Diffusion Processes on Complex Networks - Lab

Assignment 1

Janusz Szwabiński

1. Consider the undirected network defined by the following set of links:

Alice	Bob	Bob	Gail	Irene	Gail
Carl	Alice	Gail	Harry	Irene	Jen
Alice	David	Harry	Jen	Ernst	Frank
Alice	Ernst	Jen	Gail	David	Carl
Alice	Frank	Harry	Irene	Carl	Frank

   (a) Draw the network by hand.
   (b) How many nodes are there?
   (c) What is the density of the network?
   (d) Calculate the degree of each node. Who is the most central node according to this measure?
   (e) Calculate the clustering of each node and the average clustering of the network.
   (f) Calculate the closeness centrality for each node. Who is the most central node according to this measure?
   (g) Calculate the betweenness centrality of each node. Who is the most central node according to this measure?

2. For the above network:

   (a) prepare a CSV file with the edge list;
   (b) visualize the network by making use of the Gephi software;
   (c) calculate the basic network measures within Gephi.

   You may have a look at
   `https://gephi.org/tutorials/gephi-tutorial-quick_start.pdf`
   for a nice introduction to Gephi.

3. An undirected unweighted network of size N may be represented through a symmetric adjacency matrix $\mathbf{A} \in \mathrm{R}^{N\times N}$, which has $a_{ij} = 1$, if nodes i and j are connected, and $a_{ij} = 0$ otherwise. We assume that $a_{ii} = 0$, so there are no self-loops in the network.

   Let $\mathbf{e}$ be a column vector of N elements all equal to 1, i.e. $\mathbf{e} = (1, 1, \ldots, 1)^T$, where the superscript T indicates the transposition.

   Write expressions for or answer each of the following by making use of the above quantities and the matrix formalism (no sum symbol $\sum$ allowed!):

   (a) the vector $\mathbf{k}$ whose elements are the degrees k_i of the nodes $i = 1, 2, 3, \ldots, N$;
   (b) the total number L of links in the network;

(c) the matrix $\mathbf{N}$ whose element n_{ij} is equal to the number of common neighbors of nodes i and j;

(d) the number T of triangles present in the network. A triangle is three vertices, each connected by edges to both of the others (hint: trace of a matrix);

(e) how would you determine whether the network is connected only by looking at the adjacency matrix?