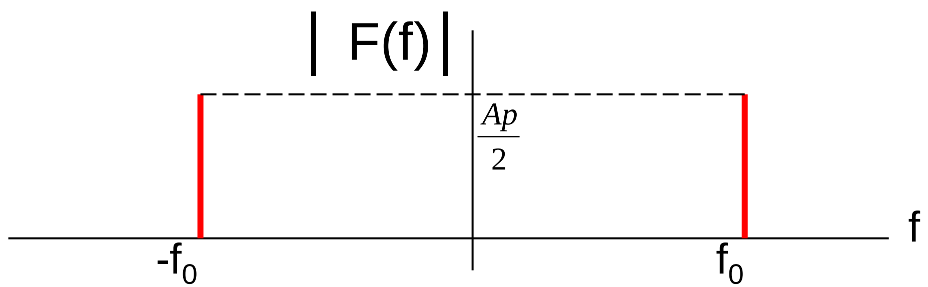
  $F(f) = (A_p/2)\ j\ [d(f - f_o) - d(f + f_o)]$
- where d is the Dirac (or Impulse) Function, defined as

  $d(0) = 1$ and zero everywhere else.
- Observing the magnitude only, and ignoring the phase, the sine wave in the frequency domain would look like this with the x axis now representing the frequency.



Sine Wave in the Frequency Domain

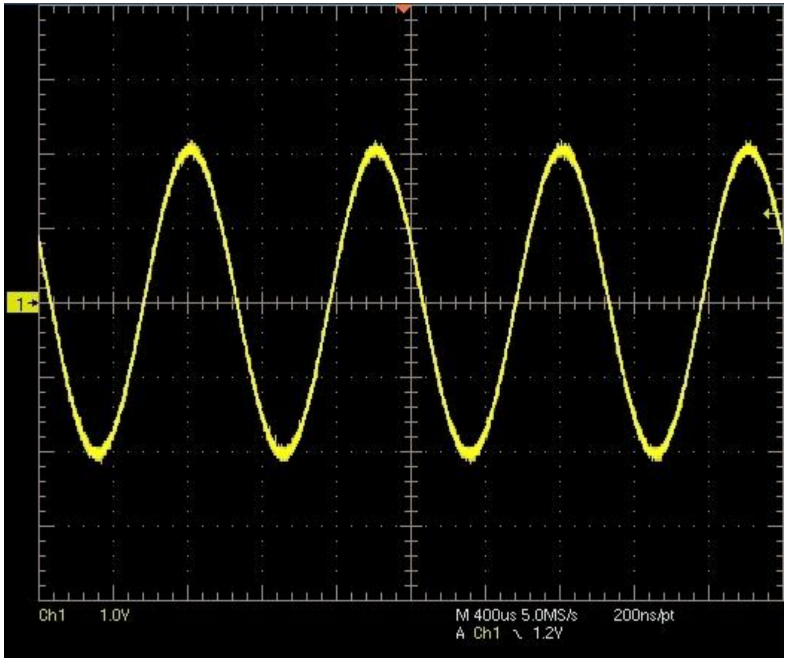
This is a mathematical representation of the sine wave in the frequency domain, which is why you will see that a negative frequency of $-f_o$ is present.

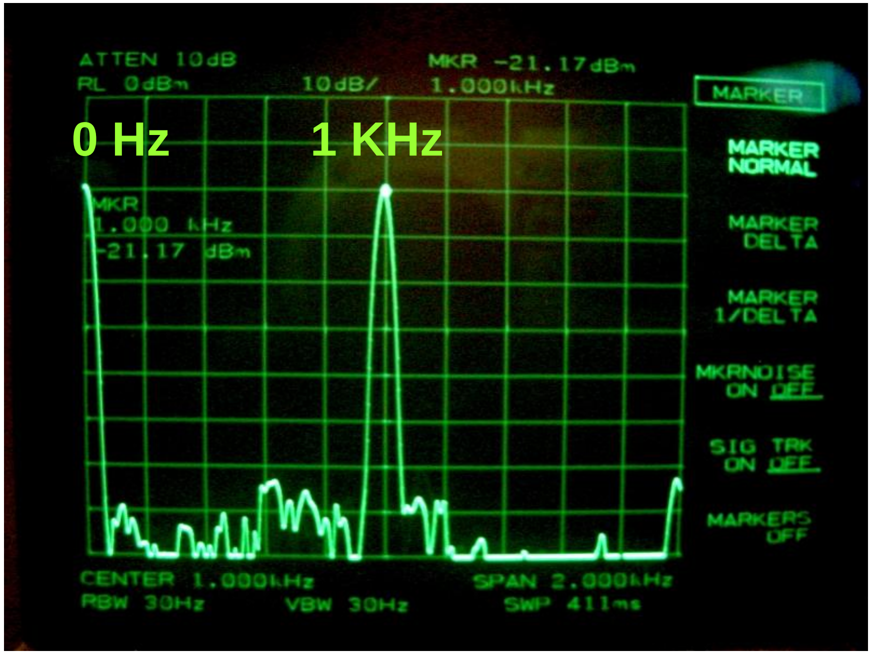


This will also be present on a spectrum analyzer, where negative frequency spikes are seen to the left of 0 Hz. Normally the negative frequencies are ignored.

Time Domain/Frequency Domain

The main thing to understand is that a sine wave will appear as a spike at the frequency f_o in the frequency domain.

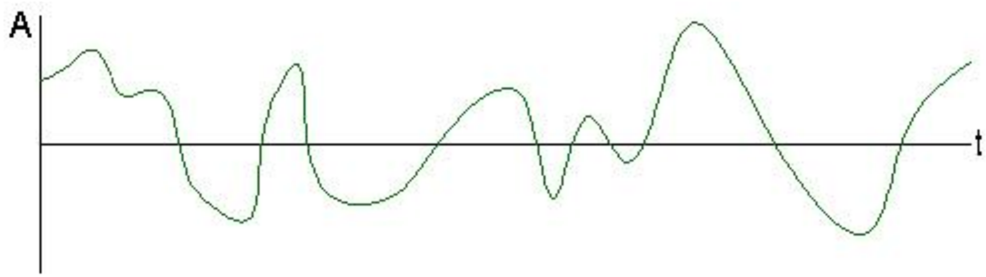




Screen shots of a 1KHz sine wave on an oscilloscope (time domain) and a spectrum analyzer (frequency domain). There is also a spike at –1 KHz (off to the left and not shown)

Multiple Sine Waves

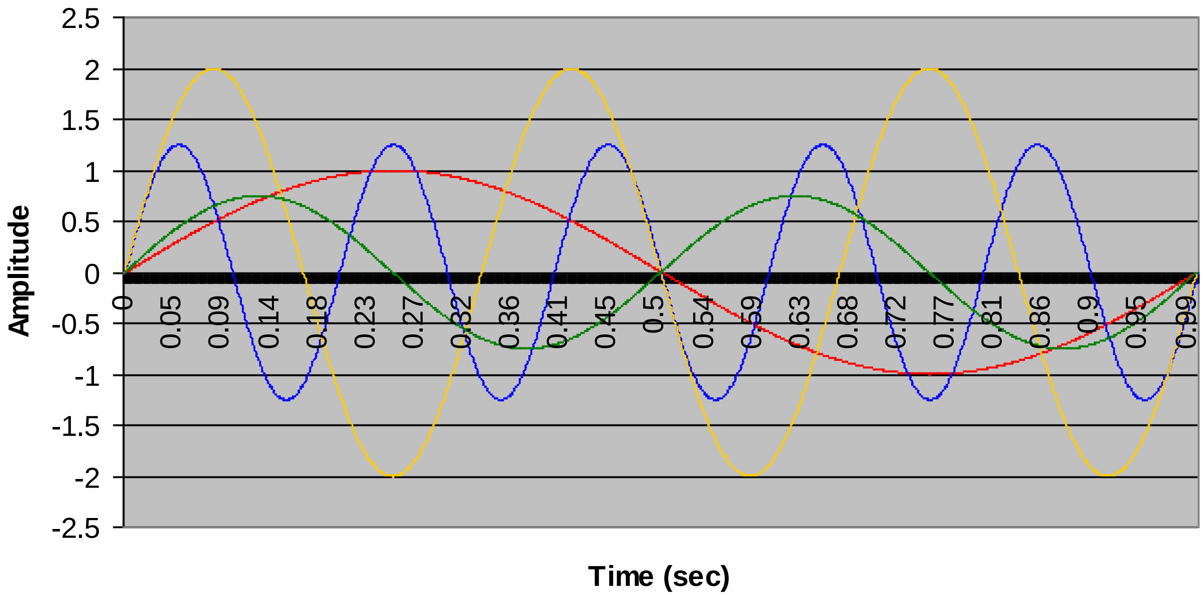
Signals that are measured on an aircraft are not pure sine waves. They contain numerous sine waves of different frequencies, amplitudes and phases. The sensor within the transducer detects the sum of all the frequency components present in the physical measurements made on the aircraft. The transducer then outputs an electrical signal which is proportional to the physical measurement.



Multiple Sine Waves

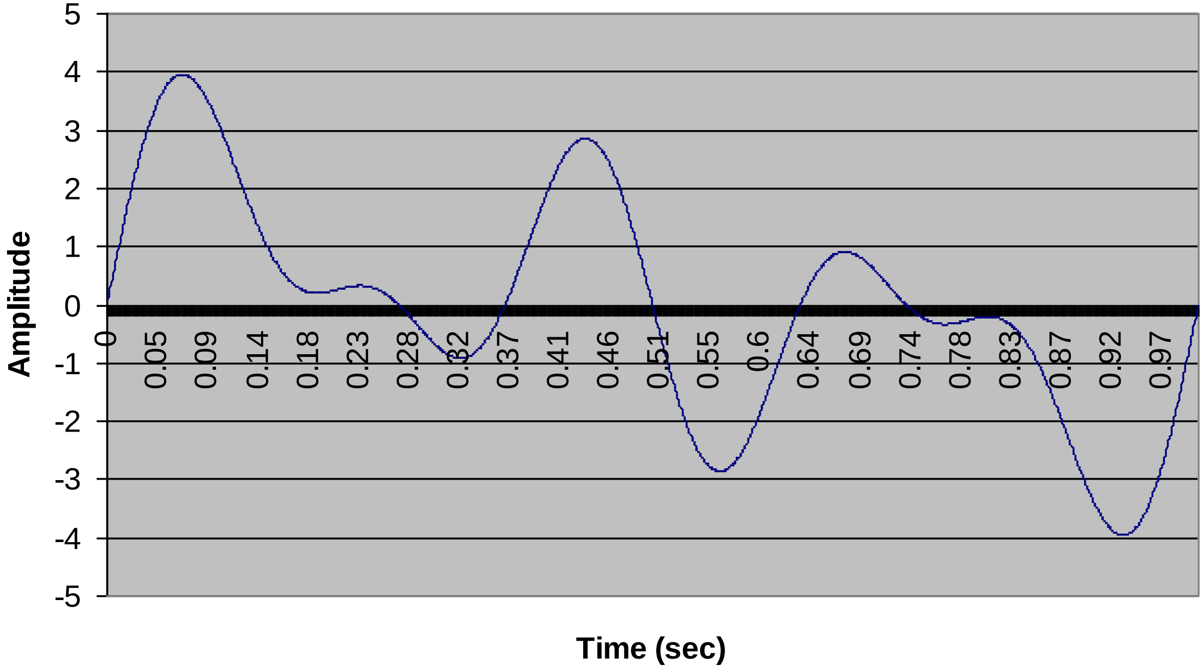
A composite signal is made up of the following sine waves:

color	amplitude	frequency	signal function
*	1.00	1 Hz	$\sin(2\pi t)$
*	0.75	2 Hz	$.75\sin(4\pi t)$
*	2.00	4 Hz	$2\sin(8\pi t)$
*	1.25	5 Hz	$1.25\sin(10\pi t)$



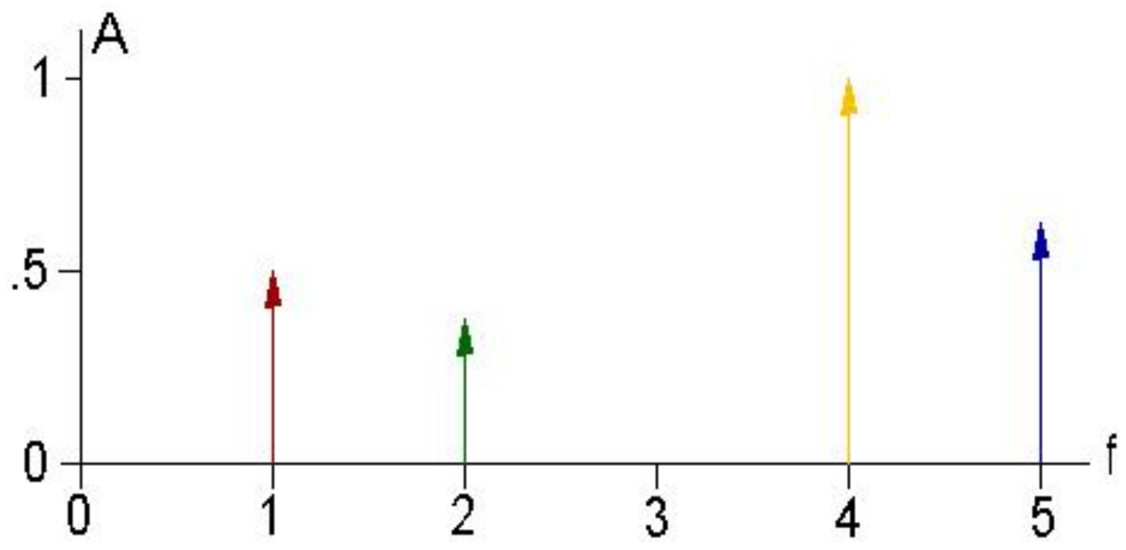
Multiple Sine Waves

Sum the four sine waves together and this is the resultant signal



Multiple Sine Waves

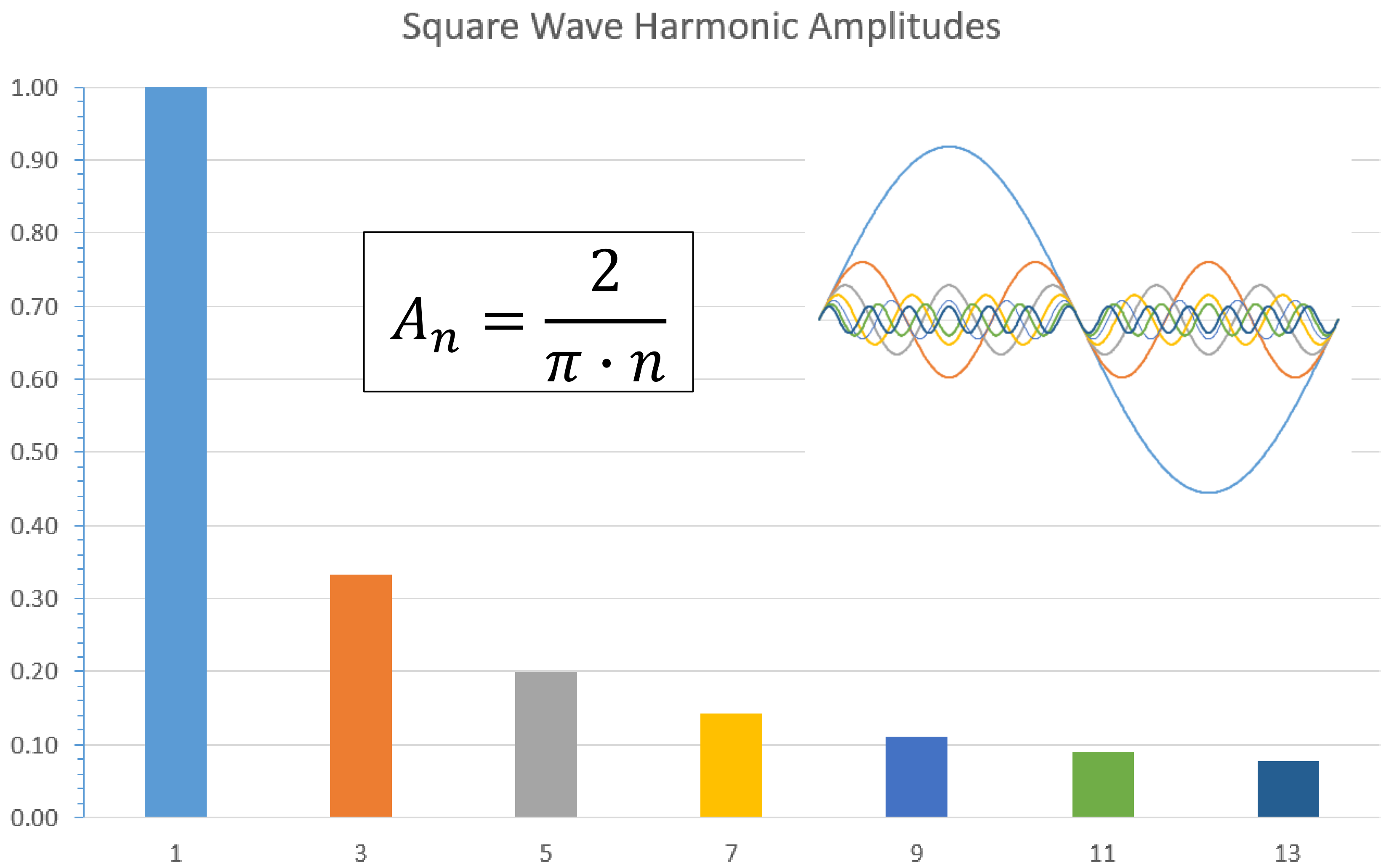
- In the frequency domain, each sine wave will have a spike at its respective frequency and amplitude as shown below.



Square Waves

- **Square waves are composed of an infinite number spectral components**
- **Square waves have spectral components at odd harmonics (i.e. multiples) of the square wave frequency**
  - **f_c, f_cx3, f_cx5, etc.**
  - **No energy at even harmonics**
- **Amplitude of harmonic levels decreases with frequency**
  - **But the number of harmonic components is infinite**

Square Waves

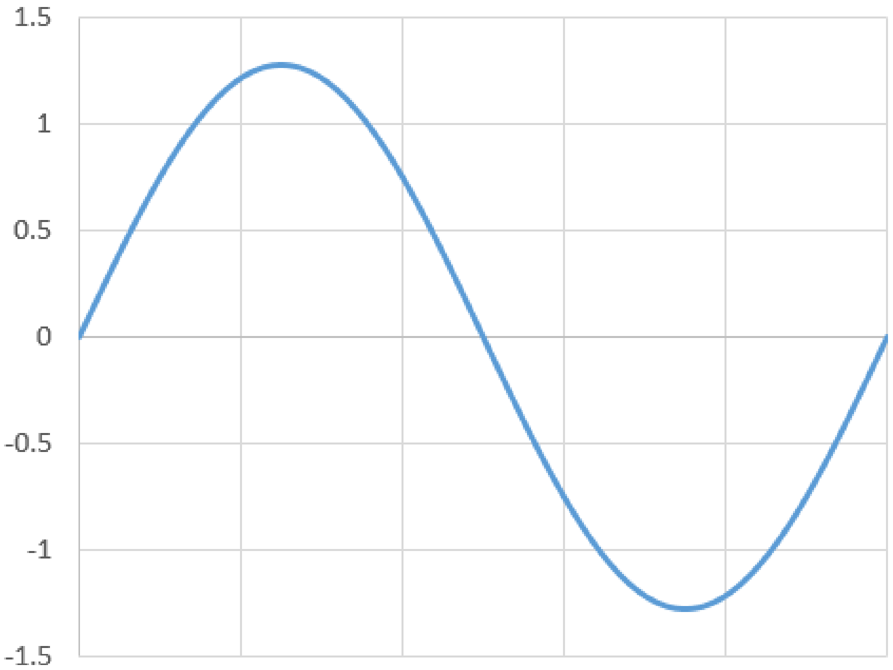


$$A_n = \frac{2}{\pi \cdot n}$$

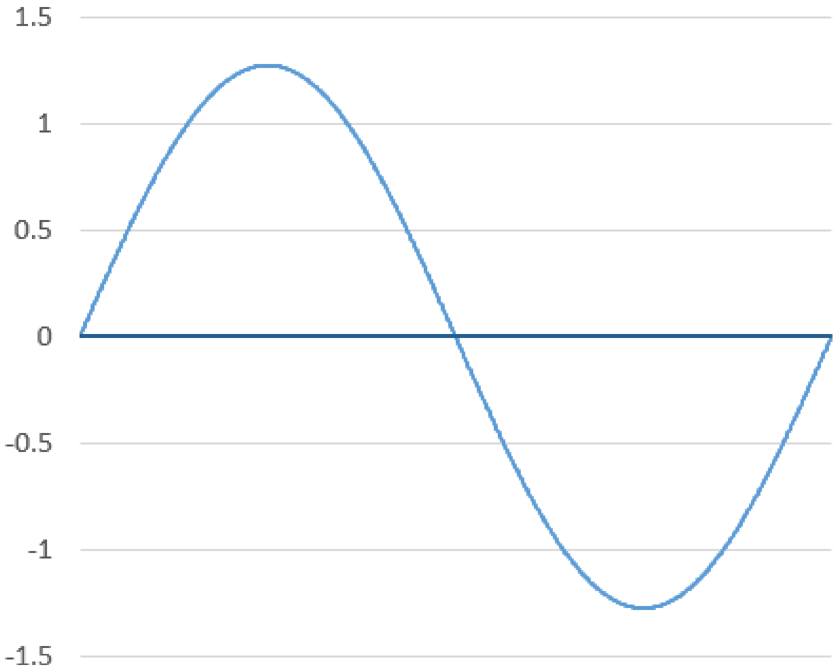
Building a Square Wave

Fundamental

Sum



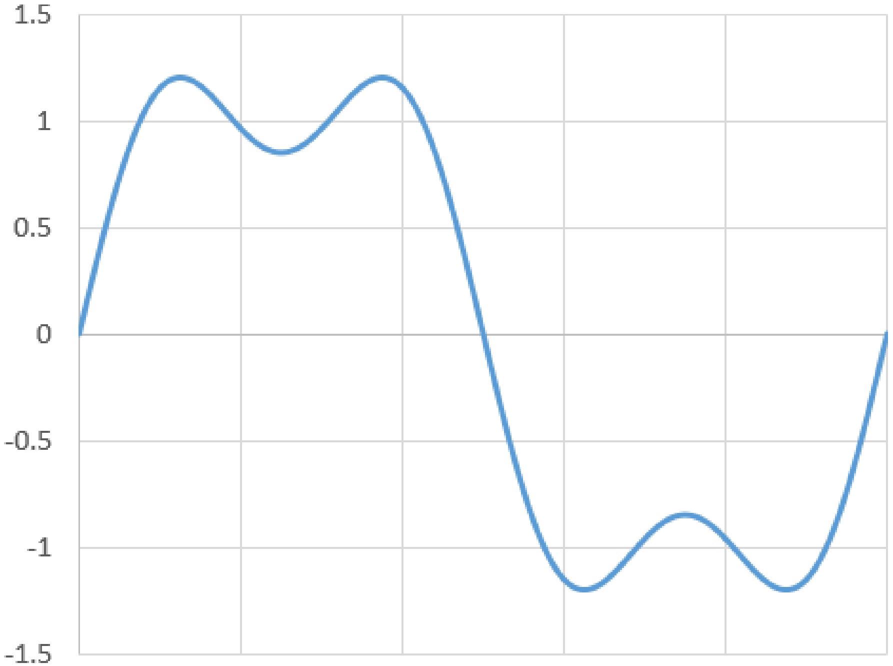
Components



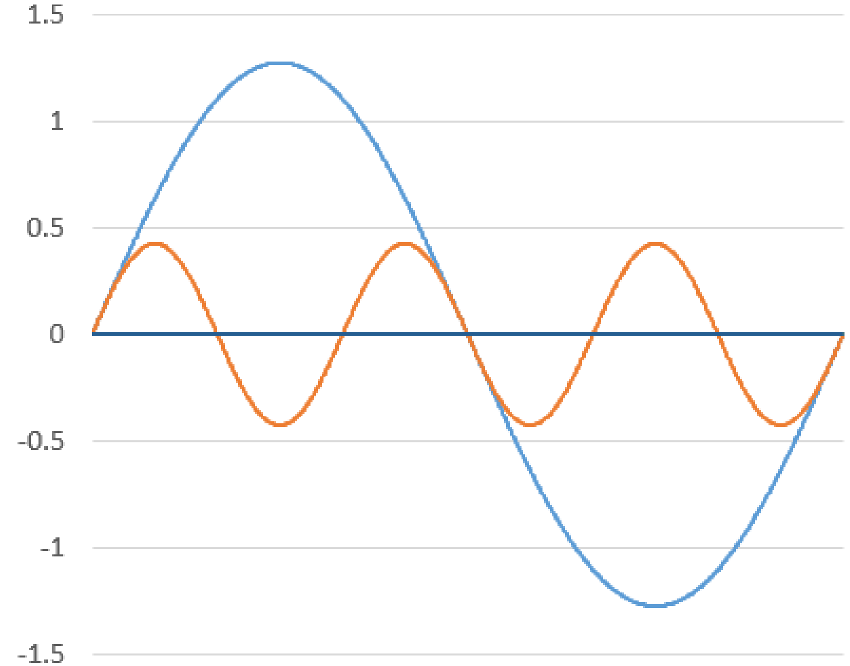
Building a Square Wave

Fundamental + 1 Harmonic

Sum



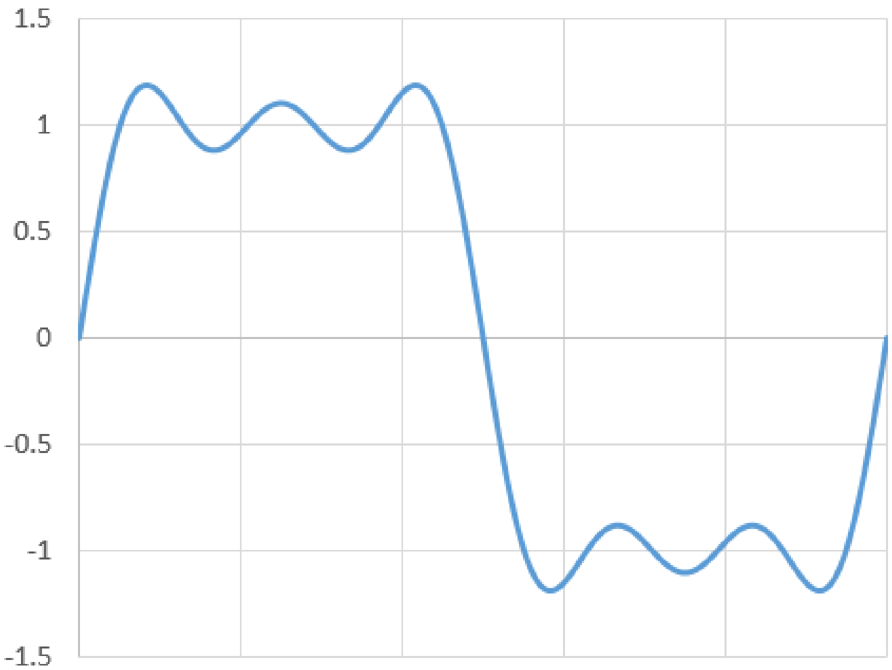
Components



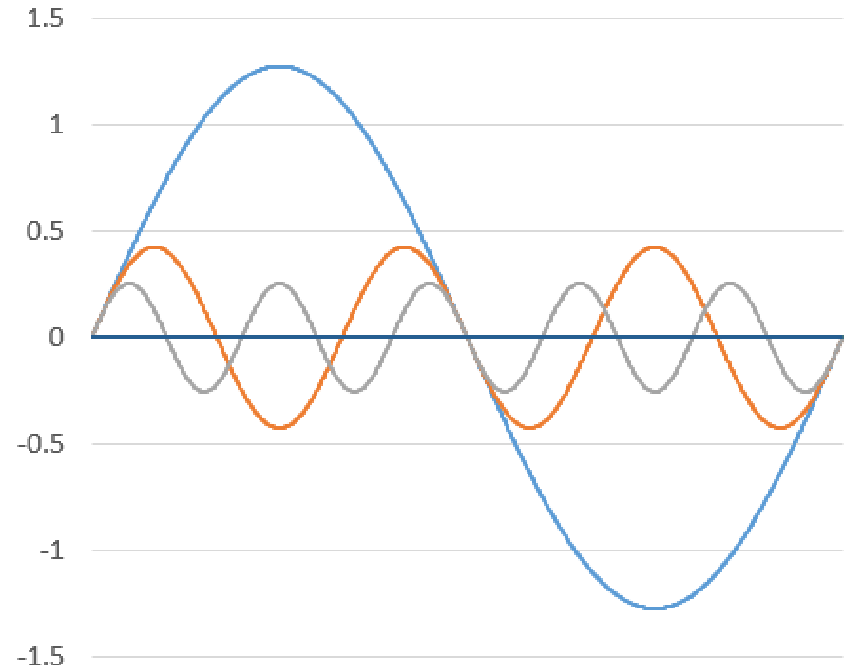
Building a Square Wave

Fundamental + 2 Harmonics

Sum



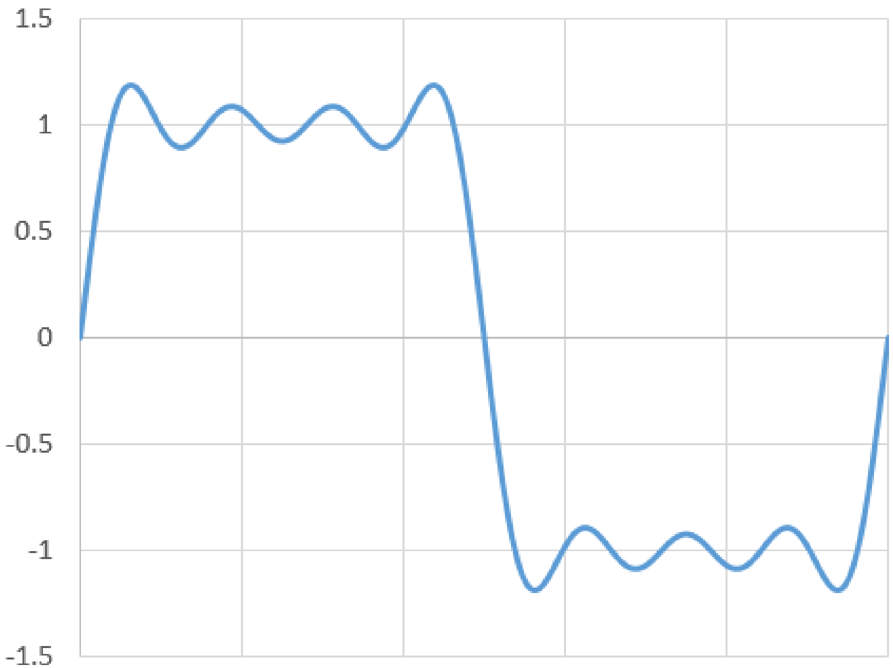
Components



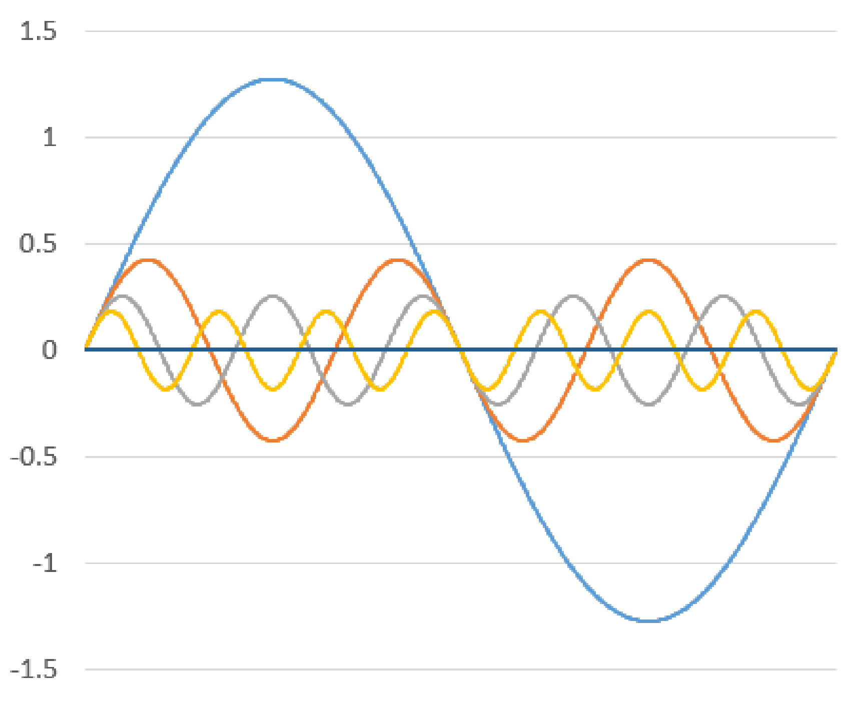
Building a Square Wave

Fundamental + 3 Harmonics

Sum



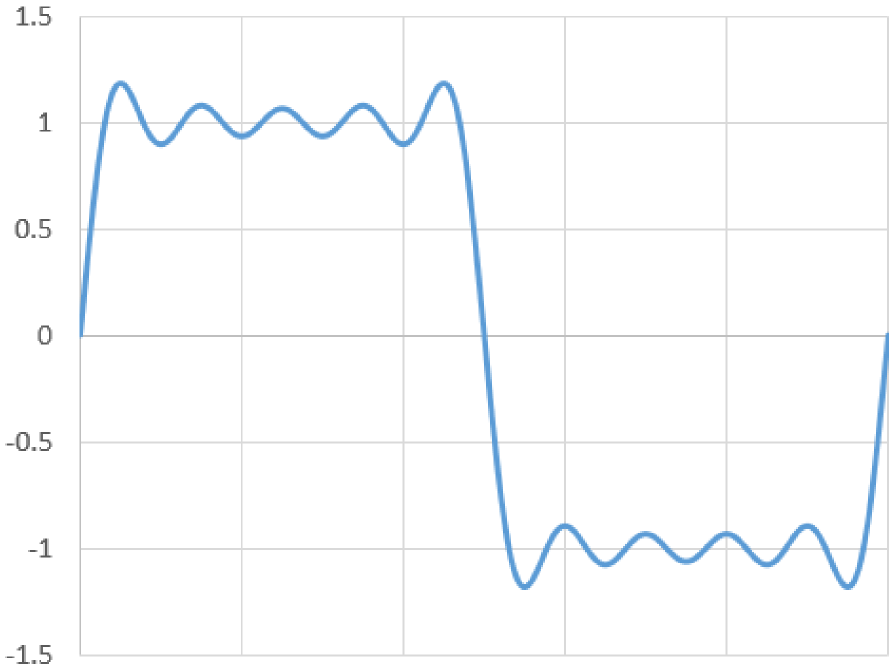
Components



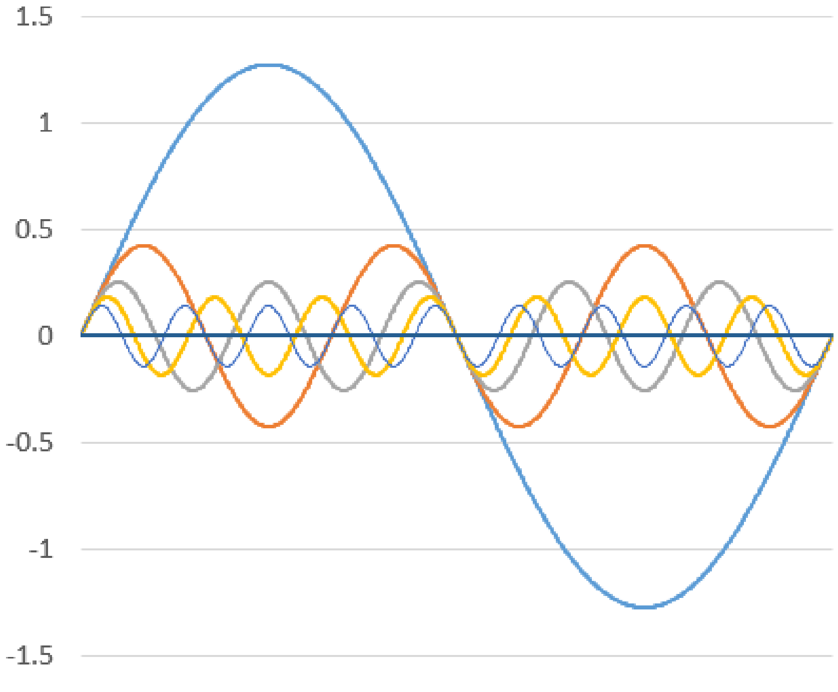
Building a Square Wave

Fundamental + 4 Harmonics

<u>Sum</u>



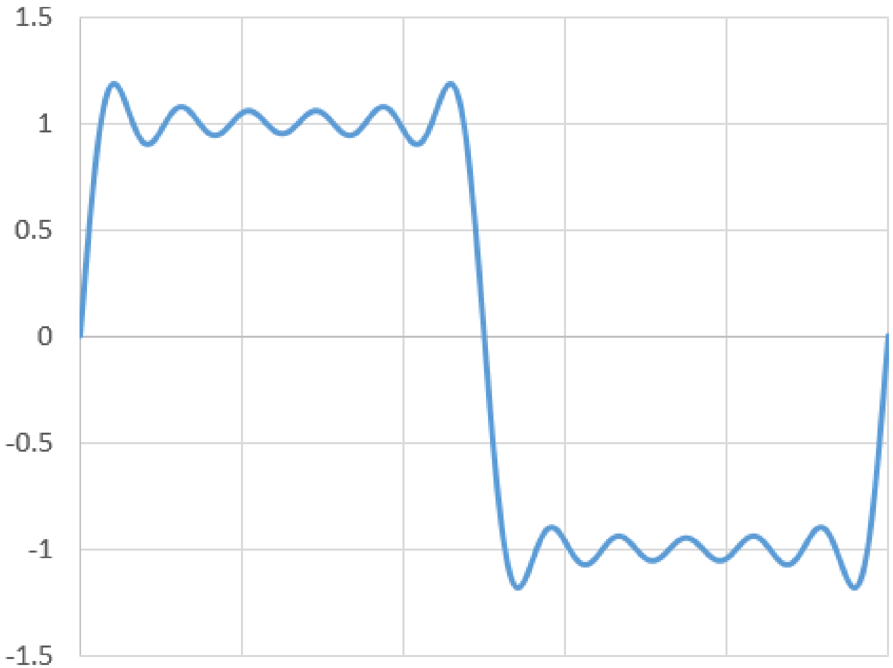
<u>Components</u>



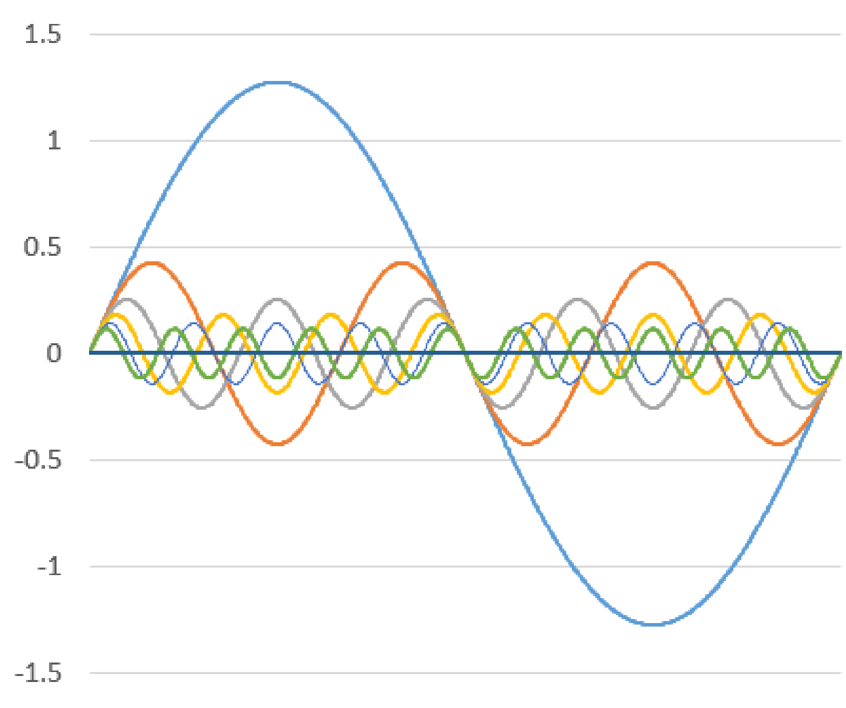
Building a Square Wave

Fundamental + 5 Harmonics

<u>Sum</u>



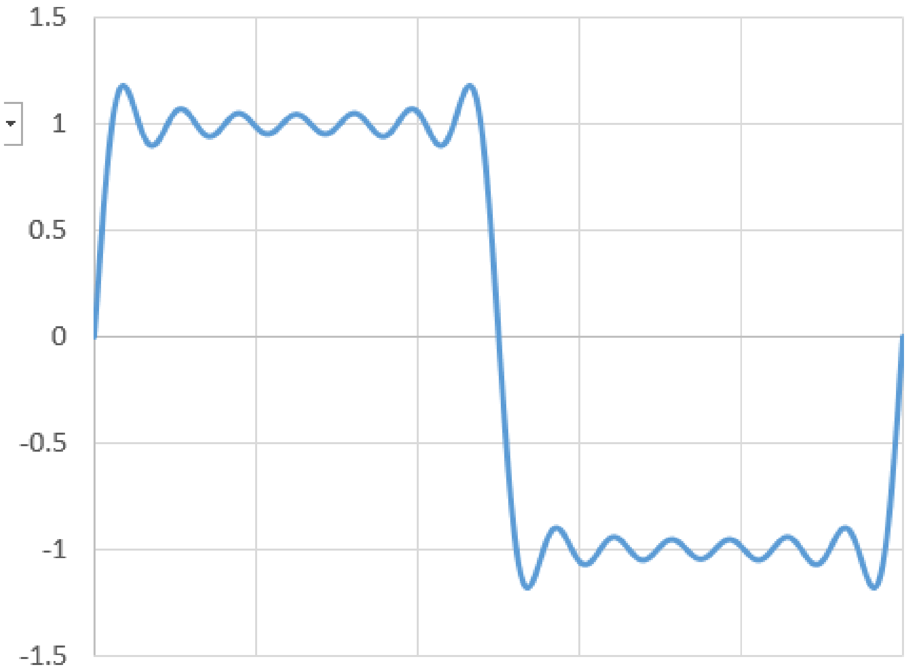
<u>Components</u>



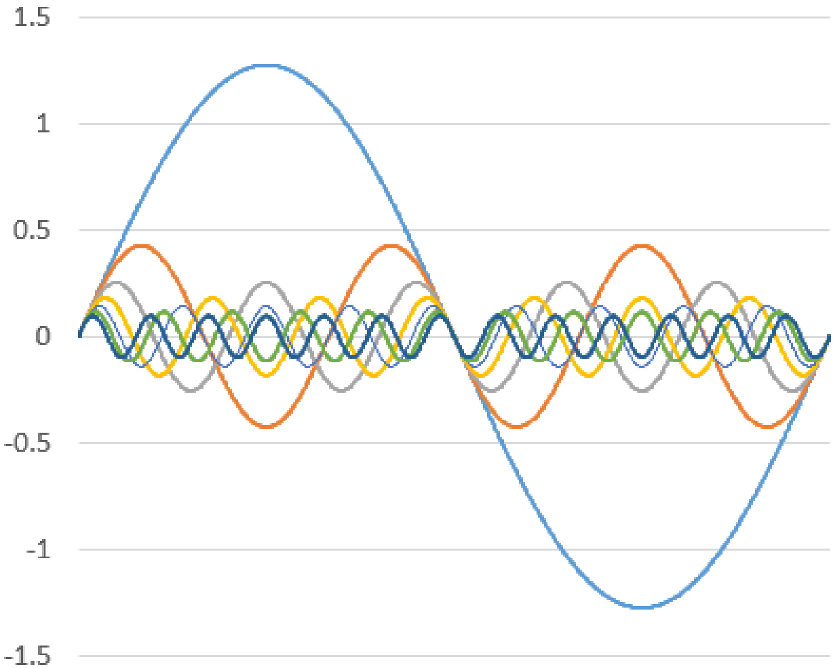
Building a Square Wave

Fundamental + 6 Harmonics

<u>Sum</u>



<u>Components</u>

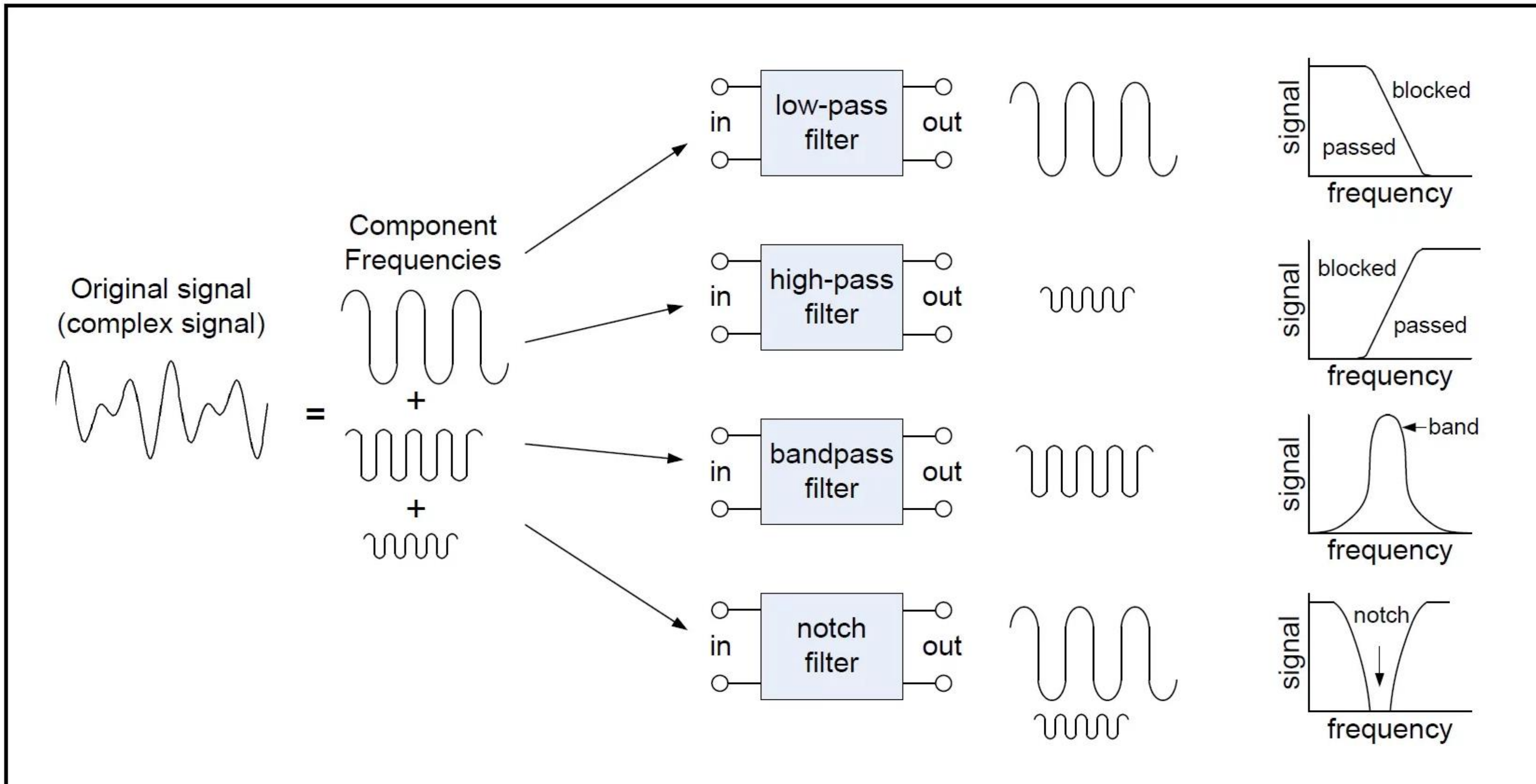


Filtering .. What is It?

- The modification of selected frequency components of the signal.
- Normally, the objective is to reduce unwanted components such as noise or other interfering signals.

- Filtering can be done in:
  - The *Analog Domain*, using filters using analog components such as resistors, capacitors, inductors and operational amplifiers
  - The *Digital Domain* using filters that operate in:
    - **The *Time Domain*.**
    - **The *Frequency Domain*.**

Major Types of Filters



Analog vs. Digital Filtering

- Analog Filtering has traditionally been used to perform Spectrum-Modification operations.
  - Anti-Alias Filters
  - Notch Filters.
- Modern digital filtering techniques have superseded many analog filtering operations because:
  - They are more accurate, reliable and predictable.
  - They can be performed with general-purpose hardware.
  - They are (becoming) less expensive.
- Exceptions:
  - Some parts of Anti-Aliasing applications
  - Very-high-speed applications.

Analog vs Digital Filters

Analog filters are made with:

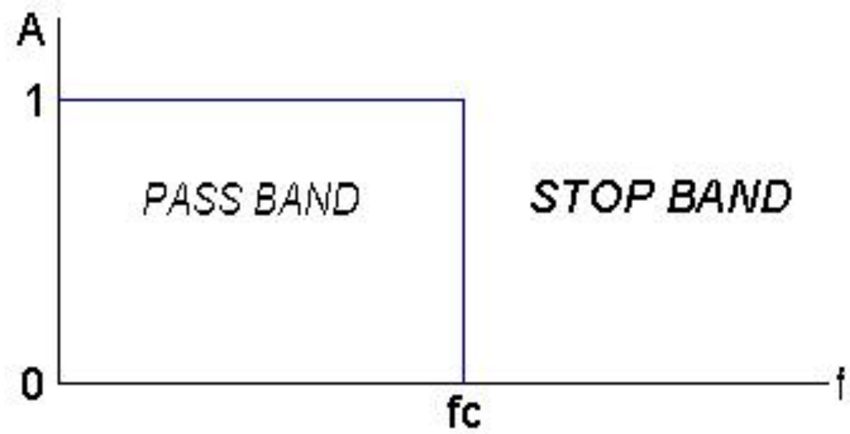
- Resistors, inductors, capacitors, amplifiers
- Other parts cause delay which cause constructive or destructive interference and thus result in frequency dependent responses.

Digital filters are made with:

- Specialized microprocessors (DSP chips) or ASIC with digital logic architectures that are optimized for digital signal processing.
- The fundamental elements of a digital filter are multiply/accumulate and delay.
- Delayed signals are added together causing constructive and destructive interference and thus frequency dependent responses.

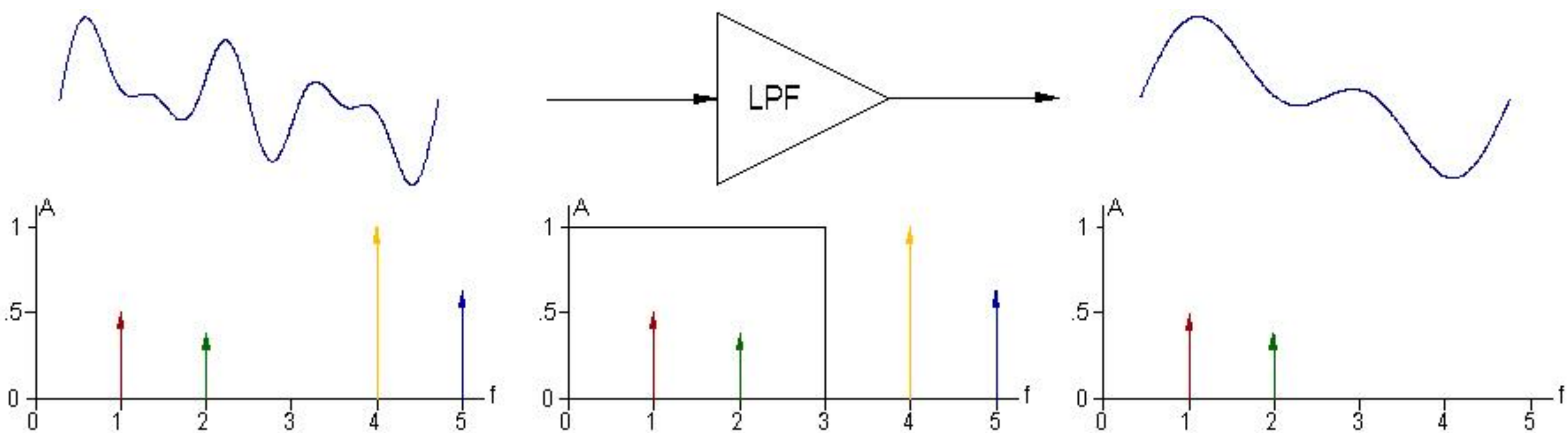
The Ideal Low-Pass Filter Example

The ideal low-pass filter has a gain of 1 in the pass band and a gain of 0 in the stop band. The cutoff frequency is $f_{c.}$ All frequencies below f_c are passed through the filter, but no frequencies above f_c are passed through. The frequency response of a low pass filter looks like the diagram below.



The Ideal Low-Pass Filter Example

First, the signal must be observed in the frequency domain in order to see the effects of the filter. Only frequency components within the pass band of the filter pass through with a gain of 1, all other frequencies are removed (gain of 0).



Real Low Pass Filters

- Ideal filters don't exist
- Filters are designed by selecting a *transfer function* that is non-ideal but has some "optimum" property
- Some transfer function optimum properties include:
  - Flat gain within the pass band
  - Steep attenuation outside the pass band
  - "Well behaved" phase characteristics
- All filter designs begin with an optimum transfer function equation
- When you select a filter type be mindful of what kind of performance you are trying to optimize

Some Concepts to Understand First: Transfer Function

- Transfer functions show the relationship between an input and an output
- Filters have one input and one output
- The function could be expressed as a function of time or as a function of frequency but in filter design it is much more natural to express as a function of frequency

$$H(s) = \frac{V_{out}}{V_{in}}$$

- Complex frequency "s" is normally used

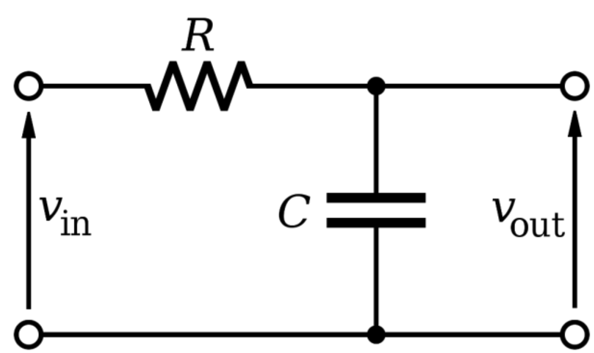
$$s = \sigma + j\omega = \sigma + j2\pi f$$

Some Concepts to Understand First: RC Filter Transfer Function Example

- In this example we start with a circuit and derive the transfer function
- In filter design we start with a transfer function and derive (or synthesize) the circuit

- An RC filter transfer function
  - Output (magnitude and phase) is defined for any and all input frequencies

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{1}{sRC + 1}$$



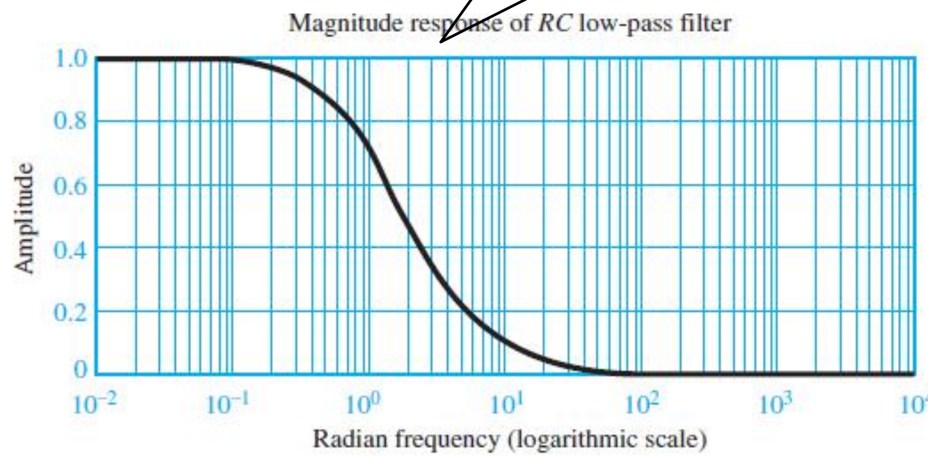
Some Concepts to Understand First: RC Filter Transfer Function

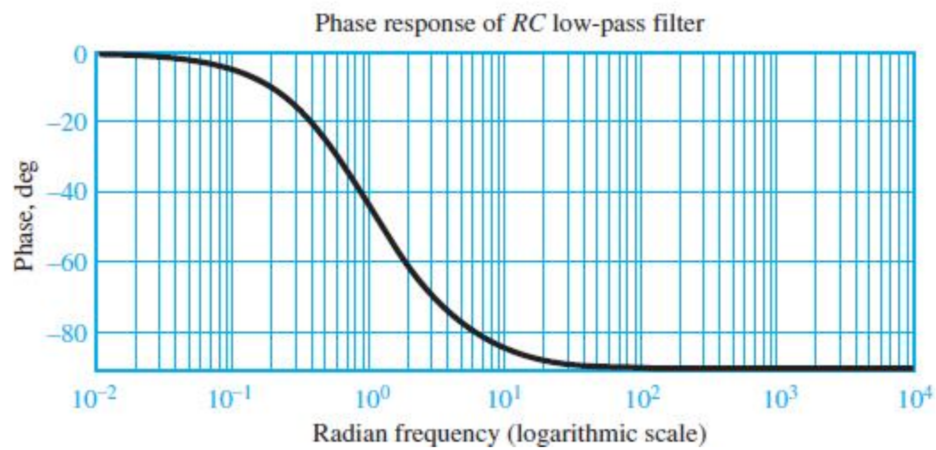
Note: Not a Bode Plot

Normally we are only interested in using the real frequency response component so *"s"* is simplified to *"f"*

For $\sigma = 0$ and $\omega = 2\pi f$

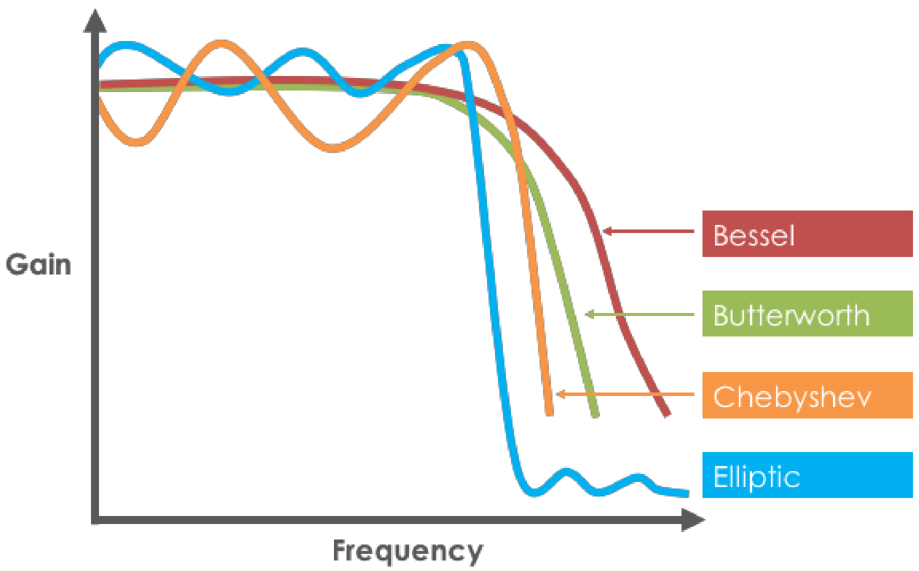
$$H(f) = \frac{1}{j2\pi f RC + 1}$$





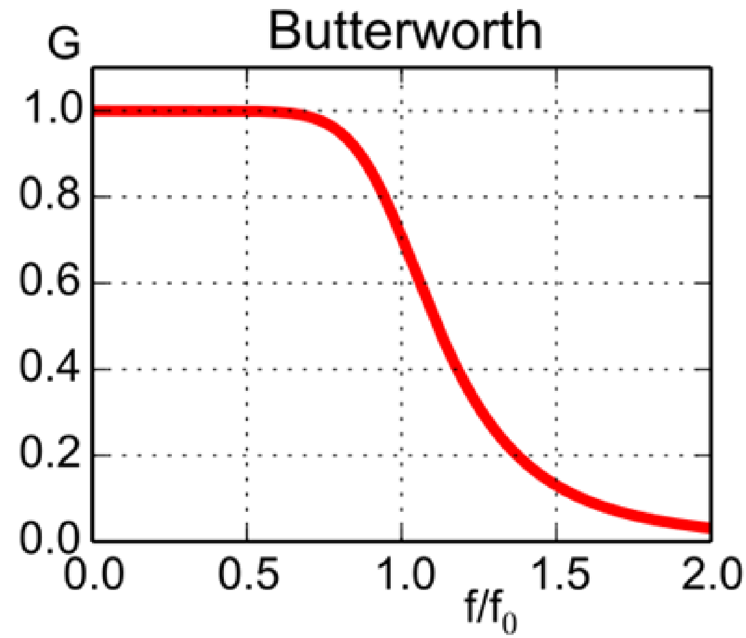
Optimum Filter Types

- Butterworth
- Chebyshev
- Bessel
- Elliptic



Optimum Filter Types - Butterworth

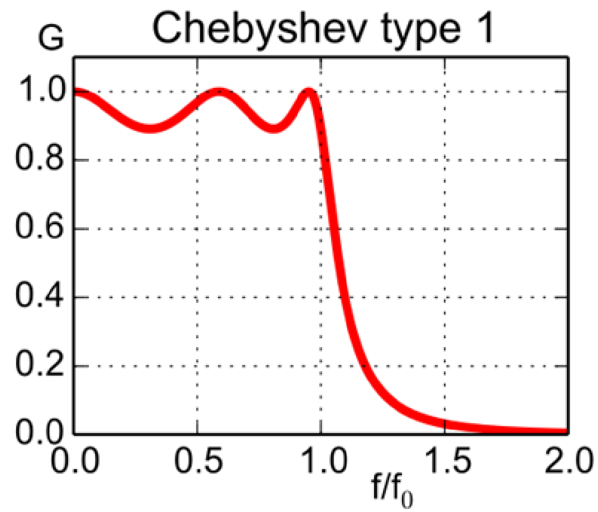
- Pros
  - Maximally flat passband
  - Moderate phase distortion
  - Easy to analyze
  - Easy to implement
- Cons
  - Slow attenuation drop off

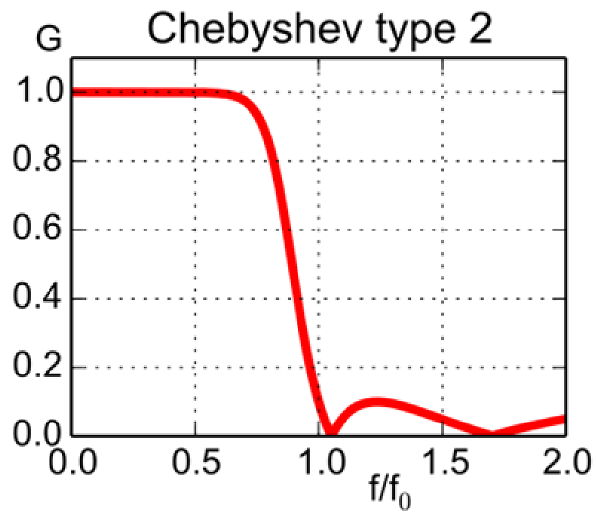


Optimum Filter Types - Chebyshev

- Pros
  - Steeper attenuation drop off
  - Easy to implement
- Cons
  - Ripples in the passband
  - Poor phase distortion

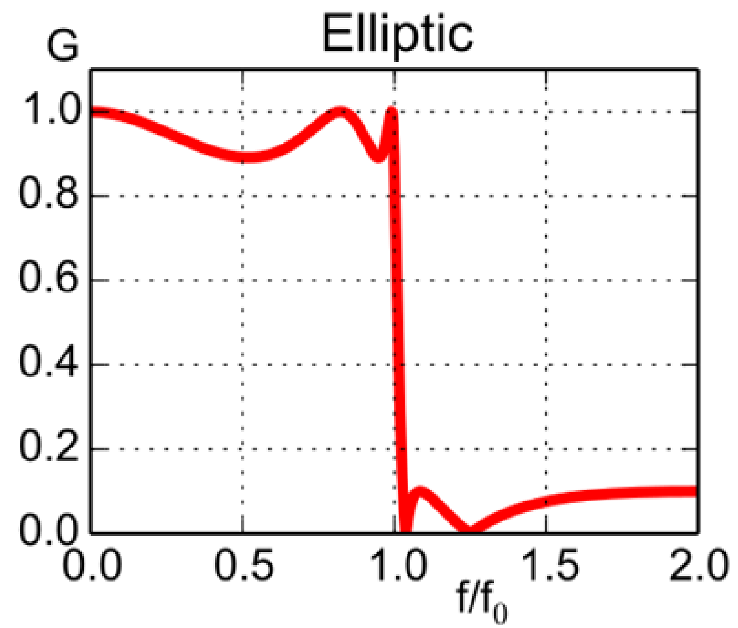
- Inverse Chebyshev has ripples in the stop band





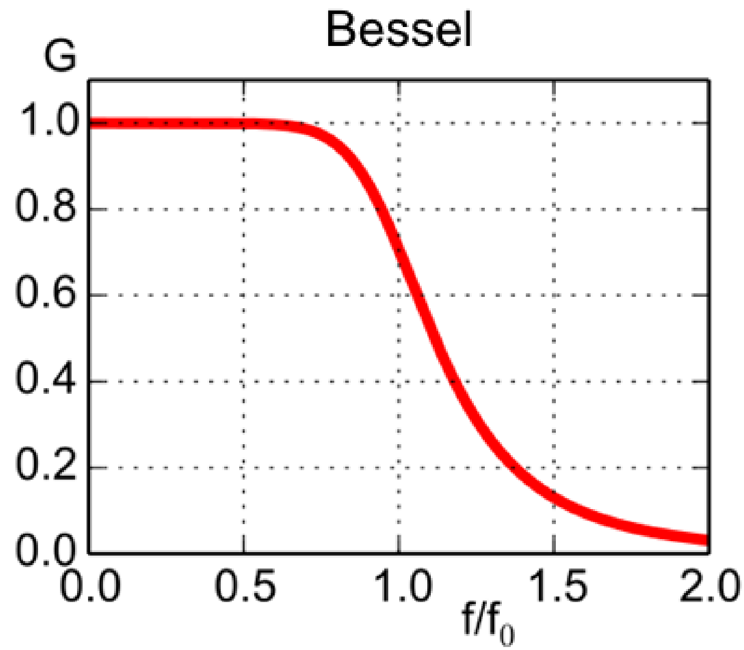
Optimum Filter Types - Elliptic

- Pros
  - Sharpest cutoff
- Cons
  - Poor phase distortion
  - More complicated implementation
  - Ripples in the pass band and stop band



Optimum Filter Types - Bessel

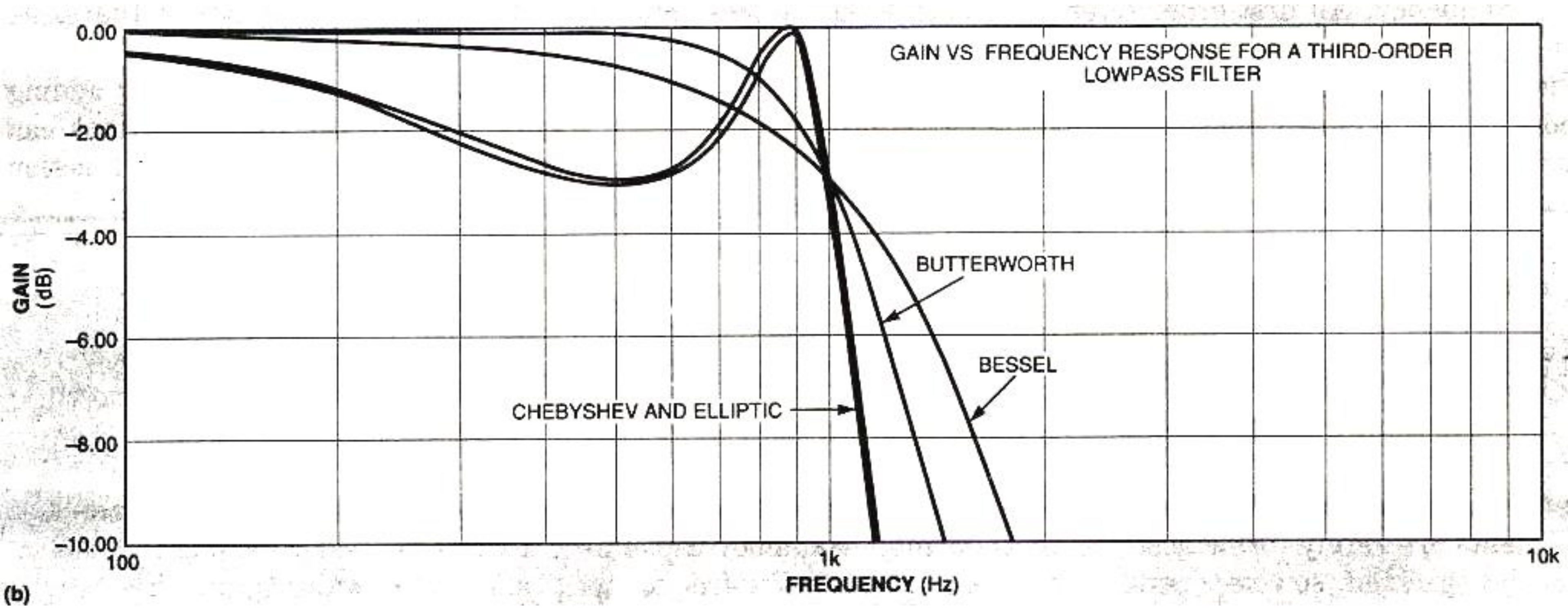
- Pros
  - Ideal linear phase response
  - Constant group (i.e. time) delay
  - Good for use with square waves
- Cons
  - Slow attenuation drop off



Because of their constant group delays, Bessel filters are commonly used for the pre-modulation filtering of digital signals.

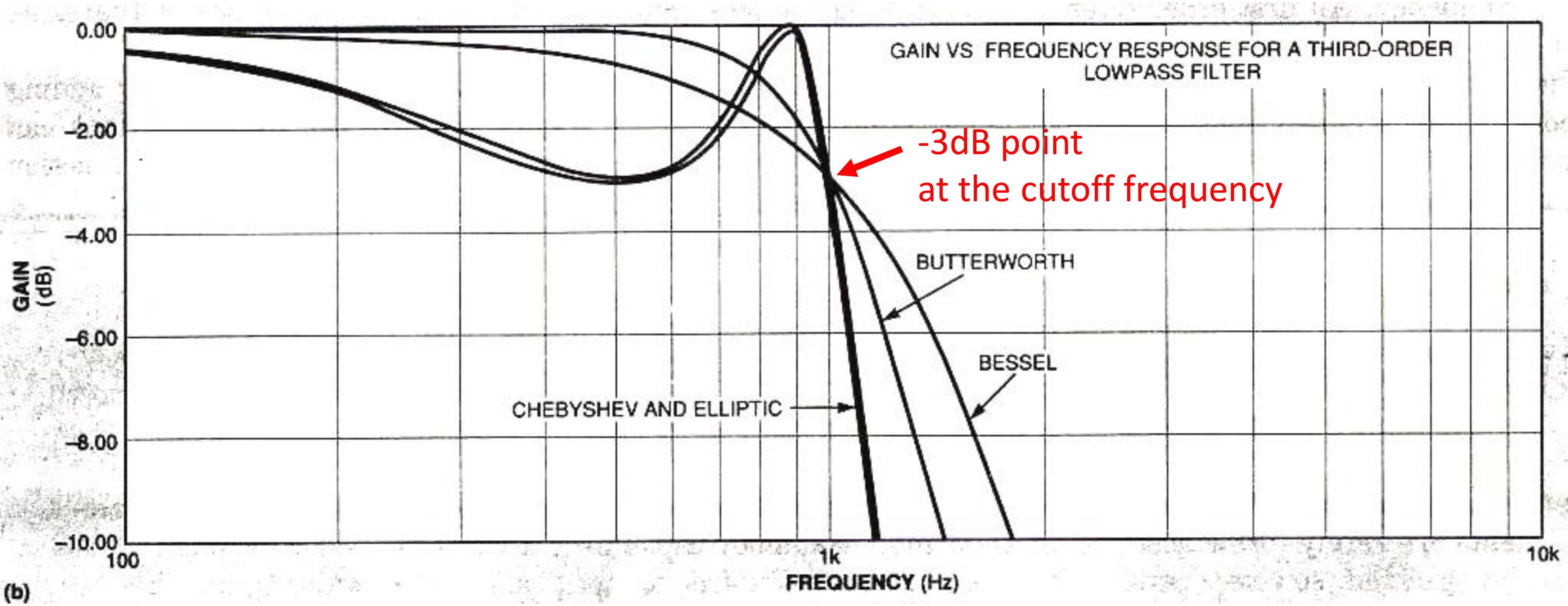
Comparison of Real LPF's

This is a Bode Plot that compares the magnitudes of a filter with a 1KHz cutoff frequency in each of the four filter representations.



Comparison of Real LPF's

Another view of the Bode plots showing more detail around the cutoff frequency defined to be the -3dB point.



Optimum Filter Transfer Functions

- **Butterworth**

$$G_n(\omega) = \frac{1}{\sqrt{1 + \omega^{2n}}}$$

- **Chebyshev**

$$G_n(\omega) = \frac{1}{\sqrt{1 + \varepsilon^2 T_n^2(\omega)}}$$

T_n is a Chebyshev polynomial of the n^{th} order

- **Bessel**

$$G_n(\omega) = \frac{\theta_n(0)}{\theta_n(\omega)}$$

$\theta_n(s)$ is a reverse Bessel polynomial

- **Elliptic**

$$G_n(\omega) = \frac{1}{\sqrt{1 + \epsilon^2 R_n^2(\xi, \omega)}}$$

R_n is the n^{th} order elliptic rational function

Some Concepts to Understand First: What is an Octave and a Decade?

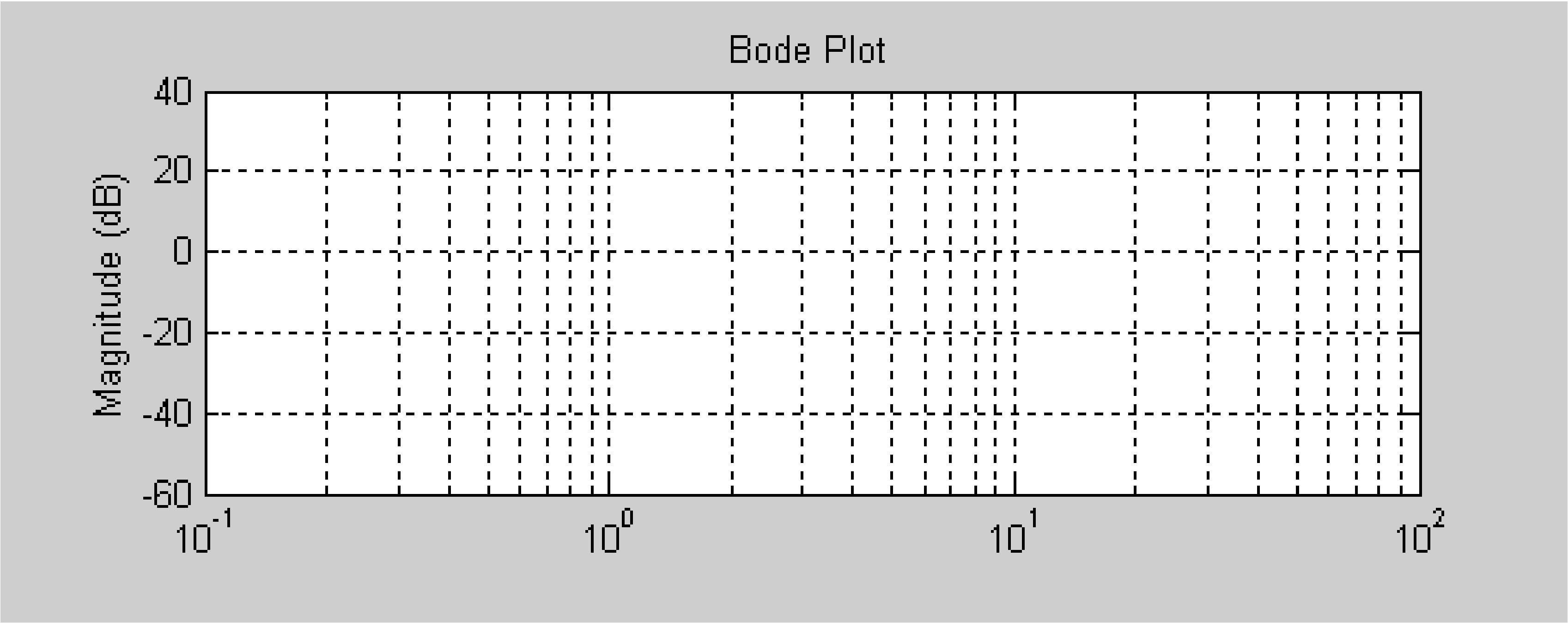
- An *Octave* is a doubling in frequency.
- 220 Hz to 440 Hz is an Octave
- 1000 Hz to 2000 Hz is an Octave

- A *Decade* is a multiplication of 10 in frequency.
- 100 Hz to 1000 Hz is a Decade
- 200 Hz to 2000 Hz is a Decade

Some Concepts to Understand First: What is a Bode Plot?

- Bode Plots are semi-log charts of the magnitude of the gain expressed in dB versus frequency on a logarithmic scale. The Bode Plot shows the characteristic features of a filter which will be shown in detail later.

Bode Plot Graph

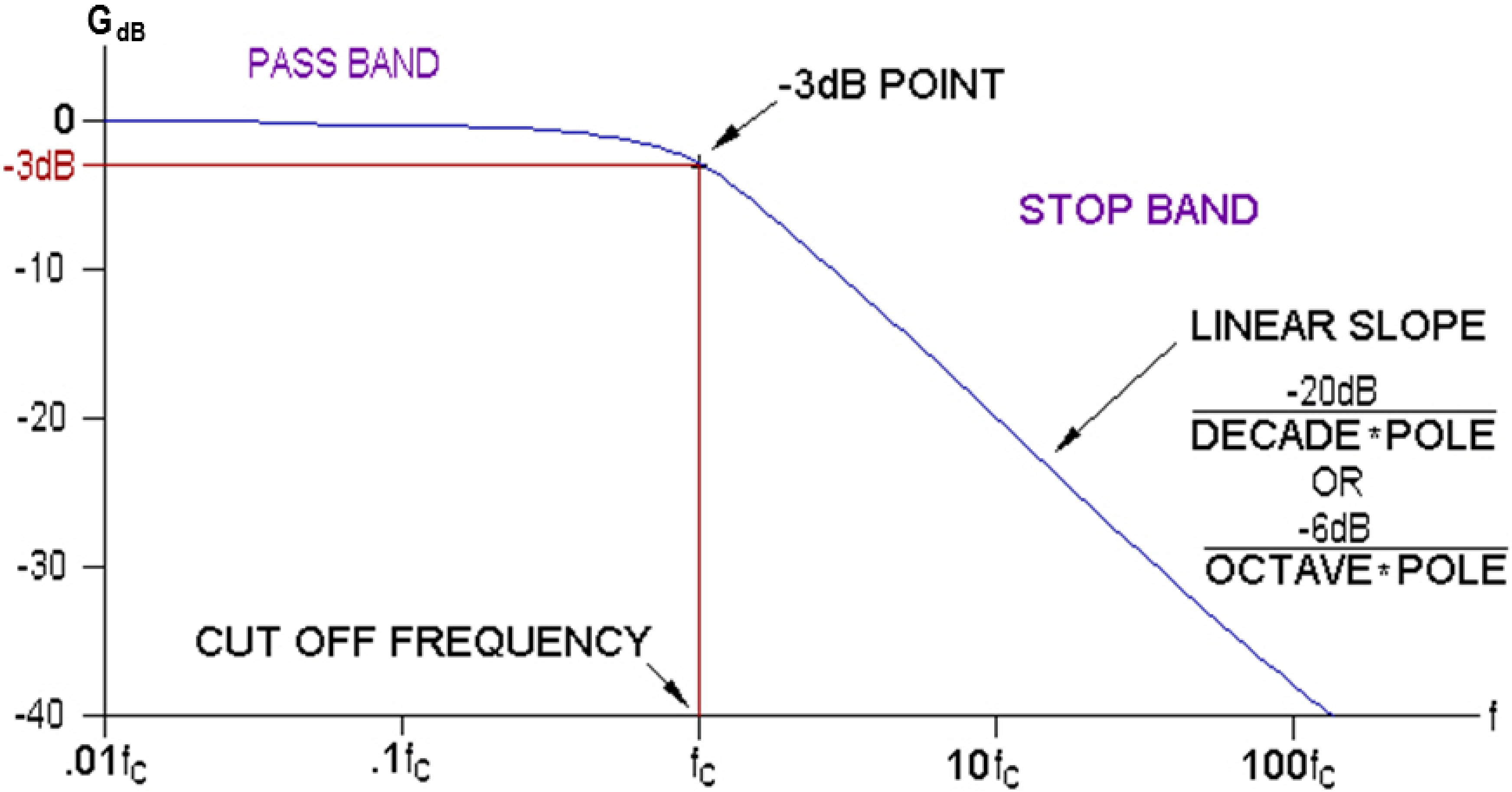


Notice that the x-axis does not go down to 0 or has negative values, why would this be?

Butterworth Filter

- The Butterworth filter is the most useful filter for most applications
  - Flat gain response across the passband
- First described by the British engineer and physicist Stephen Butterworth in 1930
  - At the time filter design was largely trial and error
- Butterworth developed a family of polynomials
  - Maximally flat in the pass band
  - Rolls off towards zero with increasing frequency
  - Increased “order” or complexity allows increased performance

Characteristics of a Butterworth Filter Magnitude of the Frequency Response (Bode Plot)

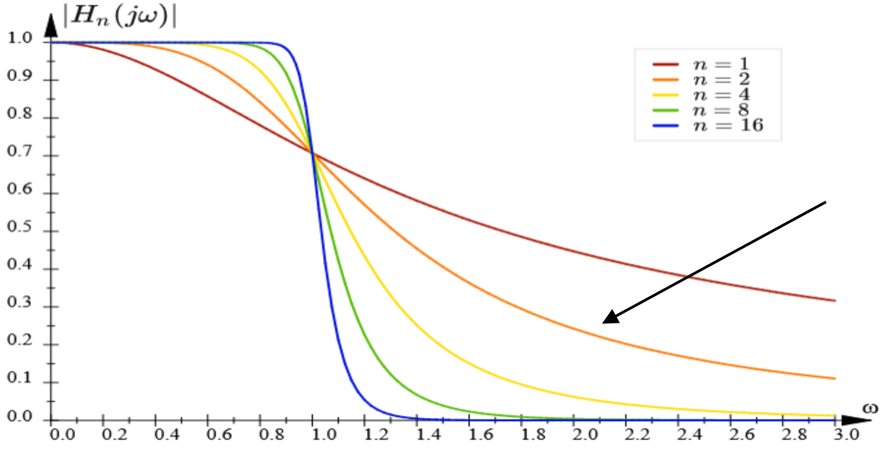


Butterworth Filter Polynomial

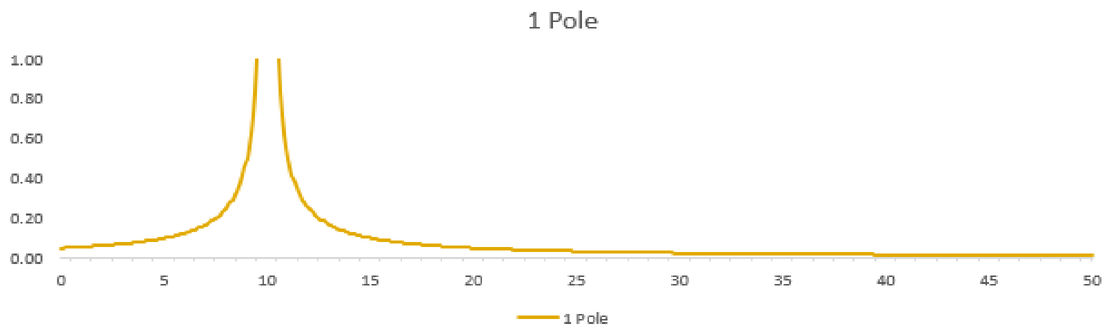
For example for a Butterworth filter with degree 2 the gain versus frequency is

$$\mathrm{H}(s) = \frac{1}{B_2(s)} \qquad \mathrm{G}(f) = \frac{1}{\sqrt{1 + (2\pi f)^4}}$$

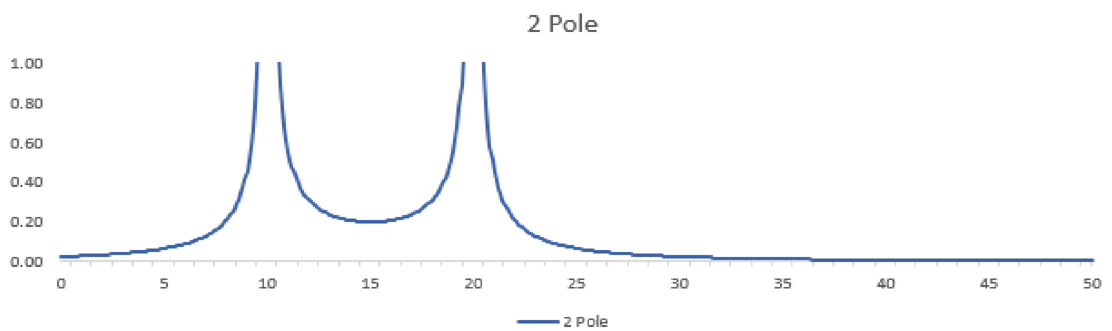
- At DC ($f = 0$) the gain is 1
- As f increases the gain decreases monotonically



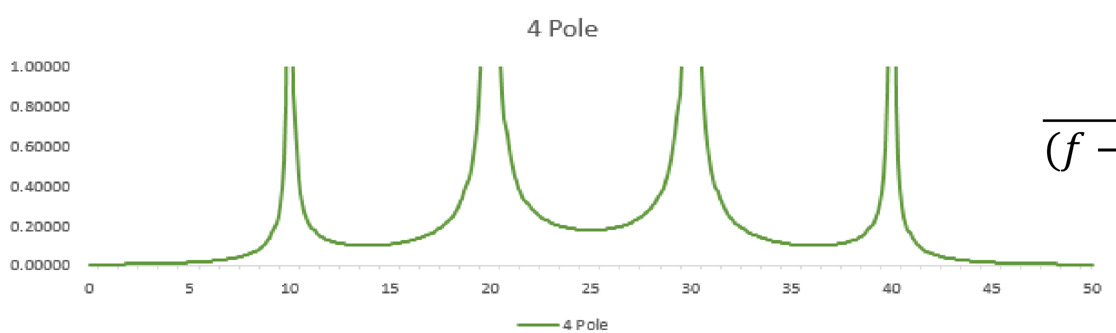
Some Concepts to Understand First: What is a Function Pole



$$\frac{1}{(f-10)}$$



$$\frac{1}{(f-10)(f-20)}$$



$$\frac{1}{(f-10)(f-20)(f-30)(f-40)}$$

Some Concepts to Understand First: What is a Function Pole

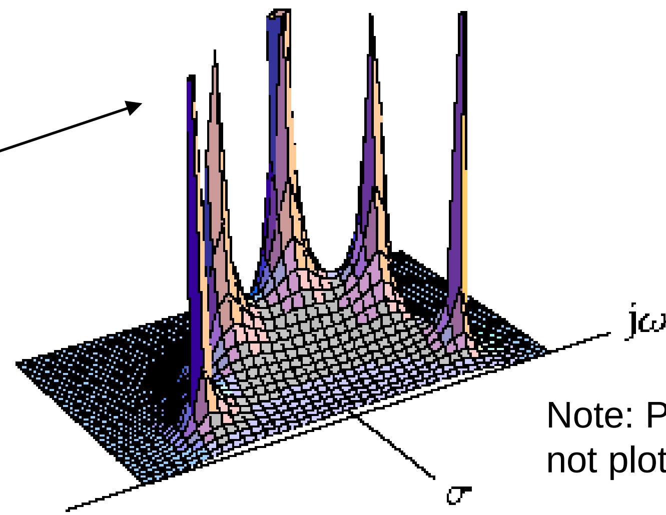
- Transfer Function Amplitude is a function of complex frequency "*s*"

$$H(s)$$

- Complex frequency "*s*" has two independent components

$$s = \sigma + j\omega \Longrightarrow \; H(\sigma + j\omega)$$

Function Poles



Note: Positive values for σ are not plotted here

Plot of $H(s)$ for values of σ and $j\omega$

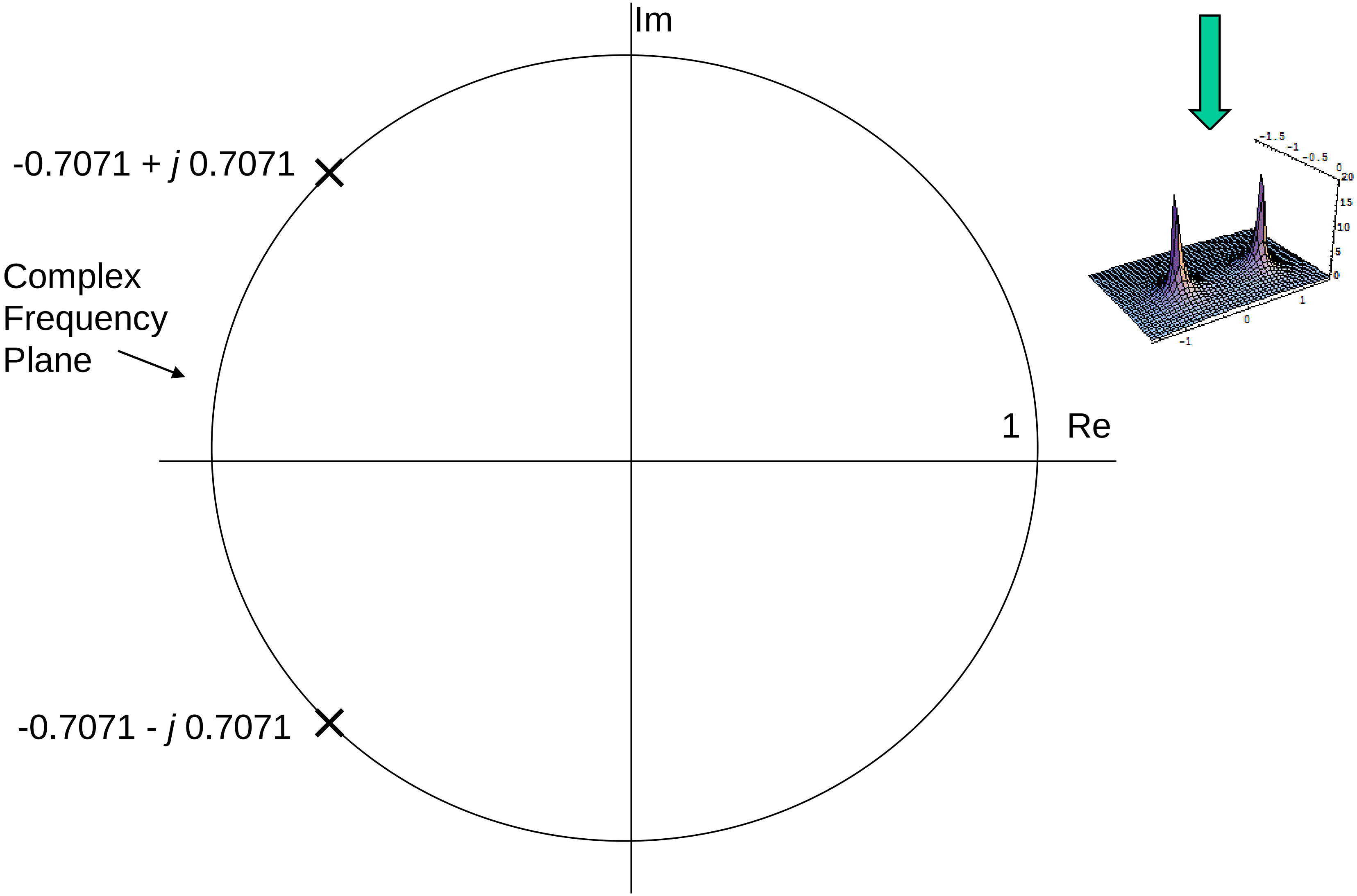
Butterworth Polynomial Factors

n	Factors of Polynomial $B_n(s)$
1	$(s+1)$
2	$(s^2+1.4142s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.7654s+1)(s^2+1.8478s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$
6	$(s^2+0.5176s+1)(s^2+1.4142s+1)(s^2+1.9319s+1)$
7	$(s+1)(s^2+0.4450s+1)(s^2+1.2470s+1)(s^2+1.8019s+1)$
8	$(s^2+0.3902s+1)(s^2+1.1111s+1)(s^2+1.6629s+1)(s^2+1.9616s+1)$
9	$(s+1)(s^2+0.3473s+1)(s^2+s+1)(s^2+1.5321s+1)(s^2+1.879s+1)$
10	$(s^2+0.3129s+1)(s^2+0.9080s+1)(s^2+1.4142s+1)(s^2+1.7820s+1)(s^2+1.9754s+1)$

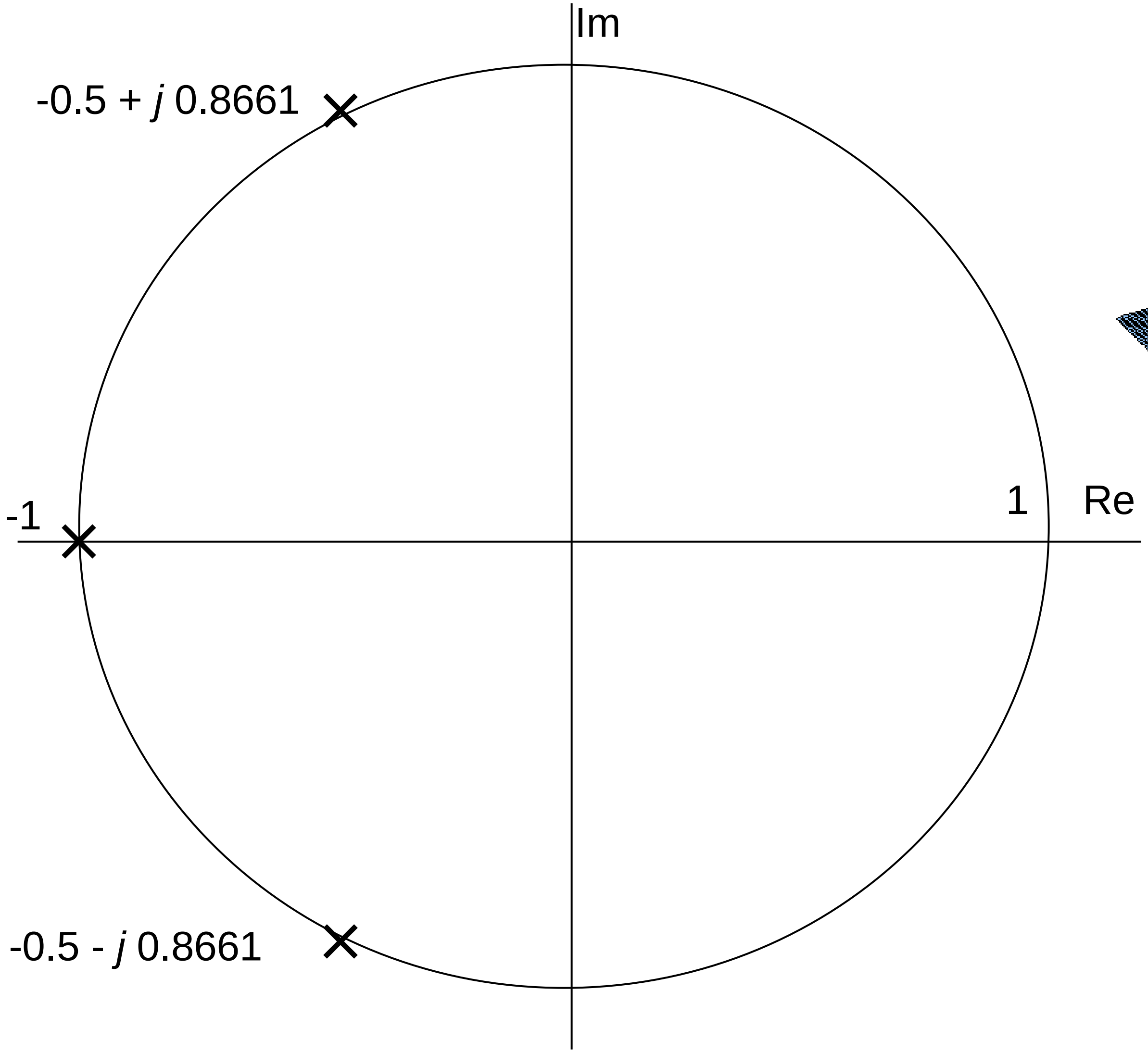
Order	Pole Location
1	$-1 \pm j\,0$
2	$-0.707 \pm j\,0.707$
3	$-1 \pm j\,0$, $-0.5 \pm j\,0.866$
4	$-0.924 \pm j\,0.383$, $-0.383 \pm j\,0.924$
5	$-1 \pm j\,0$, $-0.809 \pm j\,0.588$, $-0.309 \pm j\,0.951$
6	$-0.966 \pm j\,0.259$, $-0.707 \pm j\,0.707$, $-0.259 \pm j\,0.966$
7	$-1 \pm j\,0$, $-0.901 \pm j\,0.434$, $-0.624 \pm j\,0.782$, $-0.222 \pm j\,0.975$
8	$-0.981 \pm j\,0.195$, $-0.832 \pm j\,0.556$, $-0.556 \pm j\,0.832$, $-0.195 \pm j\,0.981$

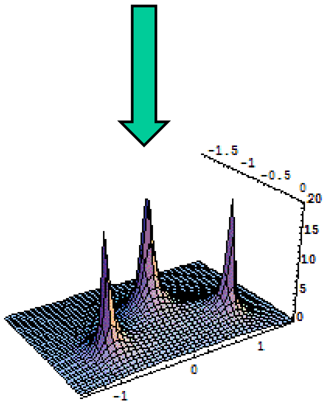
Note quadratic factors
Each will have a
complex conjugate pair

Poles of a 2-pole Butterworth Filter

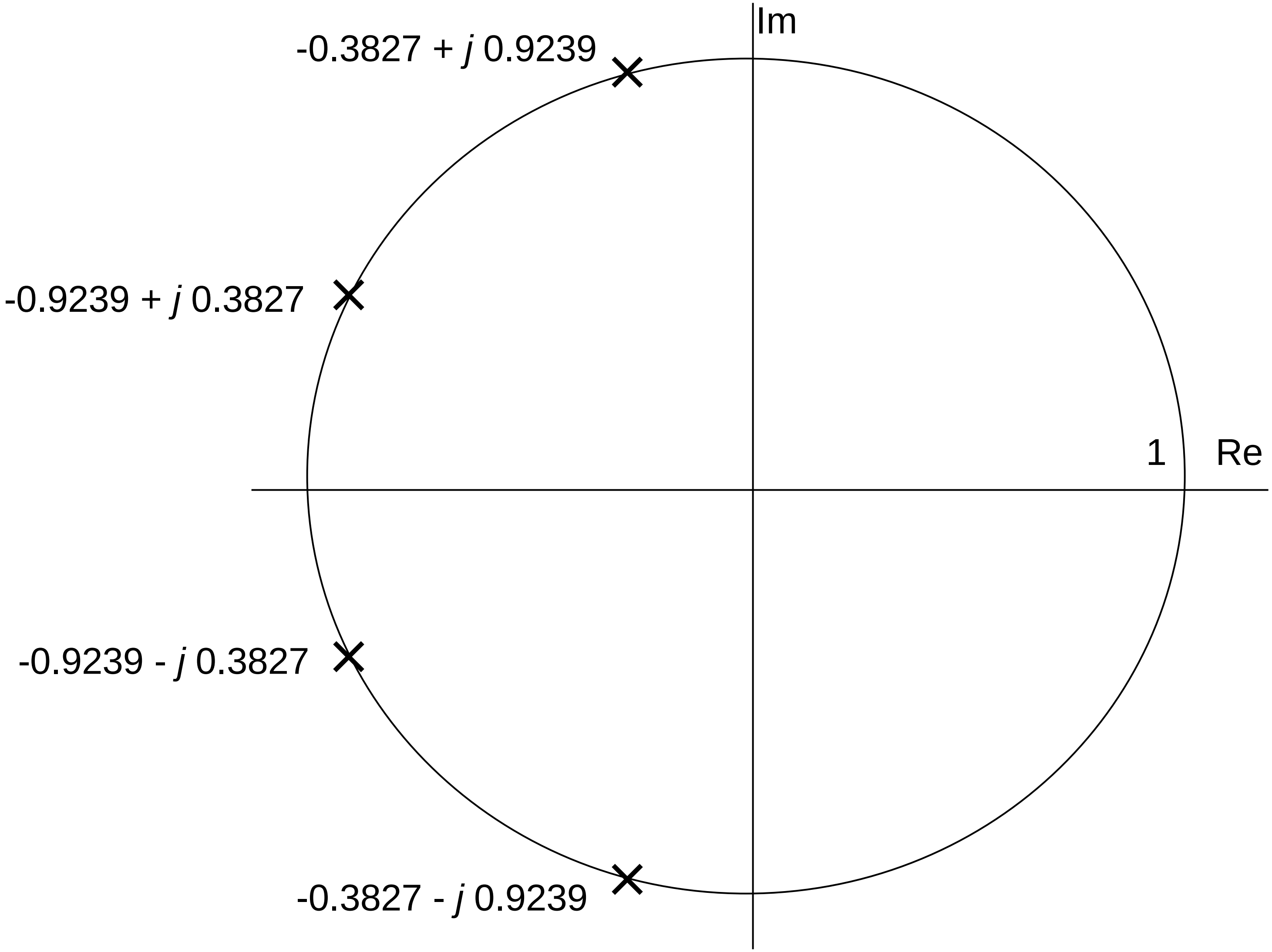


Poles of a 3-pole Butterworth Filter

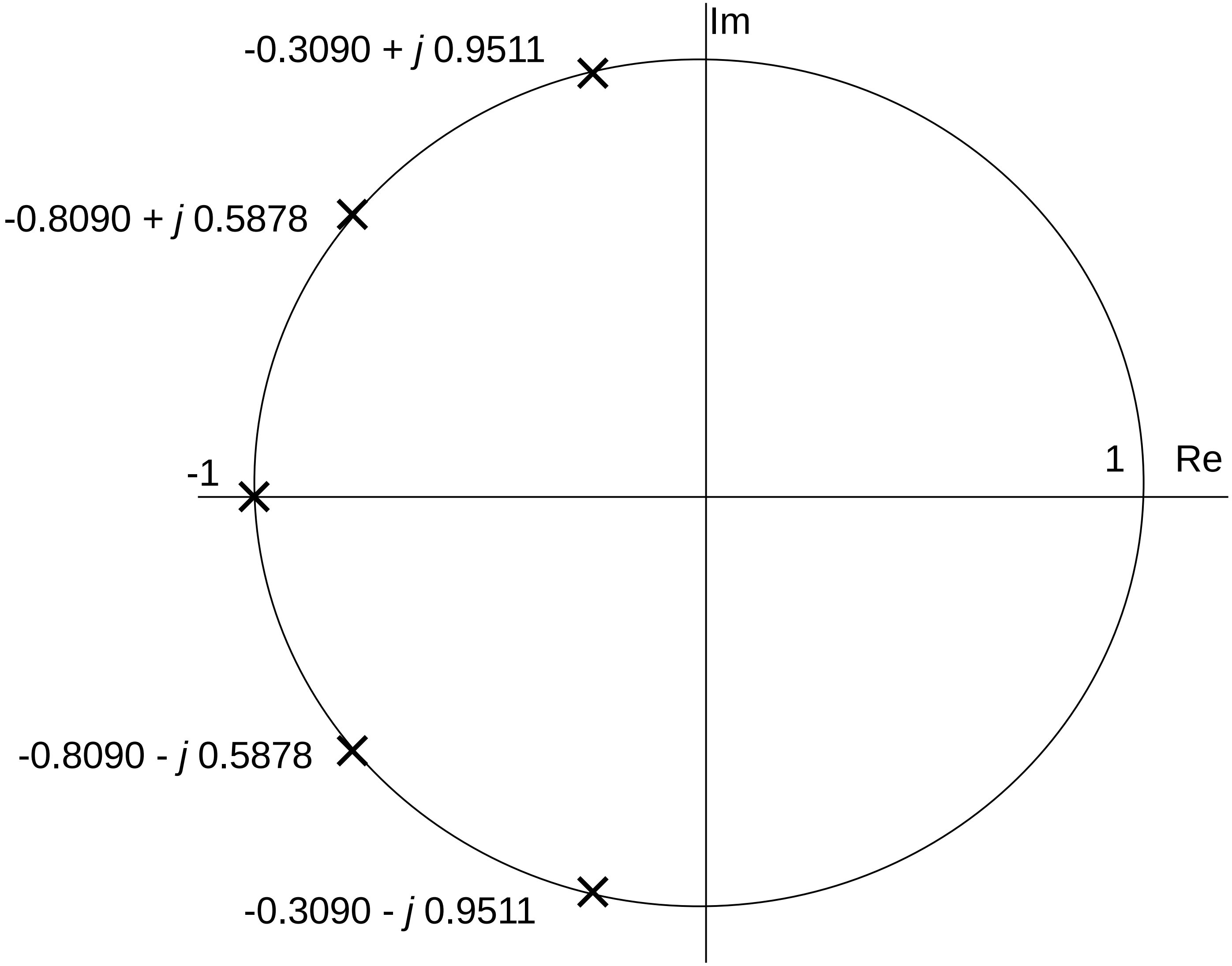




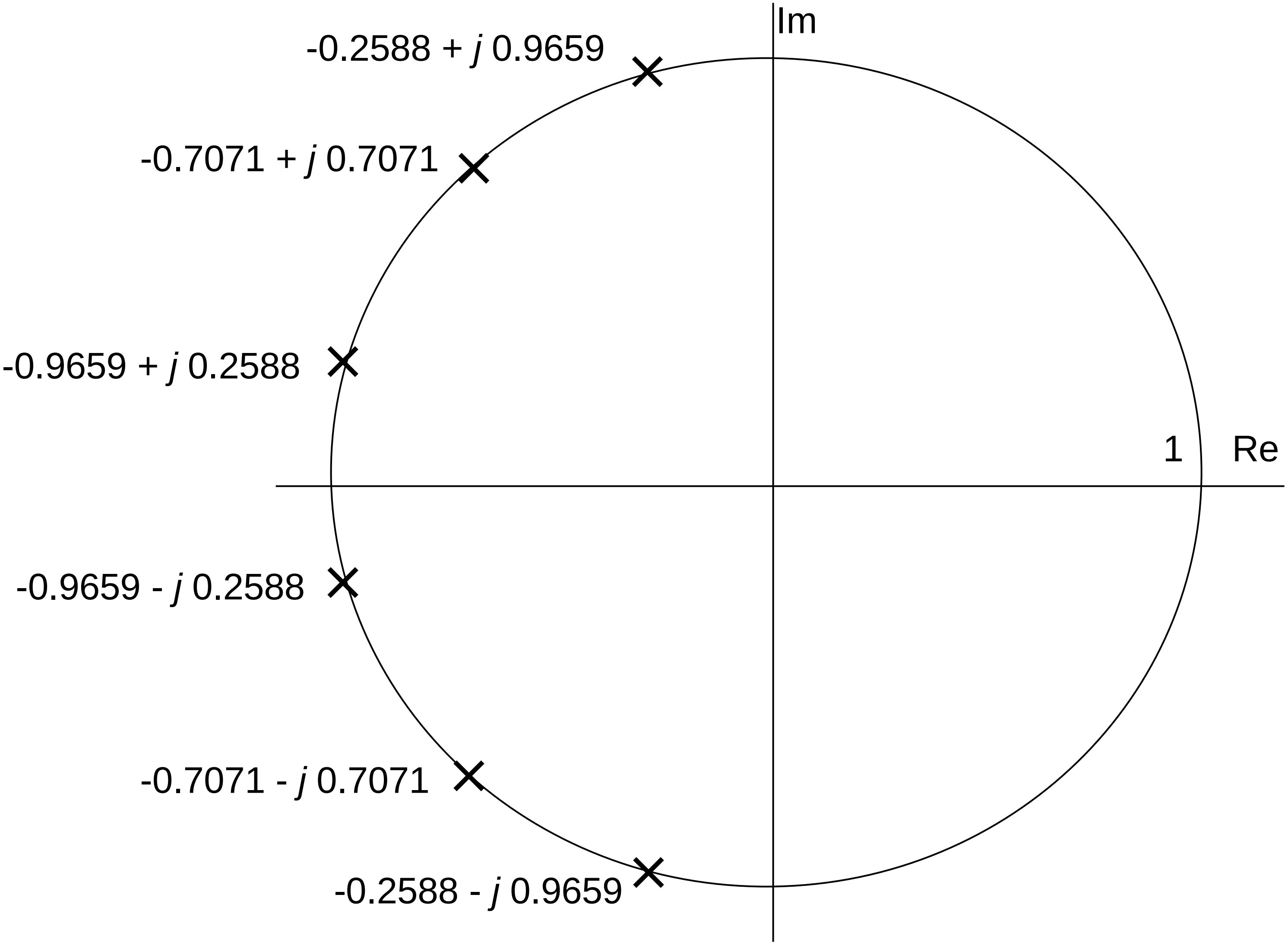
Poles of a 4-pole Butterworth Filter



Poles of a 5-pole Butterworth Filter

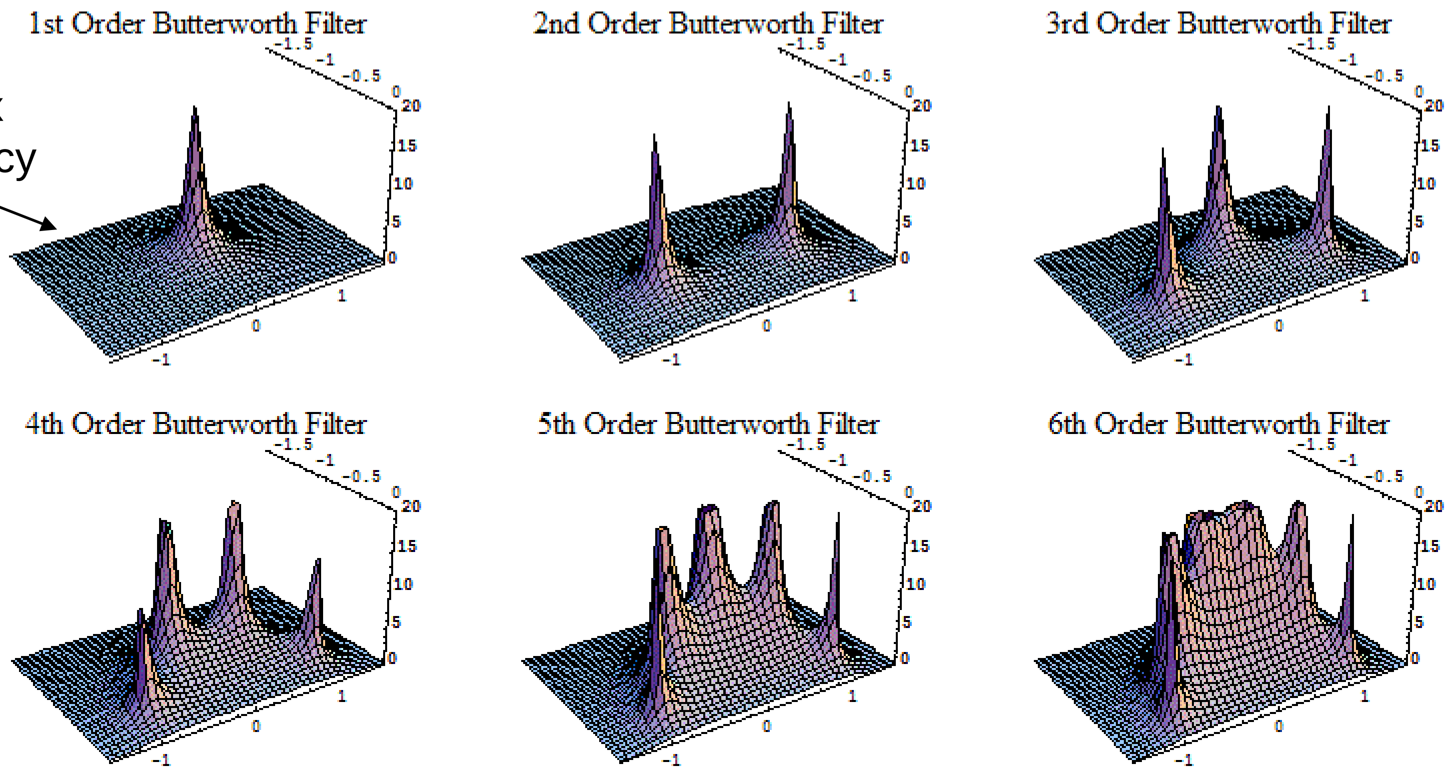


Poles of a 6-pole Butterworth Filter

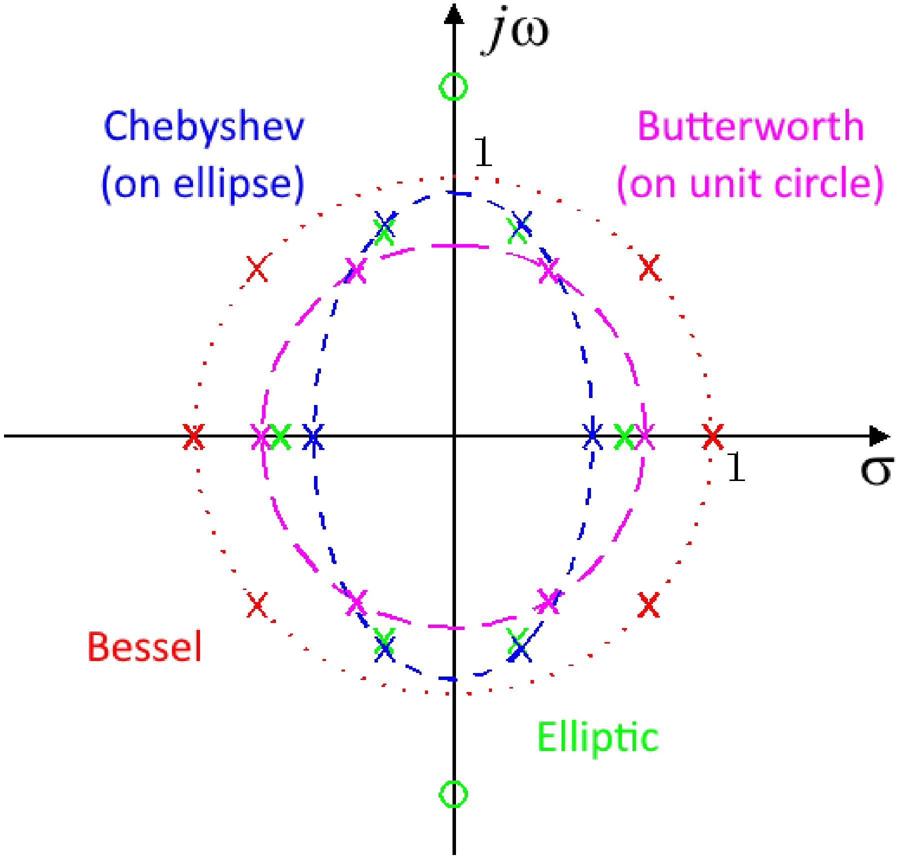


Butterworth Poles

Complex
Frequency
Plane

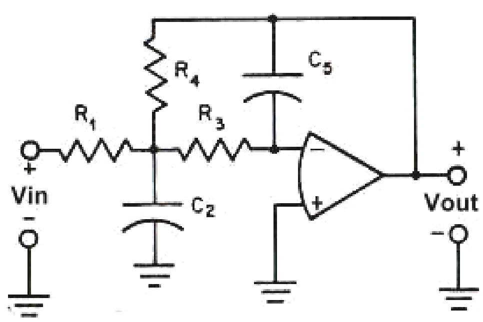


Pole (and Zero) Location for Common Filter Types

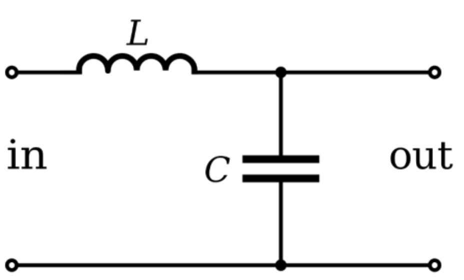


Poles and Order of a Filter

- You will hear these terms used interchangeably when describing filters.
- A 2-pole filter is the same as a 2nd order filter.
- In general the number of poles (i.e. the filter order) is the same as the number of capacitors and/or inductors in the filter circuit.

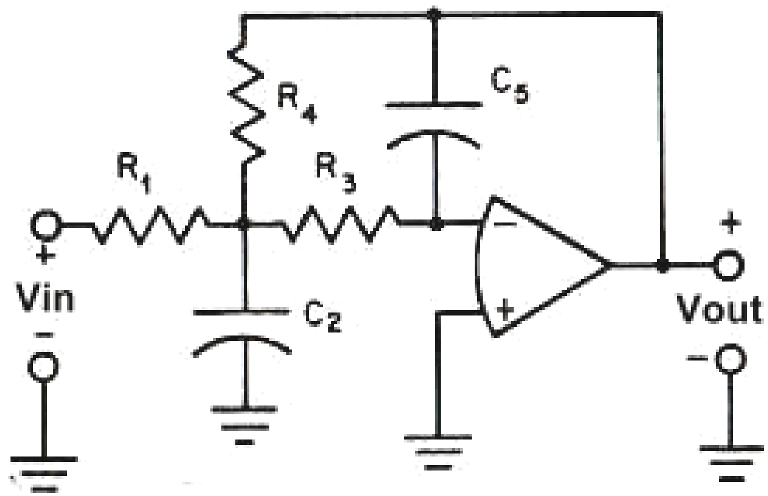


2 C’s → 2 Poles



1 L + 1 C → 2 Poles

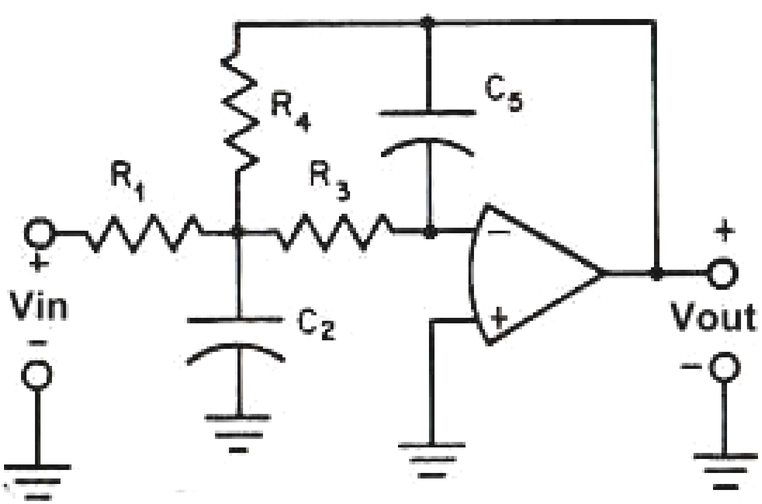
Butterworth Filters
A Practical Implementation



The classic Multi-Feedback (or Rauch) 2-pole low pass op amp filter

Butterworth Filters
A Practical Implementation

- The impedance of the capacitors used to implement a filter are dependent on frequency. Therefore, the filter's characteristics are frequency dependent.
- The diagram shows a 2-pole Butterworth filter.



The capacitors in the diagram have an impedance that is dependent on the frequency of V_{in}.

$$Z_c = \frac{1}{sC} = \frac{1}{j\omega C} = \frac{1}{j2\pi f C}$$

The varying impedance, varies the gain of the amplifier with respect to frequency.

Butterworth Filters
A Practical Implementation

The transfer function of the filter looks like the following equation containing the R and C's of the circuit. Note that the number of capacitors used in the circuit is also the number of poles in the equation.

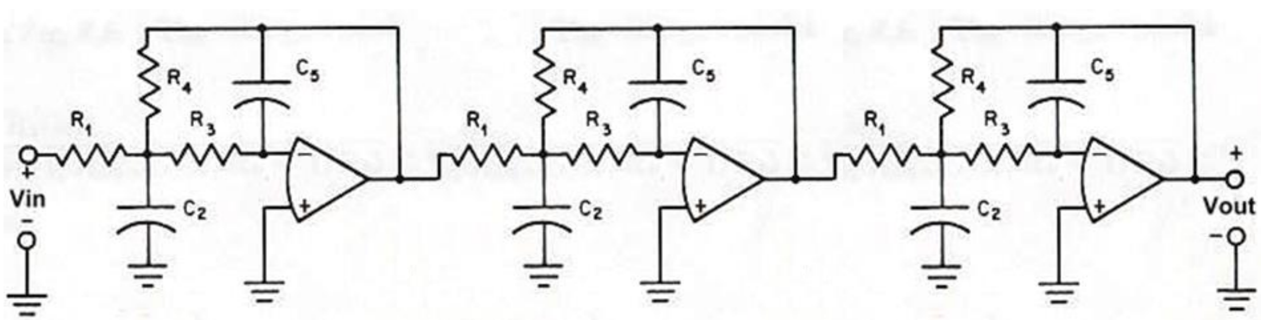
2nd order Butterworth transfer function

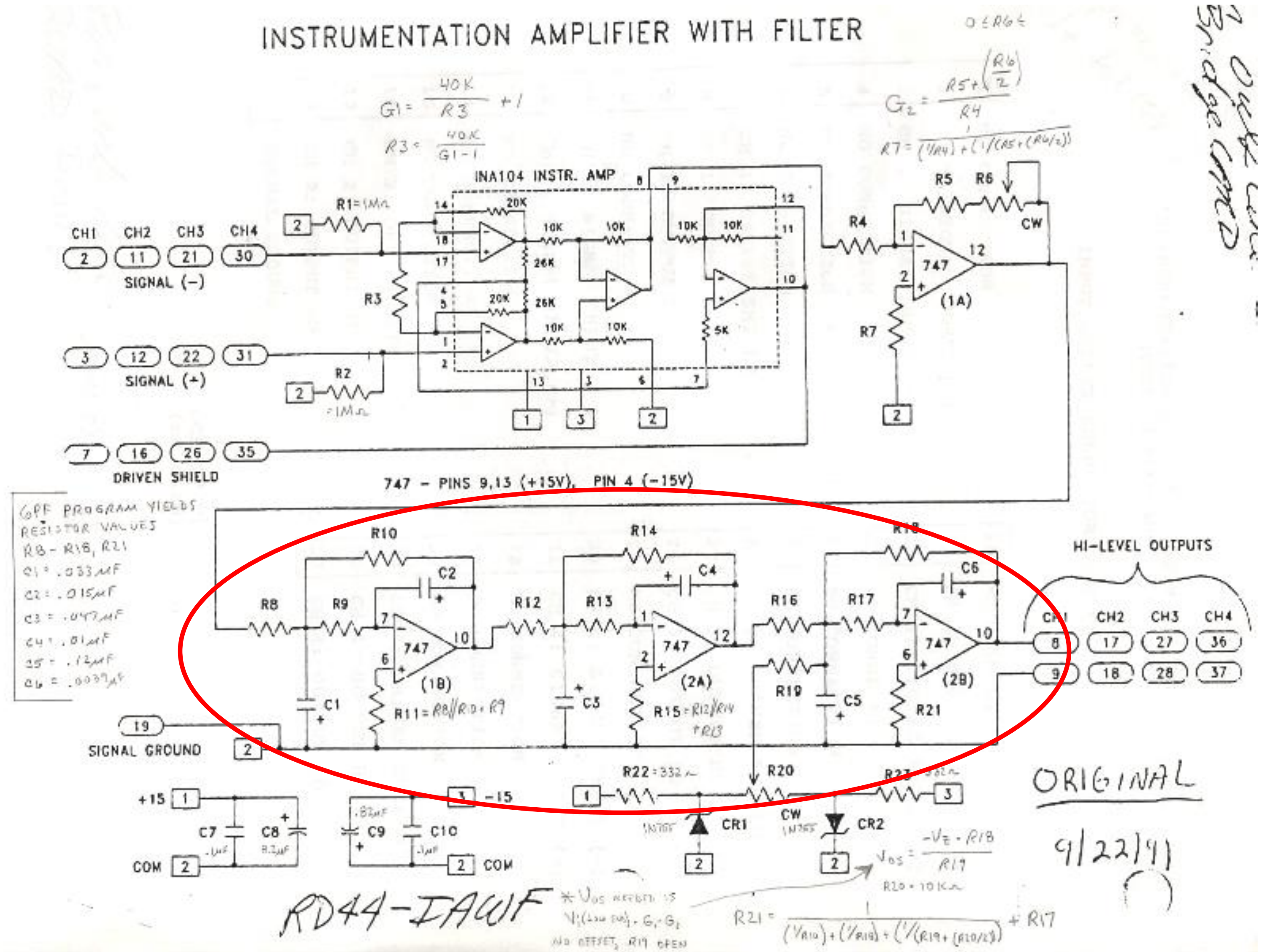
$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

$$H(s) = \frac{-1/_{R_1 R_3 C_2 C_5}}{s^2 + s\left(\frac{1}{C_2}\right)\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right) + \frac{1}{R_3 R_4 C_2 C_5}}$$

Designing Butterworth Filters With More Than 2 Poles

- The two-pole filter shown earlier is used in series to get Butterworth filters with 3, 4, 5, 6, etc . . . poles. Usually you see filters with an even number of poles.
- A 6-pole filter will have three, two-pole stages in series as shown below:





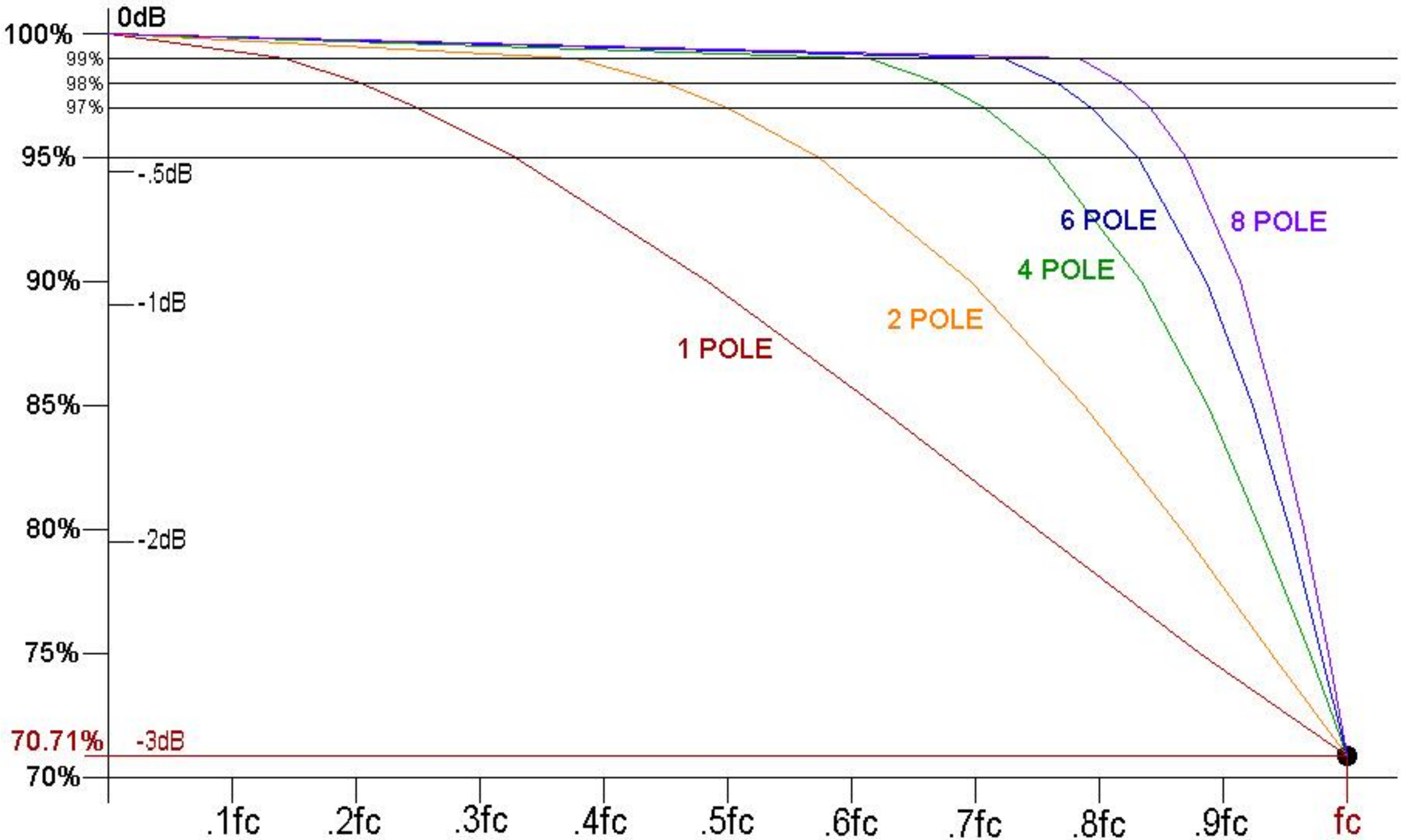
Note that our 2-pole filter stage contains an additional resistor (R11, R15, R21) which is used as an input offset bias current limitor to make the op amp look like the ideal op amp.

The Pass Band

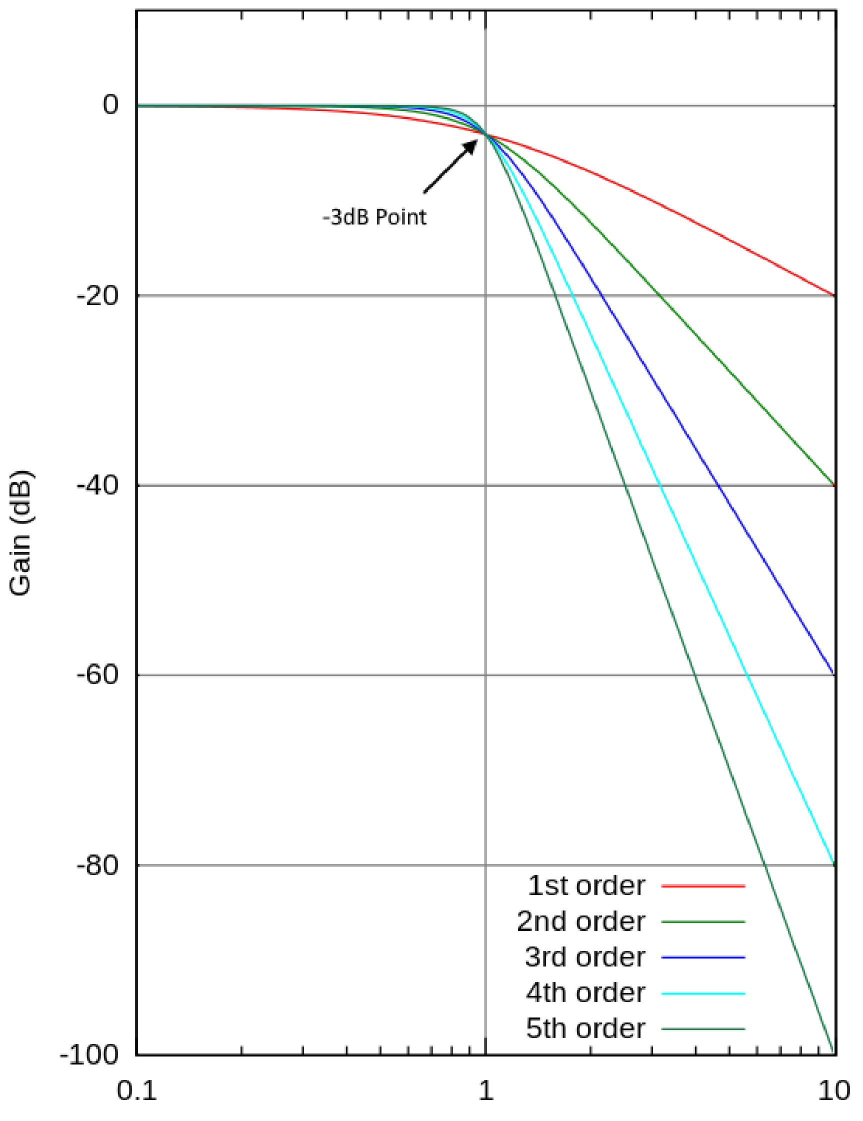
- Because the data of interest will be in the pass band, it is very important to understand what the attenuation is in that band.

- The Butterworth filter attenuates the signal even in the pass band. It is important to know by how much.

The Pass Band

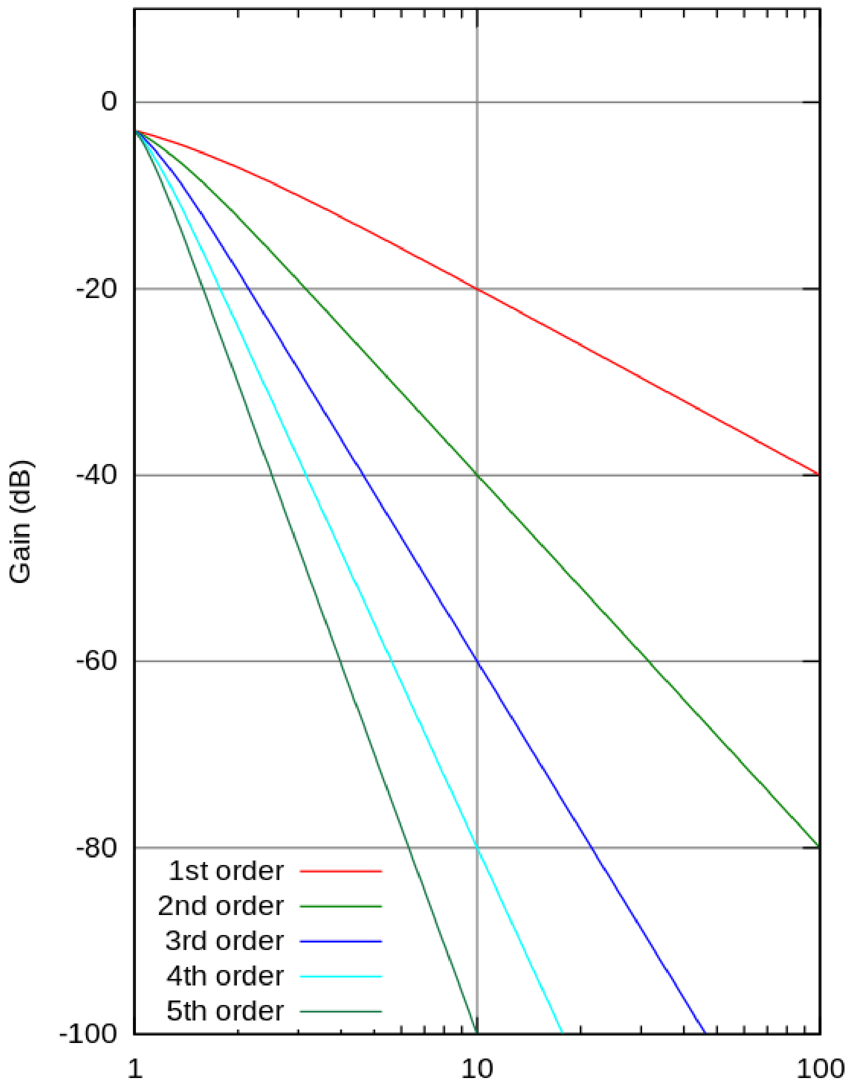


At the Cut Off Frequency



- No matter what the order of the filter is, the frequency response curve always goes through the -3dB point at the cutoff frequency
- Note that there are many definitions of bandwidth. The -3 dB point is common but not the only one

The Stop Band



The sharpness of the drop-off is dependent upon the number of poles as seen in the diagram.

Each pole contributes
-20dB/decade
or
-6dB/octave

Sampling and Aliasing

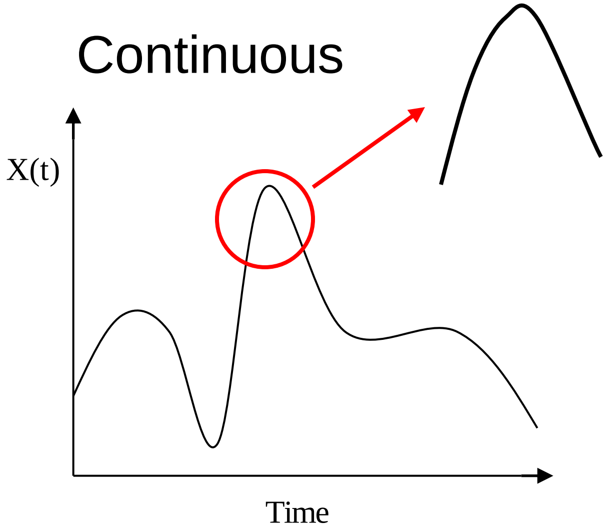
What Is Sampling?

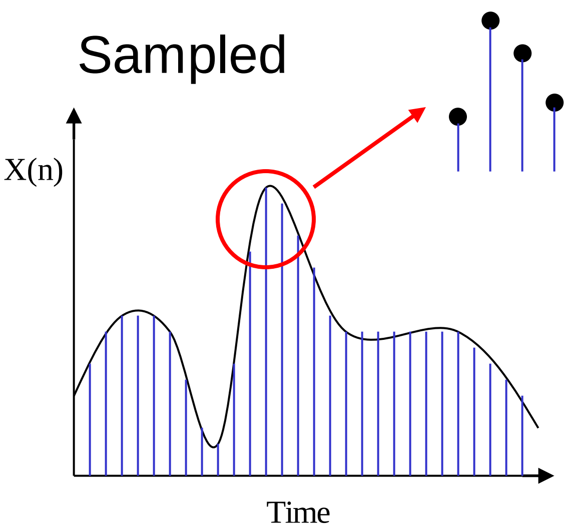
- Sampling is the act of taking a small part or quantity of something, as a sample for testing or analysis (i.e., accelerometer signal, voice, video, a piece of pie, etc.)

- The more samples you have, the more information about the signal you possess, the easier it is to determine what the original signal looked like.

What Is Sampling?

- Analog signals are continuous-time signals and have a value for every instant of time, whereas sampled signals occur at discrete intervals
- In digital data processing systems, samples of continuous-time signals are taken at discrete-time intervals to create a discrete-time signal.





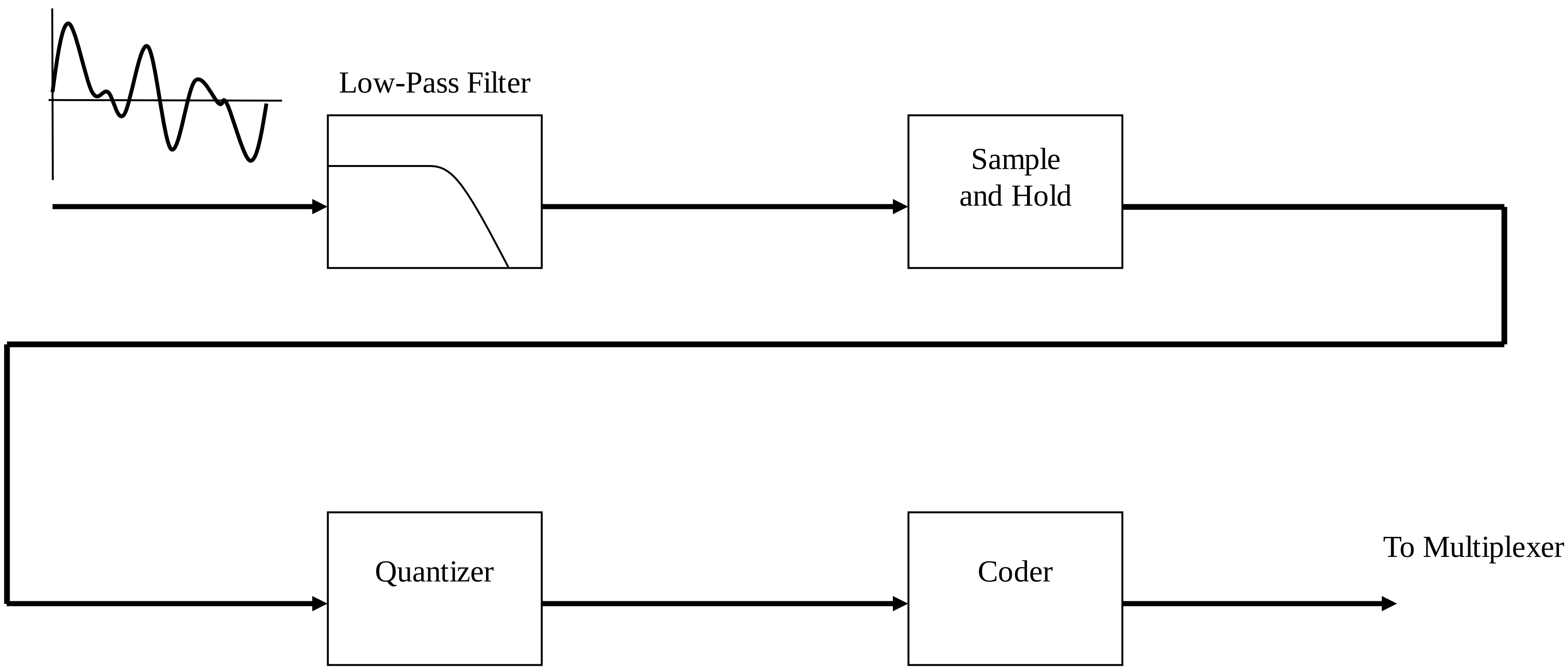
What Is Sampling?

- It is important to remember that the sampling process produces a digital signal containing partial information from the original signal.

- Basic to all digital data systems is how many samples are necessary to reproduce the original signal accurately and with minimal distortion before selecting the number of samples.

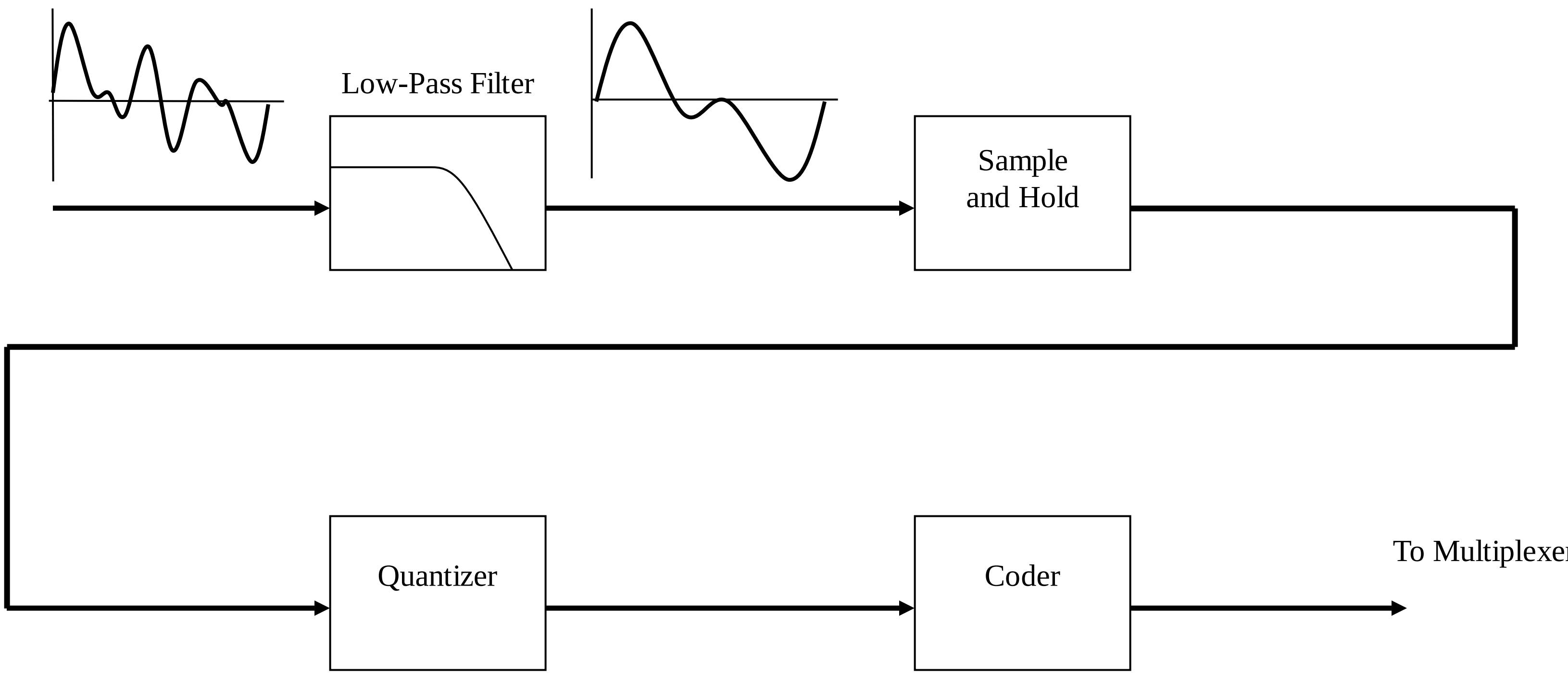
- To determine the required number of samples, you must know the frequency range of interest of the continuous-time (analog) signal.

The Sampling and Digitization Process



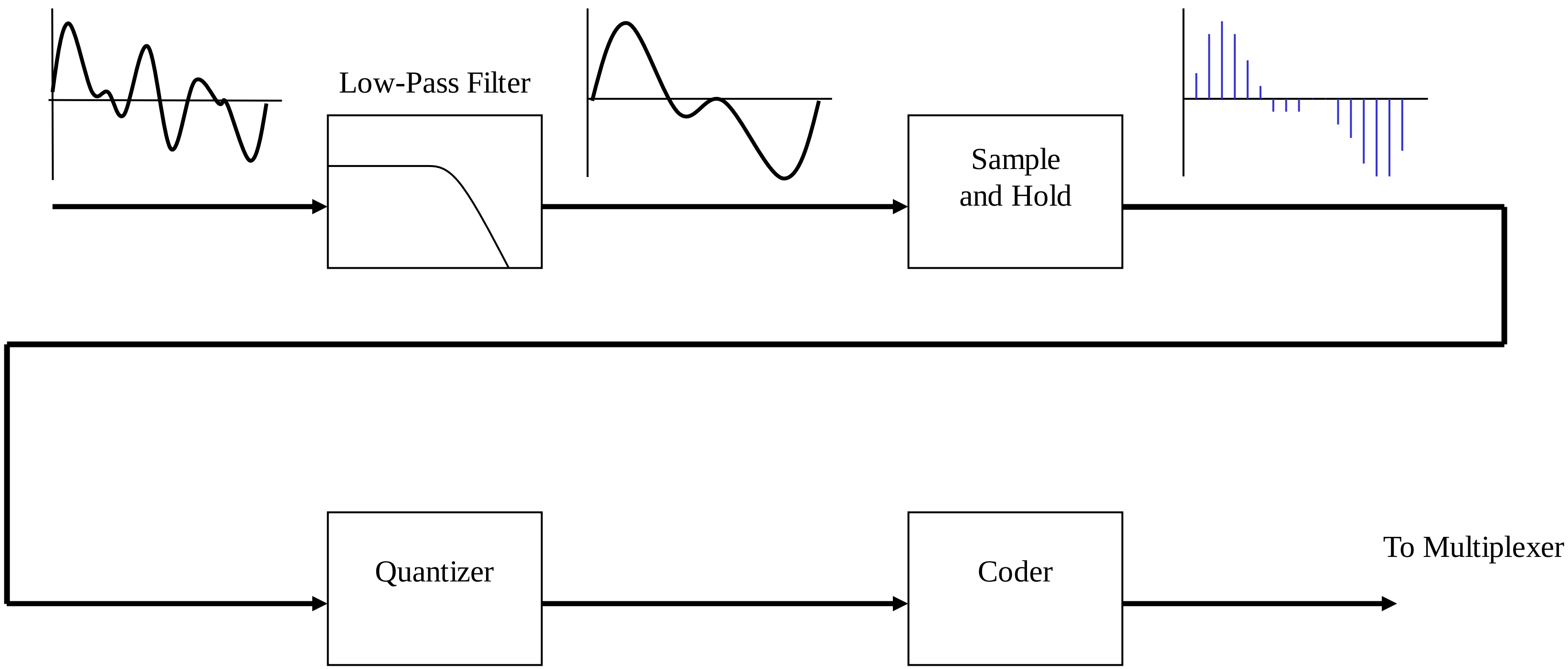
Remember that the first step before sampling is filtering. The input signal can have a wide range of frequencies – most of the times the highest frequency component won't be known.

The Sampling and Digitization Process



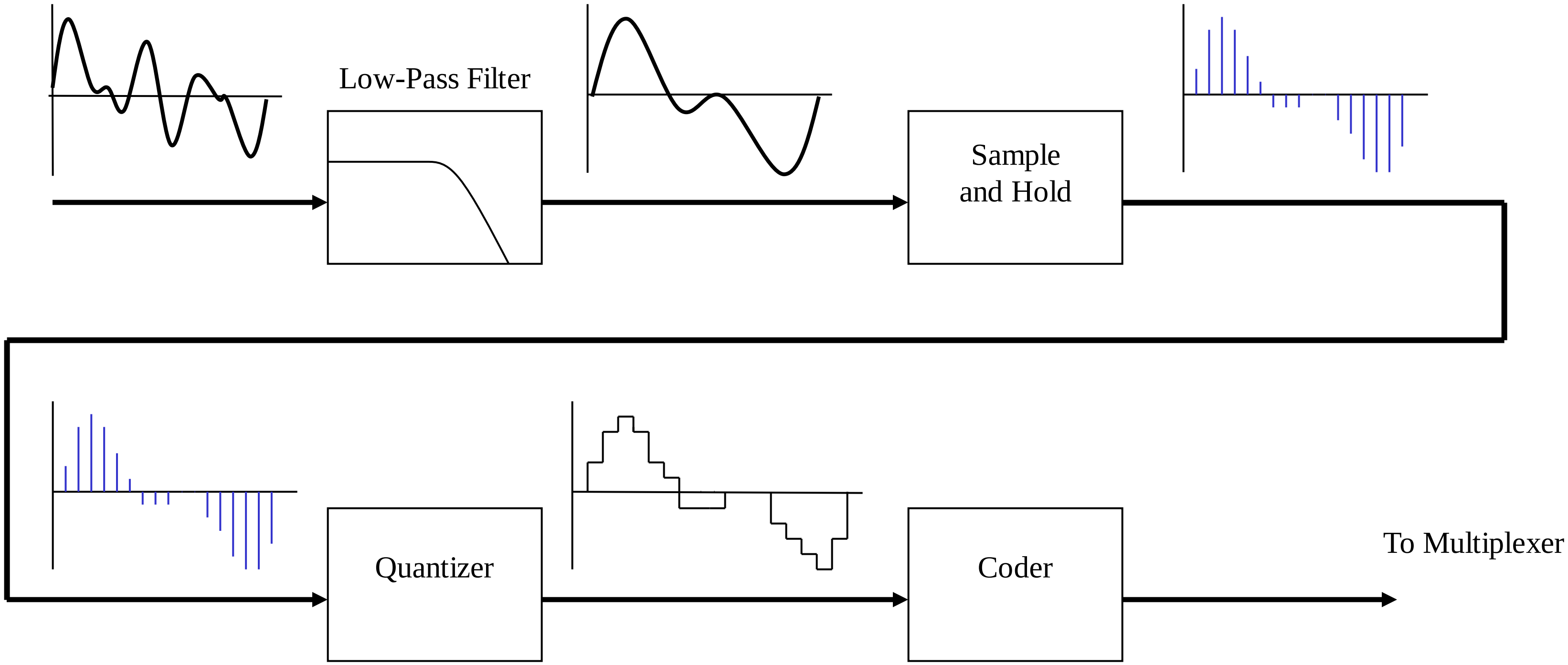
The signal is now band limited to a known range of frequencies. The sampling rate will depend on that highest frequency component. Filtering guarantees that you know what that maximum frequency will be and at what attenuation.

The Sampling and Digitization Process



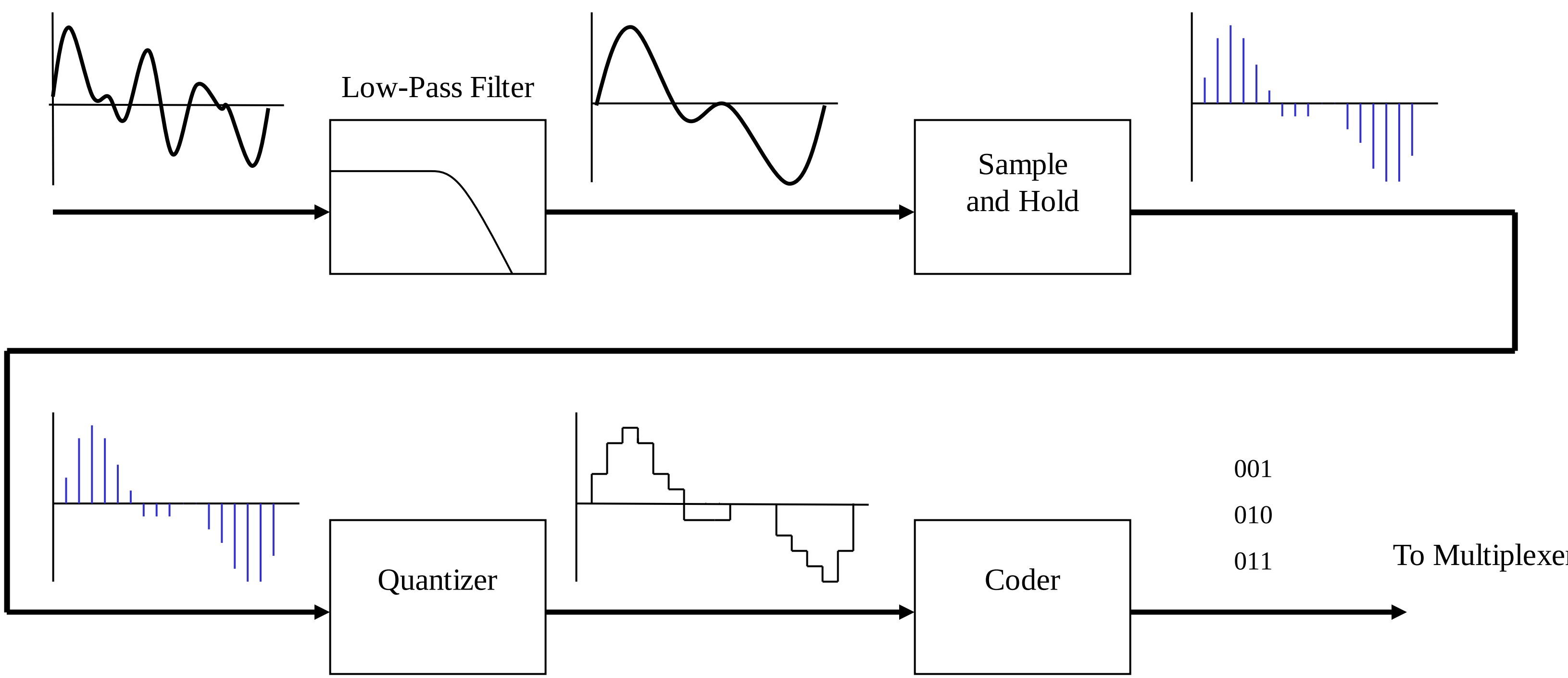
The signal is now sampled at a certain rate. For discrete instances in time, you know what the amplitude of the signal is. However, in between samples you don't. These analog amplitudes are held such that it can be digitized.

The Sampling and Digitization Process



The quantizer assigns a particular level to each of the signal amplitudes that are held. The level is determined by the amplitude surpassing a certain threshold level.

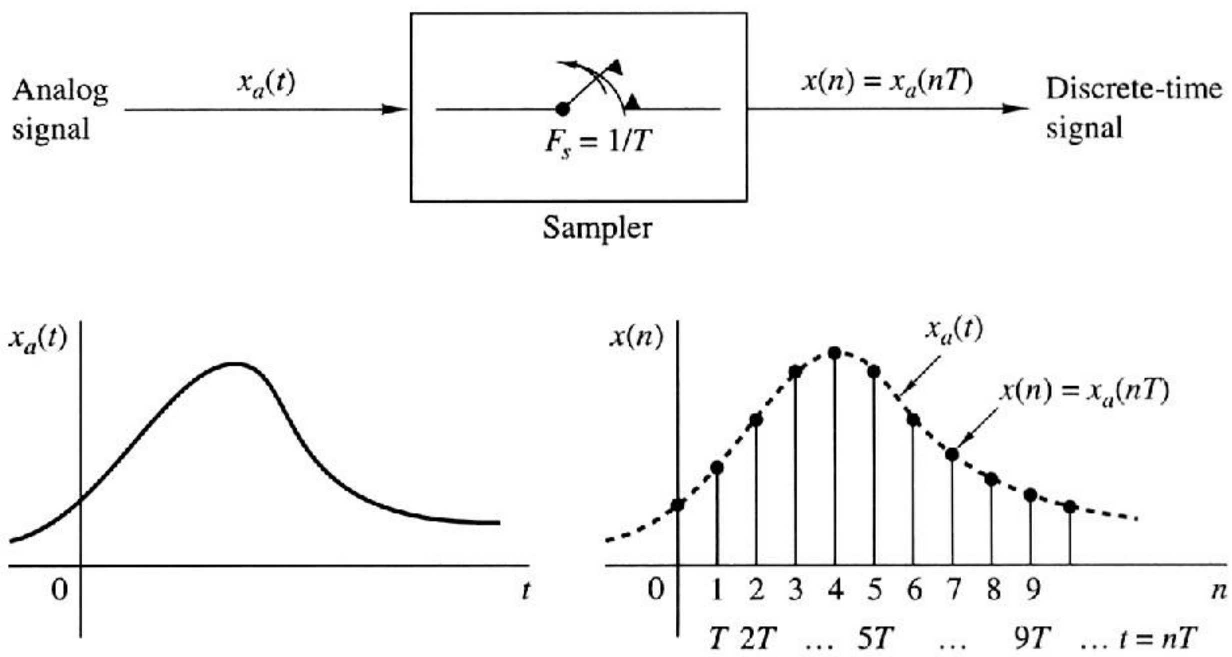
The Sampling and Digitization Process



The coder then assigned the appropriate code to each quantized level. This is the data that is multiplexed into the PCM stream.

Sampling of Analog Signals

- The discussion of sampling for this training will be limited to *periodic* or *uniform sampling*, which is the most common type of sampling used.

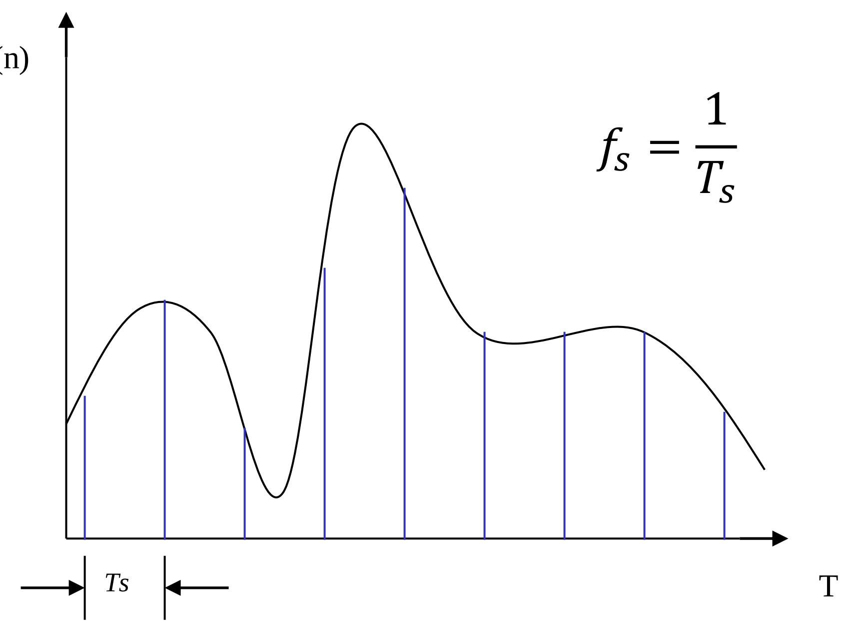


Sampling of Analog Signals

- The time interval *T* between successive samples is called the sampling period or sample interval T_s
- The reciprocal of the sampling period is called the sampling frequency (Hz) or sampling rate (samples per second).

$$T_s = 0.005\,sec$$

$$f_s = \frac{1}{T_s} = \frac{1}{0.005sec} = 200\ Hz$$



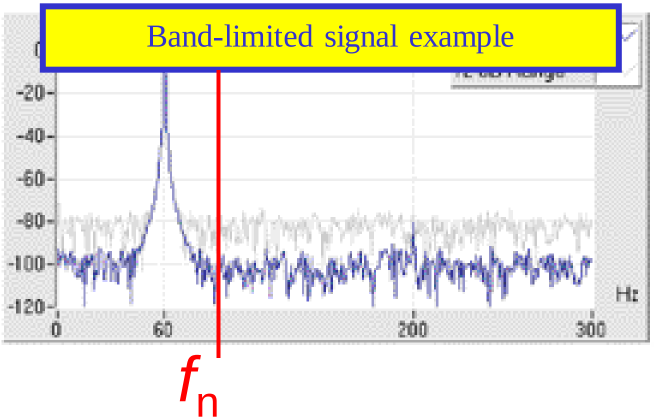
$$f_s = \frac{1}{T_s}$$

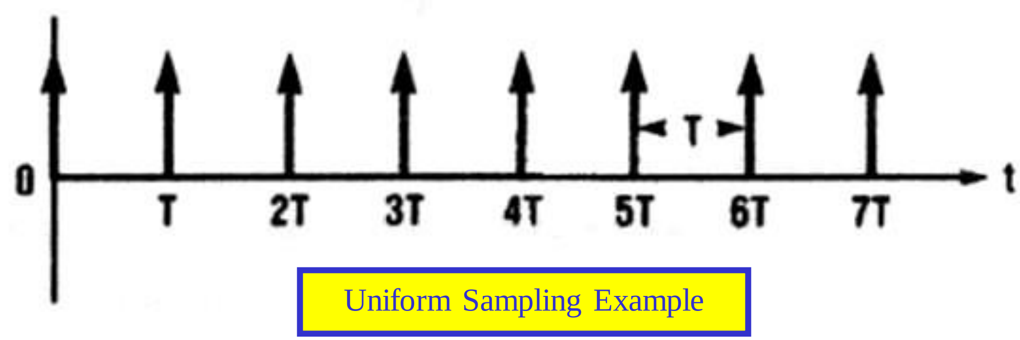
The Sampling Theorem

- There has been some debate about who to credit for the sampling theorem, Nyquist or Shannon, (most people speak about sampling in terms of Nyquist).
- For this training we will refer to the theorem as the Sampling Theorem, however terms like *Nyquist frequency*, *Nyquist rate*, and *Nyquist factor* are often used.
- There is a wealth of information about the sampling theorem and how it came about, in textbooks and on the internet if you would like to read more about it.

The Sampling Theorem

- A band-limited (filtered) signal that has no spectral components above f_n Hz , can be determined uniquely by values sampled at uniform intervals of T_s seconds, if $T_s \leq \frac{1}{2f_n}$ (*the uniform sampling theorem*).





- In other words, the sample rate (f_s = 1/T$_s$) must be at least twice the Nyquist frequency f_n.

$$f_s \geq 2f_s$$

The Sampling Theorem

- The sampling theorem simply states that in order to recover a signal function f(t), the signal must be sampled at a rate that is at least twice as high as the highest frequency component (we refer to this as the *Nyquist Factor*, where $NF=f_s/f_n$).
- We <u>must</u> use a *Nyquist Factor* greater than 2 to successfully reconstruct the original signal (this will be discussed in detail in upcoming slides)
- Sophisticated systems that employ digital signal processing and interpolation techniques can recover signals with Nyquist factors slightly higher than 2
- Sampling higher than twice the Nyquist frequency is referred to as *over-sampling*
- Sampling below twice the Nyquist frequency is referred to as *under-sampling,* and will result in <u>*aliasing*</u> .

Aliasing Is A Bad Word!

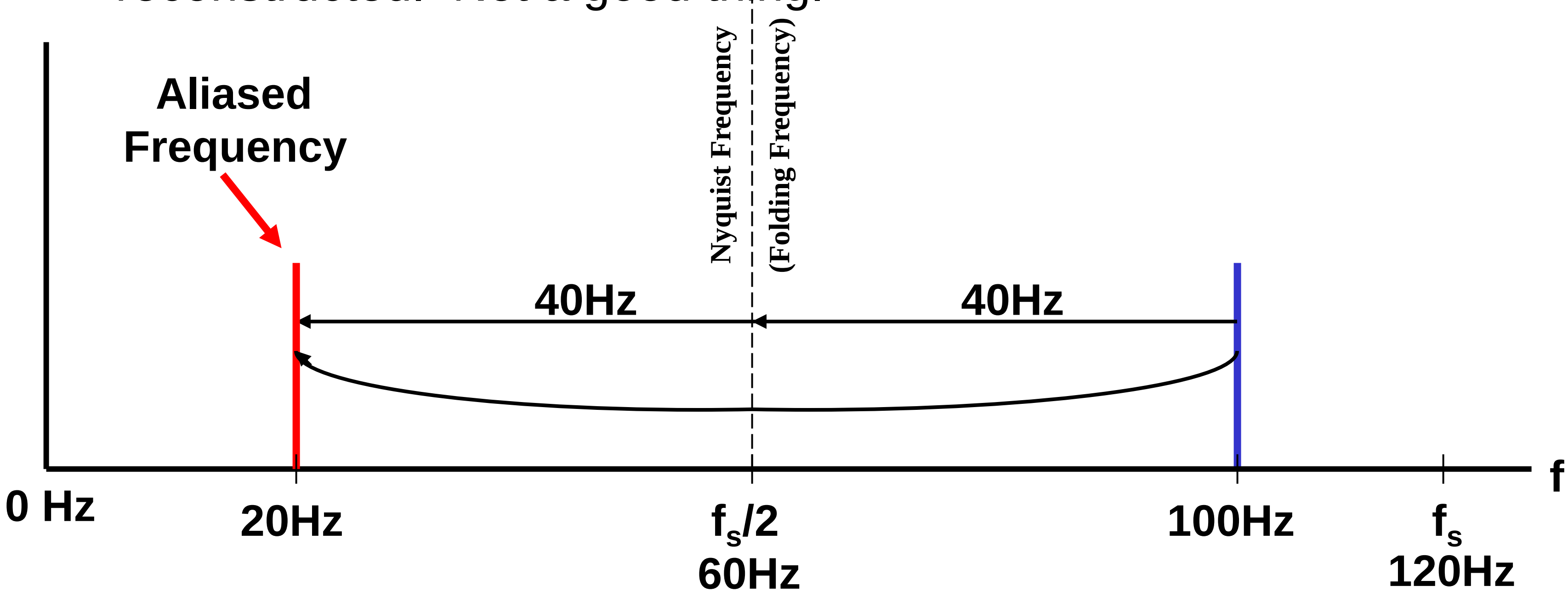
- *Alias* – A false name, otherwise known as, or Latin for, at another time
- In digital systems, it is defined as the phenomenon where high frequencies take on the identity of lower-frequencies, is called *aliasing*
- The *Nyquist Frequency* is known as the frequency which is equal to one half of the sampling frequency

$$f_{nyquist} = {}^{1}\!/_{2}\, f_s$$

- For example: if the sampling frequency is 120 Hz, then the *Nyquist frequency* is 120 Hz / 2 = 60 Hz
- Frequencies above 60 Hz will alias or fold into the 0 – 60 Hz frequency range and corrupt the data

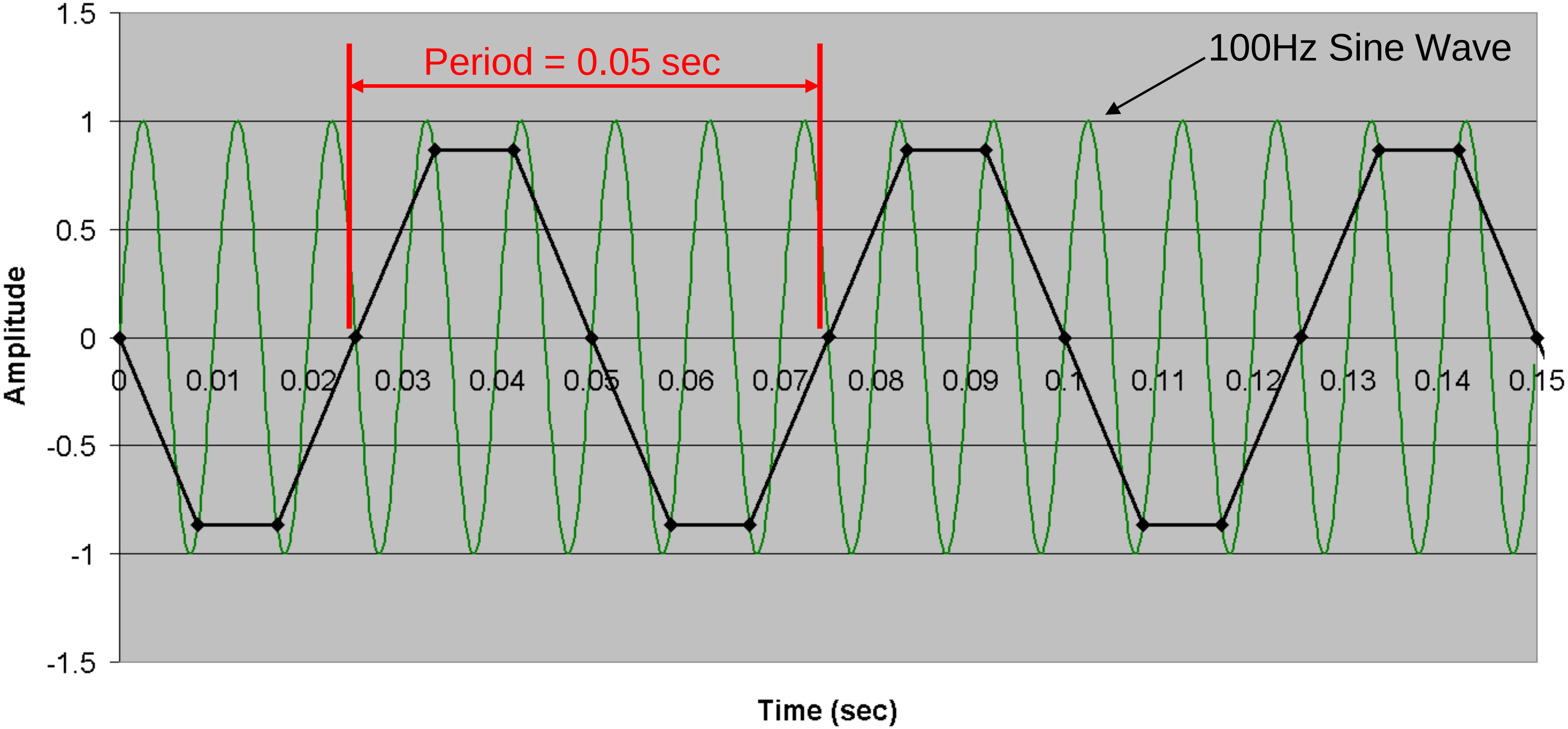
Visualization of how a Frequency is Aliased in the frequency domain

- If the sampling rate is 120 sps, then the *Nyquist frequency* is 120 Hz/2 = 60 Hz
- Frequencies above 60 Hz will alias or fold about the Nyquist frequency into the 0 – 60 Hz frequency range and corrupt the data, making it appear that there is a frequency component of the signal when one really does not exist.
- Say we have a 100 Hz sine wave sampled at 120 sps. It will appear as a 20 Hz sine wave when the sampled signal is reconstructed. Not a good thing.



Visualization of how a Frequency is Aliased in the time domain

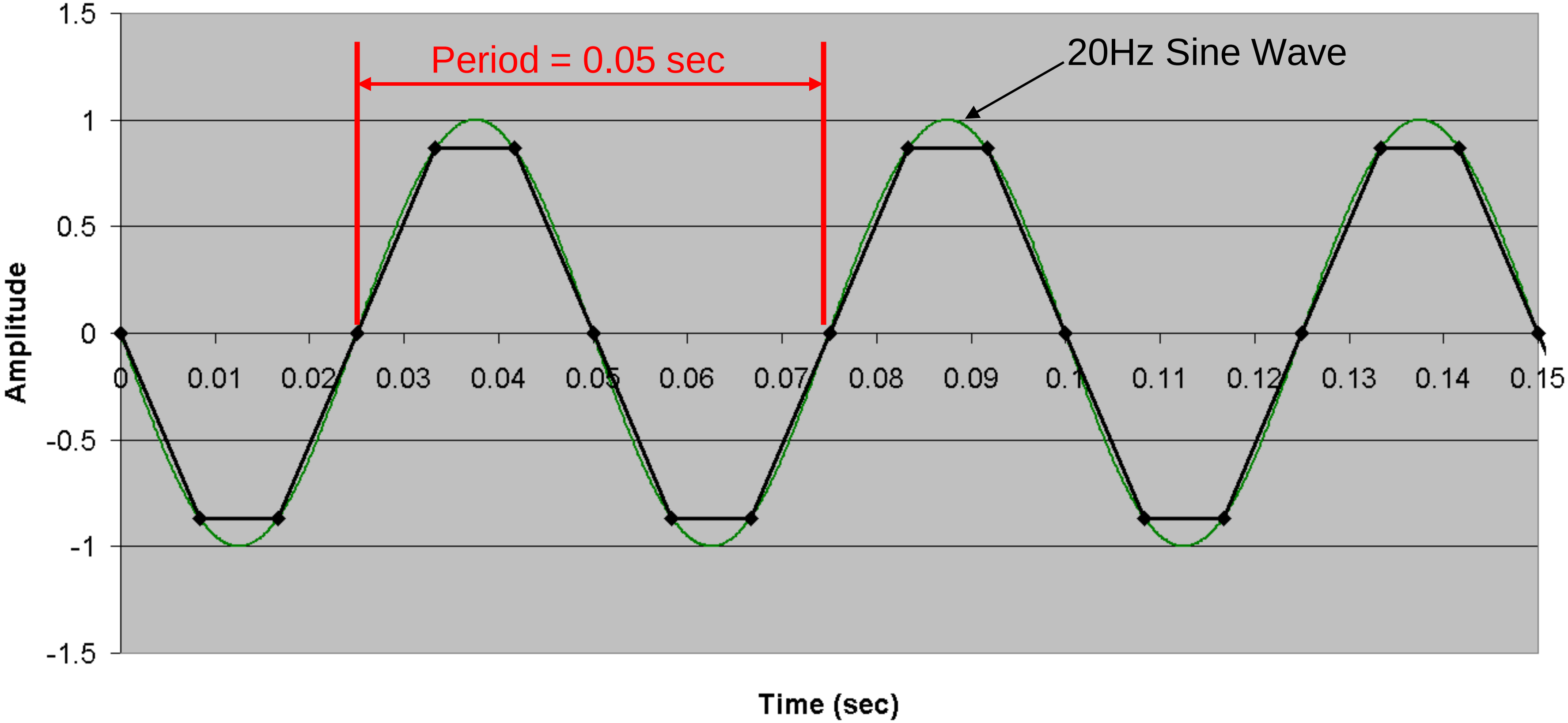
In the time domain, the reconstructed analog signal would look like a 20 Hz sine wave.



$$Frequency = \frac{1}{Period} = \frac{1}{T} = \frac{1}{0.05sec} = 20\ Hz$$

Visualization of how a Frequency is Aliased in the time domain

A flight test engineer analyzing the data would assume that a 20 Hz sine wave was present, when in actuality it was 100 Hz. This could have devastating effects if this was a test where frequencies over 75 Hz were known to cause damage to the aircraft being tested.



Aliasing

- Any frequencies present in our sampled data above the *Nyquist frequency* will fold back into our lower frequency data (below the *Nyquist frequency*) and appear as lower frequencies as shown in the example.
- Aliasing corrupts the signal in a way that cannot be fixed after sampling, it is irreversible!
- Aliased data can appear to be real data, most of the time you won't know that the data is corrupted.
- Aliasing must be dealt with in the analog world, which is why filtering/band-limiting is so important.
- Aliasing does not discriminate, it corrupts digital information of any type (i.e., PCM, video, audio, etc.).

A Classic Aliasing Example

- The wagon wheel is one of the classic examples used to help demonstrate the concept of aliasing and its effects.
- If you’ve watched a movie or TV show where a wagon wheel appears to being speeding up, slowing down, or going backwards, you have witnessed aliasing.
- At certain speeds, the frame-rate isn't fast enough to reproduce the actual rotational frequency of the wheel.
- Although the wheel is still moving forward, you observe aliasing by thinking that the wheel is going backwards.
- Things aren’t always what they seem!!!

Sampling Sine Waves

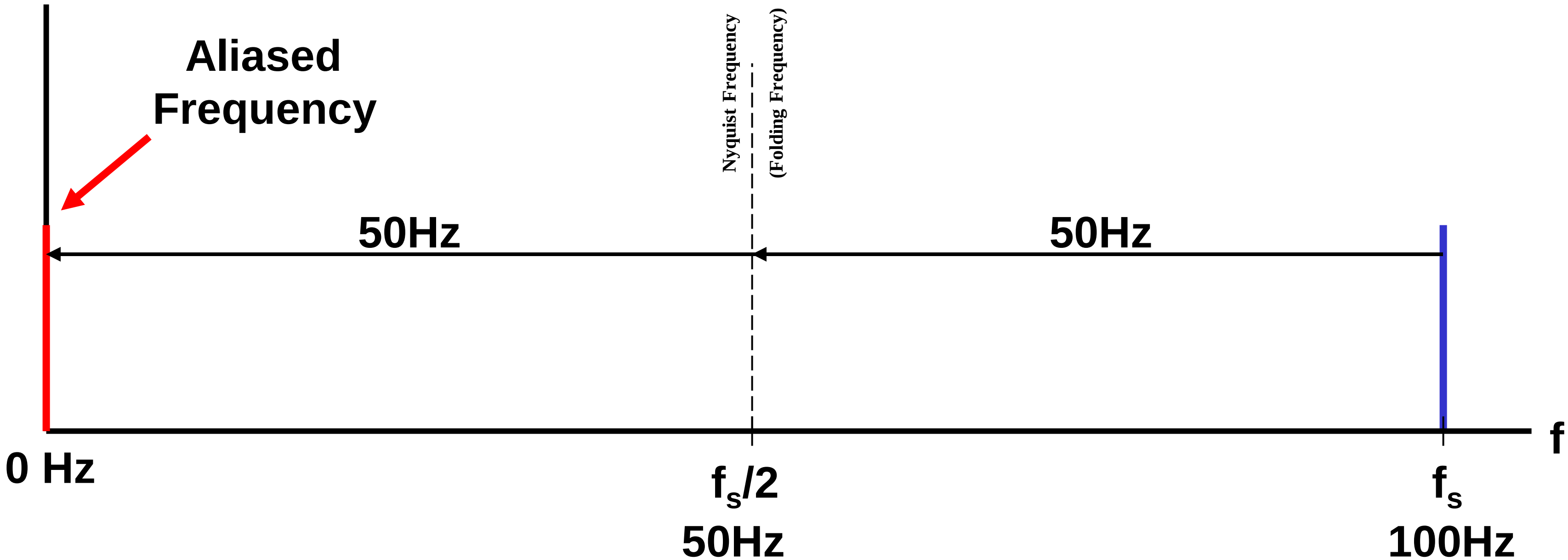
- Sampling sine waves is one of the best ways to demonstrate sampling rate and aliasing effects.
- This is basically what the sampling theorem is based on and is purely *theoretical*, meaning that if you are lucky enough to take 2 samples of a sinewave at the right time, then theoretically you should be able to reasonably reconstruct the original sine wave.
- The majority of the signals we measure are not pure sinusoids, they are more complex and are made up of multiple frequencies.
- This equates to the need for pre-sample filters and higher sampling rates (over sampling) to be able to reconstruct and best represent the original analog signal.

Sampling Sine Waves

- The following slides demonstrate the effect of increasing the sample rate of a pure 100 Hz sinusoid in the time domain and where applicable, the frequency domain.
- The sample rates will start off at 100 sps to illustrate the aliased frequencies.
- As the sample rate increases, the reproduced signal contains more information and approximates the original signal more closely
- The green trace is the original signal and the black traces are used to connect the individual samples with straight lines.

f_s=100 sps
In the Frequency Domain

Signal Freq: 100 Hz   fs: 100 sps Ts=0.01 sec   NF =100sps/100Hz = 1



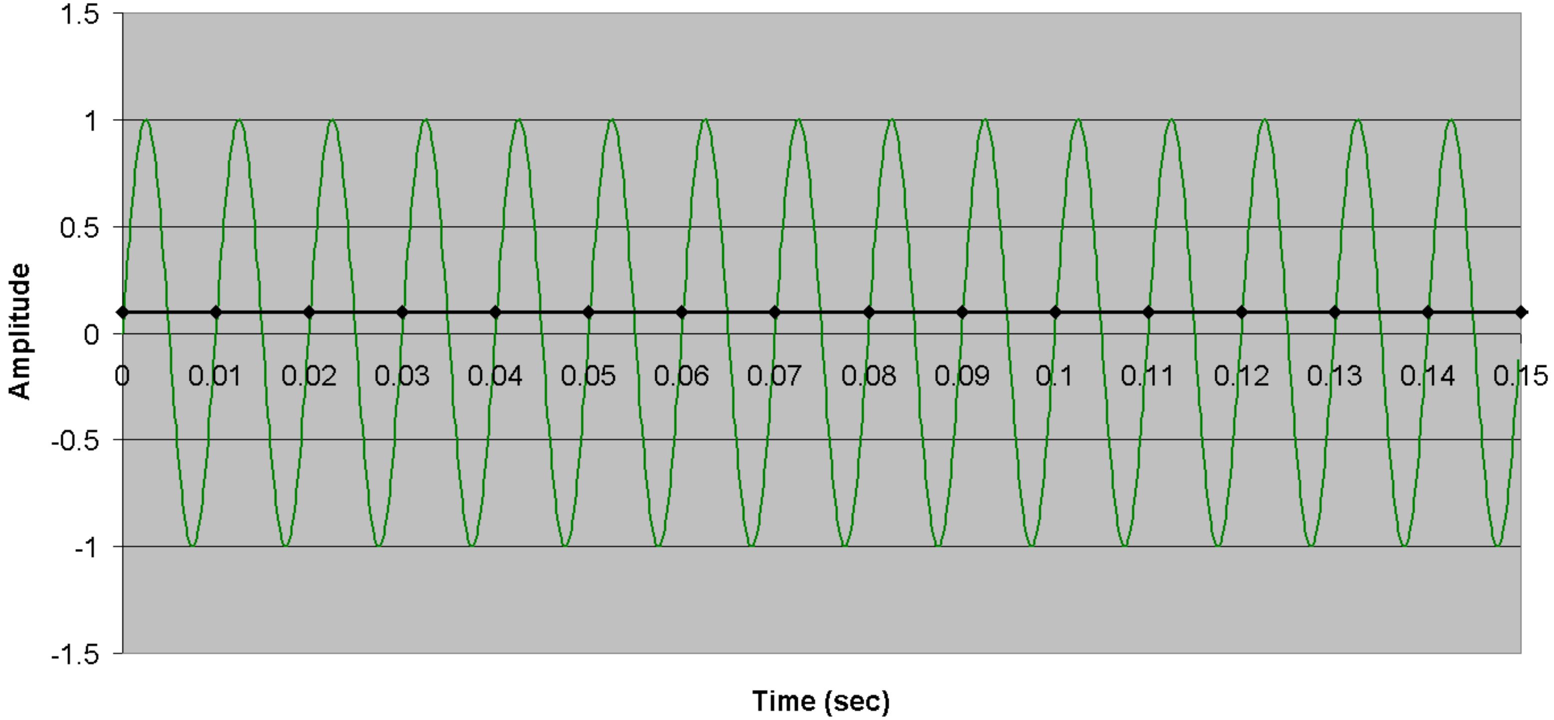
The 100 Hz sine wave will look like a 0 Hz sine wave when sampled at 100 sps. Or in other words, it will look like a DC voltage depending on where the samples happen to catch the sine wave.

Time (sec)

f_s=100 sps

In the Time Domain

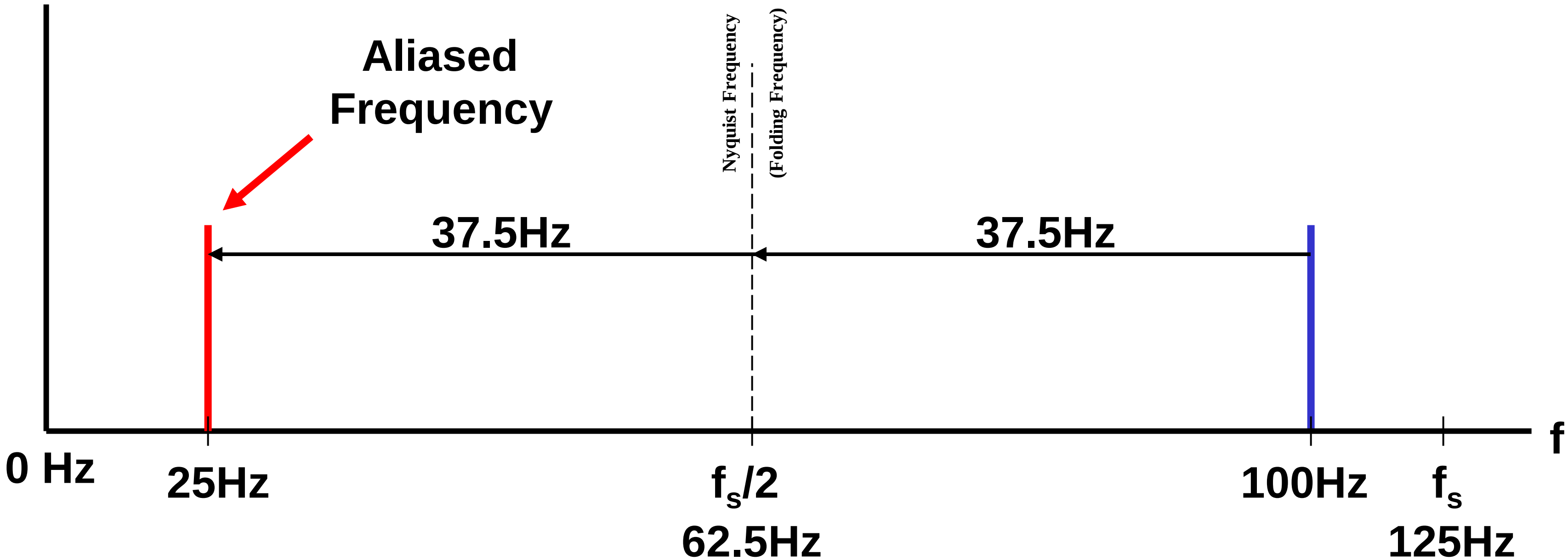
Signal Freq: 100 Hz　　fs: 100 sps　Ts=0.01 sec　　NF =100sps/100Hz = 1



In the time domain, signal has a frequency of 0 Hz. The signal looks like a DC level voltage.

f_s=125 sps
In the Frequency Domain

Signal Freq: 100 Hz fs: 125 sps Ts=8 msec NF =125sps/100Hz = 1.25

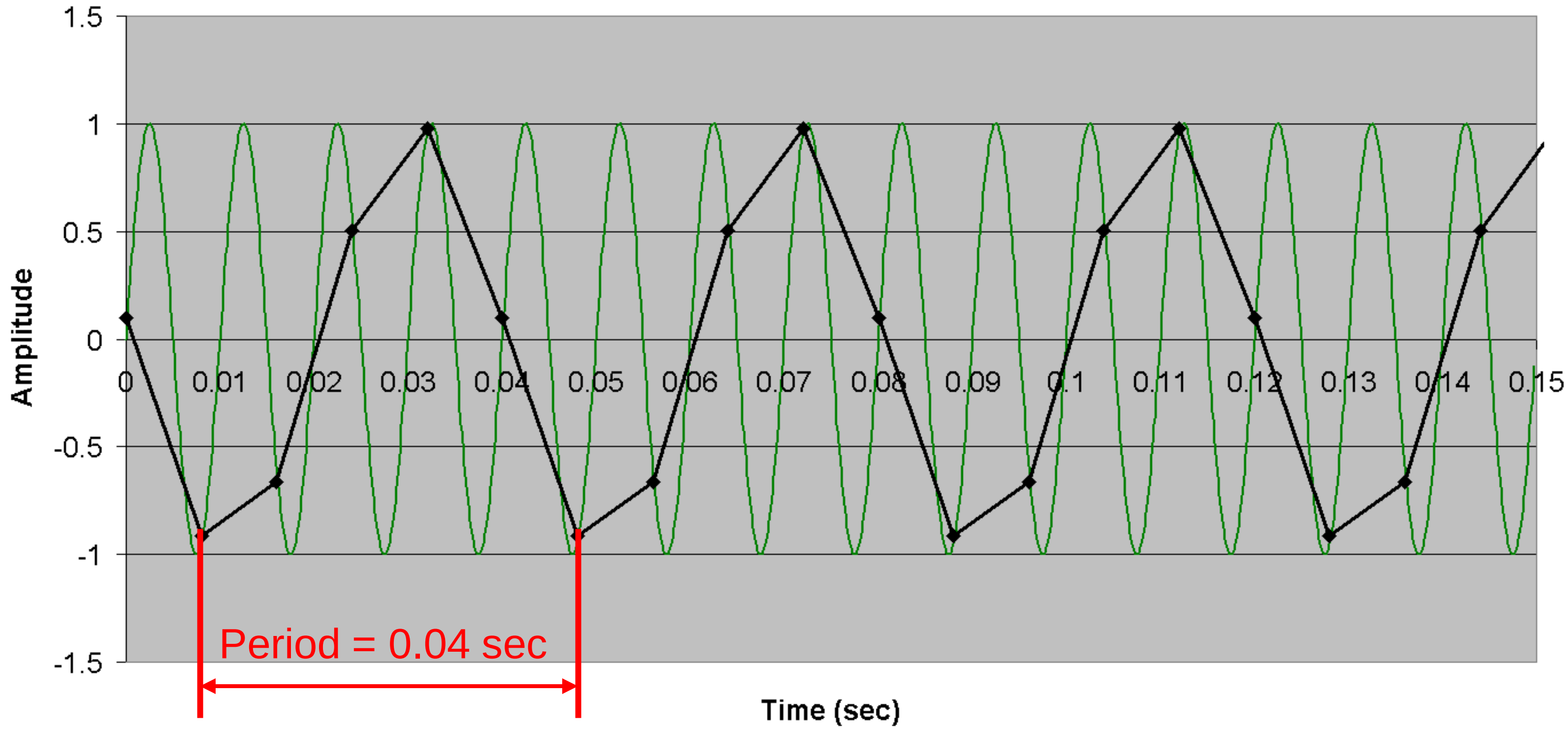


The 100 Hz sine wave will look like a 25 Hz sine wave when sampled at 125 sps.

f_s=125 sps

In the Time Domain

Signal Freq: 100 Hz fs: 125 sps Ts=8 msec NF =125sps/100Hz = 1.25



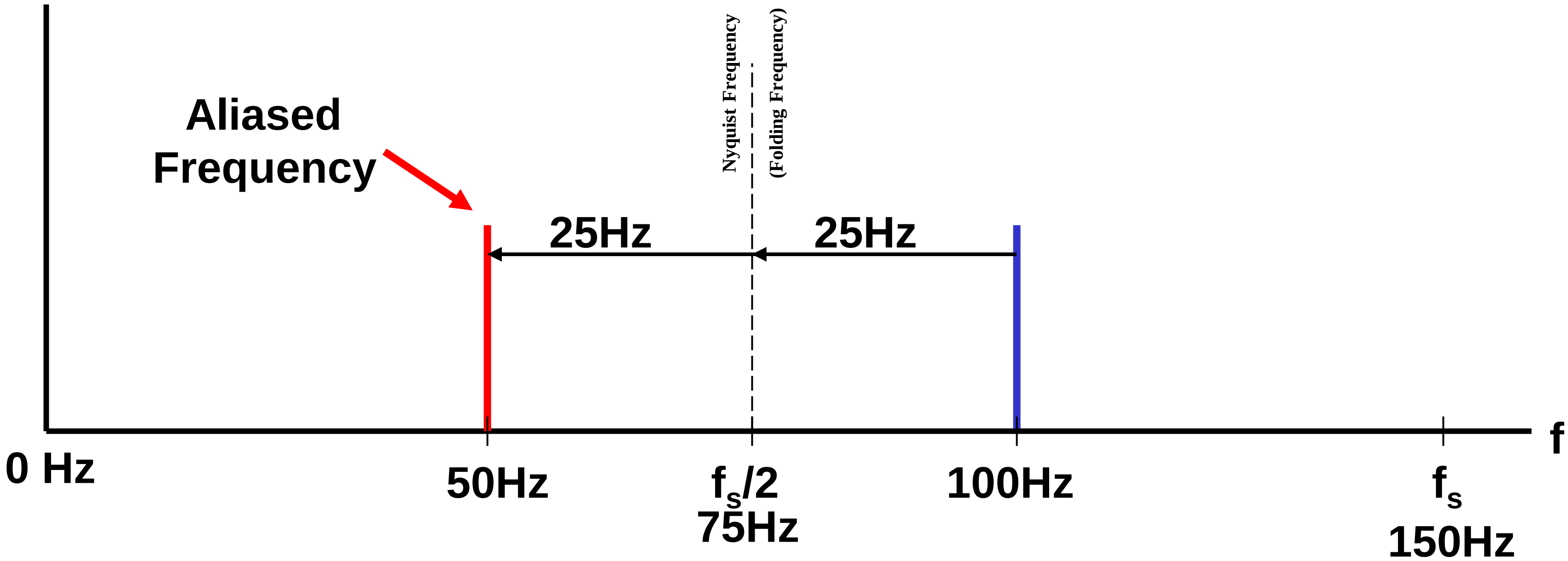
$$Frequency = \frac{1}{Period} = \frac{1}{T} = \frac{1}{0.04sec} = 25\ Hz$$

Measuring the frequency of the reproduced signal, it is determined to have a frequency of 25 Hz.

f_s=150 sps

In the Frequency Domain

Signal Freq: 100 Hz fs: 150 sps Ts=6.67 msec NF =150sps/100Hz = 1.5

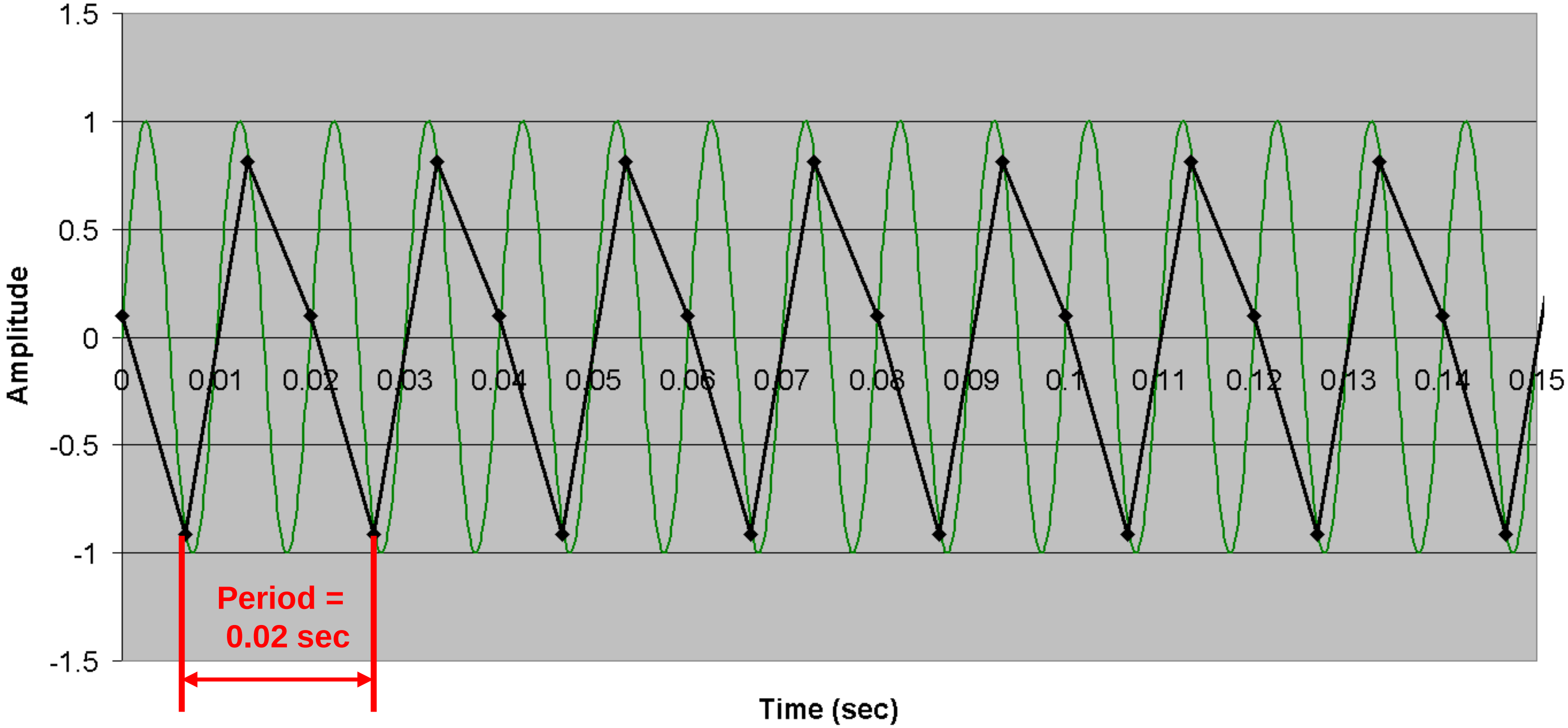


The 100 Hz sine wave will look like a 50 Hz sine wave when sampled at 150 sps.

f_s=150 sps

In the Time Domain

Signal Freq: 100 Hz fs: 150 sps Ts=6.67 msec NF =150sps/100Hz = 1.5



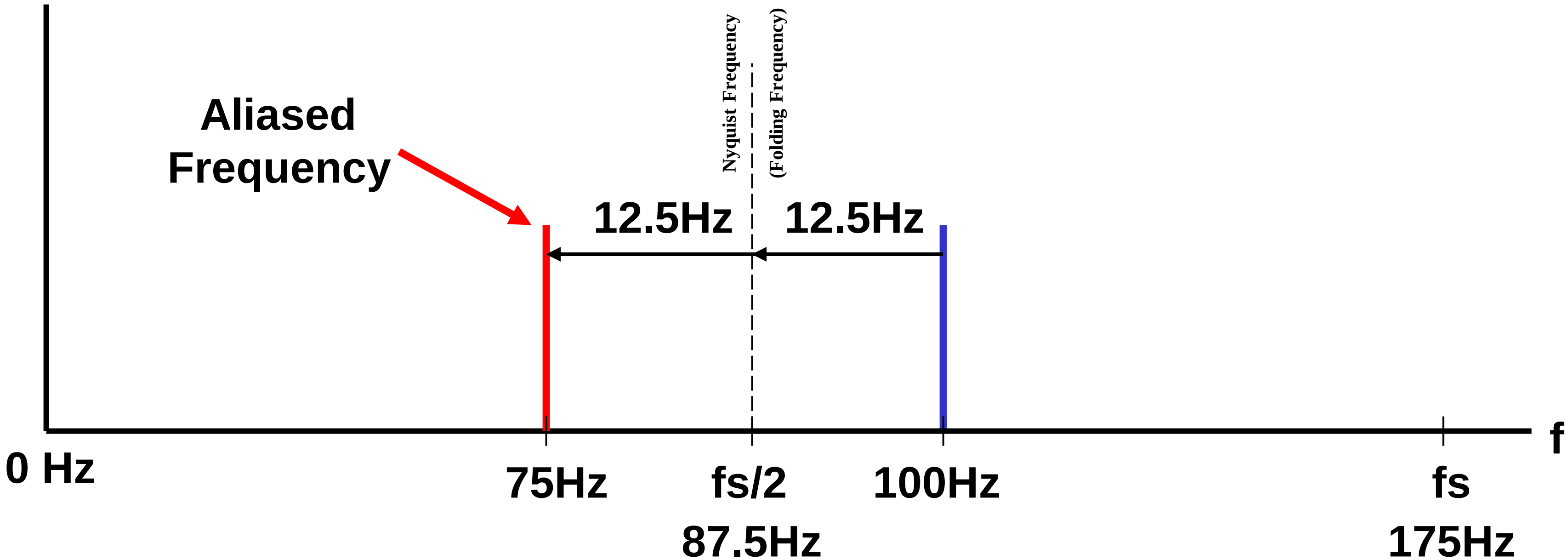
$$Frequency = \frac{1}{Period} = \frac{1}{T} = \frac{1}{0.02sec} = 50\ Hz$$

Measuring the frequency of the reproduced signal, it is determined to have a frequency of 50 Hz.

f_s=175 sps

In the Frequency Domain

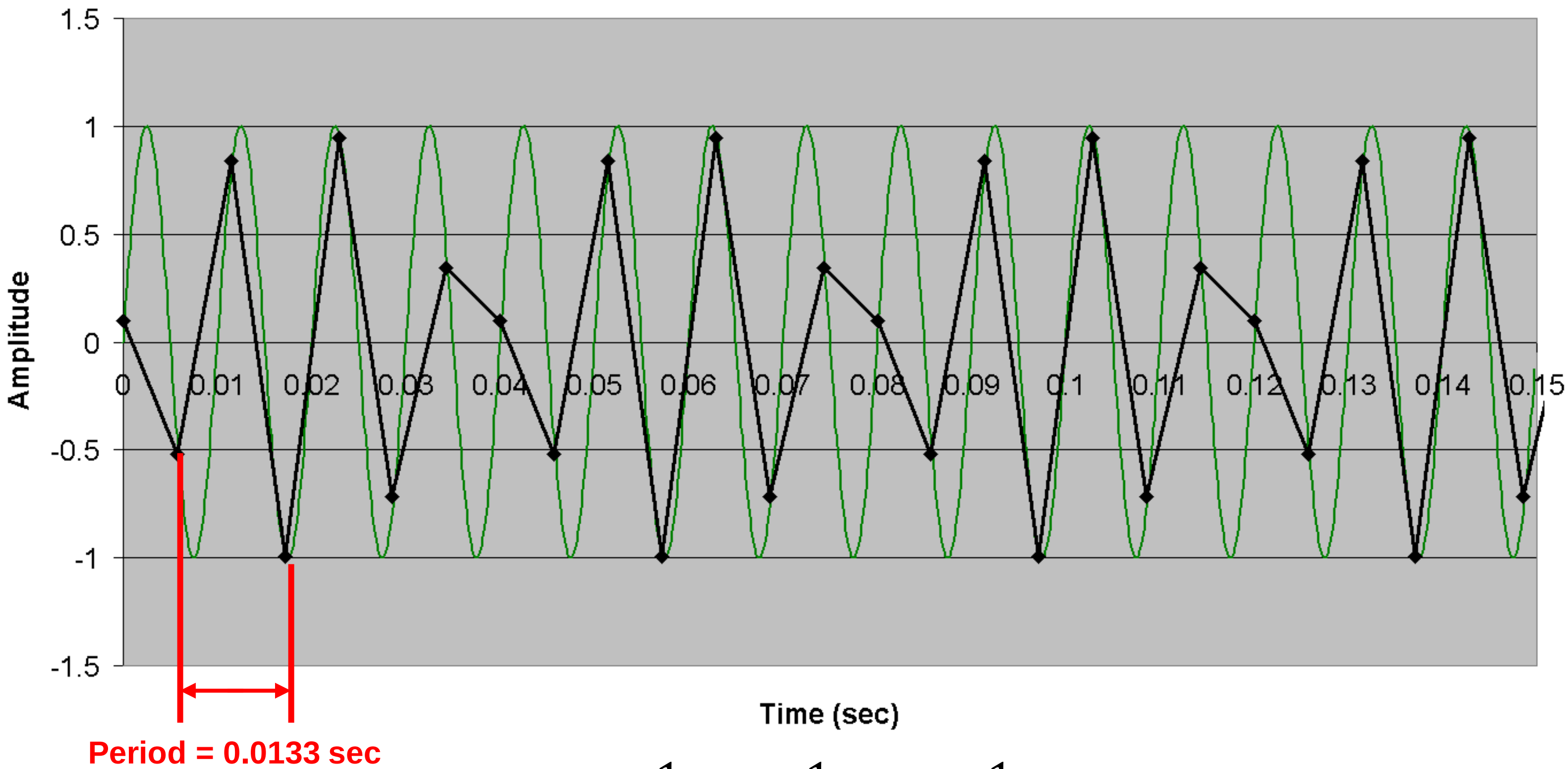
Signal Freq: 100 Hz   fs: 175 sps Ts=5.71 msec   NF =175sps/100Hz = 1.75



The 100 Hz sine wave will look like a 75 Hz sine wave when sampled at 175 sps.

f_s=175 sps
In the Time Domain

Signal Freq: 100 Hz   fs: 175 sps Ts=5.71 msec   NF =175sps/100Hz = 1.75



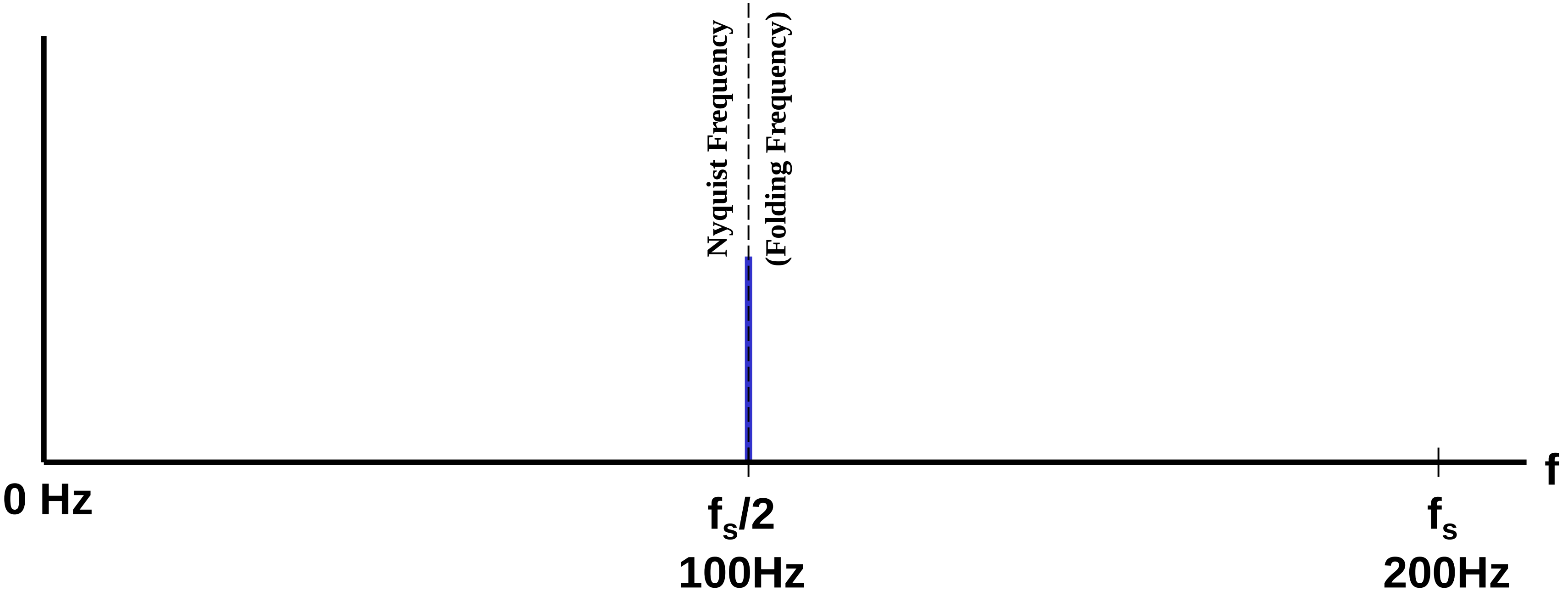
$$Frequency = \frac{1}{Period} = \frac{1}{T} = \frac{1}{0.0133sec} = 75\ Hz$$

Measuring the frequency of the reproduced signal, it is determined to have a frequency of 75 Hz.

f_s=200 sps

In the Frequency Domain

Signal Freq: 100 Hz fs: 200 sps Ts=0.05 sec NF =200sps/100Hz = 2

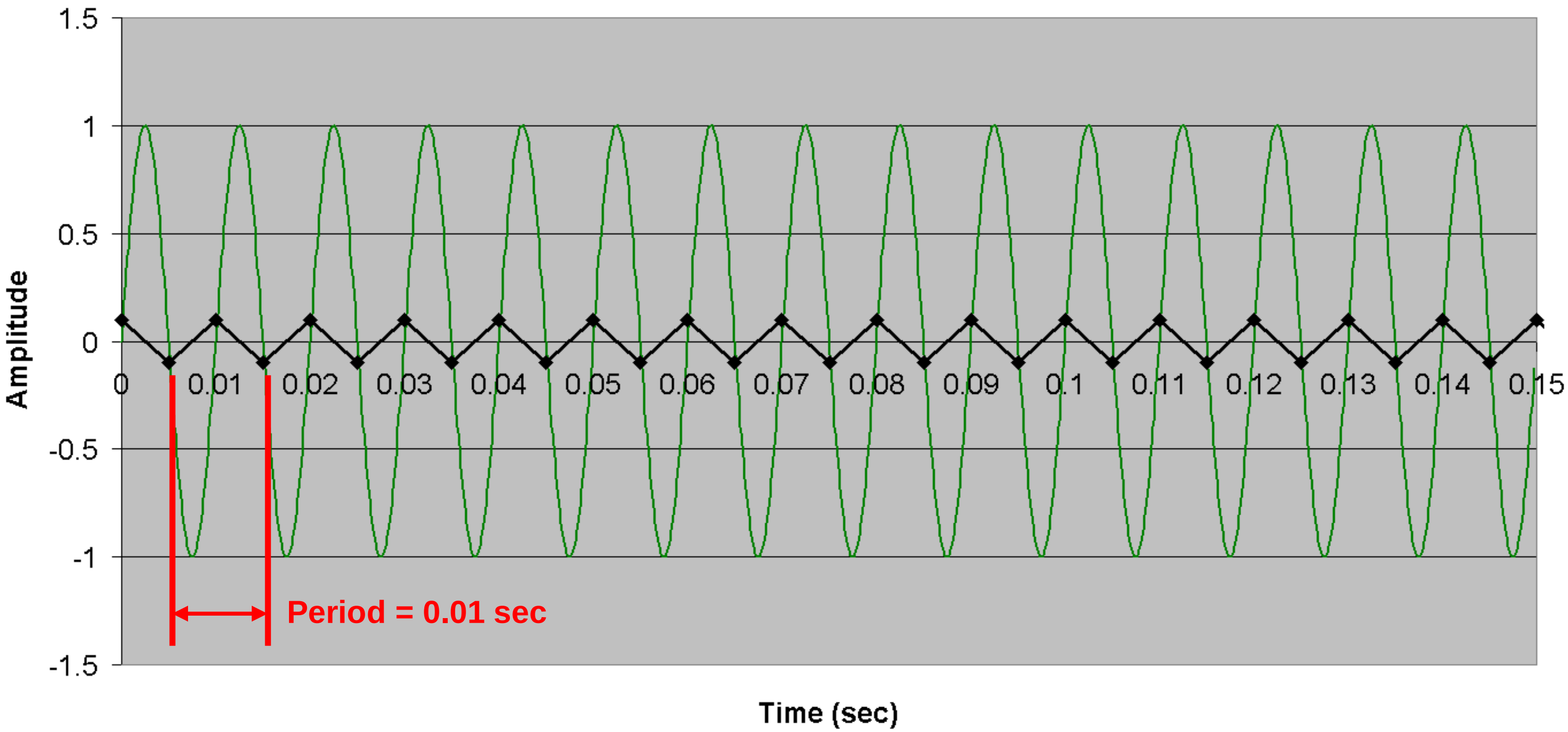


Two samples per period of a 100 Hz sine wave would yield a sample rate of 200 sps. The Nyquist Frequency will be half that or 100 Hz. The sine wave's frequency falls right on the Nyquist frequency, so theoretically you will be able to detect the frequency correctly.

f_s=200 sps

In the Time Domain

Signal Freq: 100 Hz	fs: 200 sps Ts=0.05 sec	NF =200sps/100Hz = 2



$$Frequency = \frac{1}{Period} = \frac{1}{T} = \frac{1}{0.01sec} = 100\ Hz$$

In the time domain, the frequency of the signal is correctly replicated. The Sampling Theory is mainly concerned with avoiding aliasing at the minimum rate of $f_s = 2f_n$.

Sampling to Avoid Aliasing in the real world

- Theoretically, to avoid aliasing, a sample rate of $f_s \geq 2f_n$ is needed for a signal band limited to a frequency of f_n.
- To band limit the signal, it is filtered such that unwanted frequency components of the signal are attenuated and no longer contribute to the signal.
- A frequency component is said to not contribute to a signal when it is <u>below the threshold of toggling the LSB weighting of an ADC</u> (analog to digital converter) within the band from 0 to f_c Hz (where f_c is the cutoff of the filter).
- The following slides will illustrate how the number of bits in the encoder, and the number of poles in the filter effects the sample rate needed to avoid aliasing.

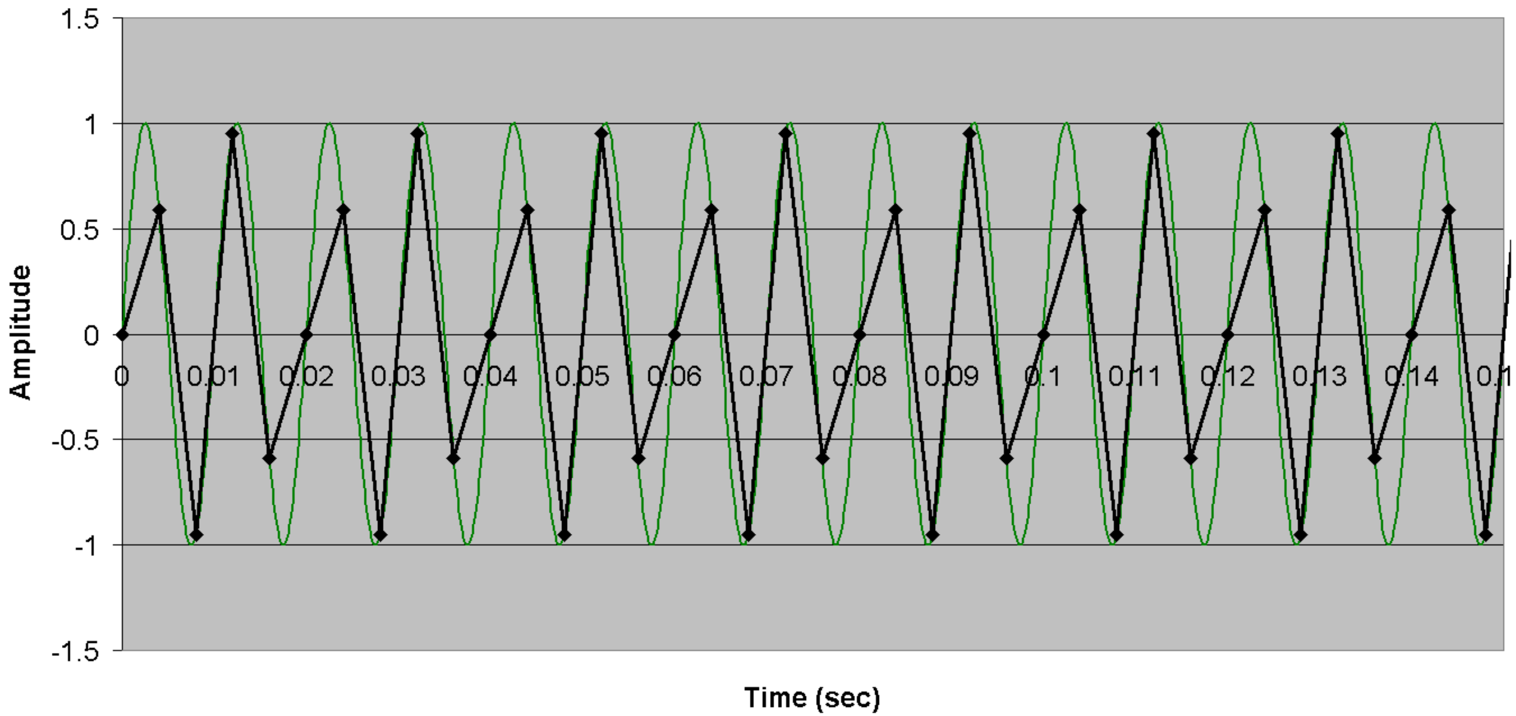
Signal Replication and Peak Detection

- The previous slides all dealt with sampling sufficiently to avoid aliasing. However we also want to accurately replicate the signal and capture the peaks.
- We will next illustrate how increasing the sample rate past $2f_n$ will better replicate the shape and amplitude of a 100 Hz sine wave.
- We will not illustrate the signal in the frequency domain because we are sampling enough not to have aliased frequencies.

f_s=250 sps

In the Time Domain

Signal Freq: 100 Hz	fs: 250 sps Ts=4 msec	NF =250sps/100Hz = 2.5

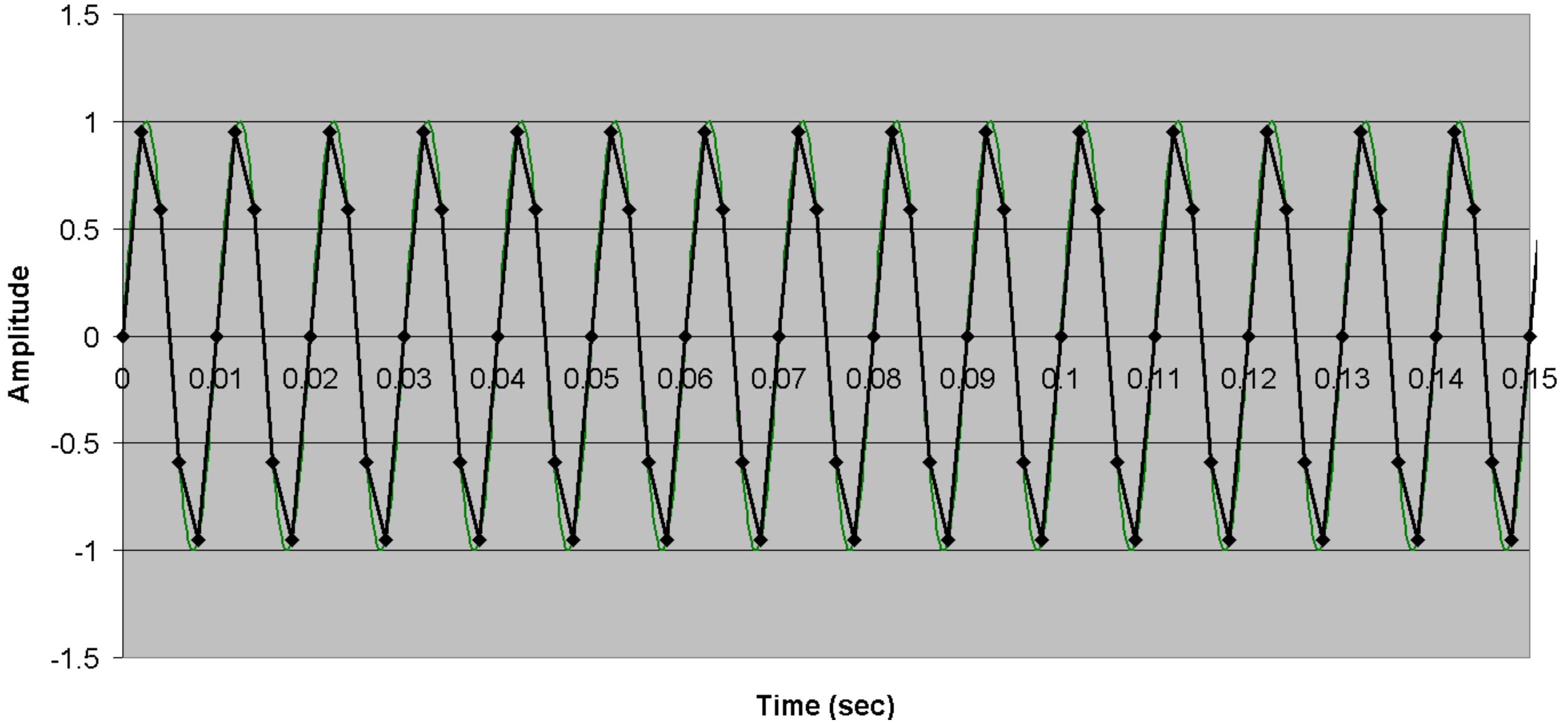


You can see the distortion of the amplitude, but the frequency content of the signal is preserved.

f_s=500 sps

In the Time Domain

Signal Freq: 100 Hz	fs: 500 sps Ts=2 msec	NF =500sps/100Hz = 5

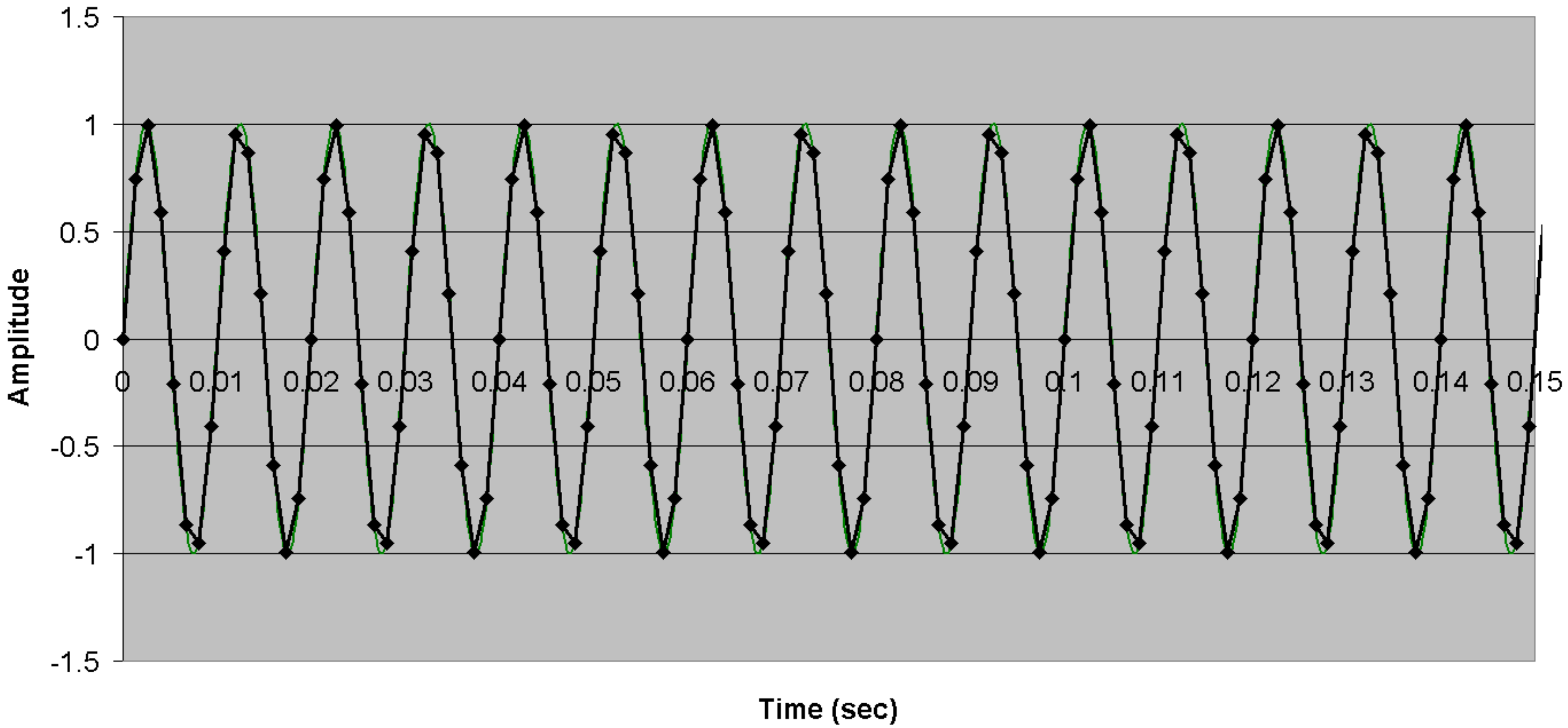


The replication of the signal is more improved as the Nyquist Factor increases to 5.

f_s=750 sps

In the Time Domain

Signal Freq: 100 Hz	fs: 750 sps Ts=1.33 msec	NF =750sps/100Hz = 7.5

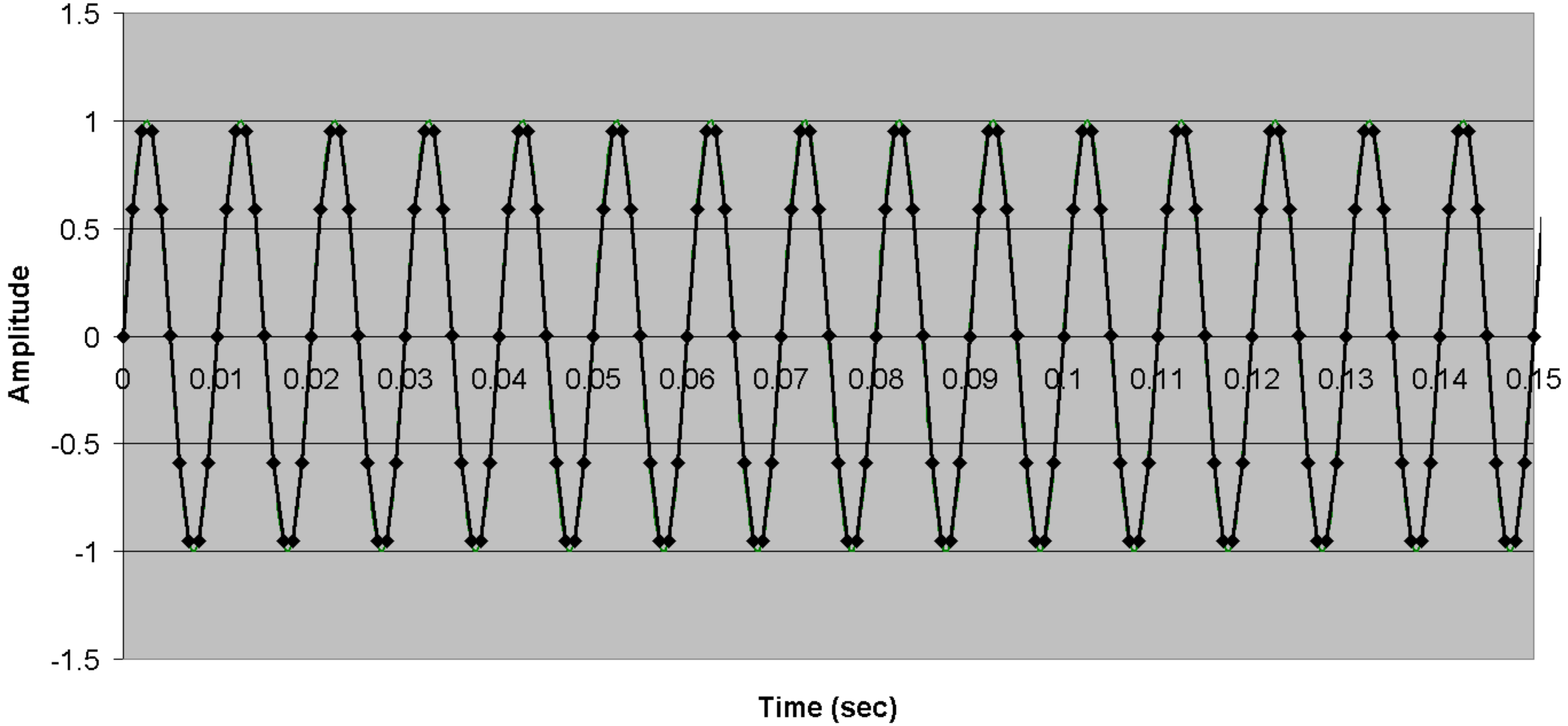


Even more improvement, but some of the peaks are being missed.

f_s=1000 sps

In the Time Domain

Signal Freq: 100 Hz   fs: 1000 sps Ts=1 msec   NF =1000sps/100Hz = 10

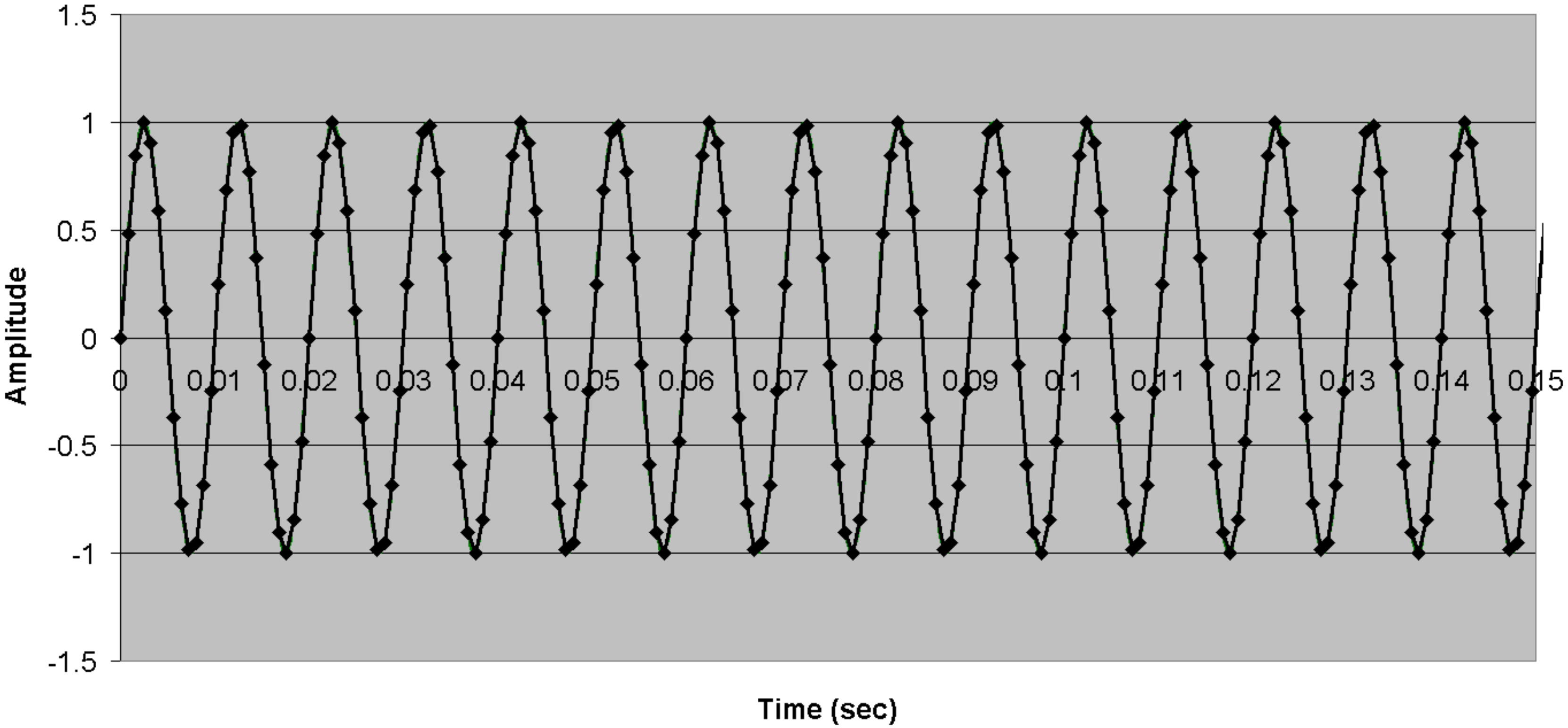


The peaks are being captured better, but not perfectly.

f_s=1250 sps

In the Time Domain

Signal Freq: 100 Hz   fs: 1250 sps Ts=0.8 msec   NF =1250sps/100Hz = 12.5



At 12.5 times the signal frequency, the peaks and shape of the signal are replicated rather well.

Steps To Successful Sampling

1. Understand the requirements of the person(s) that will be reviewing and using the data.
   - Frequency range of interest
   - Signal reproduction requirements
     - Basic frequency and amplitude
     - Max peak amplitude
     - FFT
2. Filter first – band-limit all analog signals so that they contain only the frequency range of interest, using a pre-sample/anti-aliasing filter.
3. For a p-pole Butterworth filter and an n-bit ADC, sample at $f_S \geq \left(2^{n/p} + 1\right) f_C$
4. Sample at higher rates to obtain a better amplitude reproduction of the signal. Use the chart to determine the multiplier n for a particular amplitude error. $f_s = n f_d$

Steps To Successful Sampling

5. Sampling at a rate greater than or equal to the Nyquist frequency will prevent aliasing and begin to capture frequency content of the signal

$$f_s \geq 2f_n$$

6. Sampling at a rate much higher than the twice the Nyquist frequency provides a better reconstruction of the original signal

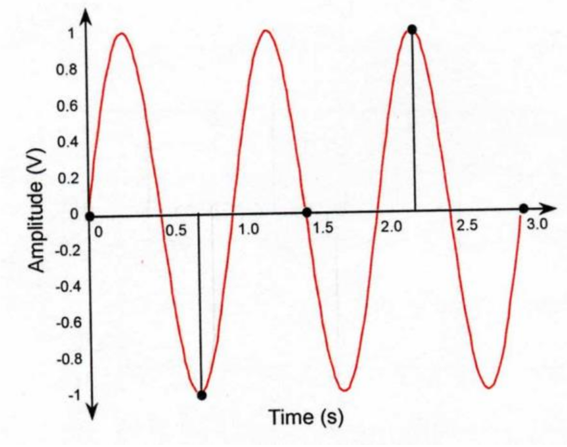
$$f_s \gg 2f_n$$

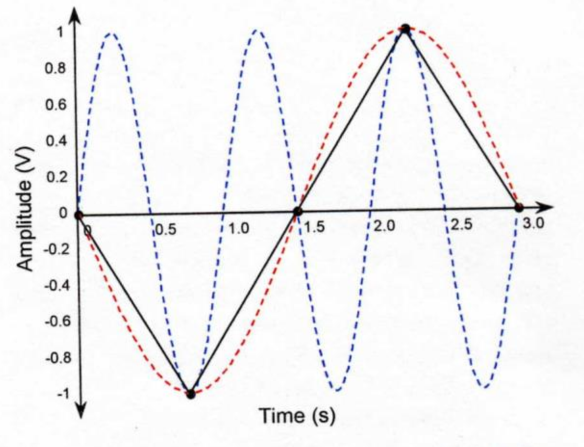
7. Peak sampling requires sampling at a rate many times higher than twice the Nyquist frequency

$$f_s \ggg 2f_n$$

Effects of Under-Sampling

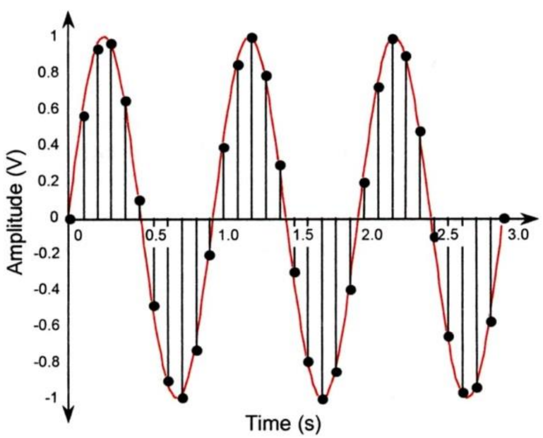
- Aliasing
- If data is aliased, it is <u>unusable</u>
- Lost data (information)
- Missed peaks (amplitude) in signal
- Difficult to reconstruct original signal accurately and with minimal distortion

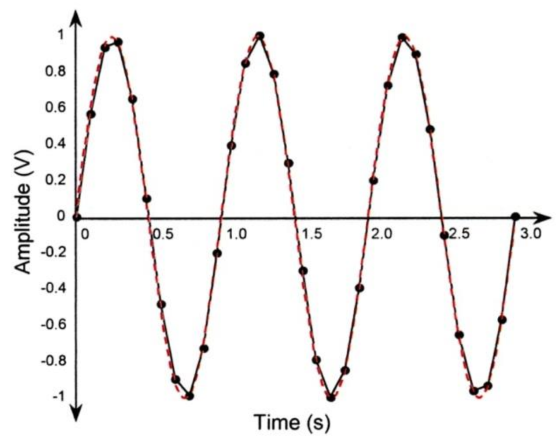




Effects of Over-Sampling

- No aliasing <u>if pre-sample filter is set correctly</u>
- More accurate data
- More data (information)
- Reconstructed signals will be more accurate with minimal distortion
- Requires more bandwidth for higher data rate
- Requires more recorder space or record time (this can be a problem with solid-state recorders)

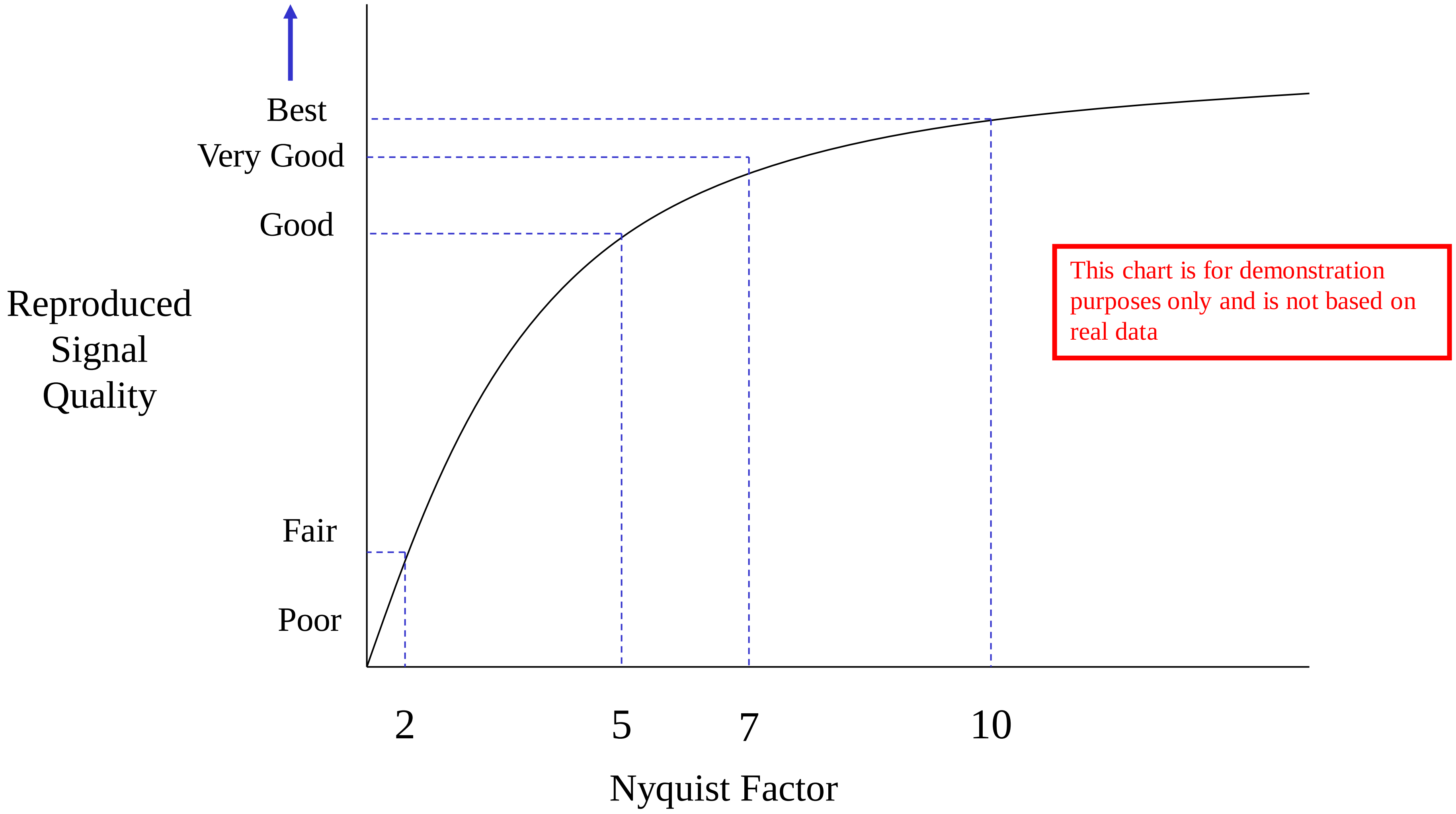




Over-sampling Bang For the Buck

- An important concept to be aware of is the fact that the relationship between Nyquist rates (over-sampling) and data quality is exponential
- Increasing the Nyquist factor from 2 to 5 provides a higher quality reproduced signal at a cost of 2.5 times the bit rate or bandwidth
- Increasing the Nyquist factor from 5 to 10 provides an even higher quality reproduced signal but the improvement for 2 times more bandwidth is not as good of a buy as increasing from 2 to 5 (bang for the $)
- The following slide illustrates this concept

Over-Sampling Bang For the Buck



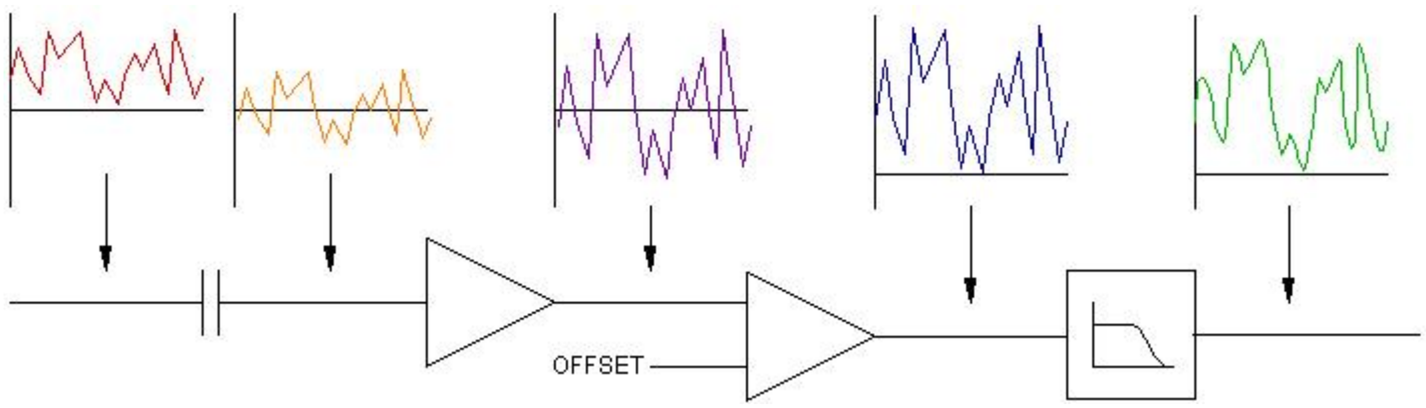
Gain and Offset

- AC/DC Coupling, Gain, and Offset are used to condition almost every analog parameter.

- Each analog measurement in a PCM encoder is digitized, so the analog input range has to be set to most efficiently use the range of the Analog to Digital Converter (ADC)

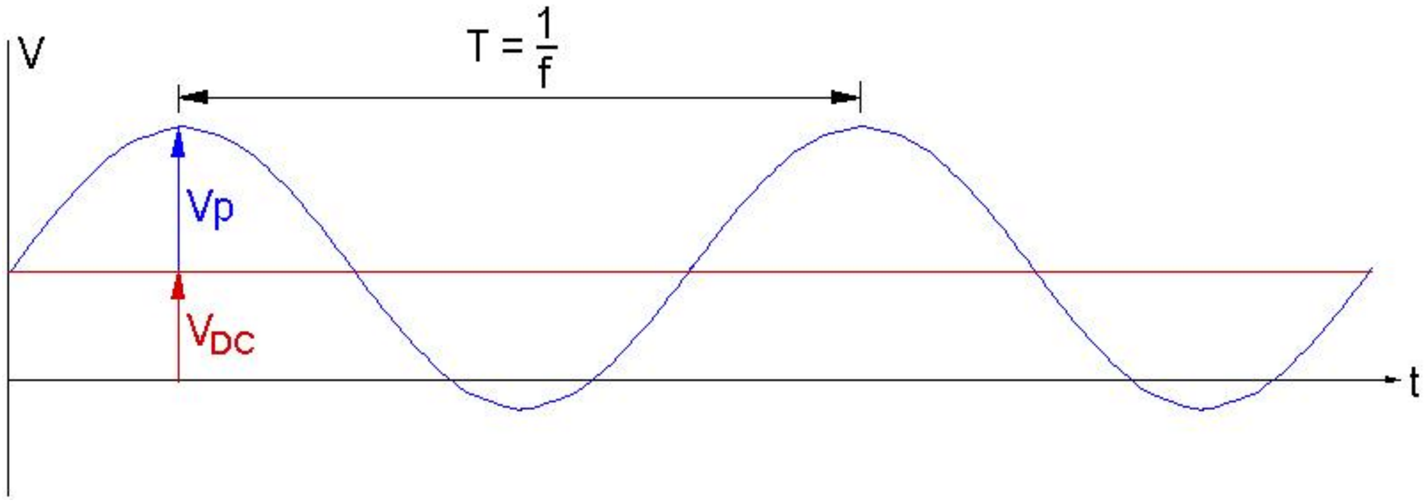
Purpose of Each Type of Conditioning

- AC Coupling - Used to remove any DC component (or offset) of a measurement that may not be desirable.
- Gain - Used to most efficiently utilize the input range of the ADC.
- Offset - Used to center the range of a measurement within the full range of the ADC.



AC and DC Component of a Signal

$$V = V_p \sin(2\pi f t) + V_{DC}$$



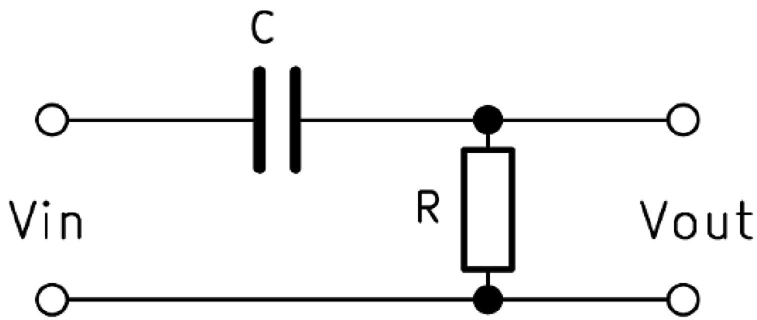
Signals not centered around 0 volts have a DC component, V_{DC}. The AC component of the signal has an amplitude of V_p.

AC Coupling

- AC Coupling is used to remove any DC component (or offset) of a measurement. This is done by placing capacitors in series with the signals. Capacitors do not pass DC signals.

- With no AC coupling (sometimes called DC coupling) both the AC and DC components of the signal are passed through to the encoder.

How AC Coupling Eliminates the DC Component of the Signal

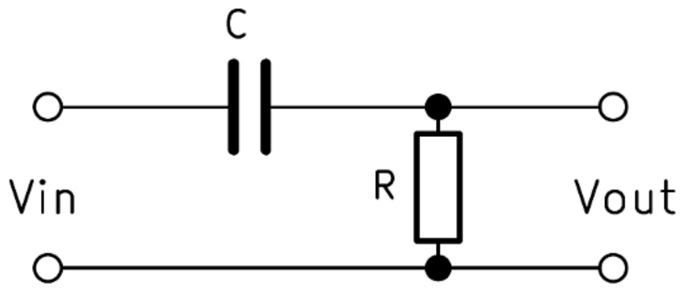


$$V_{out} = \frac{j(2\pi)fRC}{1 + j(2\pi)fRC} V_{in}$$

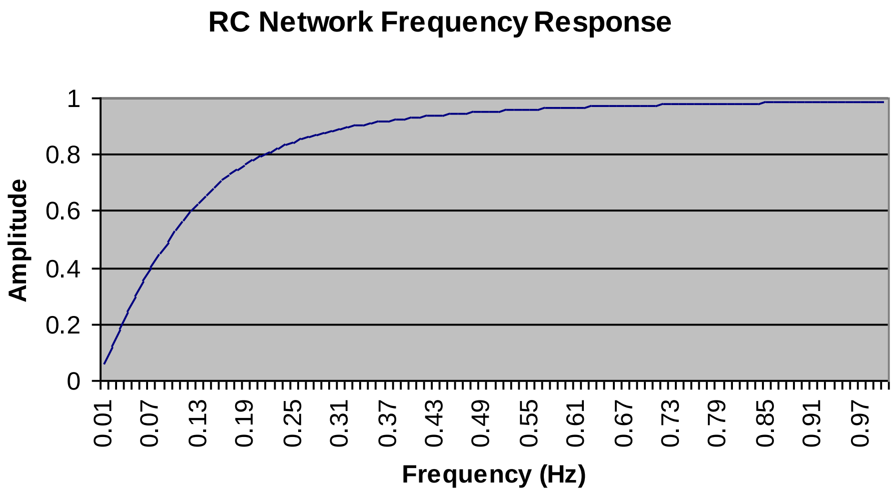
- The circuit shown above is a passive high-pass filter which uses a capacitor in series with the input signal. The DC portion of the signal does not have a frequency ($f = 0$). Substituting $f = 0$ into the equation yields $V_{out} = 0$. Therefore, no DC voltage passes through the capacitor.

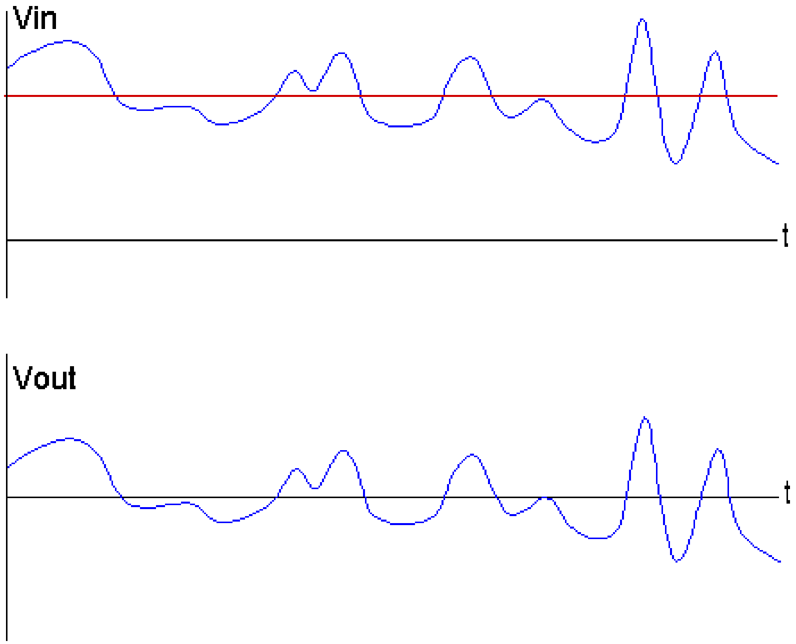
- The AC portion of the signal has a frequency. Therefore f does not equal 0, and V_{out} will have some value dependent upon the frequency.

Effect of AC Coupling



$$V_{out} = \frac{j(2\pi)fRC}{1 + j(2\pi)fRC} V_{in}$$

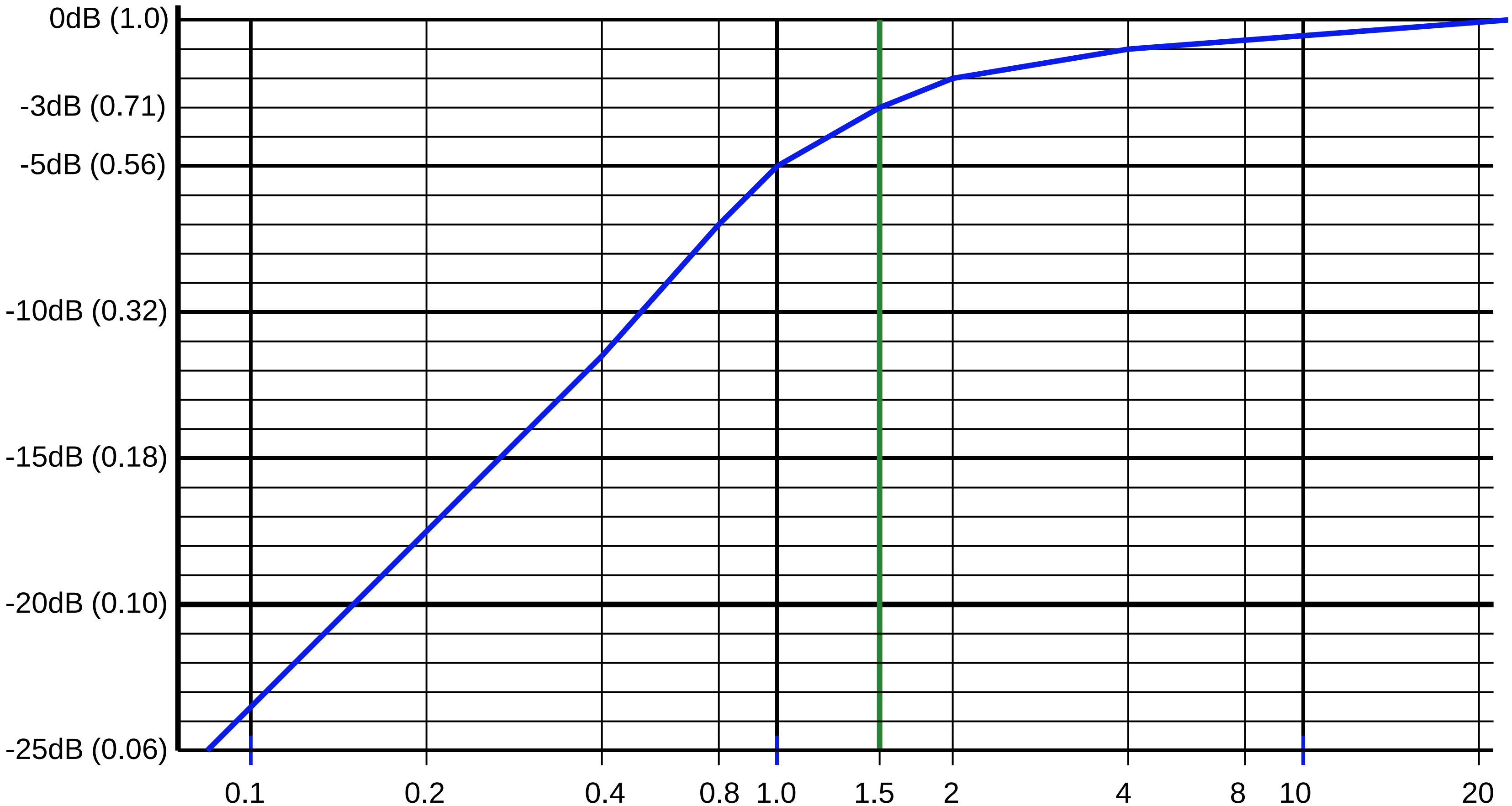




As the frequency increases, the filter's gain approaches 1.

Effect of AC Coupling

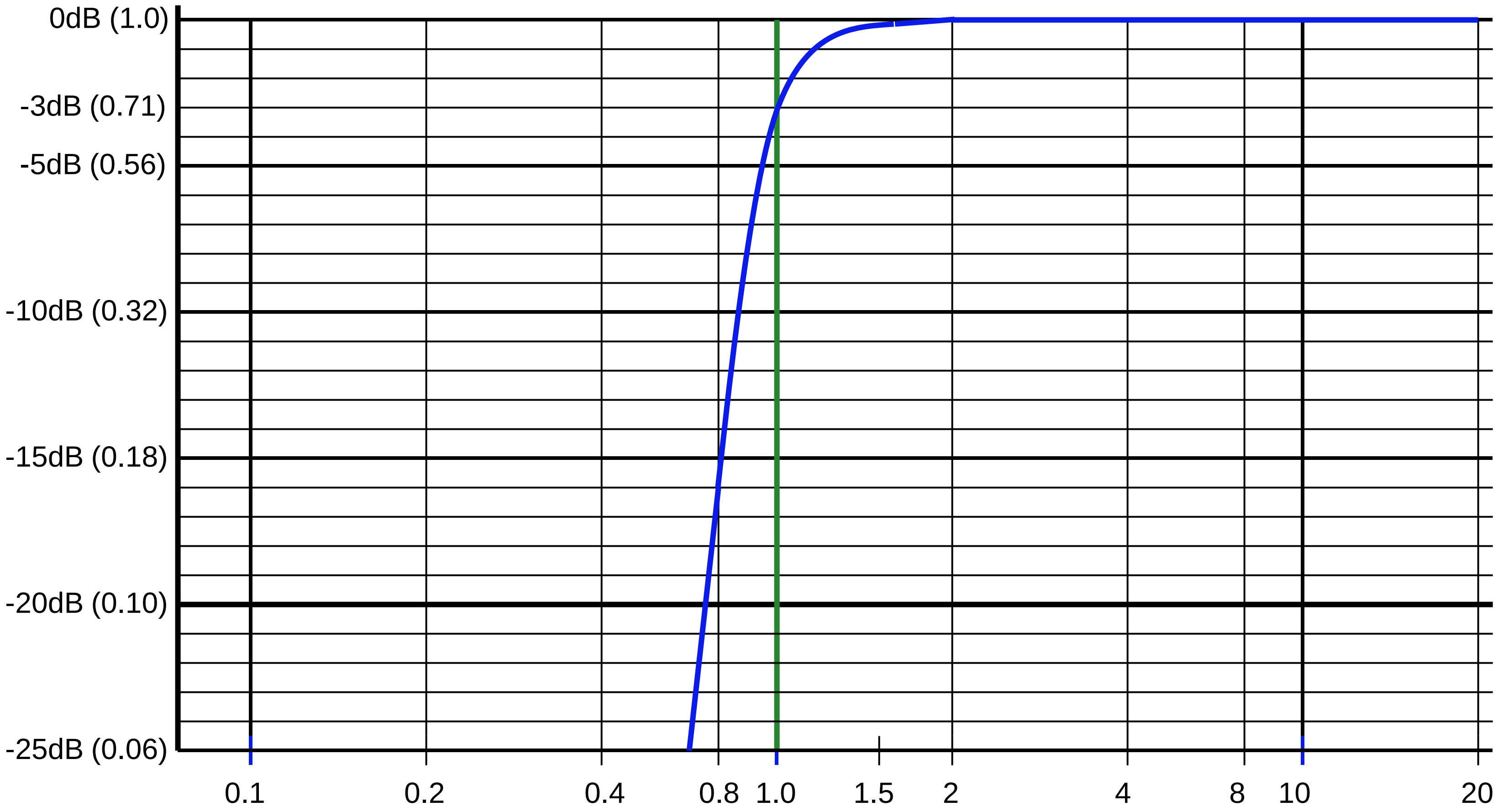
Be aware of the filtering effect of AC Coupling. Note the attenuation that occurs from 0 Hz to your lower data frequency.



This is the response for AC coupling on the SCD-608S card. It is a one-pole Butterworth with a cutoff frequency of 1.5 Hz. The attenuation does not reach 99% until 10 Hz, so frequencies below that are attenuated even more.

Lower AC Coupling Frequencies

Response of the 8-pole High Pass Butterworth filter, fc=1Hz



With the 8-pole high pass filter, frequencies at 2 Hz are not attenuated.

Why AC Couple a Signal?

- You are interested in the acceleration due to the movement of the wings during flight, but not in the 1 G acceleration due to gravity
- You are interested in the flutter or vibration response properties from a strain gage, not the static loading from the structure itself
- Sometimes a sensor has an output DC offset for now good reason

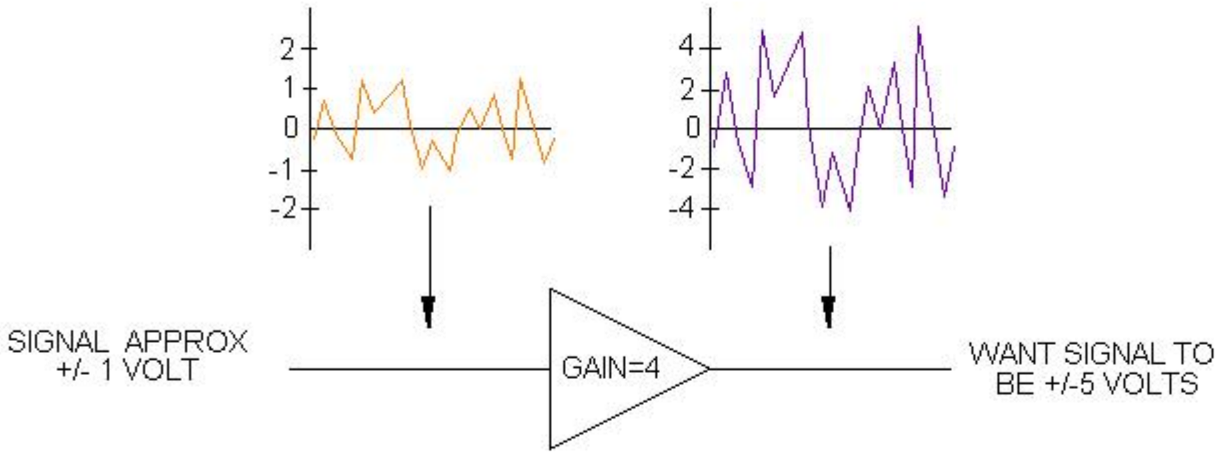
Important Note When AC Coupling

When a parameter is AC Coupled, no DC component of the signal is passed to the encoder. This poses a problem when performing calibrations where you are substituting a DC voltage to simulate the transducer.

Gain

- Gain is used in signal conditioning to expand or attenuate the measurement voltage range to maximize the usage of the range of the Analog to Digital Converter (ADC).
- In the CDAU and MCDAU, most of the input ranges of the ADC's input is ±5 volts.
- Select input signal gain such that when the ADC encodes the data (from 0-4095 counts for a 12-bit system) it is using as much of the 4095 count range as possible.

Selecting the Right Gain



A programmable amplifier has gains of 1, 2, 4, and 8. If your input signal is approximately ±1 volts and the ADC wants to see ±5 volts, then a gain of 4 should be selected to maximally utilize the input range of ±5 volts.

Offsets

- Offsets are used when the voltage of a measurement is not centered around zero volts. Many measurements such as positions, strain gages, and some high output transducers have unwanted offsets in their output.
- Since the input to the ADC is ±5 volts (centered around zero volts), offset is added or subtracted from the signal to most efficiently use that range.

Questions?