HSC Research Report

HSC/13/04

Diffusion of innovation within an agent-based model: Spinsons, independence and advertising

Piotr Przybyła*
Katarzyna Sznajd-Weron*
Rafał Weron**

* Institute of Theoretical Physics, University of Wrocław, Poland
** Institute of Organization and Management, Wrocław University of Technology, Poland

Hugo Steinhaus Center
Wrocław University of Technology
Wyb. Wyspiańskiego 27, 50-370 Wrocław, Poland
http://www.im.pwr.wroc.pl/~hugo/

Advances in Complex Systems

Diffusion of innovation within an agent-based model: Spinsons, independence and advertising*

Piotr Przybyła, Katarzyna Sznajd-Weron†
Institute of Physics, Wrocław University of Technology, Wyb. Wyspiańskiego 27, 50-370 Wrocław, Poland

Rafał Weron
Institute of Organization and Management, Wrocław University of Technology, Wyb. Wyspiańskiego 27, 50-370 Wrocław, Poland

We modify a two-dimensional variant of a two-state non-linear voter model and apply it to understand how new ideas, products or behaviors spread throughout the society in time. In particular, we want to find answers to two important questions in the field of diffusion of innovation: *Why does the diffusion of innovation take sometimes so long?* and *Why does it fail so often?* Because these kind of questions cannot be answered within classical aggregate diffusion models, like the Bass model, we use an agent-based modeling approach.

Keywords: Agent-based model; Diffusion of innovation; Word of mouth marketing; Conformity; Advertising; Spinson

1. Introduction

Modeling how a new product enters the market, also known as *diffusion of innovation*, has attracted academic interest since the 1960s [43, 29]. One of the most popular models in the field is the one developed by Bass [1]. It is a 3-parameter analytical model based on a deterministic differential equation and describes how the number of consumers having adopted to a new product (i.e., an innovation) changes in time. Although a number of other models have been proposed since 1969, the Bass model is still, in a sense, the reference model to compare with [29, 42]. While the Bass model fits many historical datasets well and is excellent at explaining what has happened, it also has several limitations, most importantly, the poor predictive power, especially early in the life cycle [30]. In fact, this is not a weakness restricted to the Bass model. All traditional analytical models are not behaviorally based, and

*This work was partially supported by funds from the National Science Centre (NCN) through grant no. 2011/01/B/ST3/00727. The Monte Carlo simulation and visualization app *The World According to Spinson* (WAS) is freely available for download from the IDEAS/RePEc repository: http://ideas.repec.org/s/wuu/hscode.html.
†Corresponding author e-mail: katarzyna.weron@pwr.wroc.pl.

do not reproduce the complexity of real-world diffusion patterns. Therefore, in the last decade there has been growing interest in applying the so called agent-based models (ABM) in the social sciences [17, 24, 46, 49, 57], particularly to understand a new product growth [18–23, 42].

A number of innovation diffusion models represent adoption behavior by means of a single dichotomous variable that represents an agent's state, i.e., agents are either in a 'potential adopter' (i.e., -1 or $\downarrow$, 'against', 'customers of the old product') or an 'adopter' state (i.e., $+1$ or $\uparrow$, 'in favor of', 'customers of the innovative product') [18–20, 29, 41]. Binary variables are also widespread in opinion dynamics models, for instance, in the voter model [33, 36], the majority model [8, 9, 32], the Sznajd model [52] or the general sequential probabilistic model [13]. Although using binary variables to describe human attitudes, opinions or behaviors is a considerable simplification, Watts and Dodds [57] argue that it can be applied to a surprisingly wide range of real world situations, including diffusion of innovation phenomena.

Interestingly, the idea of binary states can be easily linked with the seminal model of racial segregation developed by Schelling [45] in the late 1960s or even with the famous Lenz-Ising model of ferromagnetism [40] proposed already in the 1920s. Therefore, in models originating in statistical physics such dichotomous variables are called *spins*, as a reference to the magnetic properties of a system. However, from the social point of view the object of interest is a person that has one of two possible opinions, attitudes, etc. To avoid such absurd terms as 'person up' or 'person down', following [41] we call these agents *spinsons*. A spinson should be understood as a type of an agent that is characterized by only one binary trait and is a combination of an arrow (i.e., a *spin*) and a *person*, see Figures 1-4.

In this paper we modify a two-dimensional variant [56] of a two-state non-linear voter model and apply it to understand how new ideas, products or behaviors spread throughout the society in time. One of the best-established stylized facts in the field of diffusion of innovation is the S-shaped curve representing the time change of the number of consumers having adopted to a new product [29]. This means that any new model should be able reproduce this feature. Hence, we first investigate the number of adopters in time and replicate this stylized fact within our model. Then we proceed to study how factors like level of conformity or independence in the society influence the S-shaped curve, and consequently impact the success or failure of a new product adoption.

The main novelty of our model compared to the one used in [56] is the introduction of 'noise'. Previously we have considered only one type of social response – *conformity*. Therefore as a steady state one of two absorbing states was always reached: all spinsons up or all spinsons down. This was not very realistic from the social point of view. A second type of social behavior – *independence* – was introduced in [53]. With probability p an agent is acting independently of the social pressure, which introduces 'noise' to the system. The independent behavior is characterized by a spinson's *flexibility* – the higher the flexibility the more frequent are the changes of the spinson's state (or opinion). As a result of introducing in-

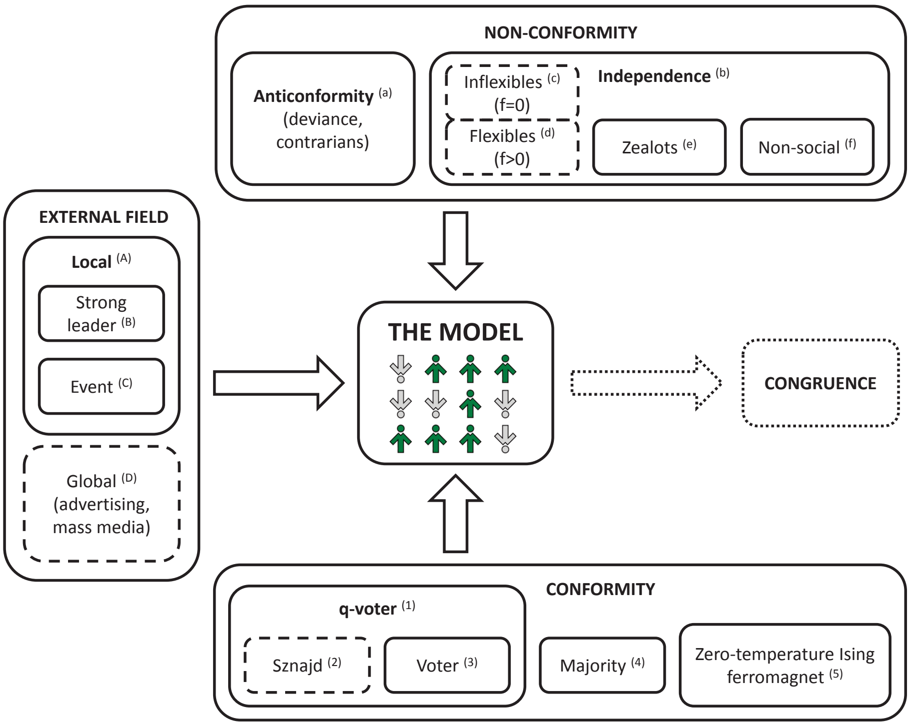

Fig. 1. Different types of social influence introduced in the literature for spinson models. Some models consist of several types and others of only the one type of social influence, see Table 1 for examples. Note that congruence is not an input, but an outcome of the model dynamics.

dependence, the system never reaches an absorbing steady state. In the stationary state there is always a certain fraction – dependent on p – of agents of the opposite sign than the majority. In this study, we use both the external field (representing advertising; introduced in [56]) and independence (proposed in [53]), see Figure 1.

The paper is structured as follows. In Section 2 we introduce the model and the Monte Carlo simulation scheme. We also discuss the position of our model in the realm of spinson-type models of social influence. In Section 3 we report on the Monte Carlo results obtained for a square lattice. In the following Section we consider our model on a complete graph and provide analytical results for the system dynamics and numerical results for the stationary values of various model characteristics. Note that these results can be treated as 'mean field' approximations for a regular lattice. In Section 5 we show that flexibility is a redundant parameter of the model and can be represented as a function of independence. Finally in Section 6 we wrap up the results and comment on possible applications of the model.

Table 1. Examples of spinson models with different types of social influence, as shown in Figure 1. For more examples we refer to the review paper [7] and the very recent book on sociophysics [16].

Conformity	Independence	External field	Applications
(2)	–	–	opinion dynamics [52]
(2)	(a)	–	opinion dynamics [34]
(2)	(c), (d)	–	opinion dynamics [53]
(2)	–	(B)	financial time series [55]
(2)	–	(D)	duopoly markets [56]
(4)	–	–	fashion phenomena [14], the wine market [5]
(4)	(e)	–	opinion dynamics [39]
(4)	(a)	–	political voting [11]
(4)	(c)	–	political voting [15]
(5)	–	(A), (D)	rational decisions [10]
(5)	–	(B), (D)	opinion dynamics [27]
(5)	(b)	(B)	opinion dynamics [28]

2. The model

2.1. *Social interactions*

We consider a set of N spinsons, i.e., agents which are described by a single binary variable: $S_i = -1$ (not adopted to the innovation) $S_i = +1$ (adopted to the innovation), $i = 1, ..., N$. At each elementary time step $dt = 1/N$, a group of four spinsons is chosen randomly and influences one of its neighbors. In general many different types of social influence are possible, as shown in Figure 1, but in this paper we take into account three of them:

- *Conformity* based on a unanimous opinion, as originally introduced by Sznajd-Weron and Sznajd [52] for a chain and later generalized to a two dimensional square lattice [51] and a complete graph [47]. In the sociophysics literature this type of influence became known as the *Sznajd model* or *Sznajd outflow dynamics* [44, 50]. In Figure 1 it is classified as conformity of type (2); for a square lattice it is illustrated in Figure 2.
- *Independence* which is a particular type of non-conformity. In this paper it should be interpreted as 'resistance to influence', like in [53] where it was originally introduced. In Figure 1 it is classified as non-conformity of type (c) or (d); for a square lattice it is illustrated in Figure 3.
- *Advertising* or *mass media* modeled by the global external field as in [56]. In Figure 1 it is classified as an external field of type (D); for a square lattice this type of influence is presented in Figure 4.

It should be stressed that although in this paper we consider only one type of nonconformity (i.e. *independence*), the second type (i.e. *anticonformity*; see Fig. 1) could easily be taken into account. We do not introduce it here for at least two reasons. Firstly, as we have shown in [41] for this type of models, at the macroscopic level anticonformity will yield qualitatively analogous results to nonconformity, i.e. it will introduce disorder to the system. A similar effect of disorder was obtained by Galam [11], who introduced contrarians to his majority model. Secondly, it is good practice to start with a simple model and expand it at a later stage. Taking into consideration anticonformity involves introducing another parameter, which makes the model more complicated and difficult to analyze comprehensively.

Furthermore, we do not treat information in mass media as a feedback created by public opinion (like in [24, 46]) but rather as advertising. Thinking of global information as the most popular opinion in a real-world society, as introduced by Shibanai et al. [46] in a generalized version of the Axellrod model, is an interesting idea but only partially suitable to model diffusion of innovation. Companies create – rather than present – reality to convince consumers to choose their product. On the other hand, surveys and news report (or at least try to report) the ‘real’ consumer preferences. Therefore, probably both types of external influence should be taken into account to fully describe reality. In this paper we focus only on advertising (as a pressure induced on consumers), but combining both would be an interesting challenge for the future work.

2.2. *Grid topology*

We investigate our model on two types of grid topology:

- A *square lattice*, where a 2×2 panel of four spinsons is chosen randomly and influences its surroundings, as proposed by Stauffer et al. [51]. Note that in this paper we use a variant introduced in [53], where in one elementary time step $dt = 1/N$ only one of the eight neighboring spinsons may change its direction, see Figs. 2-4.
- A *complete graph*, where four agents are chosen at random and they influence a fifth randomly chosen spinson, as in [53]. Note that this corresponds to the mean field approach for the square lattice and allows for analytical calculations of the system dynamics.

Certainly, using a more complicated topology – like the small-world or a scale-free network – would be more appropriate to describe social groups. On the other hand, in spite of current interest in complex networks, it is still not entirely clear what is the structure of real world influence networks [57]. Moreover, Watts and Dodds [57] show that the structure of the network impacts the macroscopic outcome to a much lesser extent than the type of interactions between agents. Therefore, to really understand the model itself, it seems reasonable to start with a simple structure. A similar approach was taken by Goldenberg et al. who first studied their model on a square lattice [18] and then investigated the same model on various complex networks [19].

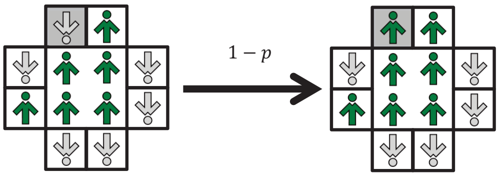

Fig. 2. Conformity in the two dimensional Sznajd model: with probability $(1-p)$ a randomly chosen spinson (the one in the gray cell; *left*) follows the opinion of the 2×2 panel in the middle (*right*), but only if it is unanimous. If the panel is not unanimous the spinson is responsive to advertising, see Fig. 4.

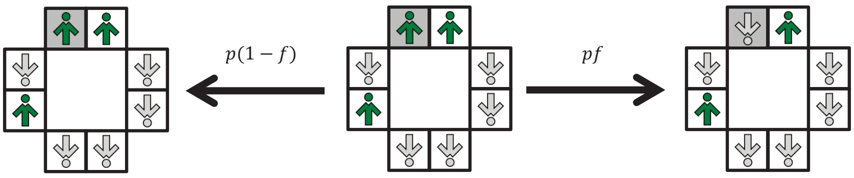

Fig. 3. Independence in the two dimensional Sznajd model: with probability p a randomly chosen spinson (the one in the gray cell; *center*) flips $S_k(t+dt) = -S_k(t)$ with probability f (*right*) or remains unchanged $S_k(t+dt) = S_k(t)$ with probability $1-f$ (*left*), independently of the state (or opinion) of the 2×2 panel.

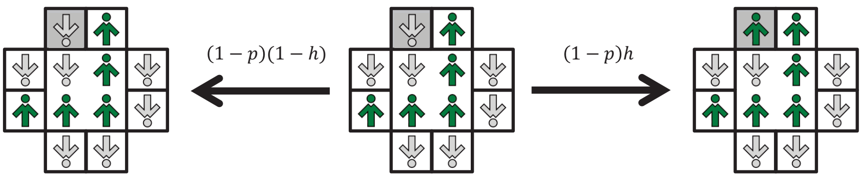

Fig. 4. Advertising in the two dimensional Sznajd model: with probability $(1-p)$ a randomly chosen spinson (the one in the gray cell; *center*) is responsive to advertising if the 2×2 panel is not unanimous. With probability h the spinson adopts to (i.e., buys) the advertised product (*right*) and with probability $(1-h)$ it remains unchanged (*left*).

2.3. *Simulation procedure*

Because in this paper we study the diffusion of innovation, at the initial stage all spinsons are down (i.e., -1, $\downarrow$, customers of the old product). Hence, $c_0 = 0$ where

c_t denotes the ratio (concentration) of adopted to the innovation at time t:

$$c_t = \frac{N_\uparrow(t)}{N} \tag{1}$$

and the number of adopted $N_\uparrow(t)$ changes in time due to social influence as discussed above.

We use a random sequential updating scheme. The time t is measured in Monte Carlo steps (MCS), which consist of N elementary time steps $dt = 1/N$. Each of the elementary time steps is composed of five simulation substeps:

Substep #1: Randomly choose one spinson S_i, which may change its orientation in this time step.

Substep #2: Check if spinson S_i will act independently (with probability p) or not (with probability $1-p$). To do this, randomly draw a uniformly distributed number $r \sim U[0,1]$. If $r < p$ then the spinson acts independently (see also Fig. 3) and goto #3, if $r \geq p$ then goto #4.

Substep #3: With probability f the chosen spinson changes into the opposite state. To check this, randomly draw a uniformly distributed number $r \sim U[0,1]$. If $r < f$ then $S_i(t+dt) = -S_i(t)$ else $S_i(t+dt) = S_i(t)$. Increase time $t \to t+dt$ and goto #1.

Substep #4: Randomly choose a 2×2 panel of spinsons that will influence S_i. If the panel is unanimous (i.e., all four spinsons have the same value) than S_i will take the opinion of a group (for a square lattice this is illustrated in Fig. 2), increase time $t \to t+dt$ and goto #1. If the panel is not unanimous than goto #5.

Substep #5: With probability h the chosen spinson S_i is responsive to adversing (see Fig. 4). To do this, randomly draw a uniformly distributed number $r \sim U[0,1]$. If $r < h$ then $S_i(t+dt) = 1$, increase time $t \to t+dt$ and goto #1.

2.4. *Similarities and differences to other diffusion of innovation models*

Before going to the results we would like to draw attention to several issues associated with the construction of the model:

- Our model is similar to many diffusion of innovation models [1, 18–22, 43] in the sense that the decisions of consumers are driven by internal (social interactions) and external (advertisement, mass media) influence.
- In contrast other studies, in our model the changes are reversible which means that spinsons can unadopt. A similar idea of rejection has been introduced by Goldenberg et al. [21]. They use four states to describe a consumer: 0 for unadopted, 1 for adopted and satisfied, -1 for adopted and dissatisfied, and 2 for a 'rejecter'. From this point of view, a down-spinson ($\downarrow$) in our model can be interpreted as a consumer who has not adopted

(which corresponds to state 0) or such who adopted but for various reasons has switched back to the old product/service (which corresponds to state 2). Note, however, that in [21] a rejecter (state 2) is an absorbing state, i.e. cannot change, whereas in our model all changes are potentially reversible.

- In our model the social interactions are assumed to be in some sense more important than the external factors. As discussed in Section 2.3, we first check whether a chosen spinson is influenced by the group and only an uninfluenced spinson is responsive to advertising. This model feature is consistent with the observation that friends have a much greater impact on decision changes than mass media [20, 46].
- At first glance, our model may resemble threshold models, in which an individual's threshold is defined as the proportion of the group needed to engage in a particular behavior [22, 25, 57]. From this perspective our model is a threshold model with the threshold equal to 1 (i.e. 100 %). However, it can easily be modified to a more general model with an arbitrary threshold [41]. A crucial difference lies in the fact that in our model always a group of four (or in a more general case – a group of q [41]) influences the spinson, while in other threshold models all neighbors influence the considered agent. This means that in our model a different group can potentially have impact on the spinson in each time step. Certainly the assumption that the group always consists of the same number of members is a simplification. On the other hand, as noticed by Galam [12], people usually discuss in small groups even if they have many friends. To randomly draw the size of the group from a certain distribution, as done by Galam, could be the next step but we leave it for future work.

3. Results on a square lattice

3.1. *Sample trajectories*

We start by presenting sample trajectories that show the time evolution of the ratio (the concentration) of the adopted individuals defined by eq. (1). In Figure 5 we plot two sample trajectories obtained from our model (*left panel*) and the historically observed percentages of household penetration for three successful innovations in the consumer-electronics market in the U.S.: television, color television and video-cassette recorder (*right panel*). Apparently in both panels the time change of the number of consumers having adopted to a new product is described by an S-shaped curve, in agreement with one of the best-established stylized facts in the field of diffusion of innovation [29]. As we have already mentioned, the S-shaped curve can be also obtained within the classical Bass model [1]. However, the Bass model, as well as other traditional aggregate models, does not reproduce the complexity of real-world diffusion patterns. For instance, it is known that in reality many innovations fail on the way from the laboratory to the market, which is often described by the 'valley of death' metaphor [58], or collapse of initially successful diffusions.

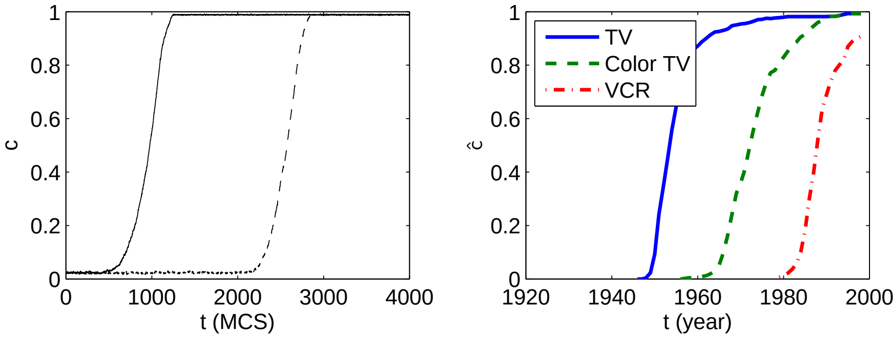

Fig. 5. *Left*: Two sample trajectories representing the time evolution of the ratio of adopted individuals c. Both trajectories start from the initial state $c_0 = 0$, are simulated on a square lattice 100×100 and have been generated for the same values of the model parameters: advertisement $h = 0.09$, independence $p = 0.1$ and flexibility $f = 0.1$. *Right*: The percentage $\hat{c}$ of household penetration for three consumer-electronics products in the U.S. from 1920 to 1998: television (TV), color television (Color TV) and videocassette recorder (VCR). Based on: Karl Hartig poster for the Wall Street Journal Classroom Edition, 1998 (data sources: A.C. Nielsen Company, Broadcasting & Cable Yearbook 1996, Electronic Industries Association, Federal Communications Commission, FIND/SVP).

These kind of phenomena cannot be explained by aggregate diffusion models.

The two sample trajectories plotted in the left panel of Fig. 5 were obtained for a 100×100 square lattice, the same initial state $c_0 = 0$ and the same set of model parameters (independence $p = 0.1$, flexibility $f = 0.1$ and advertising $h = 0.09$; these particular parameters were not chosen accidentally and we will return to this issue in a moment). Although in both cases innovation penetrates the market, the time needed for the 'take-off' is very different for the two trajectories. In the case of the solid-line trajectory about 500 MCS were enough for the 'take-off', whereas for the dashed-line trajectory this time was about four times longer. This brings us to one of the most frequently asked questions in this research area: *Why does the diffusion of innovation take sometimes so long?* Diffusion of innovation is definitely a very complex phenomenon and we certainly will not be able to fully answer this question here. However, we can at least try to understand the phenomenon within our relatively simple model.

For this purpose, it is instructive to look at the configurations of our artificial society in subsequent moments of time. Of course, each simulation will yield a unique time series of configurations. In the left panel of Figure 5 we have plotted the evolution of a macroscopic quantity, i.e., the concentration c of adopted individuals. However, one should remember that the same value of concentration c can correspond to very different spacial configurations of spinsons. In Figure 6 we present sample snapshots taken from a single time evolution of the system, described by the same set of parameters (p, f, h) as in Fig. 5. In this simulation, nothing happens in the system for a long time (ca. 500 MCS). Almost all spinsons are 'down' (light

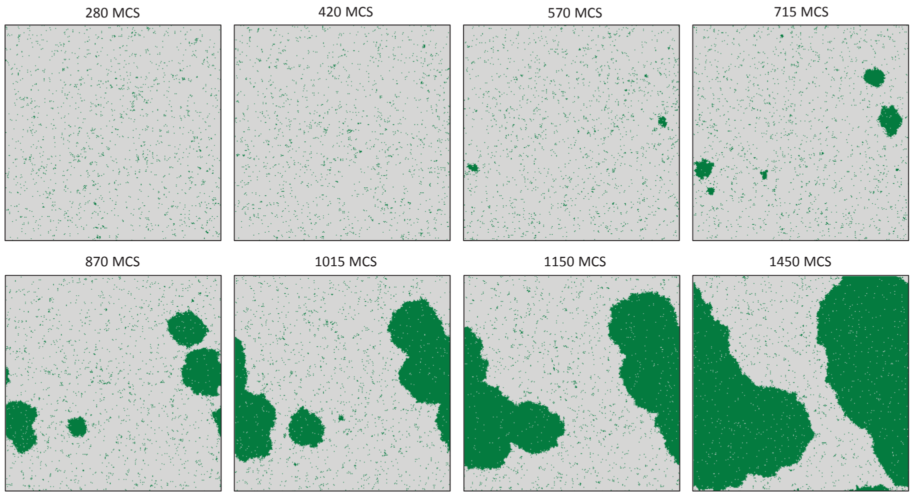

Fig. 6. Snapshots showing a sample time evolution of the system 300×300 for the initial concentration of innovators $c(0) = 0$, independence $p = 0.1$, flexibility $f = 0.1$ and advertising $h = 0.09$. Apparently the acceptance of an innovation in the society spreads like a virus (dark green clusters). Note that the parameter values (p, f, h) are not accidental – for these values the system is near the transition point, see Fig. 9. Below the transition point the innovation does not spread in the system and significantly above the transition point spreads almost immediately. *Source*: The World According to Spinson (WAS) simulation and visualization app, see footnote on page 1.

gray) except for a small number of 'up' fluctuations (dark green). However, if we look at the configuration at $t = 570$ MCS we already see two small clusters, which are seeds from which the innovation can start spreading like a virus. In the subsequent moments of time more small clusters appear and they start to grow. It seems that – at least for this particular set of parameters $(p = 0.1, f = 0.1, h = 0.09)$ – the 'take-off' starts with forming a small cluster. Then the cluster grows and eventually the innovation spreads throughout the system. This means that the so-called 'word-of-mouth' phenomena are crucial in this case – the innovation spreads through direct interactions between spinsons, i.e., through social influence. A similar effect, related to the formation of clusters, has been observed by Goldenberg et al. [20] within their stochastic cellular automata (CA) model. A successful diffusion of innovation starts with small kernels of adopters that have grown to form clusters, and later on clusters start to merge. Because in their model the transition between unadopted and adopted is irreversible, it is impossible for a cluster to disappear. However, in our model all transitions are reversible, i.e. an adopted spinson can unadopt, and consequently a cluster of adopters can disappear. This phenomenon can be related to the so-called 'valley of death' [58]. As already stressed, all results presented so far concern one particular set of parameters. It is time to examine how the results depend on the choice of these parameters.

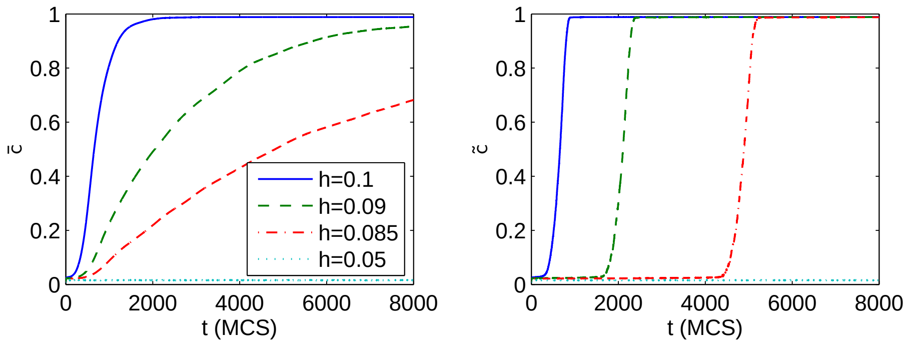

Fig. 7. The time evolution of the ratio of adopted individuals c on a square lattice 100×100 for four advertising levels $h = 0.1, 0.09, 0.085, 0.5$. The remaining two parameters independence and flexibility are set to $p = 0.1$ and $f = 0.1$, respectively. Two measures of the central tendency are presented – the mean value $\bar{c}$ (*left*) and the median $\tilde{c}$ (*right*) over 10^3 simulated trajectories.

3.2. *Parameters and the evolution of the system*

First, let us analyze the time evolution of the average value of concentration of spinsons 'up' ($\uparrow$-spinsons). As a measure of the central tendency physicists usually use the sample mean (i.e., the estimate of mean value):

$$\bar{c}(t) = \frac{1}{M} \sum_{i=1}^{M} c_i(t), \tag{2}$$

where M is the number of trajectories and $c_i(t)$ is the concentration of $\uparrow$-spinsons at time t for the i-th trajectory. This measure, although consistent with the ideas of statistical physics, can be misleading in the case of a skewed distribution. For this reason we have decided to use two measures of the central tendency (or 'average behavior') – the mean value $\bar{c}(t)$ of concentrations $c_i(t)$, see formula (2) and the left panel in Fig. 7, and the median $\tilde{c}(t)$ of concentrations $c_i(t)$ for the M trajectories, see the right panel in Fig. 7.

As expected the innovation spreads in the market faster in case of more intensive advertising, i.e., larger h. For very small values of h the innovation fails. Interestingly, the two measures of the central tendency – $\bar{c}$ and $\tilde{c}$ – yield qualitatively different pictures. Recalling Figure 5 it seems that the mean value is somehow misleading. Looking at the left panel in Fig. 7 one might get the impression that for any advertising level h, above a certain threshold value, the concentration of adopted customers grows monotonically from the starting value, only the speed of growth is h-dependent. However, looking at the single trajectories plotted in Fig. 5, it is clear that the time evolution is completely different. For a long time the innovation may not enter the market and then suddenly and rapidly may spread like a virus. This dramatic behavior is described properly on the 'average' or macro level by the median $\tilde{c}$.

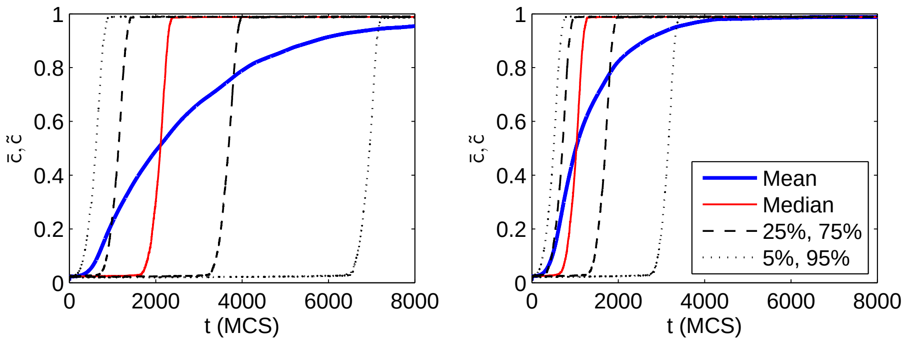

Fig. 8. The time evolution of the ratio of adopted individuals c on a square lattice 100×100 for advertising levels $h = 0.09$ (*left*) and $h = 0.095$ (*right*), independence $p = 0.1$ and flexibility $f = 0.1$. The plots include not only the two measures of the central tendency over 10^3 simulated trajectories, but also the 5%, 25%, 75% and 95% quantile lines. Note that the 50% quantile line would correspond to the median.

Differences between the mean and the median concentration can be better seen in Fig. 8. For $h = 0.095$ not only the growth of the average value of c is faster than for $h = 0.09$, but also is less the volatile – for $h = 0.09$ the distance between the corresponding lower and upper quantile lines is much wider. This means that in case of $h = 0.09$ the time needed for the 'take-off' can vary significantly from trajectory to trajectory, as has been already shown in Fig. 5. For larger and smaller values of the advertising level h the volatility is lower. Moreover, if we measure the mean value of concentration $\bar{c}_\tau$ after some evolution time τ, it occurs that the advertising level corresponding to the highest volatility is very close to the threshold value h^* below which the innovation fails and above which it penetrates the market (note the value of h for which $\bar{c}_\tau$ becomes close to one in the left panel of Fig. 9). The threshold value, in turn, depends on independence: $h^* = h^*(p)$, see the right panel in Fig. 9.

The existence of an investment threshold in the diffusion of innovation has been also found within the majority model. Galam et al. [5, 13] were able to calculate the critical value of the initial density of adopted above which the innovation penetrates the market and below which it fails. However, the initial stage of the diffusion of innovation, i.e. the process starting from no adopted, cannot be described by their model. The 'zero density of adopted' is an absorbing state of the model and no evolution is possible from this state. Therefore, Galam's model should be treated as a model which shows that there is a need of initial investment and suggests that the amount of this investment should be equal to a certain threshold value – *reaching a level below the threshold or passing the threshold is just a waste of money* [13].

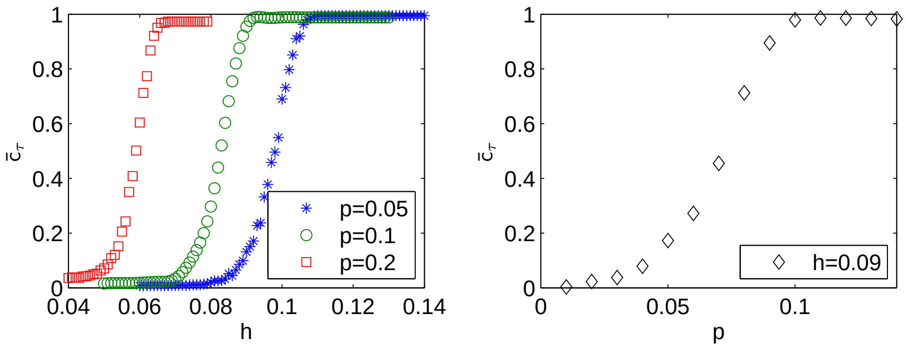

Fig. 9. The mean concentration of adopted individuals after time $\tau = 10^4$ MCS as a function of the advertising level h (*left*) and independence p (*right*). Simulations were conducted on a square lattice 100×100 for the initial concentration of adopted spinsons $c_0 = 0$ and flexibility $f = 0.1$. Note that for $p = 0.1$ (green circles), $\bar{c}_\tau$ becomes close to one for the advertising level of ca. $h = 0.09$, which is characterized by a high volatility of the success rate, see Fig. 8.

4. Results for a complete graph

Recall from eq. (1) that $N_\uparrow(t)$ represents the number of spinsons 'up' ($\uparrow$-spinsons) at time t. Similarly, denote by $N_\downarrow(t)$ the number of spinsons 'down' ($\downarrow$-spinsons) at time t. The concentration of innovators (i.e., $\uparrow$-spinsons) is given by:

$$c_t = \frac{N_\uparrow(t)}{N}, \tag{3}$$

where N is the number of spinsons in the system. The evolution of the system is described by the following equation:

$$\begin{aligned} c_{t+1} - c_t = & \underbrace{pf(1-c_t)}_{\downarrow \xrightarrow{pf} \uparrow} - \underbrace{pfc_t}_{\uparrow \xrightarrow{pf} \downarrow} + \underbrace{(1-p)c_t^4(1-c_t)}_{\downarrow \xrightarrow{(1-p)} \uparrow} - \underbrace{(1-p)(1-c_t)^4 c_t}_{\uparrow \xrightarrow{(1-p)} \downarrow} \\ & + \underbrace{(1-p)h(1-c_t^4-(1-c_t)^4)(1-c_t)}_{\downarrow \xrightarrow{(1-p)h} \uparrow}, \end{aligned} \tag{4}$$

where the first two terms on the right hand side of the above equation correspond to changes caused by independence (see Fig. 3), the third and the fourth term correspond to conformity (see Fig. 2) and the last one describes advertising (see Fig. 4). The above equation can be used to find the time evolution of c_t (see the left panel in Fig. 10) and the asymptotic behavior $c_\infty = c_\infty(p,h)$ that corresponds to fixed points $c_{t+1} - c_t = 0$ (see the right panel in Fig. 10).

As can be seen in Figure 10, there is a discontinuous phase transition caused by the change of the level of advertising h. For $h < h^*(p)$ the innovation cannot penetrate the market and for $h > h^*(p)$ almost all consumers are adopted. For $h \to h^*(p)$ the time evolution of c is very slow, see the trajectory for $h = 0.1839$ in the left panel of Fig. 10. The value of the threshold $h^*(p)$ is roughly twice the

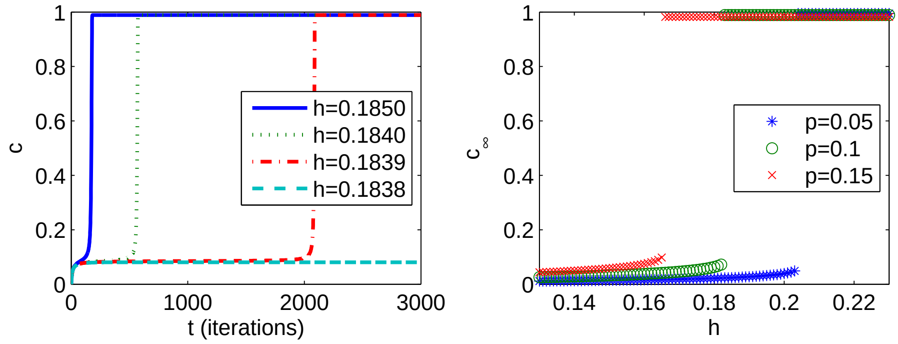

Fig. 10. The time evolution of the ratio of adopted individuals c for independence $p = 0.1$ and four advertising levels $h = 0.135, 0.1, 0.095, 0.09$ (*left*) and the stationary value c_∞ of the ratio of adopted individuals, as a function of advertising level h, for three values of independence $p = 0.05, 0.1, 0.2$ (*right*). These results are obtained from the mean field equation (4) for flexibility $f = 0.1$. Note that discontinuous phase transitions can be seen in the right panel – below the threshold value $h^*(p)$ the innovation cannot penetrate the market and above $h^*(p)$ it conquers the market.

value obtained in the previous Section for Monte Carlo simulations on a square lattice. A different observation was made in a similar context in [18]. Goldenberg et al. proposed a stochastic cellular automata model in which the probability of an individual's adoption was taken from the Bass model [1]. They simulated their model on a square lattice and compared with the (macroscopic) Bass model, which could be also treated as a mean field approximation of their cellular automata model. Results from both models were almost identical, even after the introduction of heterogeneity to their macroscopic model. The importance of the topology in models of diffusion of innovation is an open question. For instance, Watts and Dodds [57] show that the structure of the network influences the macroscopic outcome to a much lesser extent than the type of interactions between consumers.

Although the are three parameters in our model – p, f and h – up till now we have considered only two of them (independence p and advertisement h). Flexibility f was kept constant. This was not an inattention – it simply turns out that flexibility scales with independence and therefore is redundant (i.e., only two parameters are independent, see Section 5). Consequently, also the asymptotic behavior of the system depends only on two parameters: p and h. In Figure 11 we present a full phase diagram obtained using eq. 4. For small values of h and p the ratio of adopted spinsons is nearly zero (dark gray area), which means that the innovation failed. Above the transition line (white region) almost the whole population is adopted to the innovation (due to advertising). Finally, the light gray region in Fig.11 corresponds to a very high level of independence p. In this region both ordering forces – conformity and advertising – do not influence the system too much and the noise, introduced by independence, dominates.

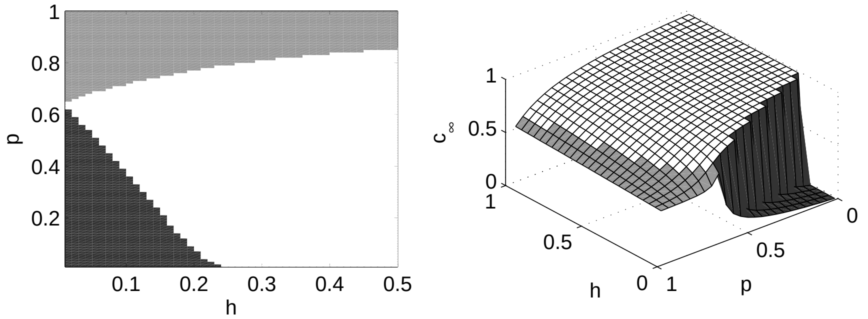

Fig. 11. The phase diagram of the stationary value c_∞ of the ratio of adopted individuals as a function of two parameters – advertising level h and independence p. Results are obtained from iterations of the mean field equation (4). For small values of h and p the ratio of adopted is nearly zero (dark gray area), which means that the innovation failed in the market. Above the transition line (white region) almost the whole population is adopted to the innovation. Light gray region corresponds to a very high level of independence p.

We should note that it is not easy to measure the independence level, as defined in this paper, in a society. However, one expects that this level should depend on the society, similarly as the level of conformity or cultural index values [26, 48, 38]. Among five of the cultural index values, *individualism* (IDV) is probably the most important for this paper. For instance, Milgram [37] found that individuals in Norway ('collectivistic culture') exhibited a higher degree of conformity than individuals in France (a more 'individualistic culture'). Similar results were reported by other researchers – in the more collectivistic cultures the level of conformity was higher [2, 4, 26].

The level of independence depends not only on the society. It was suggested by Milgram [37] that individuals conform less when the task is linked to an important issue. This suggests a higher level of independence in situations, which we perceive to be more important. Hence, the level of independence p should also depend on the type of innovation – the more important the issue the higher the independence. From Figure 11 we can conclude that in the case of an important issue (light gray region), neither global advertising nor word-of-mouth (conformity) is important.

5. Scaling

The model considered in this paper has three parameters: independence p, flexibility f and advertising level h. However, it can be shown that the evolution of the system depends only on two independent parameters. It turns out that flexibility f scales with independence p, similarly as in [53]. We can generalize the reasoning conducted in [53] to include the case of an external field (i.e., advertising). Note that at each iteration a group of spinsons is chosen to convince one of their neighbors and:

Rule I: with probability $(1-p)$ the Sznajd rule with advertising, originally introduced in [56], is applied;

Rule II: with probability pf a site is flipped ($S_i \to -S_i$), which corresponds to the notion of independence introduced in [53];

Rule III: with probability $p(1-f)$ nothing happens.

Because rule III does not change the state of the system, parameters p and f are not important as far as rule III is concerned. The state of the system changes only if either rule I or rule II is followed. Therefore the evolution of the system should depend only on the ratio of probabilities of following rules I and II:

$$r = \frac{pf}{1-p}. \tag{5}$$

We expect that if two models have parameters (p, f) and (p', f') such that $r = r'$, then they both should generate statistically similar sequences, given the same initial conditions. Therefore, we can calculate the dependence between p and p' from

$$r = r' \to \frac{pf}{1-p} = \frac{p'f'}{1-p'}. \tag{6}$$

If we want to compare the behavior of a given quantity (e.g., concentration c) in models (p, f) and (p', f') we can choose $f' = 1$ as a reference value and solve the above equation for p':

$$p' = \frac{pf}{1-p+pf}. \tag{7}$$

Indeed plotting concentration c as a function of p for various values of f we see that the curves collapse if they are rescaled according to formula (7), see Fig. 12 for mean field results and Fig. 13 for Monte Carlo results.

6. Conclusions

In the abstract we have asked two important questions: *Why does the diffusion of innovation take sometimes so long?* and *Why does it fail so often?* To answer these questions we have studied a modified version of a model originally used to investigate advertising in duopoly markets. The main novelty of the model investigated here, compared to the one used in [56], is the introduction of independence that brings ‘noise’ to the system. As a result of this, the system never reaches an absorbing unanimous steady state, which was the case in [56]. Moreover, what is even more important, independence allows to investigate the system in which initially there are no adopters, i.e., all spinsons are down. Without independence (or with flexibility $f = 0$) such a state of the system never changes, which makes the model useless in the context of the diffusion of innovation. It was shown that, although the model is described by three parameters, only two of them are independent – flexibility $f > 0$ scales with the level of independence p.

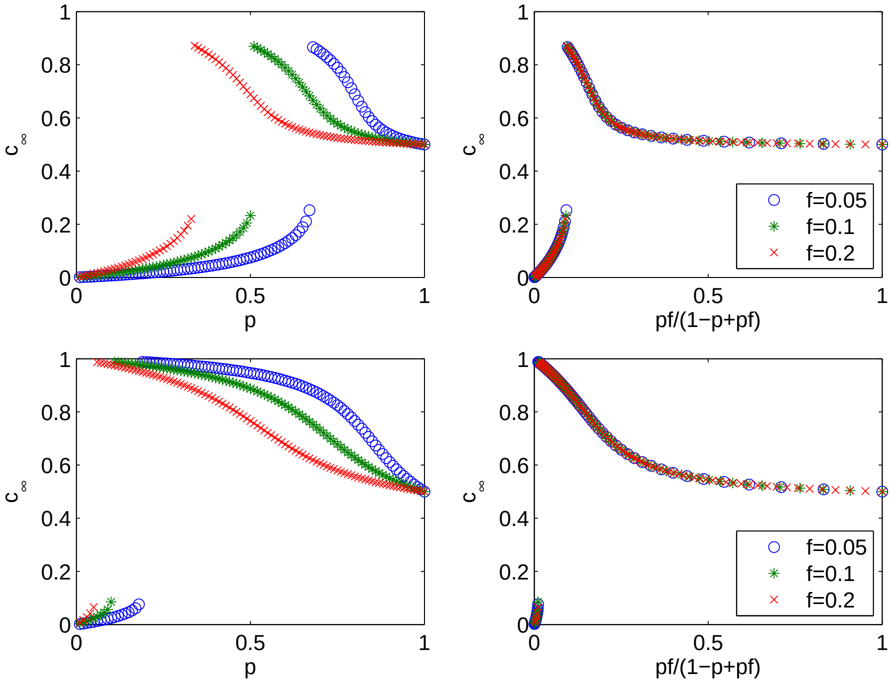

Fig. 12. *Left*: Dependence between the stationary value c_∞ of the ratio of adopted individuals and independence for three values of flexibility $f = 0.05, 0.1, 0.2$. Two levels of advertising are considered: $h = 0.05$ (*top panels*) and $h = 0.1839$ (*bottom panels*). *Right*: The same dependence rescaled according to formula (7). The results were obtained from iterations of the mean field equation (4).

One could wonder what is the rationale behind independence introduced as a probability. However, social experiments show that people are inconsistent in their behavior and simple situational factors are more powerful than individual traits in shaping human behavior, see e.g. [3] for a review. This fact is reflected in our model by the probability of independence – in each time step an agent can be independent or susceptible with probability p and its behavior changes in time. In general, independence could be heterogeneous, i.e. different for each spinson, and express the fact that agents are not identical and have individual, potentially different levels of autonomy. However, to our best knowledge the distribution of independence in real societies has not been measured and therefore in this paper we assume that p is equal for all spinsons. On the other hand, a homogenous p can be treated as a certain average value in the society [26].

The model was investigated on a square lattice using Monte Carlo simulations and on a complete graph, which allows for analytical treatment. In both approaches it was shown that there is a threshold value of independence $p = p^* = p^*(h)$ and advertising $h = h^* = h^*(p)$. Below these values the innovation fails and above them

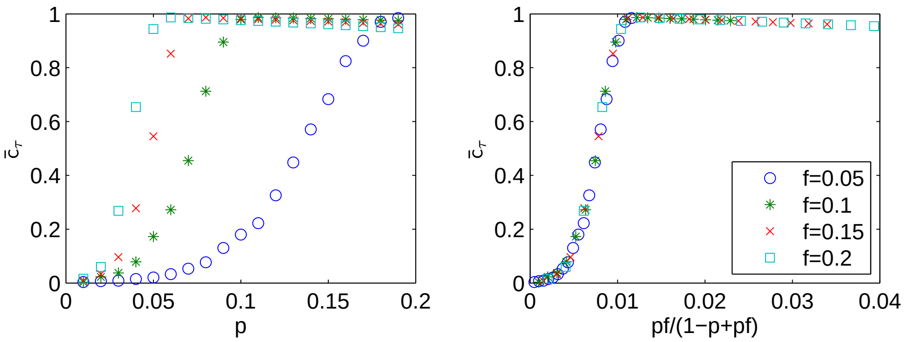

Fig. 13. *Left*: Dependence between the mean ratio of adopted individuals and independence after time $\tau = 10^4$ MCS for four values of flexibility $f = 0.05, 0.1, 0.15, 0.2$ and advertising level $h = 0.09$. *Right*: The same dependence rescaled according to formula (7). The results were obtained from Monte Carlo simulations on a 100×100 lattice and averaged over 10^3 trajectories.

it spreads in the market (society). Moreover, the time needed for the innovation to spread increases near the threshold point. By no means do we claim that exactly the same scenario functions in reality. However, our results shed new light on the problem of innovation failures.

Firstly, independence can be probably treated as a characteristic both of the society and of the product. On one hand, conformity/independence depends on the culture. On the other, it was suggested in the social science literature that individuals conformed less when the task was linked to an important issue [37]. Therefore, in each society each innovation is related to a certain value of independence p, which potentially can be measured empirically, similarly as the level of individualism (IDV) [26]. Secondly, the threshold value of advertising depends on the level of independence p, which means that the level of advertising should be adjusted to match the characteristics of the society and of the product. However, the most interesting result of our study from an applicative point of view is the existence of the threshold itself. Its existance suggests that increasing funding (or time) for advertising is not beneficial as long as it is below the threshold level $h^*(p)$. Only above $h^*(p)$ the innovation will spread in the market. On the other hand, increasing funds for advertising, when the threshold level has been exceeded, will not yield significant benefits.

For a model to be trusted it has to reproduce empirical facts. One of the best-established stylized facts in the field of diffusion of innovation is the S-shaped curve representing the time change of the number of consumers having adopted to a new product. Our model perfectly reproduces this feature. Moreover, the performed Monte Carlo simulations show that even for exactly the same parameters (p, f, h) different scenarios are possible, particularly near the threshold. Neither the time needed for the 'take-off' nor the success of the diffusion itself cannot be easily predicted. This is in contrast to classical analytical models but in line with the observed complexity of the real world. For instance, it is known that in reality many innovations fail on the way from the laboratory to the market, which is often described by the 'valley of death' metaphor [58], or collapse of initially successful diffusions. This phenomenon is observed in our model near the threshold values of p and h.

Naturally, we realize that our model has its limitations:

- The square lattice is not very realistic for social systems. However, the model can be easily generalized to arbitrary graphs, what has been recently done in [54] for Watts-Strogatz networks. We have started by analyzing the model on a square lattice and a complete graph for two reasons. First, the role of network topology in innovation diffusion models is still unclear, most likely due to the crucial role of social influence in such problems [43, 57]. Second, the complete graph allows for analytical treatment, while the square lattice correlates social interactions with physical distance, which is important in this context since social influence has been found to decay dramatically with physical distance [6, 35].
- Binary variables (spinsons) are certainly a simplification that does not take into account the complexity of human decision making. However, Watts and Dodds [57] argue that binary decisions can be applied to a surprisingly wide range of real world situations and are particularly useful for modeling innovation diffusion processes in which 'adopted' and 'not adopted' are natural states [18, 43].
- Equally sized influence groups are another model limitation. However, as noticed by Galam [12], people usually discuss in small groups even if they have many friends. To randomly draw the size of the group from a certain distribution, as done by Galam, could be the next step but we leave it for future work.

Taking all this into account we truly believe that the presented model can be used as a starting point for future, more focused research in the field of diffusion of innovation. In particular, it might prove useful for studying the diffusion of energy efficient or pro-ecological solutions in the society [58]. For instance, Kowalska-Pyzalska et al. [31] have used the model to show how personal attributes impact opinions of individual electricity consumers regarding switching from classical flat pricing schemes to innovative dynamic tariff programs. The main outcome of the paper is that currently the adoption of dynamic tariffs is virtually impossible due to the high level of indifference in today's societies. The fact that the used in this paper Monte Carlo simulation and visualization app *The World According to Spinson* (WAS) is freely available for download from the IDEAS/RePEc repository (http://ideas.repec.org/s/wuu/hscode.html) makes the model even more attractive.

References

[1] F. M. Bass, *A new product growth for model consumer durables*, Management Science **15** (1969) 215-227

[2] J. W. Berry, *Independence and conformity in subsistence-level societies*, Journal of Personality and Social Psychology **7** (1967) 415-418

[3] P. Bocchiaro and A. Zamperini, *Conformity, obedience, disobedience: The power of the situation*. In: *Psychology – Selected Papers*, G. Rossi (ed.), InTech (2012).

[4] M. H. Bond and P. B. Smith, *Culture and Conformity: A meta-analysis of studies using the Asch's (1952b, 1956) line judgement task* Psychological Bulletin **119** (1996) 111-137

[5] T. Bouzdine-Chameeva and S. Galam, *Word-of-mouth versus experts and reputation in the individual dynamics of wine purchasing* Advances in Complex Systems **14** (2011) 871-885

[6] W. M. Brucks, U. D. Reips and B. Ryf, *Group norms, physical distance, and ecological efficiency in common pool resource management*, Social Influence 2 (2007) 112-135

[7] C. Castellano, S. Fortunato and V. Loreto, *Statistical physics of social dynamics*, Reviews of Modern Physics **81** (2009) 591-646

[8] S. Galam, *Majority Rule, Hierarchical Structures and Democratic Totalitarianism: A statistical Approach*, Journal of of Mathematical Psychology **30** (1986) 426-434

[9] S. Galam, *Social paradoxes of majority rule voting and renormalization group*, J. Stat. Phys. **61** (1990) 943-951

[10] S. Galam, *Rational group decision making: A random field Ising model at* $T = 0$, Physica A **238** (1997) 66-80

[11] S. Galam, *Contrarian deterministic effects on opinion dynamics: The hung elections scenario*, Physica A **333** (2004) 453-460

[12] S. Galam, *The dynamics of minority opinions in democratic debate*, Physica A 336 (2004) 56-62

[13] S. Galam, *Local dynamics vs. social mechanisms: A unifying frame*, Europhysics Letters **70** (2005) 705-711

[14] S. Galam and A. Vignes, *Fashion, novelty and optimality: an application from Physics*, Physica A **351** (2005) 605-619

[15] S. Galam and F. Jacobs, *The role of inflexible minorities in the breaking of democratic opinion dynamics*, Physica A **381** (2007) 366-376

[16] S. Galam, *Sociophysics: A Physicist's Modeling of Psycho-political Phenomena*, New York Springer (2012)

[17] R. Garcia and W. Jager, *Special Issue on Agent-Based Modeling of Innovation Diffusion*, Journal of Product Innovation and Management 28(2) (2011) 146-318

[18] J. Goldenberg, B. Libai, E. Muller, *Using Complex Systems Analysis to Advance Marketing Theory Development: Modeling Heterogeneity Effects on New Product Growth through Stochastic Cellular Automata*, Academy of Marketing Science Review 9(3) (2001) 1-8

[19] J. Goldenberg, B. Libai, E. Muller, *Talk of the network: A complex systems look at the underlying process of word-of-mouth*, Marketing letters 12 (3) (2001) 211-223

[20] T. Garber, J. Goldenberg, B. Libai, E. Muller, *From density to destiny: Using spatial dimension of sales data for early prediction of new product success*, Marketing Science 23(3) (2004) 419-428

[21] J. Goldenberg, B. Libai, S. Moldovan, E. Muller, *The NPV of bad news*, International Journal of Research in Marketing **24** (2007) 186-200.

[22] J. Goldenberg, B. Libai, E. Muller, *The chilling effects of network externalities*, International Journal of Research in Marketing **27** (2010) 4-15

[23] J. Goldenberg, O. Lowengart, D. Shapira, *Zooming in: self-emergence of movements in new product growth*, Marketing Science **28** (2009) 274-292

[24] J. C. González-Avella, M. G. Cosenza, K. Klemm, V. M. Eguluz and M. San Miguel, *Information Feedback and Mass Media Effects in Cultural Dynamics*, Journal of Artificial Societies and Social Simulation 10(3) (2007) 9

[25] M. S. Granovetter, *Threshold Models of Collective Behavior*, American Journal of Sociology 83, (1978), 1420-1443

[26] G. Hofstede, *Culture's consequences: Comparing values, behaviors, institutions, and organizations across nations* (2nd ed.), Thousand Oaks, California: Sage Publications, Inc. (2001)

[27] K. Kacperski and J.A. Hołyst, *Opinion formation model with strong leader and external impact: a mean field approach*, Physica A **269** (1999) 511-526

[28] K. Kacperski and J.A. Hołyst, *Phase transition as a persistent feature of groups with leaders in models of opinion formation*, Physica A **287** (2000) 631-643

[29] E. Kiesling, M. Günther, Ch. Stummer and L. M. Wakolbinger, *Agent-based simulation of innovation diffusion: A review*, CEJOR **20** (2012) 183-230

[30] R. Kohli, D.R. Lehmann and J. Pae, *Extent and impact of incubation time in new product diffusion*, J. Product Innovation Management **16(2)** (1999) 134-144

[31] A. Kowalska-Pyzalska, K. Maciejowska, K. Sznajd-Weron and R. Weron, *Going green: Agent-based modeling of the diffusion of dynamic electricity tariffs*, HSC Research Report HSC/13/05, http://ideas.repec.org/p/wuu/wpaper/hsc1305.html

[32] P. L. Krapivsky and S. Redner, *Dynamics of Majority Rule in Two-State Interacting Spin Systems*, Phys. Rev. Lett. **90** (2003) 238701

[33] P.L. Krapivsky, S. Redner, E. Ben-Naim, *A Kinetic View of Statistical Physics*, Cambridge University Press (2010)

[34] M.S. de la Lama, J.M. Lopez and H.S. Wio, *Spontaneous emergence of contrarian-like behavior in an opinion spreading model*, Europhys. Lett. **72** (2005) 851-857

[35] B. Latane, J. H. Liu, A. Nowak, M. Bonevento and L. Zheng, *Distance Matters: Physical Space and Social Impact*, Personality and Social Psychology Bulletin 21 (1995) 795-805.

[36] T.M. Liggett, *Interacting particle systems* Springer, Heidelberg, (1985)

[37] S. Milgram, *Nationality and conformity*, Scientific America **205** (1961) 45-51

[38] D. G. Myers, *Social Psychology, 10ed*, McGraw-Hill (2010)

[39] M. Mobilia, *Does a Single Zealot Affect an Infinite Group of Voters?*, Phys. Rev. Lett. **91** (2003) 028701

[40] M. Niss, *History of the Lenz-Ising model 1965-1971: the role of a simple model in understanding critical phenomena*. Arch. Hist. Exact Sci. **65** (2011) 625-658

[41] P. Nyczka and K. Sznajd-Weron, *Anticonformity or Independence? – Insights from Statistical Physics*, J Stat Phys (2013) DOI 10.1007/s10955-013-0701-4

[42] W. Rand and R. T. Rust, *Agent-based modeling in marketing: Guidelines for rigor*, Intern. J. of Research in Marketing **28** (2011) 181-193

[43] Rogers, E.M. (2003) Diffusion of Innovations. Fifth Edition. Free Press, New York

[44] B. Schechter, *Push me, pull me*, New Scientist **175** (2002) 42

[45] T.C. Schelling, *Dynamic models of segregation*, J. Math. Sociol. **1** (1971) 143-186

[46] Y. Shibanai, S. Yasuno and I. Ishiguro, *Effects of Global Information Feedback on Diversity: Extensions to Axelrod's Adaptive Culture Model*, Journal of Conflict Resolution 45(1) (2001) 80-96

[47] F. Slanina and H. Lavicka, *Analytical results for the Sznajd model of opinion formation*, Eur. Phys. J. B **35** (2003) 279-288

[48] M. R. Solomon, G. Bamossy, S. Askegaard, M. K. Hogg, *Consumer behavior, 3ed*,

Prentice Hall (2006)

[49] F. Squazzoni, *The impact of agent-based models in the social sciences after 15 years of incursions*, History of Economic Ideas VVIII (2010) 197-233

[50] D. Stauffer, *Sociophysics: the Sznajd model and its applications*, Comput. Phys. Commun. **146** (2002) 93-98

[51] D. Stauffer, A.O. Sousa, M. De Oliveira, *Generalization to Square Lattice of Sznajd Sociophysics Model*, Int. J. Mod. Phys. C **11** (2000) 1239-1245

[52] K. Sznajd-Weron, J. Sznajd, *Opinion evolution in closed community*, Int. J. Mod. Phys. C **11** (2000) 1157-1165

[53] K. Sznajd-Weron, M. Tabiszewski and A. Timpanaro, *Phase transition in the Sznajd model with independence*, EPL **96** (2011) 48002

[54] K. Sznajd-Weron, J. Szwabiński, R. Weron and T. Weron, *Rewiring the network. What helps an innovation to diffuse?*, HSC Research Report HSC/13/09, http://ideas.repec.org/p/wuu/wpaper/hsc1309.html

[55] K. Sznajd-Weron, R. Weron, *A Simple Model Of Price Formation*, Int. J. Mod. Phys. C **13** (2002) 115-123

[56] K. Sznajd-Weron and R. Weron, *How effective is advertising in duopoly markets?*, Physica A **324** (2003) 437-444

[57] D. J. Watts and P. S. Dodds, *Influentials, Networks, and Public Opinion Formation*, Journal of Consumer Research 34 (2007) 441-458

[58] J. P. Weyant, *Accelerating the development and diffusion of new energy technologies: Beyond the 'valley of death'*, Energy Economics **33** (2011) 674-682

HSC Research Report Series 2013

For a complete list please visit http://ideas.repec.org/s/wuu/wpaper.html

01 *Forecasting of daily electricity spot prices by incorporating intra-day relationships: Evidence form the UK power market* by Katarzyna Maciejowska and Rafał Weron

02 *Modeling and forecasting of the long-term seasonal component of the EEX and Nord Pool spot prices* by Jakub Nowotarski, Jakub Tomczyk and Rafał Weron

03 *A review of optimization methods for evaluation of placement of distributed generation into distribution networks* by Anna Kowalska-Pyzalska

04 *Diffusion of innovation within an agent-based model: Spinsons, independence and advertising* by Piotr Przybyła, Katarzyna Sznajd-Weron and Rafał Weron