Copula and Sieve Methods in GARCH(1,1) Estimation: mortgage-backed securities, stocks, and bonds since 2007

Kieran Walsh*
Yale University

1 Introduction

This note illustrates how sieve and copula methods (in the context of GARCH models) may help us understand some key aspects and leading explanations of the recent financial crisis. Many explanations of the crisis have emphasized the role of financial frictions and collateral (both Geanakpolos (2010) and Gertler and Kiyotaki (2010) provide overviews of this literature). The story is that financial frictions or leverage effects amplify the impact that bad news or bad shocks (in, for example, the mortgage market) have on prices and real activity. Central to the "Leverage Cycle" of Geanakoplos (2010) is that bad news is accompanied by increased uncertainty (volatility). While this link is assumed in Geanakoplos (2010), Fostel and Geanakoplos (2010) provide a theoretical explanation for why bad news tends to increase volatility (in their model, good news decreases volatility).

Here, we use data from the last four years to estimate the impact of news (residuals) on the volatility of mortgage-backed security (MBS), stock, and bond market returns. Specifically, we consider daily excess returns on the Barclays MBS index, daily excess market (the daily Fama-French factor) returns, and daily excess returns on the Barclays bond index. The GARCH(1,1) model of Bollerslev (1986), which assumes volatility is a separable function of past volatility and shocks, is a framework commonly used in estimating such "news impact curves." A limitation of this method is that in its standard form the volatility reaction to shocks is quadratic and centered at zero. With such tight

*This paper grew out of research assistance work I did for Professor Xiaohong Chen. I would like to extend many, many thanks to Professor Chen for introducing me to sieve and copula estimation, for guiding me through this project, and for patiently teaching me so much about econometrics.

parametric assumptions, the data cannot tell us much about the nature of the reaction of volatility to news. However, many authors have suggested generalizations and alternate specifications (see Bollerslev, Engle, and Nelson (1994)) to allow for the possibility of, for example, asymmetry (which appears in Fostel and Geanakoplos (2010)) or reaction curves not centered at zero.

Following Chen and Shen (1998) and Chen (2007), our approach is to estimate the news reaction curve via the method of sieves: we approximate the curve with basis functions (we use B-splines) and let the data determine its shape and location. In other words, this exercise is a means by which one may quantify how and the degree to which bad shocks amplify risk in MBS, stock, and bond markets. A number of authors have performed similar exercises. Engle and Ng (1993) use piecewise linear splines (their "partially nonparametric" (PNP) model) to estimate the Japanese stock market news impact curve. Linton and Mammen (2005) show how one may use kernel methods to estimate news impact curves (and they study S&P 500 returns). Audrino and Bühlmann (2009) leave the entire volatility process unspecified (rather than just the news impact curve) and propose a B-spline estimation procedure. Finally, Engle and Rangel (2007) use splines to model slow moving, deterministic, unconditional volatility in a GARCH setting.

We also address another topic potentially important in understanding the financial crisis: risk assessment. Engle (2010) comments, "While there are many complex underlying causes [of the crisis], there are two key features of all analyses. First is the failure of many risk management systems to accurately assess the risks of financial positions and the second is the failure of economic agents to respond appropriately to these risk." Similarly, in discussing possible explanations for banks' exposure to subprime mortgages, Barberis (2010) writes, "The second explanation for banks' large holdings of subprime-linked securities can be labeled the 'bad models' view. It says that the people on the mortgage desks of banks were genuinely unaware of the risk embedded in their subprime holdings, and that this was due to faulty reasoning."

To accurately assess risk, our financial models must account for (i) the possibility of fat tails and (ii) the dependence between shocks to different assets. Copula methods are a natural means by which we may estimate such characteristics. Below, we use the technique described in Chen and Fan (2006) to explore the relationship between shocks to MBS, stock, and bond returns. In particular, we model each asset class using the "Sieve-GARCH" framework described above. Further, we assume that these processes' shocks (i) have unknown marginal distributions and (ii) are jointly distributed according to the Student's t-copula, which allows for joint fat tails. With GARCH and copula parameter estimates, we calculate value at risk (VaR) for a portfolio comprised of mortgage-backed securities, stocks, and bonds.

In summary, we use recent financial data and sieve and copula methods to estimate news impact curves, volatility, shock dependence, and VaR in MBS, stock, and bond markets. Further, we ask the question, to what extent does relaxing the standard GARCH specification affect volatility, VaR, and copula parameter estimates? In other words, does moving from GARCH to Sieve-GARCH substantially change the picture of risk?

Overall, letting the data determine the shape of news impact curves does not have much effect on volatility and VaR calculations in the sample. However, there are two extreme "good news" events for which the GARCH and Sieve-GARCH models have quite different predictions for subsequent volatility. Despite the overall similarity in predictions, that the models may substantially disagree during extreme times is evidence in favor of using the semi-nonparametric version in calculating risk.

All three news impact curves exhibit the same asymmetry: bad news increases volatility more than does good news. For mortgage-backed securities and stocks, some goods news actually decreases volatility, as in Fostel and Geanakoplos (2010). As in Linton and Mammen (2005), most good news in the stock market does not have much effect on volatility.

Moving from GARCH to Sieve-GARCH has little effect on the estimated copula parameters. In both cases, we find that (i) shocks to bonds and shocks to mortgage-backed securities are highly correlated, (ii) shocks to mortgage-backed securities and shocks to stocks are moderately negatively correlated, and (iii) shocks to bonds and shocks to stocks are also moderately negatively correlated. Moreover, bonds and mortgage-backed securities exhibit substantial tail dependence. The Sieve-GARCH method increases tail dependence estimates, but the effect is small.

2 Model

$$
\begin{aligned}
MBS\ Market &: S_t^e = c_S + \rho_S S_{t-1}^e + \beta_S M_{t-1}^e + \sigma_{S,t}\varepsilon_{S,t} \\
Stock\ Market &: M_t^e = c_M + \rho_M M_{t-1}^e + \sigma_{M,t}\varepsilon_{M,t} \\
Bonds\ Market &: B_t^e = c_B + \rho_M B_{t-1}^e + \beta_B M_{t-1}^e + \sigma_{B,t}\varepsilon_{B,t} \\
Volatility &: \sigma_{i,t}^2 = \underbrace{\omega_i + m_i\left(\sigma_{i,t-1}\varepsilon_{i,t-1}\right)}_{news\ impact\ curve} + \theta_i\sigma_{i,t-1}^2,\ i \in \{S, M, B\}\,.
\end{aligned}
$$

M_t^e, B_t^e, and S_t^e are, respectively, excess returns on the aggregate stock market, excess bond market returns, and excess MBS returns. Let $d = 3$

denote the number of assets. Let n be the sample size. $(\varepsilon_{S,t}, \varepsilon_{M,t}, \varepsilon_{B,t})'$ are independent across time but jointly distributed according to the Student's t-copula described in Chen and Fan (2006). In particular, these shocks have unknown marginal distributions $F_i^0(\cdot)$, $i \in \{S, M, B\}$, and copula density $c(\mathbf{u}; \Sigma, v)$ where Σ is the correlation matrix, v is the degrees of freedom, T_v is the univarite Student's t distribution, and

$$c(\mathbf{u}; \Sigma, v) = \frac{\Gamma\left(\frac{v+d}{2}\right)\left(\Gamma\left(\frac{v}{2}\right)\right)^{d-1}}{\sqrt{\det(\Sigma)}\left(\Gamma\left(\frac{v+1}{2}\right)\right)^{d}}\left(1 + \frac{\mathbf{x}\Sigma^{-1}\mathbf{x}'}{v}\right)^{-\frac{v+d}{2}} \prod_{i \in \{S,M,B\}}\left(1 + \frac{x_i^2}{v}\right)^{\frac{v+1}{2}},$$
$$where\ \mathbf{x} = (x_S, x_M, x_B),\ x_i = T_v^{-1}(u_i).$$

We assume that $E(\varepsilon_{i,t}) = 0$ and $E\left(\varepsilon_{i,t}^2\right) = 1$. In the standard GARCH(1,1) setting, $m_i(x) = \gamma_i x^2$ for some γ_i. In contrast to this, we estimate $m_i(\cdot)$ via the method of sieves.

Essentially, the above model generalizes the AR(1)-GARCH(1,1) copula example in Chen and Fan (2006) to include a market factor and a flexible form for $m_i(\cdot)$. In the MBS and bond market equations, the market factor is lagged to avoid endogeneity issues. The idea is to remove the predictable components of the return series. Below, we refer to $\omega_i + m_i(\cdot)$ as the "news impact curve" for series i. The news impact curve represents how unexpected return shocks affect subsequent volatility.

We include ω_i in the news impact curve to fascilitate comparison between $\omega_i + m_i(\cdot)$ and $\omega_i + \gamma_i(\cdot)^2$. Note that while volatility may not be negative, it is possible for the news impact curve to fall below zero. This corresponds to the situation in which volatility falls in response to news. Implicitly, we have defined volatility as the maximum of 0 and the above expression for volatility. However, due to the apparent high peristence of volatility and the appearance of $-1/\sigma_{i,t}^2$ in the QMLE procedure, estimated volatility stays away from zero.

3 Data and Estimation

The data are daily from March 20th, 2007 to December 31st, 2010. Stock Market and risk-free returns are from the "Fama/French Factors [Daily]" dataset on the website of Kenneth French. Mortgage-backed security market and bond market returns are log-differences of, respectively, the total return Barclays MBS index ("MBB") and the total return Barclays bond index ("AGG"). These indexes attempt to replicate the aggregate performance of their respective sectors, MBS and investment grade bonds, in the US. See http://us.ishares.com for further details.

Each m (suppressing asset subscripts for now) is approximated via five cubic B-splines basis functions:

$$
\begin{aligned}
m(x) &\approx \sum_{i=-2}^{2} \alpha_i 2^{j/2} B_3\left(2^j x - i\right) \\
B_3(u) &= \frac{1}{2} \sum_{i=0}^{3} (-1)^i \binom{3}{i} \left[\max(0, u-i)\right]^{3-1} \\
&\quad j = \max\left\{u \in \mathbb{Z} \mid 2^u \max\left(\left|\min\left(fitted\ x\right)\right|, \left|\max\left(fitted\ x\right)\right|\right) \leq 2\right\}.
\end{aligned}
$$

This choice of j ensures that for all x (all fitted residuals), $B_3\left(2^j x - i\right) > 0$ for some $i \in \{-2, ..., 0, ...2\}$. This means that all data will impact the estimation of the α's and hence m. See Chen (2007) for a general discussion of sieve spaces and sieve estimation.

First, we estimate conditional mean and GARCH parameters for each asset separately. This yields shock process estimates for each series. Second, we estimate the copula parameters. This is close to the three-step procedure of Chen and Fan (2006). The difference is that here we combine steps one and two into a single QMLE procedure.

3.1 First-Stage Estimation

First, we estimate the conditional mean and GARCH parameters. This stage is a case of sieve-M estimation with weakly dependent data. Chen and Shen (1998) provide consistency and asymptotic normality results. Each process is estimated separately, so we will suppress subscripts for now and let Y_t denote any of the three processes. Let $\varphi = (c, \rho, \beta, \omega, \theta, \alpha_{-2}, ..., \alpha_2)'$, and let $h_t(\varphi)$ denote the volatility $\sigma^2_{Y,t}$. Note that $\beta = 0$ for the stock market process. We form estimate $\widetilde{\varphi}$ of φ via QMLE:

$$
\begin{aligned}
\widetilde{\varphi} &= \arg\max_{\varphi} \frac{-1}{2n} \sum_{t=1}^{n} \left(\frac{\left(Y_t - c - \rho Y_{t-1} - \beta M^e_{t-1}\right)^2}{h_t(\varphi)} + \log h_t(\varphi) \right) \\
&= \arg\max_{\varphi} \frac{1}{n} \sum_{t=1}^{n} l_t(\varphi)
\end{aligned}
$$

where

$$
l_t(\varphi) = -\frac{1}{2} \frac{\left(Y_t - c - \rho Y_{t-1} - \beta M^e_{t-1}\right)^2}{h_t(\varphi)} - \frac{1}{2} \log h_t(\varphi).
$$

Note that for large datasets, this becomes computationally intensive: given φ, $h_t(\varphi)$ is defined recursively (letting h_0 be the sample variance of Y_t). Next, we can estimate the shock process:

$$
\widehat{\varepsilon}_{Y,t} = \frac{Y_t - \widehat{c} - \widehat{\rho} Y_{t-1} - \widehat{\beta} M^e_{t-1}}{\sqrt{\widehat{h}_t(\widetilde{\varphi})}}.
$$

The corresponding robust asymptotic covariance matrix estimate of Bollerslev and Wooldridge (1992) is

$$\widehat{A}^{-1}\widehat{B}\widehat{A}^{-1}$$

where

$$\begin{aligned}\widehat{B} &= \frac{1}{n}\sum_{t=1}^{n}\left(\frac{\partial l_t\left(\widetilde{\varphi}\right)}{\partial\varphi}\right)\left(\frac{\partial l_t\left(\widetilde{\varphi}\right)}{\partial\varphi}\right)' \\ \widehat{A} &= \frac{-1}{n}\sum_{t=1}^{n}\frac{\partial l_t\left(\widetilde{\varphi}\right)}{\partial\varphi\partial\varphi'}.\end{aligned}$$

Note that in calculating these standard errors we assume that the spline approximations are the true news impact curve specifications. The justification for this is the conjecture that a result similar to that in Ackerberg, Chen, and Hahn (2010) holds for multi-step estimation with weakly dependent data.

3.2 Second-Stage Estimation

Let α denote the vector of copula parameters. Given $\widetilde{\varphi}$, we estimate each F_i^0 with the rescaled empirical distribution of $\widehat{\varepsilon}_{i,t}$ and estimate α via maximum likelihood:

$$\begin{aligned}\widehat{\alpha} &= \arg\max_{\alpha}\frac{1}{n}\sum_{t=1}^{n}\log c\left(\widetilde{F}_{nS}\left(\widehat{\varepsilon}_{S,t}\right),\widetilde{F}_{nM}\left(\widehat{\varepsilon}_{M,t}\right),\widetilde{F}_{nB}\left(\widehat{\varepsilon}_{B,t}\right);\alpha\right) \\ \widetilde{F}_{ni}\left(x\right) &= \frac{1}{n+1}\sum_{t=1}^{n}1\left(\widehat{\varepsilon}_{i,t}\leq x\right).\end{aligned}$$

Note that this second-stage consists of two steps, estimation of the marginals and estimation of the copula parameters. Alternatively, one may use the method of Kendall's tau to estimate copula parameters. For the sake of robustness, we explore this approach in section 4.4.1 below. Chen and Fan (2006) explain that the corresponding asymptotic covariance matrix estimate is

$$\widehat{\mathcal{B}}^{-1}\widehat{\Omega}\widehat{\mathcal{B}}^{-1}$$

where

$$\widehat{\mathcal{B}} = -\frac{1}{n}\sum_{t=1}^{n}\frac{\partial L\left(\widetilde{U}_t;\widehat{\alpha}\right)}{\partial\alpha\partial\alpha'}$$

$$\widehat{\Omega} = \frac{1}{n}\sum_{t=1}^{n}\left(\begin{array}{c}\left[\frac{\partial L\left(\widetilde{U}_t;\widehat{\alpha}\right)}{\partial\alpha} + \sum_{j\in\{S,M,B\}}\widehat{Q}_{\alpha j}\left(\widetilde{U}_t;\widehat{\alpha}\right)\right] \\ \times\left[\frac{\partial L\left(\widetilde{U}_t;\widehat{\alpha}\right)}{\partial\alpha} + \sum_{j\in\{S,M,B\}}\widehat{Q}_{\alpha j}\left(\widetilde{U}_t;\widehat{\alpha}\right)\right]'\end{array}\right)$$

$$\widehat{Q}_{\alpha j}\left(U_t;\widehat{\alpha}\right) = \frac{1}{n}\sum_{s=1,s\neq t}^{n}\frac{\partial L\left(U_t;\widehat{\alpha}\right)}{\partial\alpha\partial U_j}\left\{I_{\{U_{jt}\leq U_{js}\}} - U_{js}\right\}$$

$$L\left(U_t;\alpha\right) = \log c\left(U_{St}, U_{Mt}, U_{Bt};\alpha\right)$$

$$\widetilde{U}_t = \left(\widetilde{U}_{St}, \widetilde{U}_{Mt}, \widetilde{U}_{Bt}\right)' = \left(\widetilde{F}_{nS}\left(\widehat{\varepsilon}_{S,t}\right), \widetilde{F}_{nM}\left(\widehat{\varepsilon}_{M,t}\right), \widetilde{F}_{nB}\left(\widehat{\varepsilon}_{B,t}\right)\right)'.$$

Note that the steps here are slightly different from those in Chen and Fan (2006). In their paper, the first-stage is *parametric* GARCH. Therefore, the validity of our covariance matrix rests on the extension of their result to the case with a semiparametric first-step.

3.3 Computational Notes

We use Matlab to perform the maximum likelihood computations. OLS estimates provide initial values for the conditional mean parameters. Standard GARCH(1,1) estimates provide inital values for the volatility parameters. Initial spline coefficients are chosen so that the initial news impact curve matches the quadratic GARCH(1,1) estimate. Given these initial values, we first use the derivative-free, unconstrained "fminsearch" optimization function. We use the output of this step to initialize the derivative-based, constrained optimization routine "fmincon." Nonlinear contraints ensure positive volatility estimates. Quite loose additional constraints (which do not seem to affect estimates) prevent fmincon from searching in parts of the parameter space that either prevent convergence or crash the routine. Further details and codes are available upon request.

3.4 Value at Risk Estimation

Now, consider a portfolio consisting of the stock market, the bond market, and mortgage-backed securities with respective shares α_M, α_B, and α_S ($\alpha_M + \alpha_B + \alpha_S = 1$). Let $\Pi_t \equiv \alpha_S S_t^e + \alpha_M M_t^e + \alpha_B B_t^e$ be the excess return on the portfolio.

First, note that

$$\Pi_{t+1} = C_\pi + \rho'_\pi \begin{pmatrix} S_t \\ M_t \\ B_t \end{pmatrix} + K'_{t+1} \begin{pmatrix} \varepsilon_{S,t+1} \\ \varepsilon_{M,t+t} \\ \varepsilon_{B,t+1} \end{pmatrix}$$

$$where\ C_\pi = \alpha_M c_M + \alpha_B c_B + \alpha_S c_S,$$

$$\rho_\pi = \begin{pmatrix} \alpha_S \rho_S \\ (\alpha_B \rho_M + \alpha_B \beta_B + \alpha_S \beta_S) \\ \alpha_B \rho_B \end{pmatrix},$$

$$K_{t+1} = \begin{pmatrix} \alpha_S \sigma_{S,t+1} \\ \alpha_M \sigma_{M,t+1} \\ \alpha_B \sigma_{B,t+1} \end{pmatrix}.$$

We define the value at risk at time t, VaR_t, to be the 1st percentile of Π_{t+1} (conditional on time t information) multiplied by -1,000,000. In other words, if the portfolio is worth \$1 million at time t, VaR_t is the amount of money we can be 99% certain will cover losses in the next period. We calculate "in sample" value at risk by plugging estimated parameters into the above model. Then, since C_π, ρ_π, $\begin{pmatrix} S_t \\ M_t \\ B_t \end{pmatrix}$, and K_{t+1} are known at time t, we can easily calculate VaR_t provided we have many draws from the joint $\varepsilon_{i,t}$ distribution. To do this, we have Matlab simulate many draws $\mathbf{u}_t = (u_{S,t}, u_{M,t}, u_{B,t})$ from $c\left(\mathbf{u}; \widehat{\Sigma}, \widehat{v}\right)$. Inverting these probabilities using the empirical distributions gives us the draws we need: $(\varepsilon_{S,t}, \varepsilon_{M,t}, \varepsilon_{B,t})' = \left(\widetilde{F}_{nS}^{-1}(u_{S,t}), \widetilde{F}_{nM}^{-1}(u_{M,t}), \widetilde{F}_{nB}^{-1}(u_{B,t})\right)'$. $\widetilde{F}_{ni}^{-1}(x)$ is calculated as follows. If we define

$$t^* = \arg \min_{0 \le t \le n} \left| x - \widetilde{F}_{ni}(\widehat{\varepsilon}_{i,t}) \right|,$$

then

$$\widetilde{F}_{ni}^{-1}(x) = \begin{cases} \widehat{\varepsilon}_{i,t^*} & if\ x = \widetilde{F}_{ni}(\widehat{\varepsilon}_{i,t^*}) \\ \frac{1}{2}(\widehat{\varepsilon}_{i,t^*} + \widehat{\varepsilon}_{i,t^*-1}) & if\ x < \widetilde{F}_{ni}(\widehat{\varepsilon}_{i,t^*}) \\ \frac{1}{2}(\widehat{\varepsilon}_{i,t^*} + \widehat{\varepsilon}_{i,t^*+1}) & if\ x > \widetilde{F}_{ni}(\widehat{\varepsilon}_{i,t^*}) \end{cases}.$$

It is then simple to simulate values for Π_{t+1} (conditional on time t realizations) and then get the corresponding 1st percentile excess return.

4 Results

4.1 Mortgage-Backed Securities

Below are the MBS parameter estimates. Standard errors are in parentheses. Figures displaying the news impact curves are attached. The curves are plotted over the 1st to 99th percentiles of the fitted residuals. Spline parameters are available upon request. Also attached are the estimated volatility processes.

MBS Parameter Estimates

Model	c_S	ρ_S	β_S	ω_S	θ_S	γ_S
GARCH(1,1)	.0194 (.0057)	.0754 (.0356)	.0132 (.0051)	.0007 (.0004)	.8922 (.0252)	.1022 (.0219)
Sieve-GARCH(1,1)	.0134 (.0060)	.0734 (.0376)	.0117 (.0049)	.2369 (.1724)	.9118 (.0597)	see figures

As we see in the table and plots, there are really only two major differences between the GARCH and Sieve-GARCH estimates for the MBS market. First, for negative shocks, Sieve-GARCH predicts more volatility than does GARCH, and for positive shocks Sieve-GARCH predicts less volatility than does GARCH. Moreover, with Sieve-GARCH, some good news reduces volatility, and the difference between the curves is greater in the positive region.

Second, on 9/8/2008, the MBS index experienced an over six standard deviation return of 1.7%. As we see in the figure, in the GARCH framework this led to a huge spike in volatility. In the Sieve-GARCH model, good news does not have this extreme effect, and the 9/9/2008 volatility response was much more mild.

4.2 The US Stock Market

Stock market estimates are below. News impact curves and volatility figures are attached.

Stock Market Parameter Estimates

Model	c_M	ρ_M	ω_M	θ_M	γ_M
GARCH(1,1)	.0728 (.0376)	$-.0919$ (.0311)	.0257 (.0119)	.8823 (.0140)	.1108 (.0163)
Sieve-GARCH(1,1)	.0043 (.0432)	$-.0673$ (.0357)	4.8999 (.9110)	.8954 (.0178)	see figures

In this case, conditional mean parameters differ between GARCH and Sieve-GARCH more than in the MBS example, but given the standard errors, the effect is mild. However, the GARCH and Sieve-GARCH news impact curves are quite different: for negative shocks, Sieve-GARCH predicts more volatility than does GARCH. Moreover, with Sieve-GARCH, most good news decreases volatility. As in Linton and Mammen (2005), the effect of good news in small compared to the effect of bad news. The two models provide a similar picture of volatility over the sample.

4.3 The US Bond Market

Bond market estimates are below. News impact curves and volatility figures are attached.

Bond Market Parameter Estimates

Model	c_B	ρ_B	β_B	ω_B	θ_B	γ_B
GARCH(1,1)	.0256	$-.0086$	.0237	.0014	.9232	.0618
	(.0084)	(.0344)	(.0068)	(.0009)	(.0260)	(.0185)
Sieve-GARCH(1,1)	.0197	$-.0240$	.0170	.0473	.9235	see figures
	(.0087)	(.0384)	(.0073)	(.0139)	(.0224)	

The news impact curve patterns are similar to those described in the MBS section above. Again, the two models provide a similar picture of volatility over the sample. An exception is the activity surrounding the over four standard deviation return of 1.3% on 3/18/2009. GARCH volatility experienced a large subsequent spike. Sieve-GARCH volatility actually fell on 3/19/2009.

4.4 Copula Estimates

Let $corr\,(i,j)$ denote the entry of Σ corresponding to the correlation between asset i and asset j.

Copula Parameter Estimates

Model	$corr\,(S,M)$	$corr\,(S,B)$	$corr\,(M,B)$	v
GARCH(1,1)	$-.2824$	.9178	$-.3642$	5.7645
	(.0318)	(.0061)	(.0303)	(.7240)
Sieve-GARCH(1,1)	$-.2801$	.9144	$-.3590$	5.3903
	(.0320)	(.0064)	(.0307)	(.6484)

As mentioned in the introduction, there is substantial positive correlation between MBS shocks and bond shocks. There is moderate negative correlation between MBS shocks and stock shocks and between stock shocks and bond shocks. GARCH and Sieve-GARCH provide similar estimates.

But correlation provides an only incomplete picture of financial risk. "Tail dependence," alternatively, measures the extent to which *extreme* shocks to dependent series arrive together (see Joe (1997) or Nikoloulopoulos, Joe, and Li (2009) for more details). In the t-copula case, the bivariate tail dependence between series i and series j is

$$\lambda_{ij} = 2T_{v+1}\left(-\frac{\sqrt{v+1}\sqrt{1-corr\,(i,j)}}{\sqrt{1+corr\,(i,j)}}\right).$$

Note that the Gaussian analog of λ_{ij} is zero. Plug-in estimates of tail depedence are thus (standard errors via the delta method)

Tail Dependence Estimates

Model	λ_{SM}	λ_{SB}	λ_{MB}
GARCH(1,1)	.0109	.6074	.0071
	(.0050)	(.0239)	(.0035)
Sieve-GARCH(1,1)	.0137	.6110	.0097
	(.0057)	(.0239)	(.0042)

We see that (i) MBS markets and bond markets exhibit substantial tail dependence and (ii) the Sieve-GARCH method increases tail dependence estimates, but not by much.

4.4.1 Robustness of Copula Estimates

We now describe the method of Kendall's tau, an alternate way to estimate the copula parameters.

First, we nonparametrically estimate marginals, as above. Next, following from Demarta and McNeil (2005), an estimate of the copula correlation between series i and series j is given by $\widehat{\Sigma}_{ij} = \sin\left(\frac{\pi}{2}\widehat{\rho}_\tau\left(\widehat{\varepsilon}_{i,t}, \widehat{\varepsilon}_{j,t}\right)\right)$, where

$$\widehat{\rho}_\tau\left(\widehat{\varepsilon}_{i,t}, \widehat{\varepsilon}_{j,t}\right) = \binom{n}{2}^{-1} \sum_{1\leq t_1 < t_2 \leq n} sign\left[\left(\widehat{\varepsilon}_{i,t_1} - \widehat{\varepsilon}_{i,t_2}\right)\left(\widehat{\varepsilon}_{j,t_1} - \widehat{\varepsilon}_{j,t_2}\right)\right]$$

and $\widehat{\varepsilon}_{i,t}$ are the fitted residuals for series i. Given $\widehat{\Sigma}$, we may then estimate degrees of freedom, v, via ML:

$$\widehat{v} = \arg\max_v \frac{1}{n}\sum_{t=1}^{n} \log c\left(\widetilde{F}_{nS}\left(\widehat{\varepsilon}_{S,t}\right), \widetilde{F}_{nM}\left(\widehat{\varepsilon}_{M,t}\right), \widetilde{F}_{nB}\left(\widehat{\varepsilon}_{B,t}\right); \widehat{\Sigma}, v\right).$$

The following table contains the resulting estimates

	Copula Parameter				Tail Dependence		
Model	$corr\,(S,M)$	$corr\,(S,B)$	$corr\,(M,B)$	v	λ_{SM}	λ_{SB}	λ_{MB}
Sieve-GARCH(1,1)	$-.2678$	.9161	$-.3557$	5.4341	.0141	.6133	.0091

Examining the above standard errors, we see that the method of Kendall's tau provides similar copula estimates.

4.5 VaR

We consider a portfolio with $\alpha_M = \alpha_B = \alpha_S = 1/3$. As one would expect, there was a large spike in the in sample VaR in the fall of 2008. At the height of the crisis, we estimate that there was a 1% chance of losing roughly more than 4.5% of the portfolio in one day. At the end of 2010, the number was less than 1%. The GARCH and Sieve-GARCH VaR plots, which are attached, are quite similar over the sample.

5 Conclusion

We have illustrated the usefulness of sieve and copula methods in estimating news impact curves and shock dependence, both of which are central to leading explanations of the recent crisis. Sieve-GARCH estimates for mortgage-backed security, stock, and bond market data indicate asymmetries in the response of volatility to news. However, except after extreme events, the standard GARCH and Sieve-GARCH frameworks appear to yield similar pictures of risk and volatility over the sample.

6 References

1. ACKERBERG, D., X. CHEN, AND J. HAHN (2010): “A Practical Asymptotic Variance Estimator for Two-Step Semiparametric Estimators,” *Review of Economics and Statistics*, Forthcoming.

2. AUDRINO, F., AND P. BÜHLMANN (2009): “Splines for Financial Volatility,” *Journal of the Royal Statistical Society*, 71, 655-670.

3. BARBERIS, N. (2010): “Psychology and the Financial Crisis of 2007-2008,” in preparation for *Financial Innovation and Crisis*, M. Haliassos ed., 2011.

4. BOLLERSLEV, T. (1986): “Generalized Autoregressive Conditional Heteroskedasticity,” *Journal of Econometrics*, 31, 307-327.

5. BOLLERSLEV, T., R.F. ENGLE, AND D.B. NELSON (1994): “ARCH Models,” in *Handbook of Econometrics*, volume IV, eds. R. F. Engle and D. L. McFadden, Elsevier Science, 2959-3038.

6. BOLLERSLEV, T. AND J.M. WOOLDRIDGE (1992): “Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models With Time Varying Covariances,” *Econometric Reviews*, 11, 143-172.

7. CHEN, X. (2007): “Large Sample Sieve Estimation of Semi-Nonparametric Models,” in J.J. Heckman and E.E. Leamer (eds.), *The Handbook of Econometrics*, vol. 6B. Amsterdam: North-Holland.

8. CHEN, X., AND Y. FAN (2006): “Estimation and Model Selection of Semiparametric Copula-based Multivariate Dynamic Models under Copula Misspecification,” *Journal of Econometrics*, 135, 125-154.

9. CHEN, X., AND X. SHEN (1998): “Sieve Extremum Estimates for Weakly Dependent Data,” *Econometrica*, 66, 289-314.

10. DEMARTA, S., AND A.J. MCNEIL (2005): “The t copula and related copulas,” *International Statistical Review*, 73, 111-129.

11. ENGLE, R.F. (2010): “Long Term Skewness and Systemic Risk,” Presidential Address SoFiE, 2009.

12. ENGLE, R.F., AND V. NG (1993): ”Measuring and Testing the Impact of News On Volatility,” *Journal of Finance*, 48, 1749-1778.

13. ENGLE, R.F., AND J.G RANGEL (2007): "The Spline-GARCH Model for Unconditional Volatility and its Global Macroeconomic Causes," *Review of Financial Studies*.

14. GEANAKOPLOS, J., AND A. FOSTEL (2010): "Why Does Bad News Increase Volatility and Decrease Leverage," Cowles Foundation Discussion Paper No. 1762.

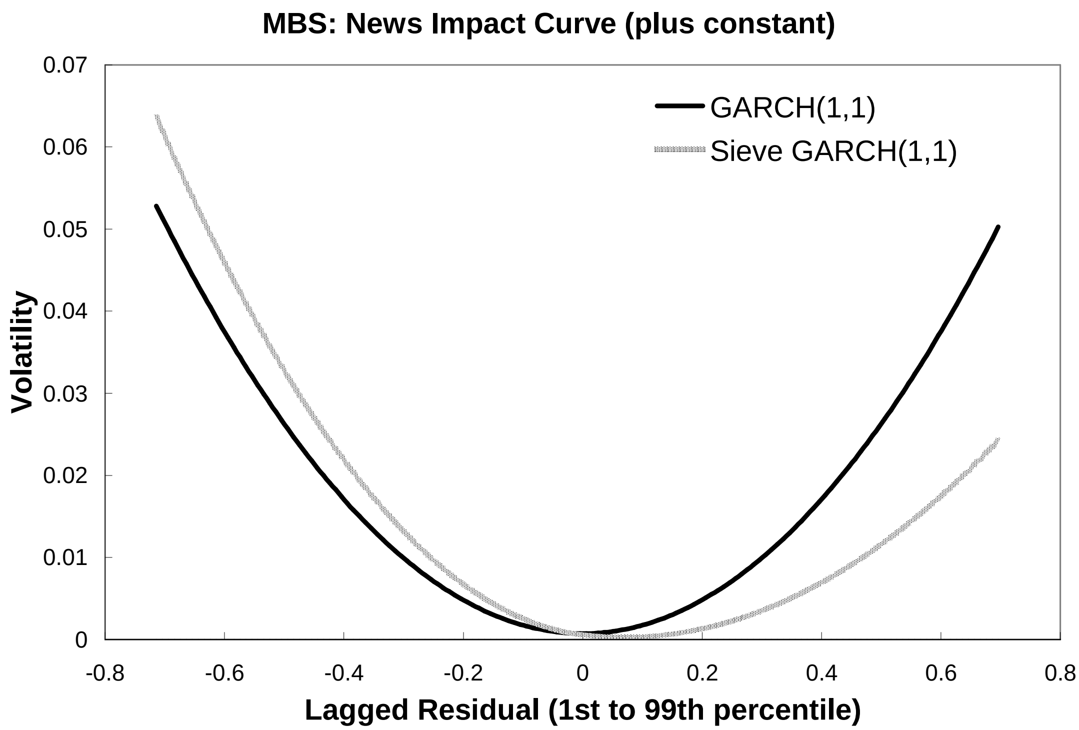
15. GEANAKOPLOS, J. (2010): "The Leverage Cycle," in D.Acemoglu, K. Rogoff, and M. Woodford (eds.), *NBER Macro-economics Annual 2009*, vol. 24, University of Chicago Press, Chicago, 2010, pp. 1-65.

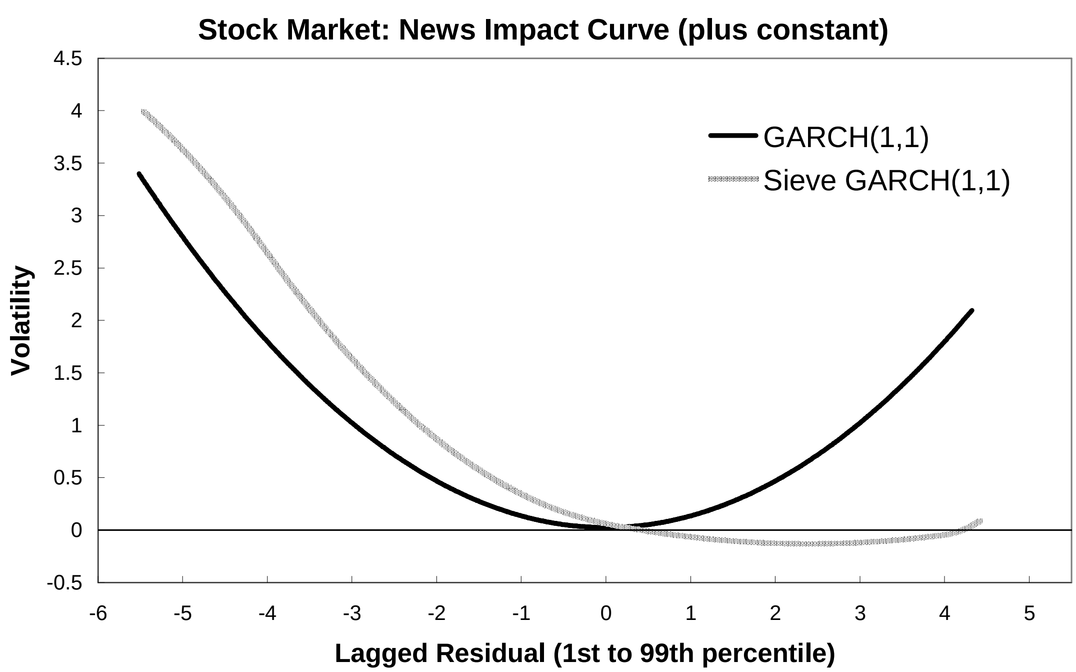
16. GERTLER, M., AND N. KIYOTAKI (2010): "Financial Intermediation and Credit Policy in Business Cycle Analysis," in preparation for the *Handbook of Monetary Economics*.

17. JOE, H. (1997): *Multivariate Models and Dependence Concepts*, London: Chapman & Hall.

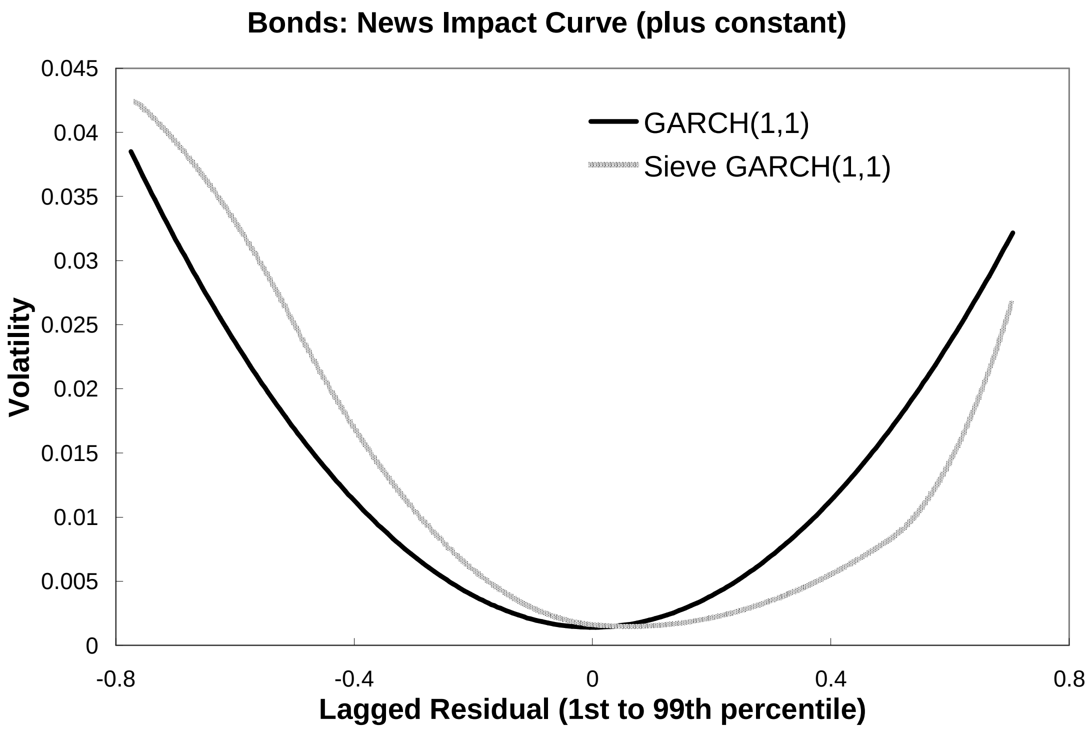
18. NIKOLOULOPOULOS, A.K., J. HARRY, AND H. LI (2009): "Extreme value properties of multivariate t copulas," *Extremes*, 12, 129-148.

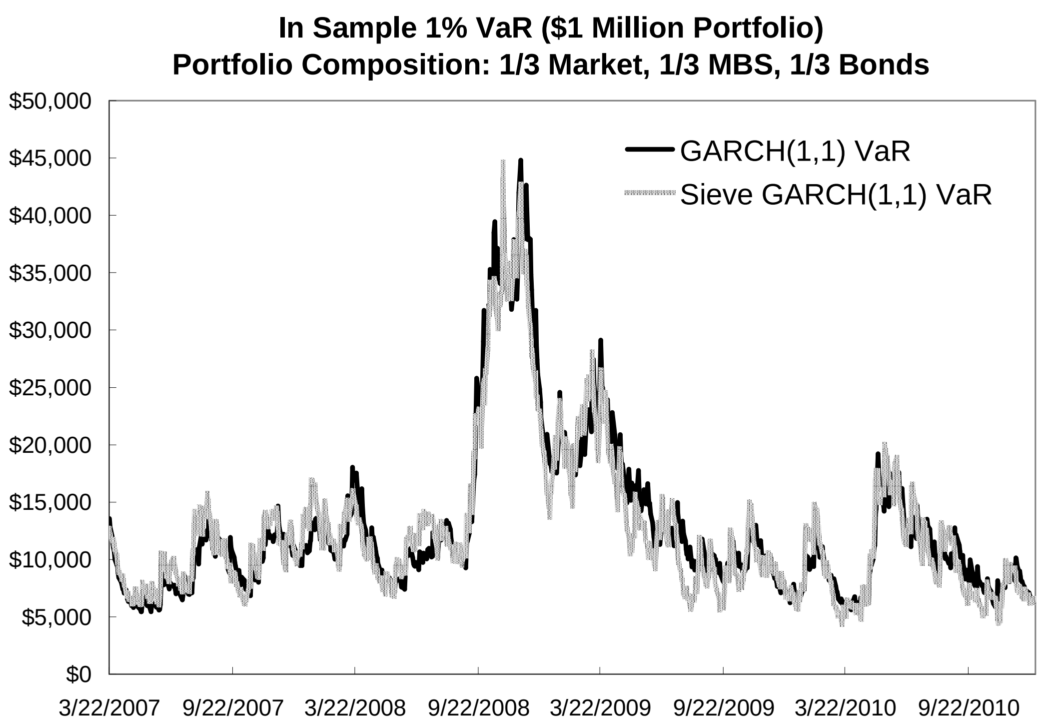
7 Figures



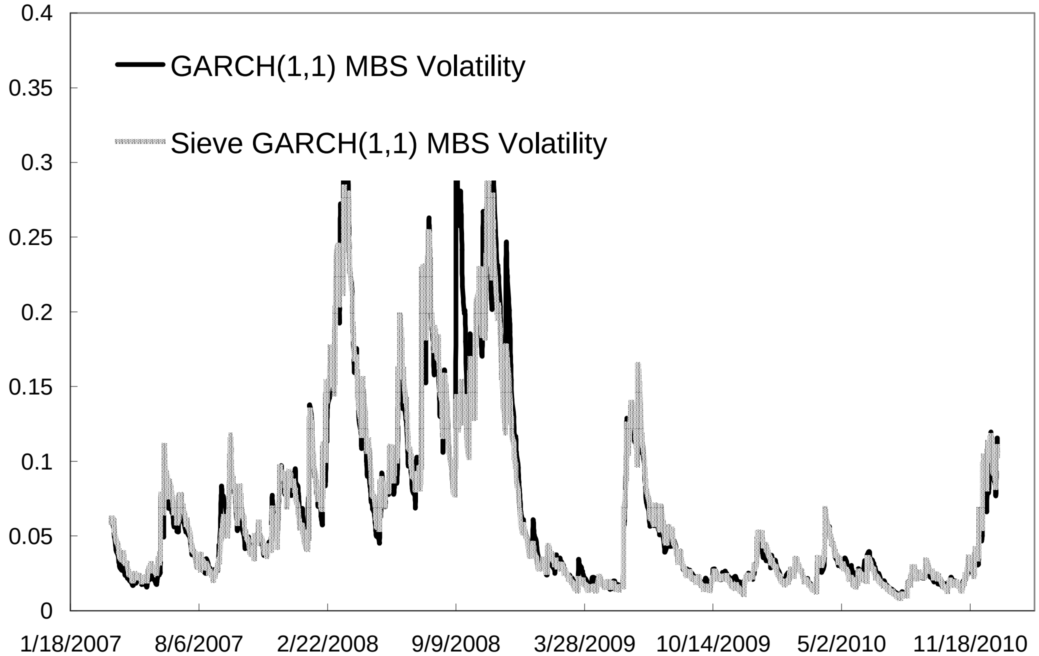


Note: Due to the lagged volatility term in the volatility equation, a negative news impact curve does not imply negative volatility. We implicitly assume that volatility is the maximum of 0 and the GARCH equation. However, the estimated process never hits this bound.

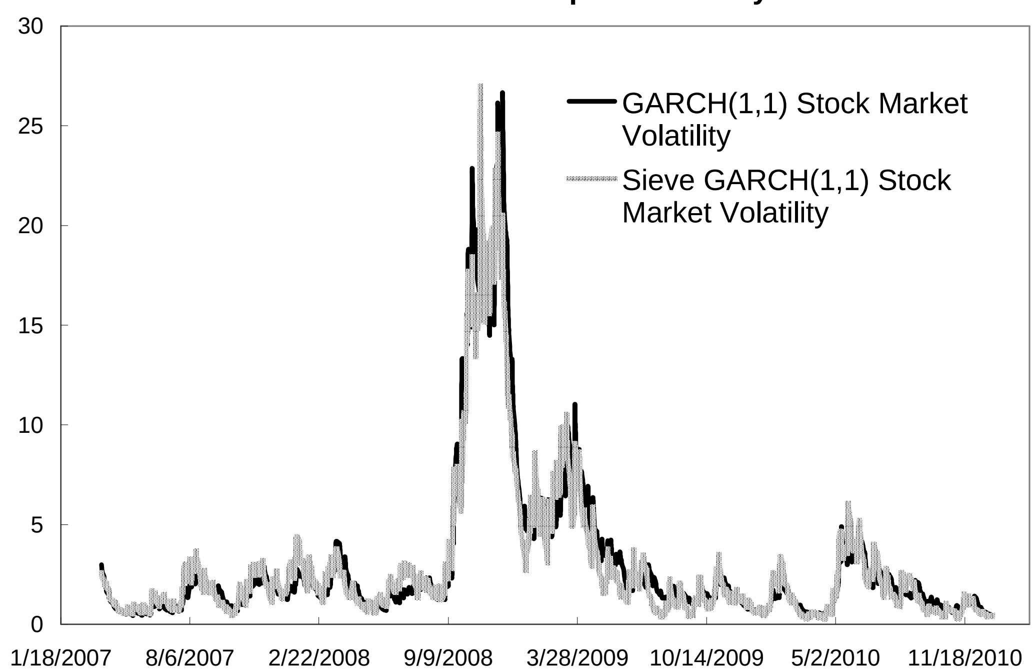

Bonds: News Impact Curve (plus constant)
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Volatility
GARCH(1,1)
Sieve GARCH(1,1)
-0.8
-0.4
0
0.4
0.8
Lagged Residual (1st to 99th percentile)

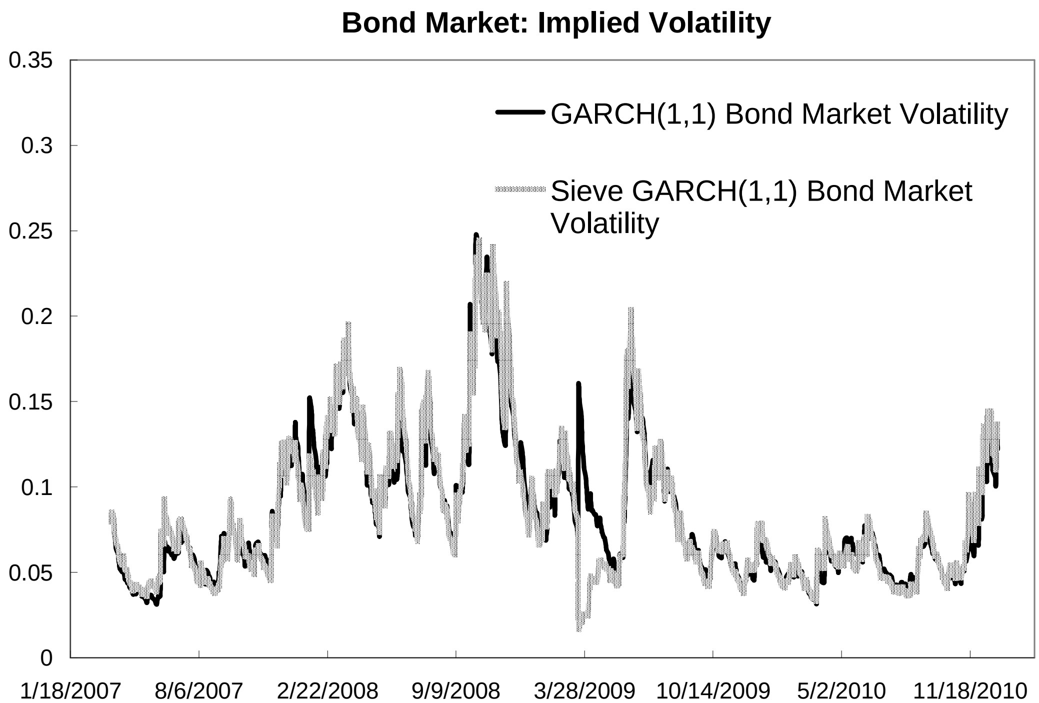

In Sample 1% VaR ($1 Million Portfolio)
Portfolio Composition: 1/3 Market, 1/3 MBS, 1/3 Bonds
$50,000
$45,000
$40,000
$35,000
$30,000
$25,000
$20,000
$15,000
$10,000
$5,000
$0
GARCH(1,1) VaR
Sieve GARCH(1,1) VaR
3/22/2007
9/22/2007
3/22/2008
9/22/2008
3/22/2009
9/22/2009
3/22/2010
9/22/2010

MBS: Implied Volatility



Stock Market: Implied Volatility




Bond Market: Implied Volatility
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1/18/2007
8/6/2007
2/22/2008
9/9/2008
3/28/2009
10/14/2009
5/2/2010
11/18/2010
GARCH(1,1) Bond Market Volatility
Sieve GARCH(1,1) Bond Market Volatility