

Modeling Natural Mortality (M)

Basic Population Dynamics

- By Numbers

$$N_{t+1} = N_t + \text{Births}_t - \text{Deaths}_t$$

- By Biomass

$$\text{Biomass}_{t+1} = \text{Biomass}_t + \text{Births}_t + \text{Growth}_t - \text{Deaths}_t$$

- Fishing impact

$$\text{Biom}_{t+1} = \text{Biom}_t + \text{Births}_t + \text{Growth}_t - \text{NatMort}_t - \text{Catch}_t$$
A diagram consisting of two arrows originating from the term 'Deaths_t' in the equation above. One arrow points down and to the left towards 'NatMort_t' in the equation below. The other arrow points down and to the right towards 'Catch_t' in the equation below.

- Continuous Case by numbers

$$\frac{dN}{dt} = b(t) - (M(t) + F(t))N(t)$$

Basic Population Dynamics

- By Numbers, age-structured

$$N_{a,t+1} = N_{a,t} + \text{Births}_{a,t} - \text{Deaths}_{a,t}$$

Noting that Births is zero except when a is zero

- Continuous Case

$$\frac{dN}{dt} = r(N, t) - (M + F)N$$

- Noting, within a year, particularly for $a > 0$:

$$\frac{dN_a}{dt} = -(M + F)N_a$$

– Noting also that this means M and F can be confounded in modeling and estimation

Natural Mortality (M)

- Key parameter in population dynamics
- M is continuous/instantaneous rate of mortality from causes other than fishing (F is rate of fishing mortality)
- We often model M as constant not only within year and age, but across ages.
 - This is a simplification – M is typically higher for younger and smaller fish, as well as for the oldest fish.

Natural Mortality (M)

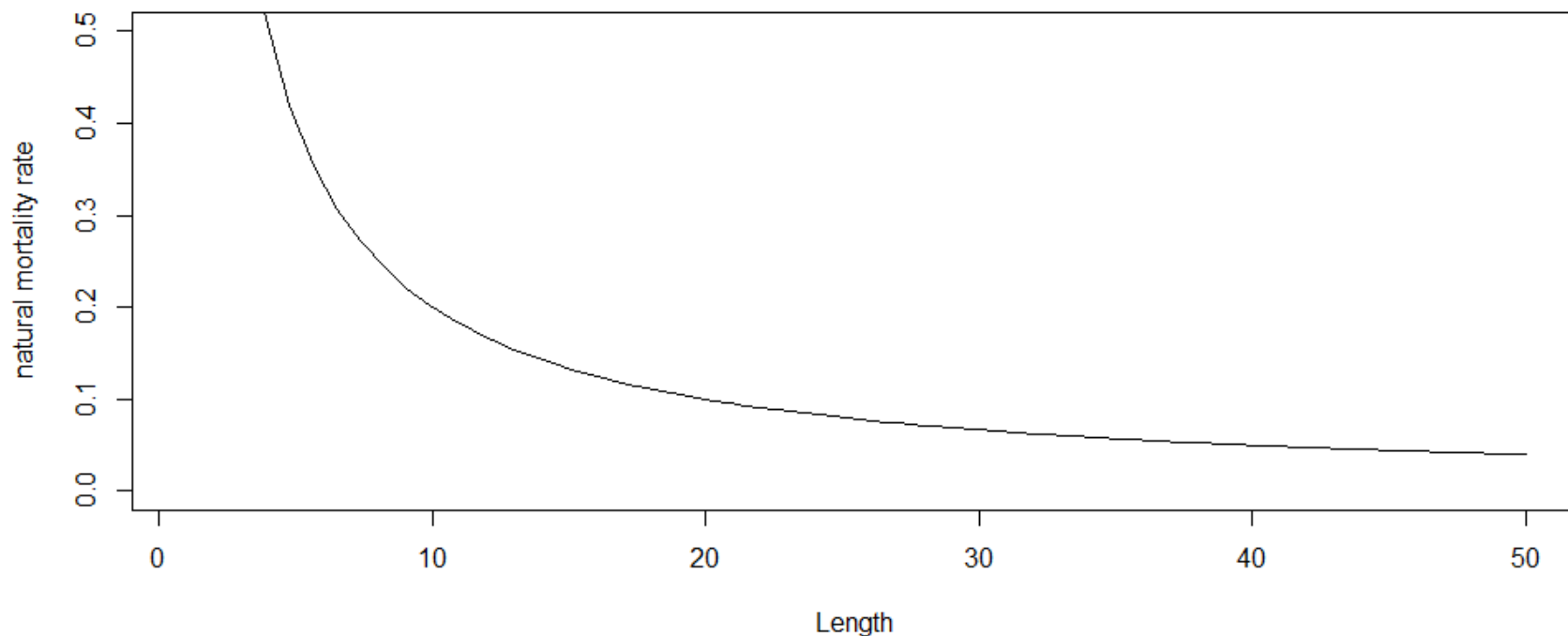
- In fact, Natural Mortality varies by:
 - Age
 - Sex
 - Time
 - Density
 - Environmental drivers
 - Predation
 - Prey availability
 - Disease/parasites
 - Competition
 - Temperature
 - Etc...

Natural Mortality (M)

- However, we often model M in our assessments as:
 - $M = \text{constant}$,
 - or
 - $M_{\text{female}} = C_1$; $M_{\text{male}} = C_2$
- For species that mature and show up in fisheries closer to maximum size, this can be reasonable

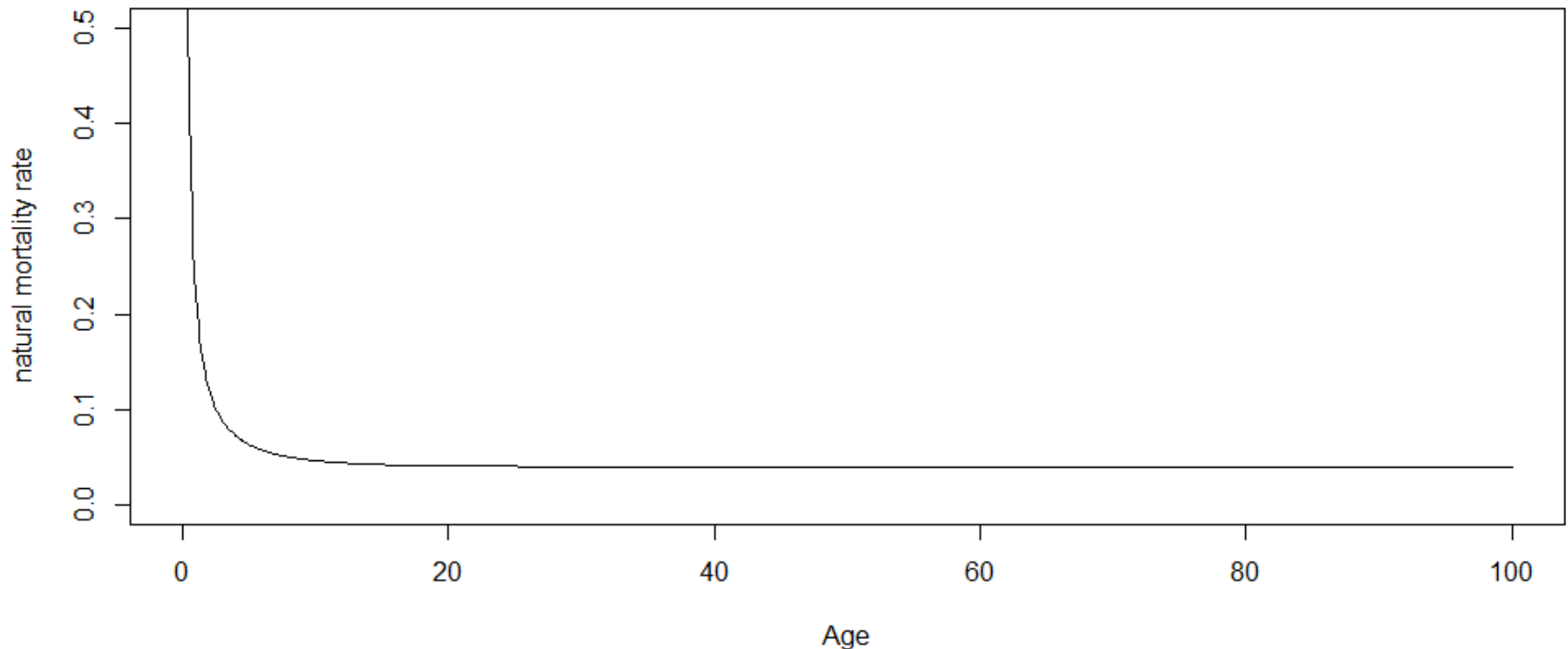
Natural Mortality (M)

- To restate, We often model M as:
 - M = constant, even though there is evidence for M being inversely related to length (on average) (e.g. Lorenzen 2022; Lorenzen et al. 2022)



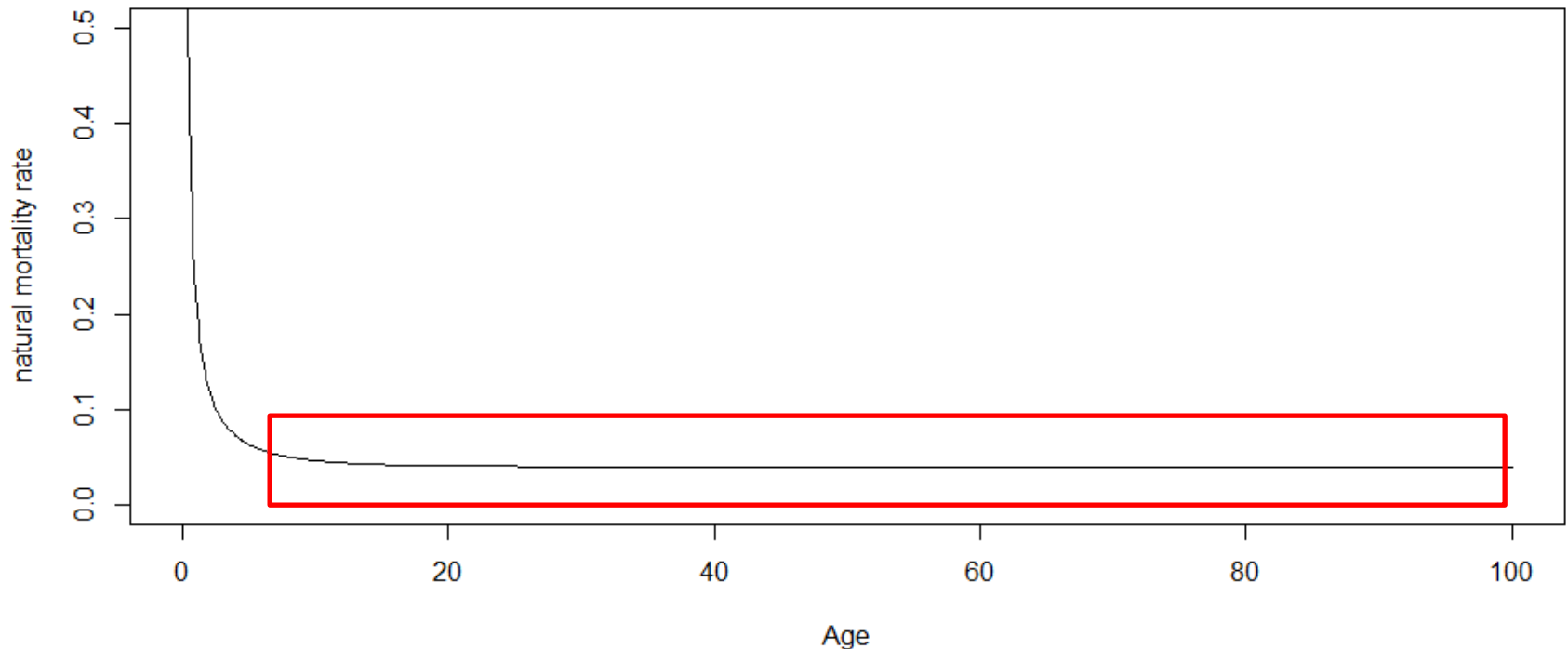
Natural Mortality (M)

- However, we often model M as:
 - $M = \text{constant}$: looks more reasonable with M at age



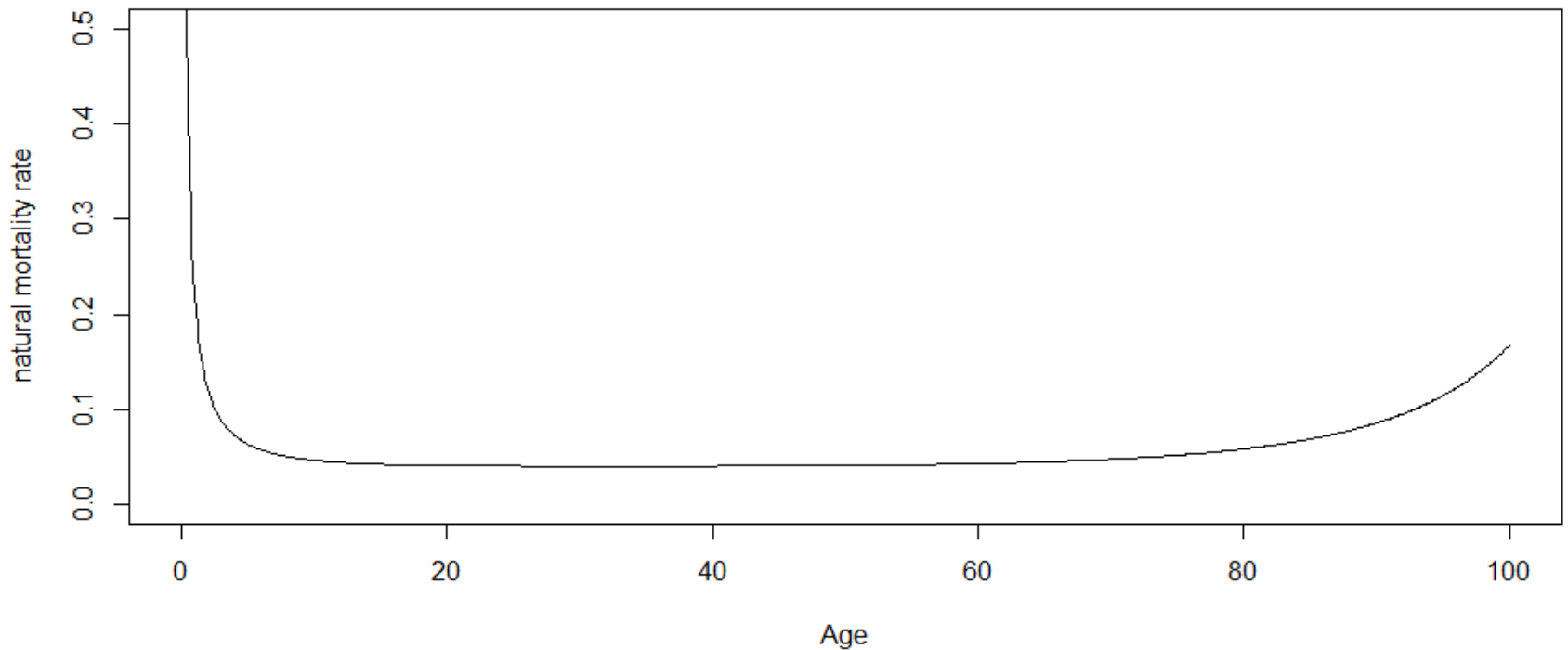
Natural Mortality (M)

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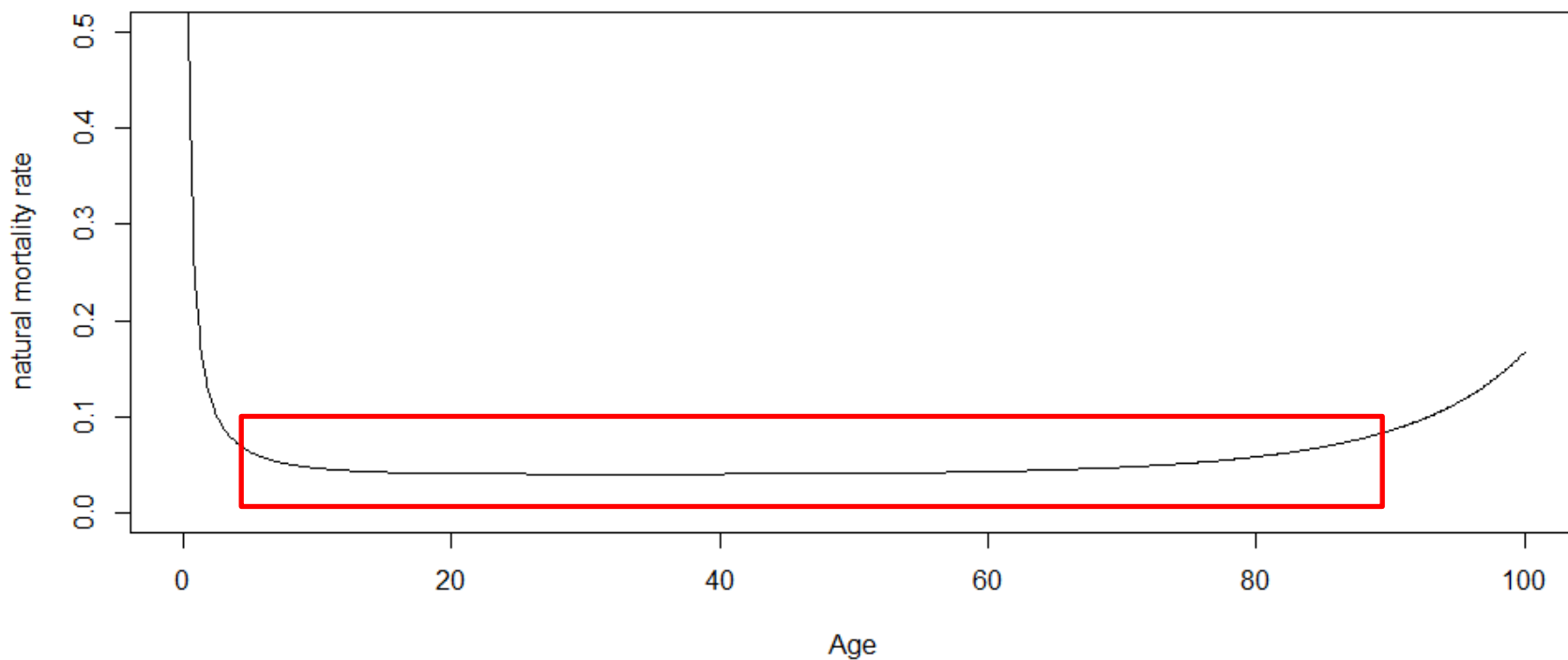
Natural Mortality (M)

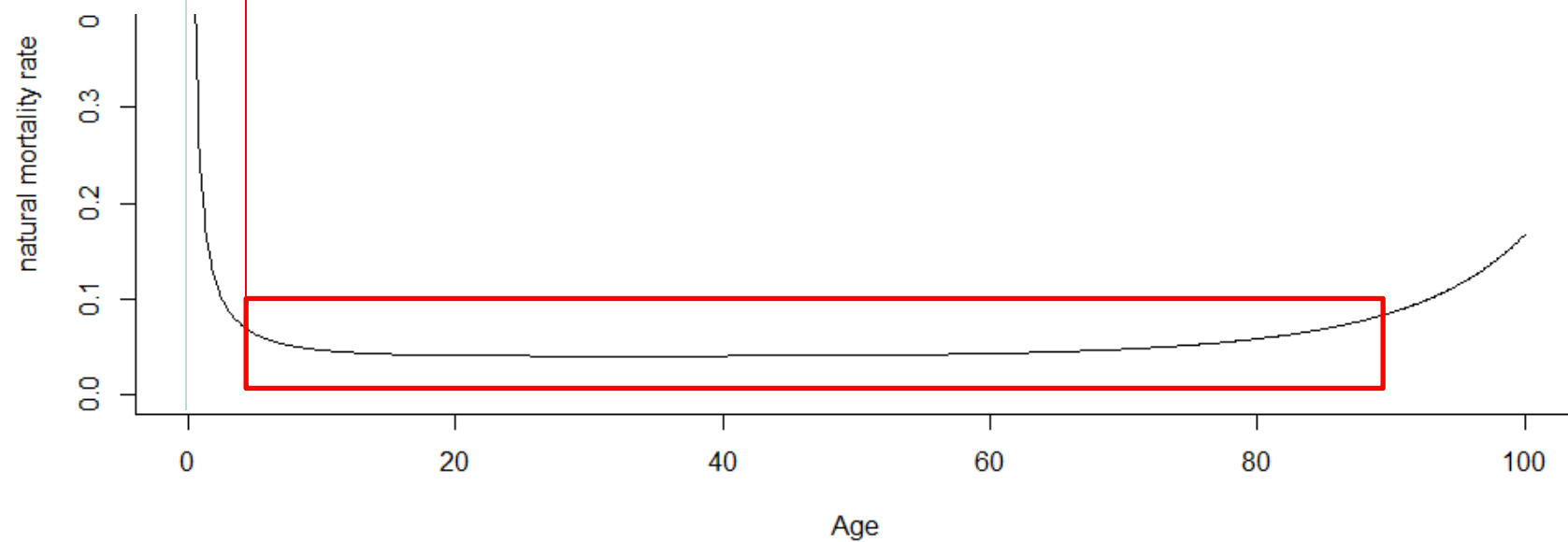
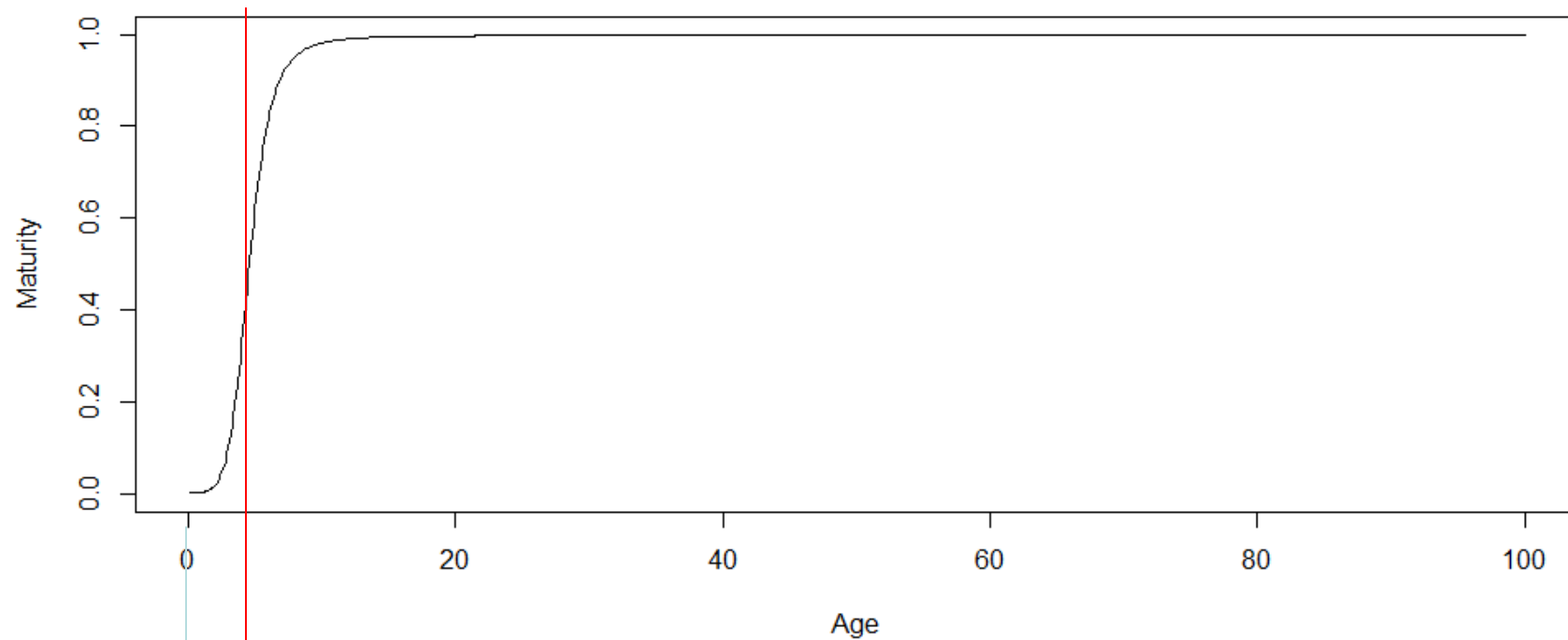
- However, we often model M as:
 - M = constant: looks more reasonable with M at age
 - (even with senescence)



Natural Mortality (M)

- However, we often model M as:
 - M = constant: looks more reasonable with M at age
 - (even with senescence)





Natural mortality

- Constant M at age OK
 - We won't accurately model numbers at age for youngest fish, but we aren't catching them either, so all wrapped up in recruitment values
 - Maybe we are overestimating the impact of fishing on the youngest fish – but for long-lived fish, F is small, so not a large error
 - Sensitivity analyses could include a Lorenzen relationship between length and M

Natural mortality

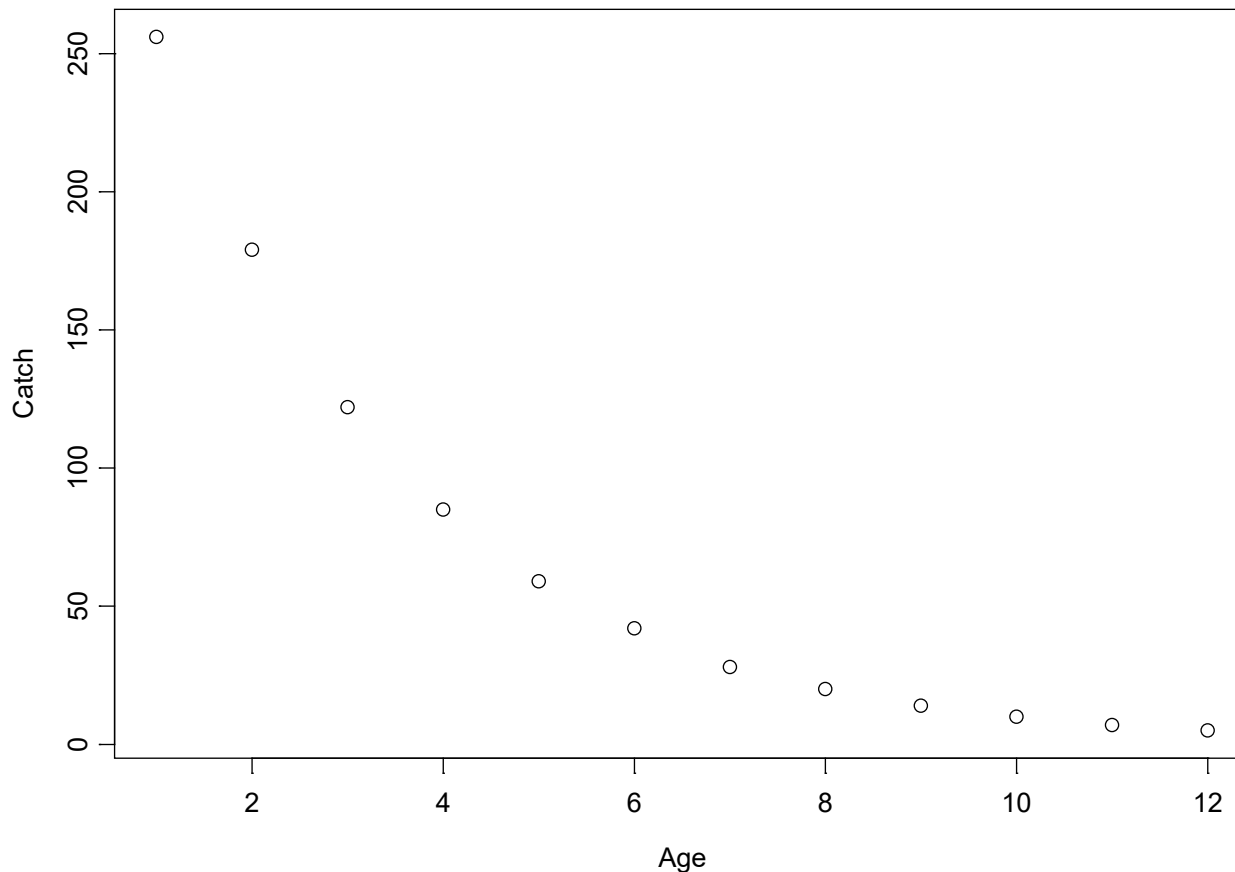
- How do we estimate M ?
 - Directly from data or in assessment (rare)
 - From meta-analyses with other life history parameters (e.g.:
 - maximum age (longevity),
 - growth rate,
 - size,
 - environmental temperature,
 - etc.)

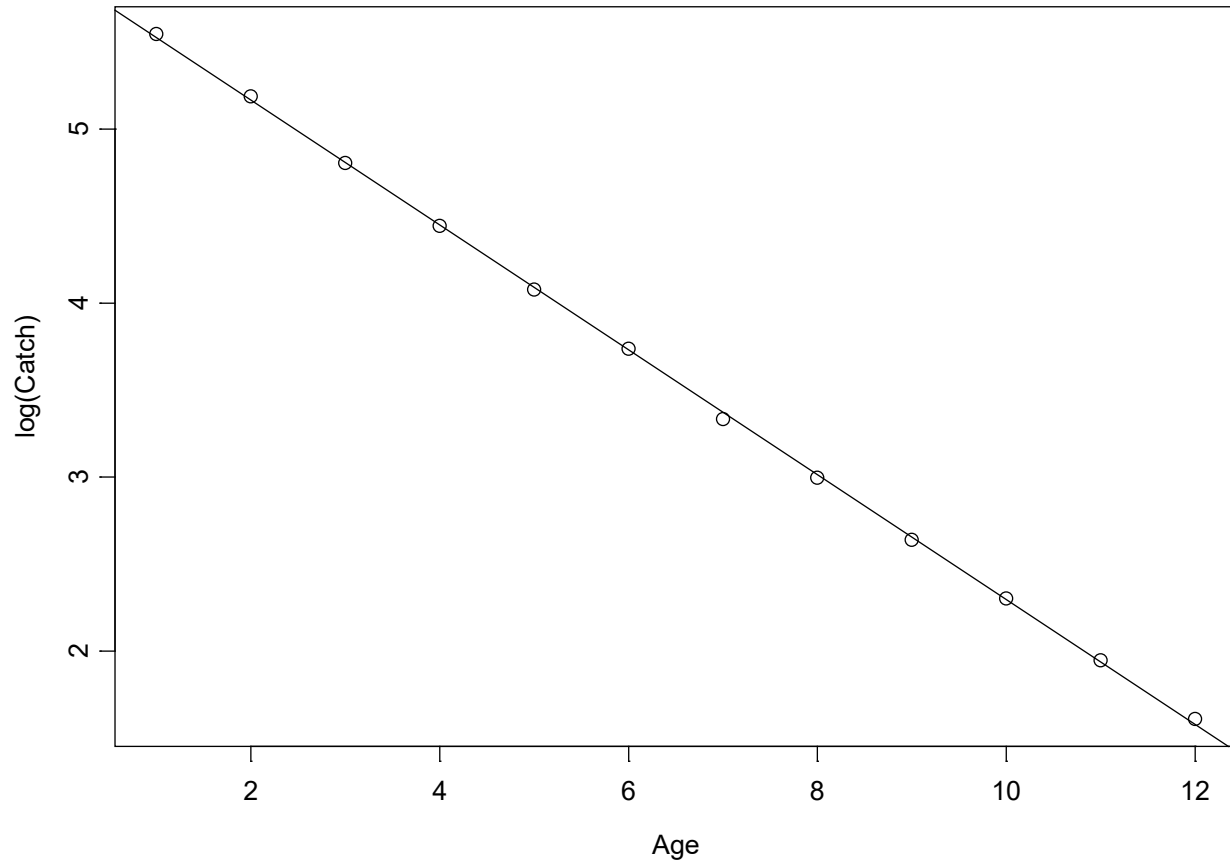
How to estimate M ?

- Beverton and Holt (1957):
 - Catch curve analysis
 - Ratio of abundance or catch of a single year class in successive years (with very low F)
 - Ratio abundance or catch of adjacent age groups in a sample
 - Mark-Recapture
 - Great, if can get adequate (lots) and reliable data
 - Biases due to change in fish behavior/survival/capture due to marking, incomplete reporting by fishermen, migration.

How do we estimate M ?

- Ideally, one has a nice catch curve...





$$M = 0.359$$

But...

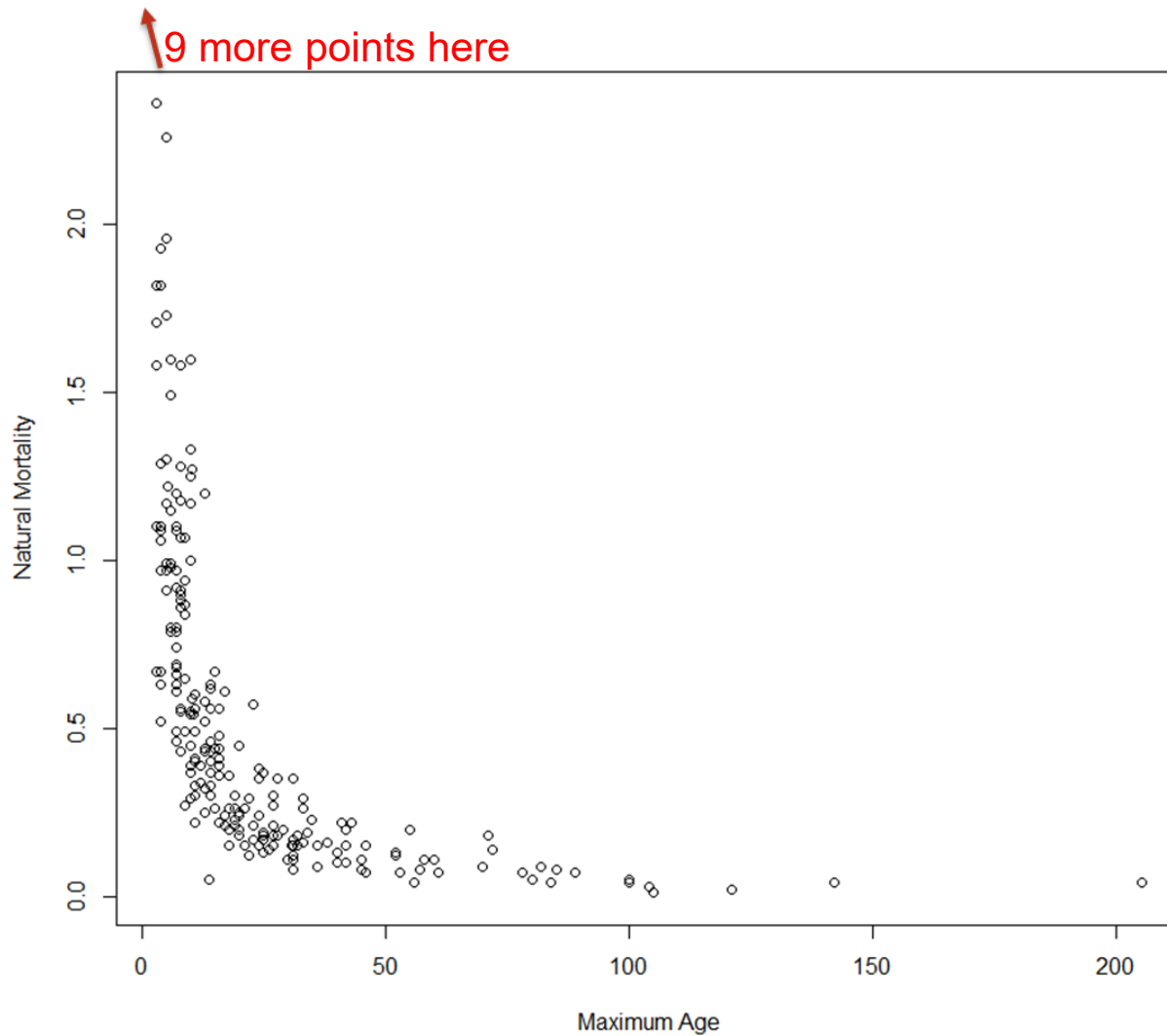
- This never happens
- There are issues with:
 - Ageing error
 - Selectivity
 - recruitment variability
 - age and time varying parameters
 - etc.

Natural mortality

- How do we estimate M ?
 - Directly from data or in assessment (rare)
 - From meta-analyses with other life history parameters (e.g. maximum age (longevity))

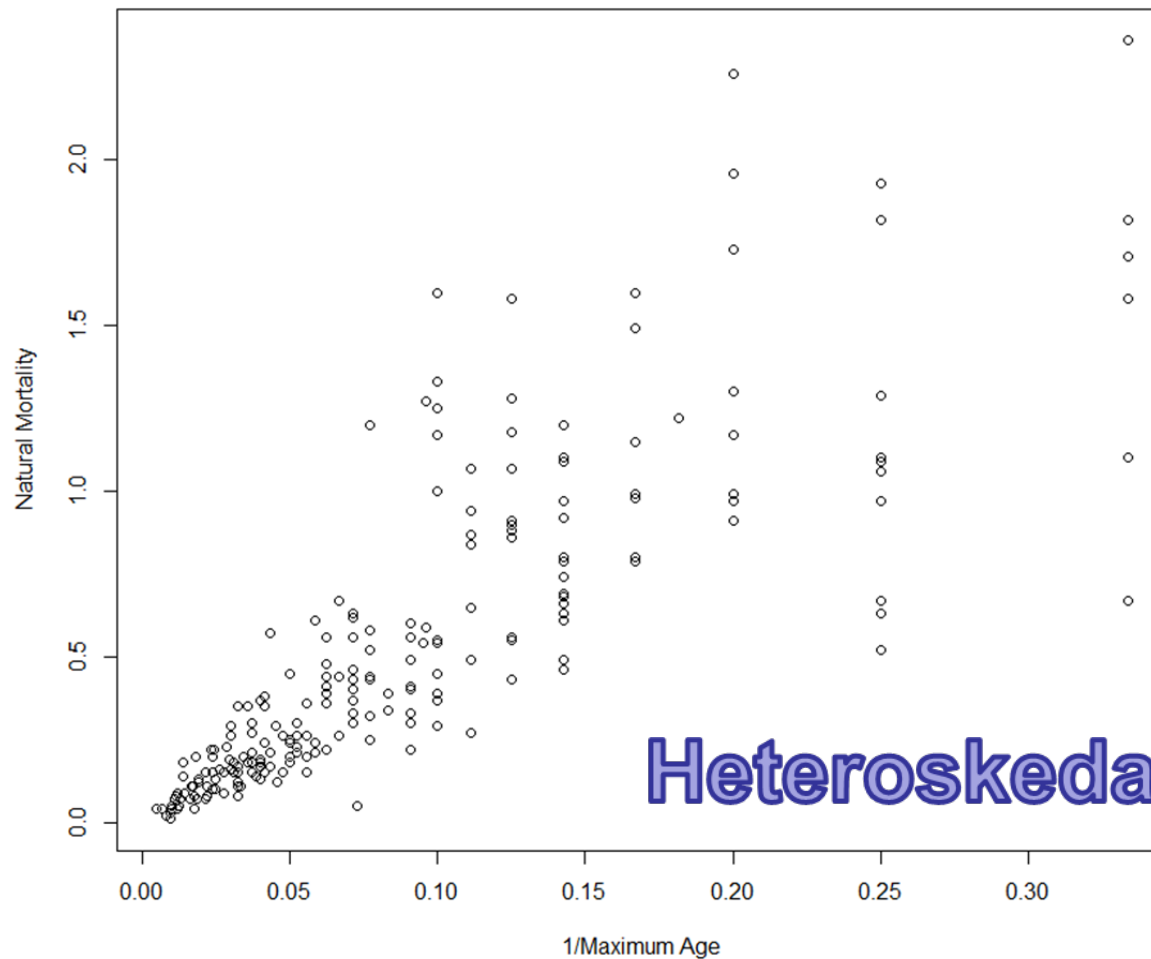
Then et al. 2015 Data Set

M vs. Maximum Age

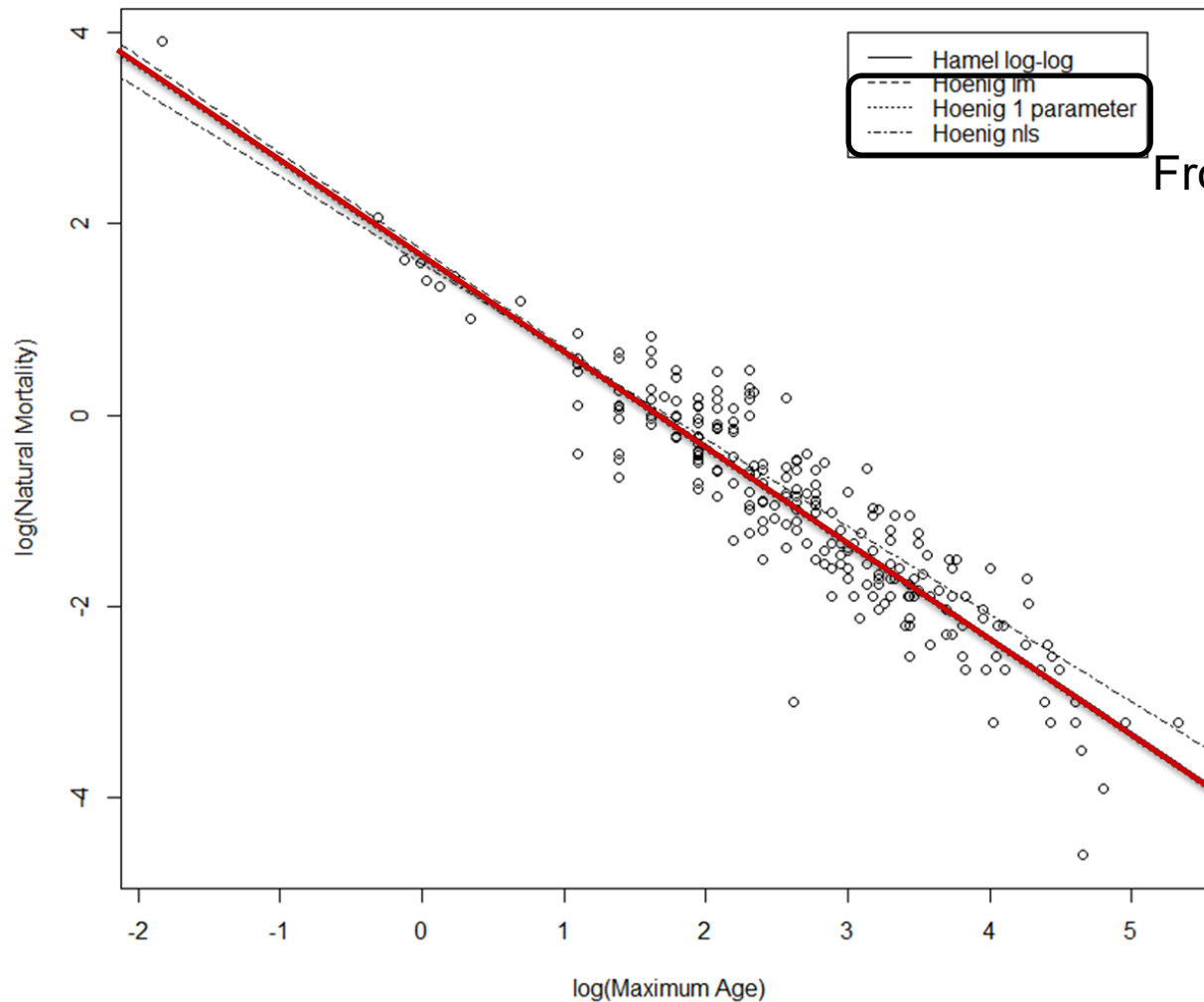


Then et al. 2015 Data Set

M vs. 1/Maximum Age



Hamel and Cope 2022

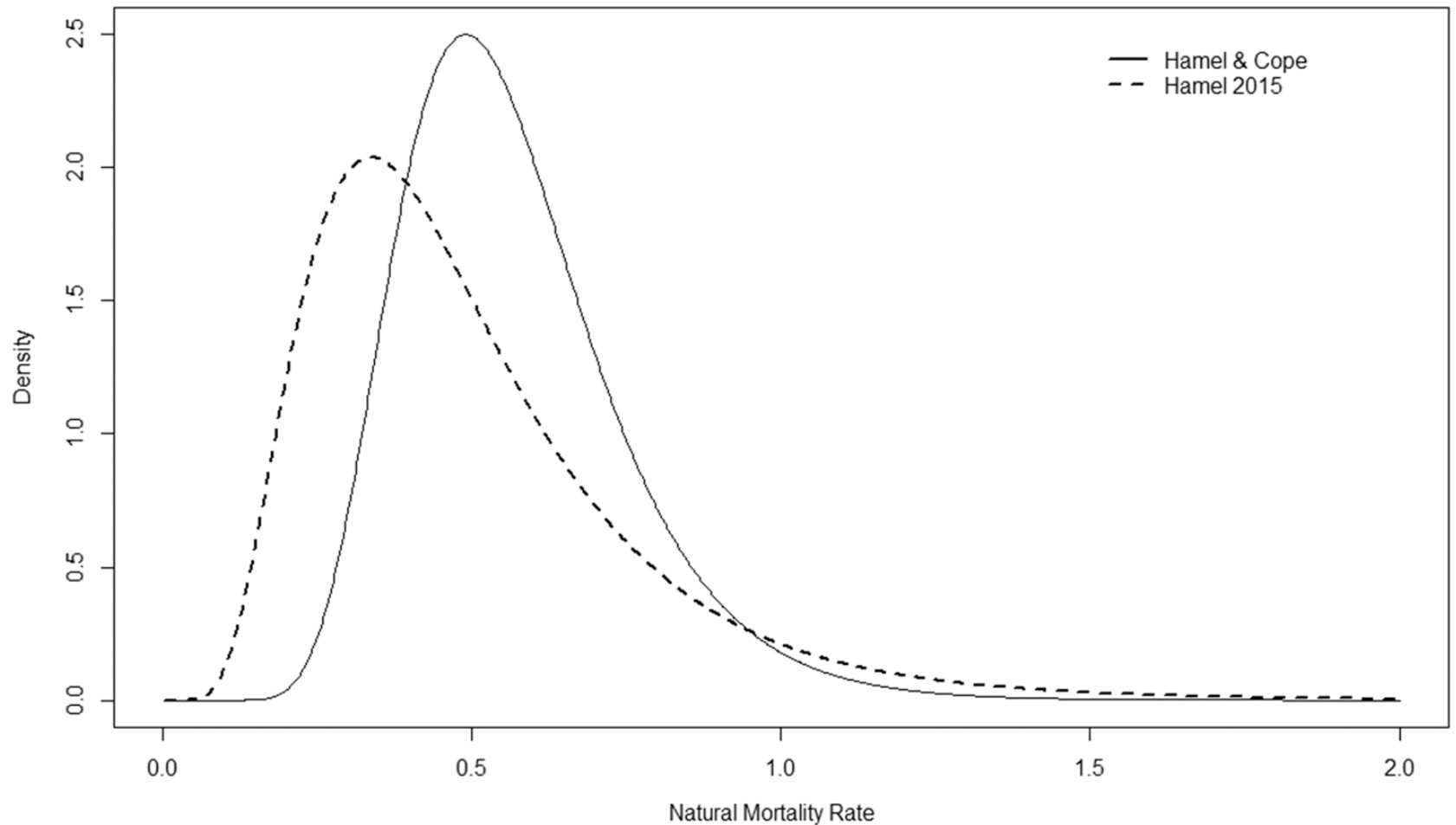


From Then et al. 2015

Prior/point estimate

- $M = \frac{5.40}{A_{max}}$ (we usually fix this in assessment)
 - Value above is median (= log space mean)
 - Risk neutral (half of probability density above, half below)
 - In contrast to using mean for e.g. weight at length
- For Prior:
 - log-space sd =0.31
 - This is an improvement over previous work (e.g. Hamel 2015, Then et al. 2015)

Prior on M (for max age = 10)



References

- Cope, J.M. and Hamel, O.S. 2022. Upgrading from M version 0.2: An application-based method for practical estimation, evaluation and uncertainty characterization of natural mortality. *Fisheries Research* 256, 106493.
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