Competition, Tasks and Earnings Inequality

PABLO FORERO*

Since 2000 developed countries like the US have seen a polarization of earnings. This paper explores a novel mechanism that can explain the recent trends in earnings inequality. I build a tractable general equilibrium model which differentiates between three different types of tasks: manual, routine and cognitive. Workers are heterogeneous in skill and self select into the type of task they perform. Those performing cognitive tasks take a managerial job and are susceptible to a superstar phenomenon. I use this model to look at the effects on labour composition and the earnings structure of an increase in competition between varieties, brought about by participating in a more globalized economy. I find that this increases income inequality, especially for top earners, due to a superstar effect. At the same time, there is a trickle down effect, increasing the number of manual jobs and reducing those of cognitive tasks, which then produces a polarization of earnings.

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1 INTRODUCTION

Earnings inequality has steadily increased since the 1960's in most developed economies, especially among top earners. The number of hypotheses vying to explain the observed trend in earnings has increased correspondingly. Much of the formal discussion has revolved around the consequences of international trade, skill-biased technological change and shifts in the demand for different types of tasks.[1] This paper proposes an additional explanation by combining a superstar effect, similar to Rosen (1981), and changes in the demand for each task (Autor and Dorn, 2013).

The economics of superstars was introduced in Rosen (1981), where a small number of highly skilled workers take a non-proportional share of total sales in a given industry. The superstar effect arises when workers are not perfect substitutes and they have access to a technology which allows them to reach a large market. This phenomenon is common in the music, sport and book publishing industries. In a superstars' economy, an increase in demand will lead to a more skewed earnings distribution, disproportionately favouring the most talented individuals. While the mechanism proposed by Rosen (1981) is widely understood, it has seldom been used in the trade and tasks literature to explain the increase of earnings inequality. The goal of this paper is to fill this gap.

This paper develops a tractable general equilibrium trade model with heterogeneous workers and different tasks. I use this framework to explore observed changes in earnings and the labour composition of tasks under alternative hypotheses, such as automation and skilled-biased technological change. I develop the analysis further by introducing the effects of international trade, by increasing the competition between varieties, as a relevant factor behind these changes. As firms find it easier to reach more consumers, this will create a superstar phenomenon. But by formalizing this superstars mechanism in a general equilibrium context I show that, not only it will skew the earnings distribution towards top earners, but that it will also have a trickle down effect on labour composition, which can polarize earnings.

More formally, the model features a population of workers with heterogeneous skills who self select the type of task they perform in line with the span-of-control model proposed by Lucas (1978). The economy produces two types of goods: a non-tradable homogeneous good and a tradable heterogeneous good. The homogeneous good is produced in a competitive market using a technology exclusively

1 Typically defined as Manual, Routine and Cognitive, following the descriptions in Acemoglu and Autor (2011).

based on manual tasks where production does not depend on workers'skills. By contrast, the heterogeneous good is produced by a continuum of heterogeneous firms and is characterized by monopolistic competition as in Melitz (2003). Each firm has one owner-manager, whose cognitive skill determines the productivity of the firm, and employs an endogenous number of routine workers.

Cognitive tasks are the main driver of the mechanism of the model. According to the literature on tasks, as described in Acemoglu and Autor (2011), cognitive tasks exhibit two very special characteristics. First, they are scale invariant. The effort required to perform a cognitive task is independent of the scale. For example, the effort needed to write a book remains the same independent of whether it sells ten copies or ten million copies. Second, the skill is non-additive. By adding the effort of two cognitive workers to the same task, the final outcome will be the equivalent of that performed by just the best of the two workers. This is the reason why books are seldom written by more than one author.

These specific features of cognitive tasks have a large overlap with those that give rise to the superstar phenomenon. Superstars need to be able to mass produce and reach a large market when goods are not perfect substitutes. The scale invariance of cognitive jobs provides the technology for mass production. The fact that skill is non-additive for cognitive workers means that you can never substitute a good produced by a high skill worker with a good produced jointly by two lesser skilled workers. Thus, jobs which require a large share of cognitive tasks, like writing a book, become more susceptible to the superstar phenomenon.

According to the model, an increase in the substitutability of different varieties -say, due to globalization will lead to a superstar effect in the earnings of cognitive workers. On the one hand, an increase in competition implies a higher demand of the goods produced by the most skilled cognitive workers, skewing the production curve towards such workers. On the other hand, through the self-selection of workers into jobs, the shift in demand in the heterogeneous good sector, has a knock on effect on the demand for the homogeneous good, the earnings distribution and the occupational choices of all workers in the population.

In effect, as a result of earnings becoming skewed towards the *superstars*, workers trickle down from the higher earnings tasks to less paid tasks. The least skilled owner-managers may find that the outside option of performing a routine or manual job exceeds their current income after the shift in demand. Hence, as they change their occupation, the share of cognitive workers gets reduced, while increasing production and the relative demand of the heterogeneous good gets concentrated among a smaller number of highly productive firms.

When complementarities between the homogeneous and heterogeneous good are present the demand for the homogeneous good goes up as well, and therefore the demand for manual tasks increases with it. This leads to a rise in the earnings of manual workers. Finally, workers performing routine tasks - who are around the median of the skill distribution - see a relative decrease in earnings compared to those in the lower tail. The overall effect of the increase of competition between cognitive workers is not only an increase of earnings inequality between top earners and the rest, but also a simultaneous polarization of earnings.

There are several advantages to modelling the increase in inequality through a superstar phenomenon in a general equilibrium setup. First, this setup reconciles the overall increase of earnings inequality and the earnings polarization in only one mechanism. Second, it can also explain other facts that the literature focusing on job polarization has struggled to reconcile. The polarization of earnings caused by the superstar phenomenon is more robust and stronger than those achieved by job polarization. Further, job polarization requires an increase in the share of cognitive jobs in total occupations, while the superstar phenomenon requires a reduction. Interestingly, the predicted reduction in the number of cognitive tasks is consistent with the findings of Beaudry et al. (2016) where they document that since 2000, in the US, cognitive tasks have been decreasing creating a trickle down in the jobs ladder.

Summing up, the methodological contribution of this paper is twofold. First, by introducing the superstars effect in a formal general equilibrium setup, I can explore its role as an alternative driver of changes in both earnings inequality and jobs composition. Second, I introduce a new simple and tractable analytical framework which builds upon the existing trade and tasks literature. In this fashion, the model could be used to address further interesting issues, such as the impact of educational systems or offshoring, on labour composition and earnings.

1-1 LITERATURE REVIEW

This paper extends a standard heterogeneous firms model, along the lines of Melitz (2003). There is a large literature that extends these types of models to study earnings inequality (Yeaple (2005), Helpman et al. (2010)). In particular, Manasse and Turrini (2001) introduces a production model similar to the heterogeneous good sector proposed here. Their paper, in contrast to mine, endogenizes the number of firms, occupational choice and the fraction of workers by skill, besides allowing for the homogeneous goods sector.

The trade literature that looks at earnings inequality focuses on the same mechanism: the effects of opening to trade when there is a fixed cost to exporting. Accordingly, higher trade implies that the most productive firms and high skill managers gain access to a larger market share. While this standard mechanism leads to an increase in inequality, it can not account for the polarization of earnings.

To explain the polarization of earnings, the mechanism proposed here relies on an increase in the degree of substitutability between different varieties of the heterogeneous good. It creates a realignment of production which skews towards the firms with the highest skilled cognitive workers/managers. This realignment is exploited in Mayer et al. (2016) where it is interpreted as being akin to positive transitory demand shocks explaining fluctuations in exports and production in economies which become more open to trade. I differ from Mayer et al. (2016) by considering these shocks as being more permanent, and analyzing how it affects jobs composition and earnings in the long run.

The models most closely related to this paper in the trade literature are Monte (2011) and Egger and Kreickemeier (2012). One the one hand, Monte (2011) considers an economy with heterogeneous firms and workers, to analyze the effects of skill-biased technological change and trade integration on the earnings structure. Workers self select to be managers whose skill determines firms' productivity- or production workers, and the number of firms is endogenous.[2] Although the heterogeneous skills of managers could give rise to a *superstar* effect, this is not explored in his paper. The framework I use departs by allowing production workers to also be heterogeneous in skill. I also introduces an additional non-tradable good which utilizes a third type of labour, manual tasks, making it more tractable with respect to the literature on tasks.

On the other hand Egger and Kreickemeier (2012), using again a model similar to Monte (2011), focus on the role of a fair-wage mechanism. When opening to trade the most skilled managers benefit the most from it due to the fixed cost to exporting. The existence of a a fair-wage mechanism guarantees that the increase of inequality is not just limited between managers, but also between production workers. Even if production workers are homogeneous, their wage depends on the wage of the manager. Therefore production workers in more productive firms will see a larger increase in earnings, due to opening up to trade, than those in less

2 Production workers are homogeneous in skill.

productive firms. Like their paper, I focus on with-in group inequality.

The literature on tasks has focused on identifying the determinants underlying the polarization of jobs and earnings. Most of these studies try to explain polarization of jobs in terms of a reduction of routine jobs due to automatization, as in Autor and Dorn (2013) and Lee and Shin (2016), or offshoring (Autor et al., 2013). By contrast, this paper shifts the focus to earnings polarization rather than job polarization. While those models can get some earnings polarization by assuming large complementarities between managers and manual workers, the mechanism presented in this paper leads to a much stronger earnings polarization without making such a strong assumption. It also adapts to Beaudry et al. (2016) findings that cognitive jobs have been decreasing, instead of increasing, since the beginning of the new century.

The paper is organized in the following manner. In Section 2 I describe the model. In Section 3 I show how changes in labour composition of tasks changes the earnings inequality between and with-in groups. In section 4 I propose the superstar effect as an increase in competition between different varieties in the heterogeneous good sector. The mechanism can create and increase of earnings inequality and a polarization of earnings. In Section 5 , I analyze the effects of SBTC and automatization on the economy and show how they can not be a leading cause of earnings inequality in this framework. Section 6 provides some concluding remarks.

2 THE MODEL

The economy produces two types of good: a homogeneous good produced using manual labour; and a continuum of heterogeneous goods produced by a mass M of firms, which are differentiated by the cognitive skill of the owner-manager of the firm. Production of the heterogeneous good uses routine labour. There is a population of mass 1 with heterogeneous skill which self-selects the type of labour it supplies given its skill.

2-1 DEMAND

2-1.1 Aggregate

Consider an economy with a population of mass 1. There are two types of goods: a homogeneous good and a heterogeneous good. The heterogeneous good consists of a continuum of differentiated varieties, while there is only one type of the homogeneous good. The preferences over both types of goods are given by the constant elasticity of substitution (CES) utility function.

$$U = [\gamma Q_0^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma)Q^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}}$$

Where Q_0 is the total aggregate of the homogeneous good, and Q is the CES-aggregate of all available varieties of the heterogeneous good. The elasticity of substitution between both aggregates is given by $\epsilon \in (0, \infty)$, and $\gamma \in (0, 1)$ is a distribution parameter which determines how important the two goods are for the consumers. As the product market for both types of goods are competitive, market clearing implies their relative prices have to satisfy:

$$\frac{P}{P_0} = \frac{1-\gamma}{\gamma}\left(\frac{Q}{Q_0}\right)^{-\frac{1}{\epsilon}} \tag{1}$$

The price of the homogeneous good is given by P_0 and the price index for the CES-aggregate of all the existing varieties of the heterogeneous good is given by P.

2-1.2 Heterogeneous Good Sector

The heterogeneous good sector consists of a continuum of differentiated varieties. The aggregate heterogeneous good Q can be given as a CES aggregate of all available intermediate varieties indexed by ω:

$$Q = \left[M^{(\rho-1)} \int_{\omega\in\Omega} q(\omega)^{\rho} d\omega\right]^{\frac{1}{\rho}}$$

Where the measure of the set Ω represents the mass of available intermediate varieties M. The goods are substitutes such that $\rho \in (0.5, 1)$[3] and the elasticity of substitution between any two goods is given by $\sigma = \frac{1}{1-\rho} > 2$.

3 While the model still works for $\rho \in (0, 0.5]$ $(1 < \sigma \leq 2)$ it implies that those with the highest skill perform routine jobs instead of cognitive jobs, and vice versa. While this scenario is plausible in some uncompetitive economies, its potential implications are not the main focus of this paper. Therefore I limit myself to the parameter range that better describes the current situation observed by the literature.

Normalizing the price of the CES aggregator of the heterogeneous good to one without loss of generality and by profit maximization of the aggregate good, I obtain the demand function for the intermediary goods:

$$q(\omega) = \frac{Q}{M} p(\omega)^{-\sigma} \tag{2}$$

Where $p(\omega)$ is the price of intermediate variety ω.

2-2 PRODUCTION

2-2.1 Homogeneous Good

The homogeneous good is produced by employing a constant returns to scale (CRS) technology where manual labour is the only input. All firms have access to the same technology, and all workers performing manual tasks produce the same amount of manual labour. The aggregate production of the homogeneous good is given by:

$$Q_0 = \beta L_m$$

The productivity of the technology of the homogeneous good is given by $\beta \in (0, \infty)$, and L_m is the aggregate of manual labour employed, which is the population share employed in manual tasks. All agents provide the same amount of manual labour, therefore the wage paid to all workers employed in the service sector is a fixed wage w_m. The price of the aggregate homogeneous good is then given by:

$$P_0 = w_m / \beta$$

2-2.2 Heterogeneous Good

The market for intermediate heterogeneous varieties consists of a continuum of monopolistically competitive firms, each producing a unique variety. Each firm requires an owner-manager, whose skill defines the productivity, or quality, of the good and an endogeneous amount of routine labour l_r. The production function of a firm is given by $q = l_r \varphi$. In this case the productivity parameter $\varphi > 1$ is given by the cognitive ability of the owner and is different for each firm. Under this set-up it is easy to see how cognitive workers benefit from economies of scale. And by imposing only one owner-manager for each different firm, the non-additive characteristic of cognitive tasks is preserved. Firms facing the demand curve in (2) choose a profit maximizing mark-up $\frac{1}{\rho}$ and therefore the price of each intermediate good is:

$$p(\varphi) = \frac{c(\varphi)}{\rho} \tag{3}$$

The cost function $c(\varphi)$ is the cost each firm faces for producing each unit given the productivity of the firm and the cost of each unit of routine labour w_r. The marginal cost function each firm faces is given by:

$$c(\varphi) = \frac{w_r}{\varphi}$$

Given the demand and the pricing rule, in equilibrium the demand each firm faces, their revenue and their profits are given by:

$$\begin{aligned} q(\varphi) &= \frac{Q}{M}\left[\frac{c(\varphi)}{\rho}\right]^{-\sigma} \\ r(\varphi) &= \frac{Q}{M}\left[\frac{c(\varphi)}{\rho}\right]^{1-\sigma} \\ \pi(\varphi) &= \frac{Q}{M\sigma}\left[\frac{c(\varphi)}{\rho}\right]^{-\sigma} \end{aligned}$$

The wage of routine labour is the same for all firms. Therefore the ratio of the output, revenue and profits of two firms will depend only on the ratio of their productivity levels:

$$\begin{aligned} \frac{q(\varphi_1)}{q(\varphi_2)} &= && \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma} \\ \frac{r(\varphi_1)}{r(\varphi_2)} &= \frac{\pi(\varphi_1)}{\pi(\varphi_2)} = && \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1} \end{aligned} \tag{4}$$

The model departs from the standard Melitz (2003) in that there are no fixed costs, the mass of available goods M is endogenous as well as the aggregate of routine labour supplied L_r, and the cost of labour w_r.

2-3 LABOUR

Labour is supplied by population of mass 1 of agents. Agents are heterogeneous in skill φ which affects their productivity while performing cognitive and routine tasks. Workers draw their skill from a common distribution $g(\varphi)$, which has positive support over $[1, \infty)$ and has a continuous cumulative distribution $G(\varphi)$.[4]

An assumption maintained this paper is that all jobs require only *one* type of task to be performed. The O*NET database, which quantifies how much time is spent in each job performing different tasks, shows that it is very rare to find a job with such characteristics. It is more common that a worker will perform all three tasks, but the time spent in each task varies.

Agents choose the type of labour they provide depending on their skill level. They can become the owner-manager of a firm, performing a cognitive task, with the productivity of the firm being equal to their skill φ. They can also work in the production of the heterogeneous good with the agent providing φ effective units of routine labour. Or they can forgo their skill and produce a homogeneous good using the common technology and earning a fixed wage w_m. Agents will self select their job in a manner similar to Lucas (1978).

The labour market will be characterized by two skill cut-off points. One represents the skill $\underline{\varphi}$ in which a worker is indifferent between performing a routine task or a manual task. The other cut-off $\overline{\varphi}$ is the skill of the worker who is indifferent between working a routine job or a cognitive job.

$$
\begin{aligned}
\underline{\varphi}: \quad w_m &= w_r \underline{\varphi} \\
\overline{\varphi}: \quad w_r \overline{\varphi} &= \pi(\overline{\varphi})
\end{aligned}
$$

Lemma 1 *For all workers with a skill level lower than $\underline{\varphi}$ it must be that $w_r\varphi < w_m$. For all workers with a skill level lower than $\overline{\varphi}$ it must be that $\pi(\varphi) < w_r\varphi$.*

In other words:

$$
\begin{aligned}
w_r\varphi &\geq w_m \quad \forall\, \varphi \geq \underline{\varphi} \\
\pi(\phi) &\geq w_r\varphi \quad \forall\, \varphi \geq \overline{\varphi}
\end{aligned}
$$

4 The $(\sigma - 1)$th moment of that distribution must exist.

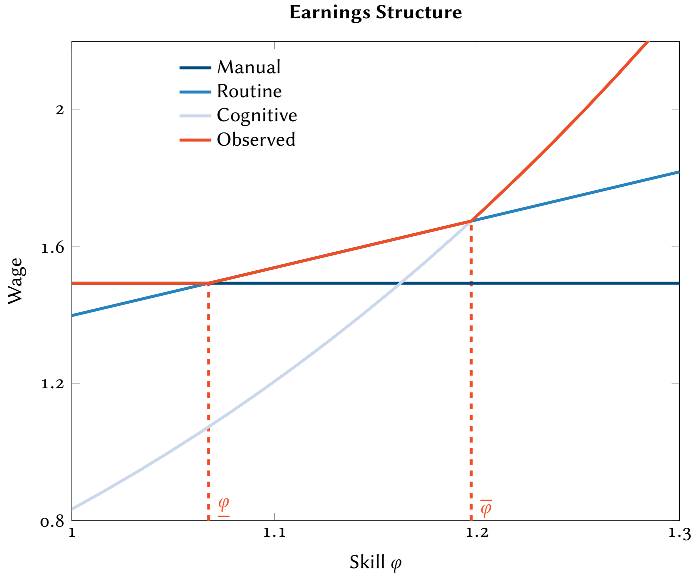


FIGURE 1: Wage self selection

The first part is straightforward, as $w_m = w_r\underline{\varphi}$. Therefore any worker with a skill lower than the routine/manual cut-off point will be worse off doing a routine task than a manual task. The latter statement can be inferred from the ratio of profits of firms (ie earnings of cognitive jobs) in (4). The profits of a firm with a cognitive skill lower than $\overline{\varphi}$ would be:

$$\pi(\varphi) = \left(\frac{\varphi}{\overline{\varphi}}\right)^{\sigma-1} w_r\overline{\varphi}$$

With the assumption of $\sigma > 2$, because $\varphi < \overline{\varphi}$ it is simple to see that this will be lower than $w_r\varphi$.

These cut-off points characterize the labour composition in the economy. In equilibrium all workers with a skill parameter smaller than $\underline{\varphi}$ will perform manual tasks. For workers with a skill higher than $\overline{\varphi}$ they will all be owner-managers of a firm performing cognitive tasks. The remaining workers with skill levels between both cut-off points will be employed in the production of the heterogeneous good providing routine labour.

The composition of labour in the economy will be given by the share of workers performing manual tasks (λ_m), routine tasks (λ_r) or cognitive tasks (λ_c):

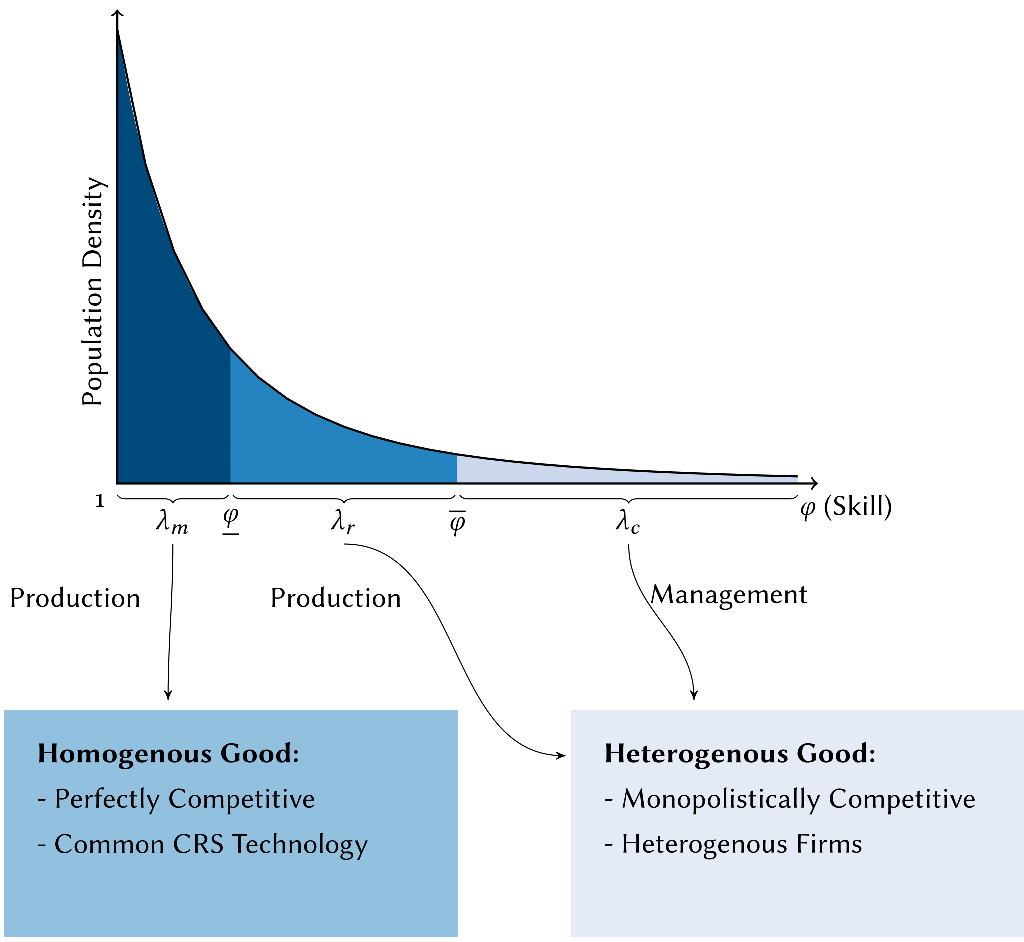
$$\begin{aligned}\lambda_m &= G(\underline{\varphi}) \\ \lambda_r &= G(\overline{\varphi}) - G(\underline{\varphi}) \\ \lambda_c &= 1 - G(\overline{\varphi})\end{aligned}$$

This will imply the total supply of manual and routine labour, and the mass of existing firms will be:

$$\begin{aligned}L_m &= \lambda_m \\ L_r &= \lambda_r \tilde{\varphi}_r \\ M &= \lambda_c\end{aligned}$$

Where $\tilde{\varphi}_r$ is the weighted average skill of routine workers.

FIGURE 2: Summary of Labour Selection



2-4 AGGREGATION

The aggregate heterogeneous good Q can be characterized by using the weighted average productivity of existing firms:

$$\tilde{\varphi} = \left[\frac{1}{M} \int_{\overline{\varphi}}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}$$

Given this particular productivity, the production of the firm with an owner-manager with skill $\tilde{\varphi}$ will equal the output per firm for the heterogeneous good market. From the normalization of $P = 1$ we have that $p(\tilde{\varphi}) = 1$ and $R = Q = Mr(\tilde{\varphi})$. This makes it possible to characterize the aggregate heterogeneous good market given the cut-off skill.

For routine labour it is convenient to assume that firms hire a bundle of routine labour workers, with each bundle having the same average productivity. In order for this to hold, the productivity of each bundle of workers must equal the weighted average productivity of routine workers $\tilde{\varphi}_r$, which is given by:

$$\tilde{\varphi}_r = \frac{1}{\lambda_r} \int_{\underline{\varphi}}^{\overline{\varphi}} \varphi g(\varphi) d\varphi$$

2-5 FACTOR ALLOCATION IN EQUILIBRIUM

When the heterogeneous market is in equilibrium, the wage for each unit of routine labour can be obtained using the pricing rule in equation (3) and the normalization of the price of the representative good to one. Therefore, the routine wage is equal to:

$$w_r = \frac{\tilde{\varphi}(\sigma - 1)}{\sigma} \tag{5}$$

From the mark-up pricing rule it must be that aggregate profits of the heterogeneous good are $(1 - \rho)Q$, and therefore the profit of the representative firm is $\pi(\tilde{\varphi}) = (1 - \rho)Q/M$. At the same time, the least productive manager must be indifferent between the firm's profits and its wage performing a routine job from which I obtain $\pi(\tilde{\varphi}) = (\tilde{\varphi}/\overline{\varphi})^{\sigma-1} w_r \varphi$. From the normalization of the routine wage in Equation (5), I obtain that the aggregate production of the heterogeneous good from the perspective of only cognitive workers,[5] which depends only on the upper cut-off skill ($\overline{\varphi}$) and the substitutability of goods σ. Therefore the *cognitive production* (**CP**) of the heterogeneous good is given by:

$$Q_c = \frac{M\pi(\tilde{\varphi})}{\sigma}$$

On the other hand, total production will also depend on routine labour. If the average productivity of heterogeneous firms is $\tilde{\varphi}$, the total production of heterogeneous good must be the total amount of routine labour in the economy times the average productivity. This is the *routine production* (**RP**) of the heterogeneous good:

$$Q_r = L_r\tilde{\varphi}$$

In order for the heterogenous good market to be in equilibrium (that is without accounting for the relative demand for the homogeneous good), it must be that the aggregate cognitive production equals the aggregate routine production.

Lemma 2 *Given any $\underline{\varphi}$ there exists a unique $\overline{\varphi} \geq \underline{\varphi}$ such that the cognitive production equals the routine production.*

This occurs because **CP** is decreasing in $\overline{\varphi}$ while **RP** is increasing. If $\overline{\varphi} = \underline{\varphi}$ total routine labour supplied is zero, and therefore **RP** is zero. And as $\overline{\varphi}$ tends to ∞ the **CP** of the heterogeneous good goes to zero as the mass of firms goes to zero. This implies that both functions are single crossing, and there exists a unique $\overline{\varphi}$ for each $\underline{\varphi}$ such that the heterogeneous good market is in equilibrium.

Given this, there exists a function $\overline{\varphi}(\underline{\varphi})$ which is one to one and continuous for all $\underline{\varphi}$ with a positive support in $g(\varphi)$, which provides the upper skill cut-off $\overline{\varphi}$ where the heterogeneous good market is in equilibrium given any lower skill cut-off.

Corollary 1 *The function $\overline{\varphi}(\underline{\varphi})$ is increasing in $\underline{\varphi}$ and $Q(\overline{\varphi}(\underline{\varphi}), \underline{\varphi})$ is decreasing in $\underline{\varphi}$.*

5 For example, if the cut-off skill goes to infinity $\overline{\varphi} \rightarrow \infty$, it must be that there are no firms and therefore they do not produce any goods, independent of the amount of routine labour available in the economy.

Proof: See Appendix.

2-6 EQUILIBRIUM

In this economy Equilibrium is characterized by the skill cut-off values $\underline{\varphi}$, $\overline{\varphi}$ such that:

- Aggregate demand of the homogeneous and heterogeneous good is in equilibrium.
- Each firm producing a different variety of intermediate heterogeneous goods is profit maximizing.
- Workers are employed in the job that maximizes their income.
- Total production, labour composition and skill of workers clear.

It can be shown that an equilibrium exists and is unique for any distribution $g(\varphi)$ such that there is a finite average productivity for the heterogeneous good. Equilibrium is characterized by an equilibrium in the heterogeneous market, where **CP** equals **RP**, and the market for both the homogeneous good and the heterogeneous good market clears.

Using the ratio between firm's profits, and the indifference between routine labour and cognitive labour at $\overline{\varphi}$, with some rearranging, the condition for heterogeneous market equilibrium can be written as

Heterogeneous Market clearance (**HM**):

$$\frac{\overline{\varphi}^{2-\sigma}}{\sigma}\int_{\overline{\varphi}}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi = \frac{\sigma}{\sigma-1}\int_{\underline{\varphi}}^{\overline{\varphi}}\varphi g(\varphi)d\varphi \tag{6}$$

The market clearance for both the homogeneous and heterogeneous good in equation (1) can be easily rewritten as:

Market Clearance (**MC**):

$$w_r \underline{\varphi} G(\underline{\varphi})^{\frac{1}{\epsilon}} = \beta^{(1-\frac{1}{\epsilon})} \frac{\gamma}{1-\gamma} Q^{\frac{1}{\epsilon}}$$

The equilibrium of (6) comes from Lemma 1. If **HM** is in equilibrium, we get that the left hand side of **MC** is increasing in $\underline{\varphi}$, and the right hand side is decreasing. This guarantees that the $\underline{\varphi}$ where **MC** is in equilibrium, given that **HM** is already in equilibrium, is also unique.

3 EARNINGS INEQUALITY

Earnings inequality is given by the ratio between the earnings of different skilled workers. Their earnings will depend on the cut-off skills.

The ratio of earnings between workers performing the same task is given by:

- **Manual:**
  $\frac{w(\varphi_1)}{w(\varphi_2)} = \frac{w_m}{w_m} = 1$
- **Routine:**
  $\frac{w(\varphi_1)}{w(\varphi_2)} = \frac{w_r \varphi_1}{w_r \varphi_2} = \frac{\varphi_1}{\varphi_2}$
- **Cognitive:**
  $\frac{w(\varphi_1)}{w(\varphi_2)} = \frac{\pi(\varphi_1)}{\pi(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$

It can be easily observed that within group earnings inequality does not depend on any parameter for manual and routine workers. And for cognitive workers within-group earnings inequality only depends on σ.

The earnings inequality between groups is given by:

- **Routine/Manual:**
  $\frac{w(\varphi_1)}{w(\varphi_2)} = \frac{w_r \varphi_1}{w_m} = \frac{\varphi_1}{\underline{\varphi}}$

- **Cognitive/Routine:**
  $\frac{w(\varphi_1)}{w(\varphi_2)} = \frac{\pi(\varphi_1)}{w_r \varphi_2} = \frac{\overline{\varphi}}{\varphi_2} \left(\frac{\varphi_1}{\overline{\varphi}}\right)^{\sigma-1}$

- **Cognitive/Manual:**
  $\frac{w(\varphi_1)}{w(\varphi_2)} = \frac{\pi(\varphi_1)}{w_r \underline{\varphi}} = \frac{\overline{\varphi}}{\underline{\varphi}} \left(\frac{\varphi_1}{\overline{\varphi}}\right)^{\sigma-1}$

Between group earnings inequality depends on the cut-off skills. Therefore as these change, the inequality of workers between groups change. An increase of $\underline{\varphi}$ will *decrease* earnings inequality between routine[6] and manual workers. Similarly, an increase of $\overline{\varphi}$ will also decrease inequality between a worker performing a cognitive job and a routine job.

Within group inequality for routine and manual workers can be achieved by relaxing the assumption that each job requires only one type of task. If the assumption is that higher skilled routine worker will spend more time doing cognitive tasks, and a low skilled routine worker does more manual tasks, it can be extrapolated so movements on the cut-off skills can cause some within group inequality for routine workers.[7]

Proposition 1 *Job polarization, understood as the proportion of workers undertaking cognitive and manual, leads to a polarization in earnings.*

Job polarization, as a decrease in routine tasks and an increase in both manual and cognitive tasks, is represented by an increase in $\underline{\varphi}$ and a decrease in $\overline{\varphi}$. From the between group ratios, it can be seen that the movements in the cut-offs increases the earnings inequality between cognitive and routine worker, while decreasing the earnings inequality between manual and routine workers. As cognitive and manual jobs are performed by workers on the upper and lower tails of the distribution, respectively, and routine workers are between them, the final effect will be a polarization of earnings.

Polarization in the US and other developed countries is seen as a polarization of jobs and a polarization of earnings. The hypotheses that aim to explain it, such as automatization or offshoring, rely on the destruction of routine jobs as well as an increase in cognitive and manual jobs. The evidence on the increase of low skill

6 And also cognitive if we assume $\overline{\varphi}$ is fixed
7 Same intuition can apply to manual workers.

manual jobs is unequivocal, and can be observed in Autor and Dorn (2013) and Beaudry et al. (2016). Where both of these papers depart from each other is in the trend of high skill cognitive tasks. The mechanism proposed in this paper is closer to the findings in Beaudry et al. (2016) with a decrease in cognitive jobs. But it is important to note that under the framework of this paper, a polarization of jobs, with the number of cognitive jobs increasing, is still consistent with a polarization of earnings.

4 COMPETITION

This paper proposes an increase in competition between different varieties as an alternative hypothesis for the increase of earnings inequality and polarization of earnings. If the substitutability between different varieties of the heterogeneous good increases it will give rise to a superstar phenomenon. Heterogeneous goods are differentiated by the skill of cognitive workers. The good produced by each seller are imperfect substitutes, but each firm can increase its production at no additional cost for the cognitive owner manager. Therefore the heterogeneous good market, and especially the role of cognitive tasks, make it a superstar economy. An increase in competition between different varieties, by an exogenous increase in the substitutability of varieties, will intensify the superstar effect, skewing the production distribution towards the most productive firms.

The last half century has seen a more global and interdependent economy. Products created and designed in one country have to compete with other similar products created on the other side of the planet. Local differences in tastes and culture are assimilated into a more global culture. Local firms have access to larger markets as transport costs, tariffs and cultural barriers are reduced. New products sold globally employ the best engineers in the world for their design, a good example being Apple's iPhone. The best local tastes have also risen in prominence on a global scale, with the wide availability of Italian and Japanese food being a testament to this.

The trend towards a more globalized market place can be seen as an increase in substitutability between differentiated goods, as they have to compete on a more equal footing, where local differences matter less. As the economy becomes more globalized, the goods produced by the highest skilled cognitive workers will gain a larger market share, giving rise to a superstar phenomenon. Local superstars turn into global superstars, intensifying the skewness of earnings between cognitive workers. Those who are at the upper tail on a global scale will benefit form the superstar effect. Ex-local superstars that do not pass the new global cut will instead have to look for other type of jobs.

In order to explore the effect of a rise of the superstar phenomenon I look at the effects of an exogenous increase of the substitutability of heterogeneous goods (σ). This effect can be micro founded in different ways, but the paper abstains from this in order to preserve parsimony. A reduction of iceberg costs for the heterogeneous market would lead to a rise in the substitutability of goods due to the Chamberlinian equivalence in the monopolistic competition of the heterogeneous good market following Dixit and Stiglitz (1977). An increase in market size in a monopolistic competitive set-up with varying elasticities of substitution, as in Bertoletti and Epifani (2014) and Mayer et al. (2016), will also lead to an increase in the elasticities of substitution.

The effects of an increase of $\sigma > 2$ on the economy is non linear. Its general equilibrium effect can be quite different depending on where the parameters are on the (σ, ϵ) space. I will first analyze its effect on the heterogeneous good sector before looking at the general equilibrium effect.

Lemma 3 *The function $\overline{\varphi}(\underline{\varphi})$ is increasing in σ if the skill distribution $g(\varphi)$ does not have most of its mass very close to one.*

Proof: See Appendix

The increase in competition creates a reallocation of production towards the most productive firms, increasing the cognitive production (**CP**) of the heterogeneous good market. To compensate the increase in production from the cognitive side, an increase of routine labour is necessary for the market to clear, therefore $\overline{\varphi}$ increases, given the same $\underline{\varphi}$. This implies:

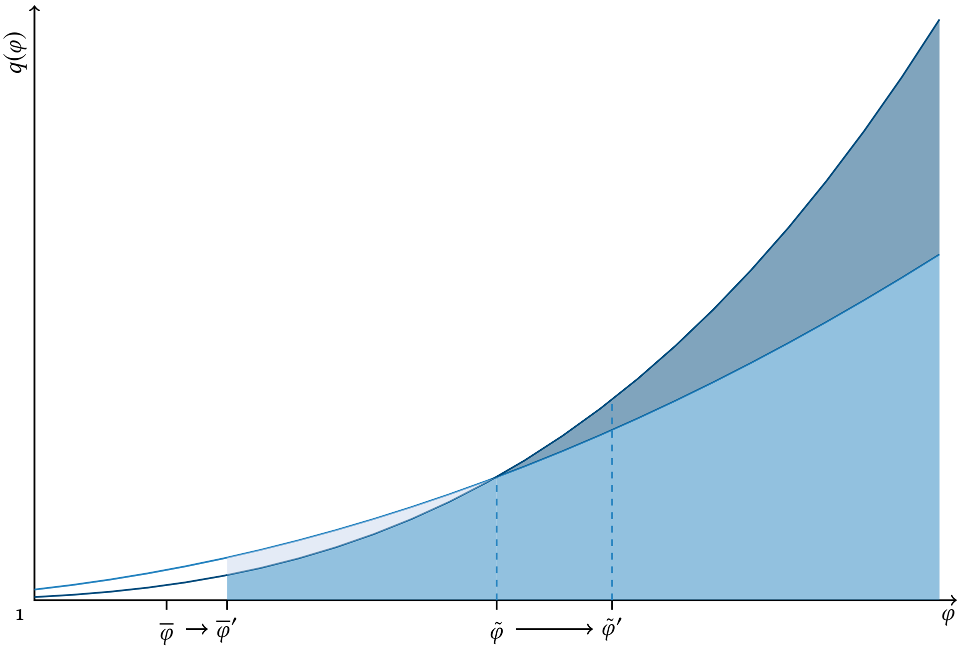
Corollary 2 *The equilibrium L_r in the heterogeneous good market is increasing in σ.*

The effect is only partial equilibrium, as it does not take into account the movement of $\underline{\varphi}$. Following Corollary 2, within the partial equilibrium of the heterogeneous market, an increase in σ implies an increase in Q, w_r and $\tilde{\varphi}$.[8]

8 There are some theoretical cases when this will not hold. Most of these occur when $\tilde{\varphi}$ tend to infinity and the share of cognitive workers are close to zero. I ignore them as it is not an interesting case.

The reallocation of production from the increase in σ is accompanied by the least productive managers abandoning cognitive tasks entirely. This means that the representative productivity of firms ($\tilde{\varphi}$) increases. As the weighted mean productivity and the routine labour employed by the heterogeneous sector increases, it has to be that the overall production of the heterogeneous good increases as well. The increase of the wage for each effective unit of routine labour w_r increases from equation (5).

Figure 3: Partial Equilibrium Effect of increase in Competition on the Heterogeneous Sector



These are only the partial equilibrium effects. The general equilibrium effects will depend form the market clearance of the homogeneous and the heterogeneous good. By rearranging **MC** so all variables depending on the heterogeneous market are on the same side of the equation as follows:

$$\underline{\varphi} G(\underline{\varphi})^{1/\epsilon} \beta^{1/\epsilon - 1} \frac{1-\gamma}{\gamma} = Q^{1/\epsilon} w_r^{-1}$$

The derivative of the right hand with respect to sigma side is:

$$\frac{\partial \text{RHS}}{\partial \sigma} = -\frac{Q^{1/\epsilon - 1}}{\epsilon w_r} \left[(\epsilon - 1) L_r \tilde{\varphi}' + \tilde{\varphi} \left(\frac{\epsilon L_r}{\sigma(\sigma - 1)} - L_r' \right) \right]$$

It can be easily observed that if $\epsilon \rightarrow 0$ the derivative is positive. While as ϵ goes to ∞ it is negative. The general equilibrium effect will be determined by the effect that the increase in demand for the heterogeneous good has on the demand for the homogeneous good. Their substitutability is given by ϵ. That is why when $\epsilon \to 0$ the increase in production of the heterogeneous good increases the demand for the homogeneous goods, as they are gross complements. On the other hand, if they are strong gross substitutes ($\epsilon \to \infty$), the more productive heterogeneous good will reduce the demand of the homogeneous good.

Thinking in terms of gross substitutes and complements is useful to understand the intuition, but they do not determine the general equilibrium effect. The skill distribution and the substitutability of the heterogeneous goods have a large role determining the final effect. For this reason, in order to understand the mechanism I divide the general equilibrium effect into three cases, depending on the general equilibrium effect. But it is perfectly plausible that with the same distribution and ϵ, the effect on an increase in ρ can be any of the three cases, depending on the current value of ρ.

Strong Substitutes

This is the only case where the share of cognitive tasks increases. It is characterized by a decrease of both $\underline{\varphi}$ and $\overline{\varphi}$. The increase of the heterogeneous good is accompanied by a decrease of the homogeneous good. The labour share of workers performing manual tasks will be decreasing, while those performing routine tasks and cognitive tasks will be increasing. This case is characteristic of economies where both goods are very strong substitutes, while different varieties are still imperfect substitutes. If the goods are close to perfect substitutes, then the increase in demand for the more productive heterogeneous good is offset by a lesser need for varieties.

Weak Substitutes

If the increase in the relative demand of the heterogeneous good is dominated by the decrease in demand for variety, the scenario is the Weak Substitutive case. This case occurs when only the labour share of routine jobs increases. It is characterized by a decrease of $\underline{\varphi}$ and an increase of $\overline{\varphi}$. The increase of the heterogeneous good is still accompanied by a decrease of the homogeneous good. The labour share of workers performing manual and cognitive tasks will be decreasing, while those performing routine tasks will be increasing. They occur when the drop in $\underline{\varphi}$ in **MC** is less than the increase of $\overline{\varphi}$ in **HM**.

Complements

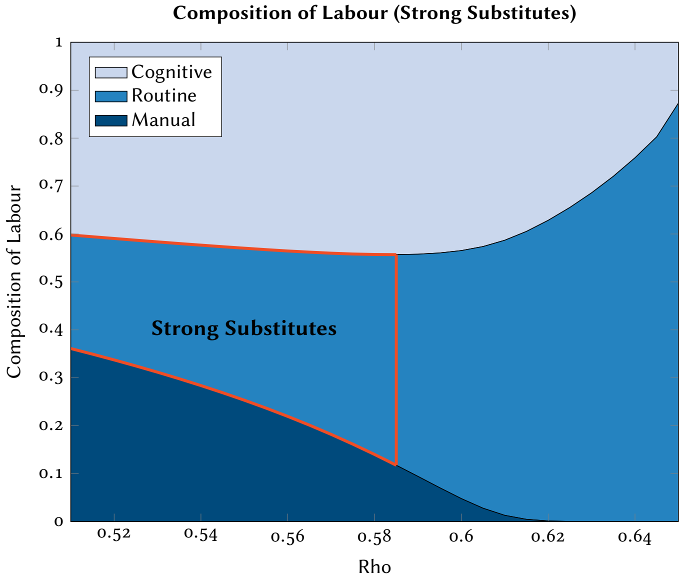


FIGURE 4: Composition of Labour under Strong Substitutes

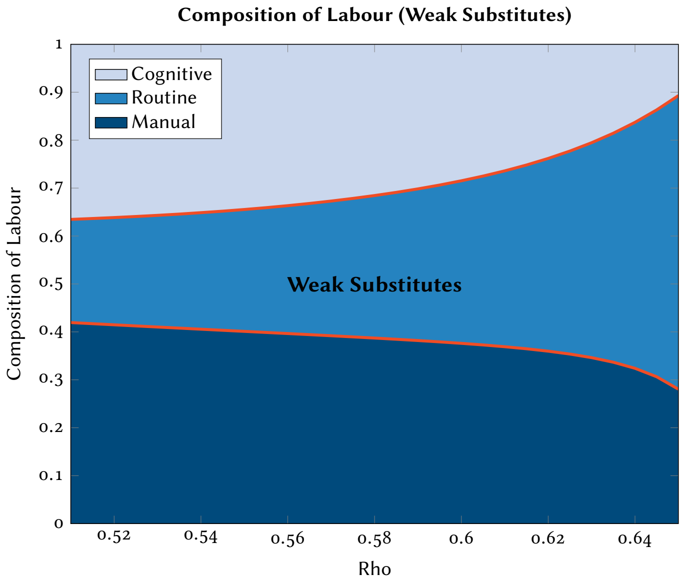


FIGURE 5: Composition of Labour under Weak Substitutes

The literature on tasks assumes that manual tasks and cognitive tasks are complementary. The production of more bottles of Coca-Cola creates a demand for more waiters. When the increase in production in the heterogeneous good sector is large enough to increase the demand for the homogeneous good, and therefore manual tasks, the general equilibrium effects are those of the Complements case. Under this case, the production of the homogeneous and heterogeneous good increases, while the share of cognitive jobs decrease and those of manual jobs increase. This case is the one that resembles the current trend in the US since 2000, as described in Beaudry et al. (2016).

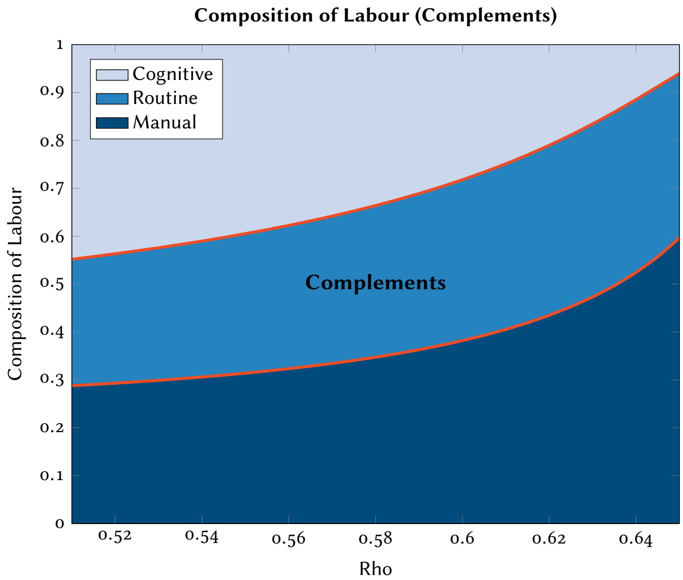


Figure 6: Composition of Labour under Complements

While the general equilibrium effect on the increase in competition between varieties of the heterogeneous goods depends on the parameter value, the model still predicts how they change from one state to another. As competition increases, an economy can shift from one case to another. When such a shift happens, it is from the Strong Substitutes to the Weak Substitutes case, and the Weak Substitutes to the Complements case.

As the substitutability increases, the skewness of the production curve steepens. The most productive firms take a larger share of production. The increase in productivity of the representative firm and the total output of the heterogeneous good start to increase in an exponential manner. As this happens the complementarities with the homogeneous good will change the trend and increase the demand for the homogeneous good, causing a gradual shift towards the Complements case.[9]

4-1 EARNINGS INEQUALITY UNDER COMPETITION

The ratios representing earnings inequality, presented in Section 3 depended on the cut-off skills and σ. If the cut-off did not move, an increase in competition between varieties will increase the with-in group inequality of cognitive workers. This is the superstar phenomenon skewing production, and therefore profits, to-

9 It can happen that it shifts from the complements case, to the weak substitutes. But this is true only when the homogeneous and heterogeneous good are gross substitutes, and $\tilde{\varphi}^{\sigma-1}$ is close to infinity. Then the economy turns to one cognitive worker, a few routine workers, and over 99% of the working populations performs manual tasks.

wards the highest skill workers. The steeper earning structure of cognitive workers leads to an increase of earnings inequality between cognitive workers and the rest of the population.

In order to see the overall effect it is necessary to take into account the general equilibrium effect of the changes in cut-offs according to the three possible cases:

Strong Substitutes

The strong substitutes case is characterized by an overall increase in earnings inequality. The increase of earnings inequality for cognitive workers caused by the increase in σ is accompanied by the fall of both cut-off skills in the strong substitutes effect. The reduction in cut-offs further increases the earnings inequality between groups. The reduction of manual jobs increases the inequality between manual and routine workers. The increase of cognitive jobs, on the other hand, increases the inequality between cognitive and routine workers.

This case can be attributed to the experience of the U.S. between 1950 and 1980. In Autor et al. (2008) they find that this period is characterized by a reduction of the share of employment of service occupations (which are generally manual tasks). During the same period the share of employment of managerial and professional jobs (which rely more on cognitive tasks) are increasing. That same period had a sharp increase of with-in group and overall wage inequality, as documented first in Katz and Murphy (1992). Therefore, the movements in the share of employment in different jobs and the earnings structure in that period are consistent with those found in the case of strong substitutes.

Weak Substitutes

The Weak Substitute case will seem to have a very similar effect on earnings inequality as that of the Strong Substitutes case. The fall of the cut-off $\underline{\varphi}$ means that the inequality between routine and manual workers is increasing. But the case for cognitive and routine workers is no longer clear. The increase of $\overline{\varphi}$ would decrease the inequality between cognitive and routine workers. But the skewness of earnings distribution between cognitive workers, brought by the superstar phenomenon, off sets that effect. The majority of cognitive workers would therefore actually *increase* their earnings with respect to routine workers. But a small number of low skill cognitive workers, close to the cut-off, would observe a decrease in earnings.

Figure 8 seems similar to the Strong Substitutes case of Figure 7. A more detailed

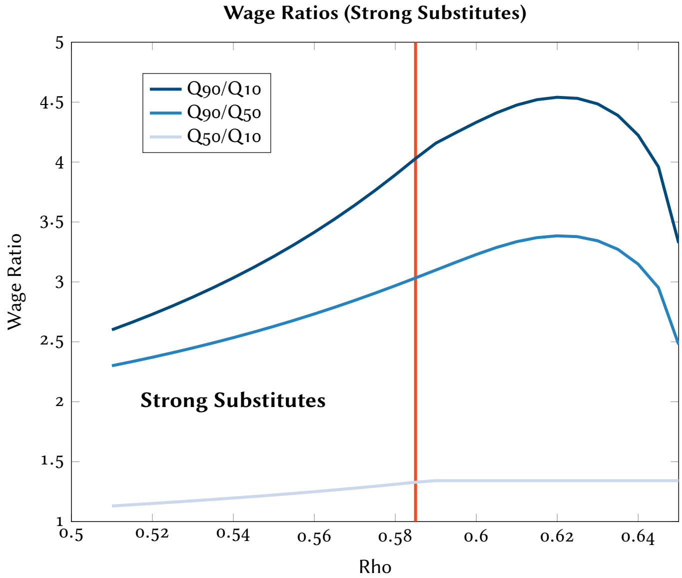


Figure 7: Wage Ratios under Strong Substitutes

look at different cognitive workers shown in 9 shows that the effect is not the same. The earnings inequality of cognitive workers is increasing in most of the Weak Substitutes case, but there is a peak, and then it decreases. Cognitive workers in lower parts of the distribution peak with lower values of σ, and after peaking they eventually change their occupation to routine jobs.

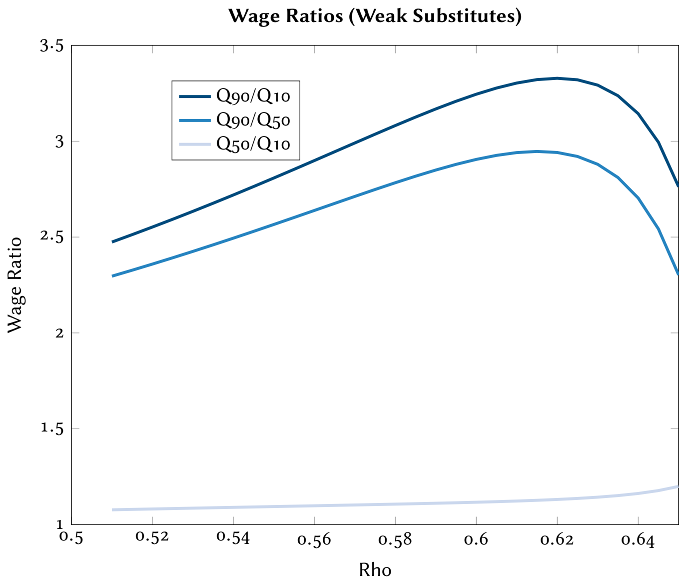


Figure 8: Wage Ratios under Weak Substitutes

Complements

When the general equilibrium effect of an increase in competition increases the share of manual jobs, it results in a polarization of earnings. The increase in the

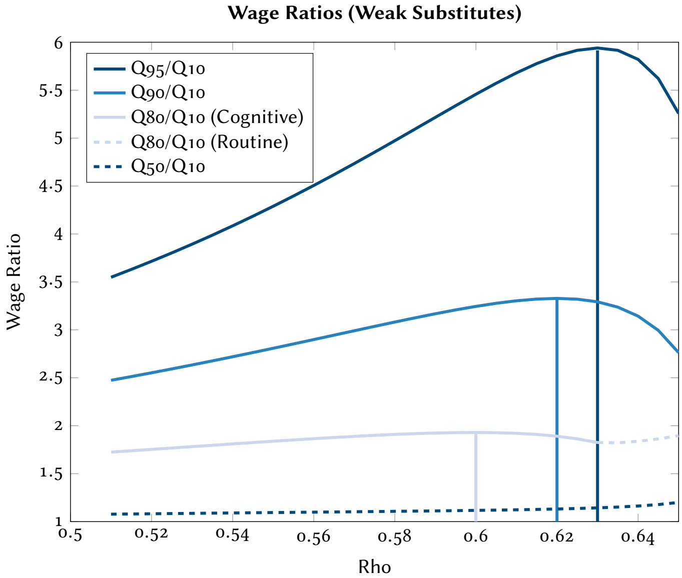


FIGURE 9: Wage Ratios under Weak Substitutes (Detail Cognitive Tasks)

upper cut-off $\overline{\varphi}$ is offset by the superstar phenomenon between cognitive workers, as in the weak substitutes case. The routine workers, around the median of the distribution, observe an increase in inequality with respect to the high skill cognitive workers in the upper part of the skill distribution.[10] At the same time, the increase in $\underline{\varphi}$ from the increase demand of manual tasks, reduced the earnings inequality between routine workers and manual workers, who are in the lower part of the distribution.

The observed effect is a polarization in earnings, where the demand for manual tasks increases, while for cognitive tasks it decreases. This case is consistent with the changes in labour composition and the earnings polarization observed since 2000. Explaining the earnings polarization using the superstar phenomenon in this set-up is advantageous for numerous reasons.

An increase in competition between varieties only requires that the demand for manual tasks increases. As long as the economy is in the Complements case, which is the only case where the labour share of manual tasks increases, earnings will polarize. No further assumption about the distribution of skills or complementarities between the homogeneous and the heterogeneous good are required. Making the result robust to any assumptions about the parameters.

10 The cognitive workers who see a reduction of earnings inequality with respect to routine workers are those closer to the median. Therefore even low skill cognitive workers are consistent with earnings polarization.

While job polarization models that rely on automatization and offshoring can create earnings polarization, the increase in inequality between the manager and routine workers is less than the reduction in inequality between routine and manual workers. The superstar phenomenon in this paper, which benefits the workers in the higher part of the earnings distribution, means that the most skilled cognitive workers - say, in the 90th percentile - see a larger increase in earnings inequality with respect to the rest of workers.

Figure 10 showcases how high skill cognitive workers benefit more from the increase in competition than low skill manual workers. The graph shows the change in log wages after an increase in competition (under the Complements case) given the worker's skill percentile. Low skill manual workers, in the lower quartile, observe a larger increase in earnings than routine workers around the median. At the same time, the highest skill workers, who perform cognitive tasks observe the largest increase in earnings from the increased competition.

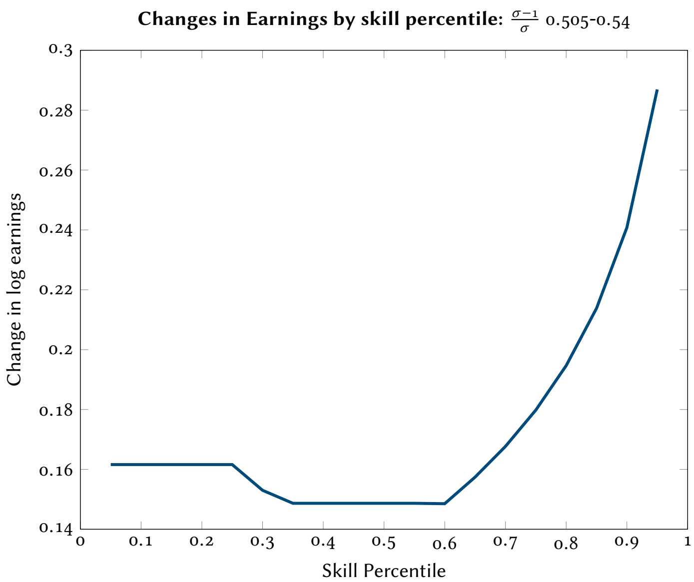


Figure 10: Change in Earnings under Complements

An interesting aspect of the mechanism is that the general equilibrium effects of an increase of competition between varieties can explain the trends in shares of employments and earnings from various periods. The strong substitutes case fits with the pre-1990s overall increase in earnings inequality, with the increase in cognitive jobs. The complements case, on the other hand, fits better the post-2000 period, with a polarization of earnings and an increase in manual tasks, and decrease in cognitive tasks.

The non monotone change from the strong substitute case to the complements case is consistent with the hypothesis that the changes in earnings inequality are caused by a global increase in competition. The trends in labour composition and earnings do not necessarily need to be monotone with an increases in substitutability, therefore the superstar phenomenon can explain the trend during both periods, and the shift between both periods.

The mechanism proposed does a good job at explaining earnings polarization, but falls short in explaining job polarization. The mechanism does not require the fall in routine labour which is observed in the job polarization. To explain the reduction of routine jobs, other complementary hypothesis can do a better job. Automatization, offshoring or skill-bias technological change are mechanisms better suited to explain this reduction, as they focus directly on the root causes. The framework presented in this paper allows us to study some of these alternative hypothesis.

5 AUTOMATIZATION / SBTC

In Autor et al. (2008) they describe how changes in technology through information and communication technology are complementary to cognitive tasks, and substitutive of routine tasks. Skilled-Biased Technological Change (SBTC) can be modelled as a technology that increases the productivity of firms, given the same skill level of the cognitive workers. Therefore the productivity of firm φ is now determined by $\alpha_s\varphi$, and an increase of SBTC is an increase in α_s.[11] It will increase the productivity of firms, and therefore cognitive tasks, without directly affecting the demand for cognitive skills in the heterogeneous good market.

Automatization can be modelled as a technology which, given the same population producing routine tasks, the total amount of routine labour L_r provided for production of the heterogeneous good is increased. Therefore by introducing in the model a parameter α_a such that $L_r = \lambda_r \alpha_a \tilde{\varphi}_r$, an increase in α_a can be seen as automatizing routine labour, as there is need for less effective units of routine labour to produce the same amount of heterogeneous goods.

Lemma 4 *The effects of an increase in SBTC and an increase in automatization are equivalent.*

11 An uncommented assumption in the basic model is that the skill between routine and cognitive tasks is one to one. By introducing α this is no longer necessary, and can be used to rescale the skill of cognitive and routine workers better. In equilibrium α only affects total output of Q without affecting the **HM** equilibrium. As what matters is the ratio of output between Q and Q_0, this difference can be equally captured in the basic model by using only β.

It does not matter which approach is chosen, either increasing the productivity of cognitive workers, or increasing the amount of routine labour produced with each unit of effective labour performed by workers. Both changes are captured by an increase in the wage of routine labour w_r, and do not affect the cut-off point for equilibrium in the **HM** function. They both increase w_r and the aggregate of the heterogeneous good Q by the change in α.[12] Therefore an increase in SBTC or automatization will have no effect on **HM** as a change in α has no effect on $\overline{\varphi}(\underline{\varphi})$. The effect of automatization and SBT will only change the equilibrium of **MC**.

Proposition 2 *If the heterogeneous good and the homogeneous good are gross complements, an increase of SBTC and/or automatization will lead to a decrease in the share of employment doing routine tasks and an overall reduction of earnings inequality.*

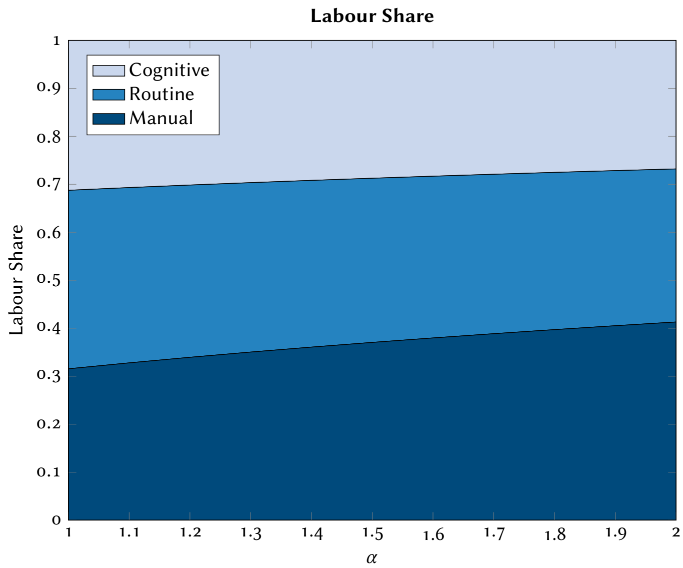


FIGURE 11: Labour Composition under Automatization ($\epsilon < 1$)

An increase in STBC/automatization increases Q and w_r by the change in α in the equilibrium factor allocation of the heterogeneous good market. Aggregate output of the heterogeneous good, and the wage for each unit of effective labour are now given by:

$$\begin{aligned} Q &= \alpha L_r \tilde{\varphi} \\ w_r &= \alpha \tilde{\varphi} \frac{(\sigma-1)}{\sigma} \end{aligned}$$

By looking at **MC** it can be seen that if $\epsilon < 1$ the increase of the ratio between the heterogeneous and the homogeneous good is larger than the change in the

12 I will refer to α for the parameter for SBTC and Automatization from now on.

ratio between prices, given the new manual wage. In order to restore equilibrium, more of the homogeneous good needs to be produced by increasing the share of workers performing manual tasks (increase in φ). This in turn increases $\overline{\varphi}$ as well in the new equilibrium. The earning ratios of Section 3 showed that an increase in φ led to a decrease in earnings inequality between manual and routine workers. And an increase in $\overline{\varphi}$ decreased the earnings ratio between cognitive and routine workers. Leading to an overall decrease in earning inequality among all workers.

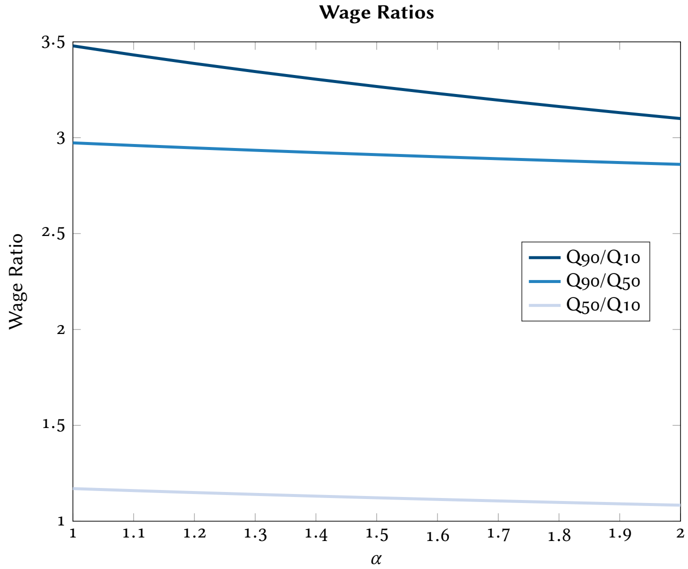


FIGURE 12: Wage Ratios under Automatization ($\epsilon < 1$)

The reduction of firms producing the heterogeneous good (ie increase in $\overline{\varphi}$) from the new equilibrium in the **MC** function, means that the total production of the heterogeneous good in the new equilibrium is less than the one that would have been if **MC** hadn't been adjusted. The increases in both cut-off skills means that the average skill of routine and cognitive workers have also increased. Therefore, the population share of workers performing routine tasks must have been reduced.

The automatization hypothesis is not robust under this framework for simultaneous reduction of cognitive workers, and an increase for manual and cognitive workers. As it does not affect the **HM** through the equilibrium function $\overline{\varphi}(\varphi)$ an increase of manual tasks should lead to a decrease of cognitive workers, and vice-versa. This is still consistent with the idea that cognitive jobs are decreasing instead of increasing. Therefore it contradicts the hypothesis that automatization is a leading cause for the reduction of cognitive jobs. But within this framework it is surely not robust in explaining the increase of earnings inequality or earnings polarization.

The canonical model of SBTC shows that as the most skilled workers benefit from a technology which only makes them more productive, overall earnings inequality increases. In this set-up, the increase in the productivity of cognitive workers α, does not create the same result. SBTC assumes that the technology can be used only by high skilled workers, increasing their value. The framework of this paper departs from that idea by allowing all workers to benefit from the new technology. Some however, do not benefit directly from it (ie manual workers) is because they prefer not to change to a cognitive job.

To give an example, a student who gets a 10th percentile score in their SATs writing skills, can still try to write a book. It won't be a very good one. But this student can still benefit from technology changes in the sector such as electronic word processing, self publishing and e-books in the same manner as a Nobel prize winning writer. The reason the bad writer does not go into book publishing and benefit from these productivity increases, is not a lack of ability to use these new technologies, but the lack of ability to write a book good enough to earn more money than from the outside option.

The SBTC hypothesis is blind to the possibility of workers effortlessly going from low-skill to high-skill work, without having to make a large investment in education or changing jobs and losing with-in job knowledge. It is the friction from high-skill and low-skill that drives the decrease of earnings of manual and routine workers from SBTC, even if they do not benefit directly from the increase in productivity. And for this reason under in the model put forward within this paper SBTC and automatization are not valid hypotheses for explaining the increase in earnings inequality.

6 CONCLUSION

In this model I have presented a tractable model with two sectors where labour is differentiated between different tasks. I use the natural characteristics of these different tasks to exploit their differences, and study how there can be changes in labour composition and earnings inequality between and within group. In order to explain the trends in wages during the last 35 years SBTC and automatization have often been cited as playing an important role. I have shown that while automatization and SBTC can lead to a reduction of routine jobs, the main cause of job polarization, under this framework within the model of the paper they cannot explain the increase of earnings inequality and earnings polarization.

I propose an alternative mechanism in order to explain earnings polarization, an increase in competition between differentiated varieties. Workers use their cognitive skills to differentiate between goods, and the increase substitutability between goods gives rise to a superstar economy. As this effect intensifies, the earnings structure skew more to the highest skilled workers. Meanwhile, complementarities between heterogeneous and homogeneous goods increase the demand and wages of manual workers. The result is a polarization of earnings. The mechanism is robust to the increase in manual tasks and decrease of cognitive tasks observed in the US, and the different trends observed since the 1960s. The mechanism does not explain all trends, such as the reduction of routine jobs, and in order to obtain the full picture it would be of interest to develop the model further to consider the role of factors such as offshoring.

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APPENDIX

PROOF OF LEMMA 2:

Equilibrium in the Heterogeneous Market is given by a $\overline{\varphi}$ such that given a $\underline{\varphi}$ Equation (6) equalizes, and the production of Q given the Routine Labour $\overline{\mathbf{RP}}$ equals the total sales of the firms **CP**. Therefore equilibrium in the heterogeneous market is defined by a function $\overline{\varphi}(\underline{\varphi})$ that guarantees the market is in equilibrium.

Equation (6) can be re-written as:

$$\frac{\overline{\varphi}^{2-\sigma}}{\sigma}\int_{\overline{\varphi}}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi = \frac{\sigma}{\sigma-1}\int_{\underline{\varphi}}^{\overline{\varphi}}\varphi g(\varphi)d\varphi$$

Limits:

$$
\begin{aligned}
\lim_{\overline{\varphi}\to\underline{\varphi}} \text{LHS} &= \frac{\underline{\varphi}^{2-\sigma}}{\sigma}\int_{\underline{\varphi}}^{\infty}\sigma^{\sigma-1}g(\varphi)d\varphi \\
\lim_{\overline{\varphi}\to\infty} \text{LHS} &= 0 \\
\\
\lim_{\overline{\varphi}\to\underline{\varphi}} \text{RHS} &= 0 \\
\lim_{\overline{\varphi}\to\infty} \text{RHS} &= \frac{\sigma}{\sigma-1}\int_{\underline{\varphi}}^{\infty}\varphi g(\varphi)d\varphi
\end{aligned}
$$

Derivative:

$$
\begin{aligned}
\frac{\partial \text{LHS}}{\partial\overline{\varphi}} &= \frac{1}{\sigma}\left[(2-\sigma)\overline{\varphi}^{1-\sigma}\int_{\overline{\varphi}}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi - \overline{\varphi}g(\overline{\varphi})\right] && \leq 0 \\
\frac{\partial \text{RHS}}{\partial\overline{\varphi}} &= \frac{\sigma}{\sigma-1}\overline{\varphi}g(\overline{\varphi}) && \geq 0
\end{aligned}
$$

Therefore LHS is decreasing in $\overline{\varphi}$ and tends to zero, and RHS is increasing with respect to $\overline{\varphi}$ and starts at zero. Therefore both functions are single crossing in the $(Q,\overline{\varphi})$ space.

PROOF OF COROLLARY 1:

Two properties are crucial of the function $\overline{\varphi}(\underline{\varphi})$ guaranteeing equilibrium in the Heterogeneous Goods market. The upper skill cut-off $\overline{\varphi}$ is increasing in $\underline{\varphi}$, while the total production of the heterogeneous good Q is decreasing in $\underline{\varphi}$.

1. $\frac{\partial\overline{\varphi}(\underline{\varphi})}{\partial\underline{\varphi}} \geq 0$

The lower cut-off point $\underline{\varphi}$ only affects the **RP** Routine Production. Keeping $\overline{\varphi}$ fixed, an increase of $\underline{\varphi}$ will mean a reduction of the Routine labour L_r supplied, and therefore a reduction of **RP**. Therefore in order for the market to be in equilibrium, the fall off routine labour must be compensated, with an increase of routine labour through an increase of $\overline{\varphi}$ and/or a decrease of **CP**, which is also achieved by increasing $\overline{\varphi}$. Therefore any increase in $\underline{\varphi}$ will lead to an increase of $\overline{\varphi}$.

2. $\frac{\partial Q}{\partial\underline{\varphi}} \leq 0$

The derivative of the LHS of (6) with respect to $\overline{\varphi}$ is negative, therefore it must be that **CP** is decreasing in $\overline{\varphi}$. Following from the previous property it must be that Q is decreasing in $\underline{\varphi}$.

PROOF OF EQUILIBRIUM:

Given that $\overline{\varphi}(\underline{\varphi})$ grantees equilibrium in **HM**, and its properties from Corollary 1, the following proves that there exists a unique solution for **MC** to be in equilibrium.

Equilibrium is obtained with (**??**) is equalized. Taking $\varphi_{\text{min}} > 0$ as the infimum of all φ that have a positive support in $g(\varphi)$ $\varphi_{\text{min}} \inf\{\varphi : g(\varphi) > 0\}$. The function can rewritten as:

$$w_r \underline{\varphi} G(\underline{\varphi})^{\frac{1}{\epsilon}} = \beta^{(1-\frac{1}{\epsilon})} \frac{\gamma}{1-\gamma} Q^{\frac{1}{\epsilon}}$$

Limits:

$$\begin{aligned}
\lim_{\underline{\varphi} \to \varphi_{\text{min}}} \text{LHS} &= 0 \\
\lim_{\underline{\varphi} \to \infty} \text{LHS} &= \infty \\
\lim_{\underline{\varphi} \to \varphi_{\text{min}}} \text{RHS} &= \beta^{(1-\frac{1}{\epsilon})} \frac{\gamma}{1-\gamma} \left[\frac{\sigma}{\sigma-1} \int_{\varphi_{\text{min}}}^{\overline{\varphi}(\varphi_{\text{min}})} \varphi g(\varphi) d\varphi \right]^{\frac{1}{\epsilon}} \\
\lim_{\underline{\varphi} \to \infty} \text{RHS} &= 0
\end{aligned}$$

Derivative:

$$\begin{aligned}
\frac{\partial \text{LHS}}{\partial \underline{\varphi}} &\geq 0 \\
\frac{\partial \text{RHS}}{\partial \underline{\varphi}} &\leq 0
\end{aligned}$$

The derivative of the LHS comes from w_r being increasing in $\overline{\varphi}$. While the derivative of the RHS comes from the property 2. of $\overline{\varphi}(\underline{\varphi})$. Therefore RHS is decreasing in $\underline{\varphi}$ and tends to zero, and LHS is increasing with respect to $\underline{\varphi}$ and starts at zero. Therefore both functions are single crossing.

PROOF OF PROPOSITION 2:

Automatization is the production of Routine Labour using machines, making it more efficient. Its effect on the economy, can be seen by adding a parameter α such that increases the the productivity of routine labour. Therefore in the new version:

$$L_r = \alpha \int_{\underline{\varphi}}^{\overline{\varphi}} \varphi g(\varphi) d\varphi$$

The new production function facing the firm would be

$$q(\varphi) = \alpha l_r \varphi$$

Note that from here we can see that the effects would be equivalent to an increase of the managers productivity. From the assumption that $p(\tilde{\varphi}) = 1$ we can see that the whole effect moves into the wage of routine labour:

$$w_r = \frac{\alpha \tilde{\varphi}(\sigma - 1)}{\sigma}$$

α DOES NOT CHANGE THE HM FUNCTION:

We can re-write (6) to include the parameter of productivity of routine labour α such that it is written in the following manner:

$$\alpha \frac{M}{\sigma} \left(\frac{\tilde{\varphi}}{\overline{\varphi}} \right)^{\sigma - 1} \overline{\varphi} = \frac{\sigma}{\sigma - 1} \alpha \int_{\underline{\varphi}}^{\overline{\varphi}} \varphi g(\varphi) d\varphi$$

It can be seen that equilibrium in this market is not affected by α, as both sides of the equation are multiplied by α. Therefore the equation $\overline{\varphi}(\underline{\varphi})$ is unaffected by changes in α. But what is affected is $Q(\overline{\varphi}(\underline{\varphi}), \underline{\varphi})$ and w_r, as an increase in α increases the total output with the same cut-off points.

PROOF OF PROPOSITION 3:

The Routine productivity parameter α affects both, the total output of the Heterogeneous Good Q and the routine labour wage w_r. This means that some changes occur when rearring the market clearance equation so it looks like this:

$$\alpha^{1-\frac{1}{\epsilon}}\frac{\sigma-1}{\sigma}\underline{\varphi}G(\underline{\varphi})^{\frac{1}{\epsilon}}=\beta^{1-\frac{1}{\epsilon}}\frac{\gamma}{1-\gamma}L_r^{\frac{1}{\epsilon}}\tilde{\varphi}^{\frac{1}{\epsilon}-1} \tag{7}$$

It can be seen that the changes the effects on the market clearance equilibrium of α will depend on the value of ϵ.

$\epsilon < 1$

If the Heterogeneous Good and the Homogeneous Good are gross complements, we can see that an increase of α would reduce the LHS of (7). This implies that in order to restore equilibrium we need the cut-off point $\underline{\varphi}$ to increase. As the equation $\overline{\varphi}(\underline{\varphi})$ is the same as in the base model, it has the same properties, and $\overline{\varphi}$ will also increase. The increase of $\underline{\varphi}$ means that the total output of the homogeneous good Q_0 will also increase. And as Q and Q_0 are gross complements, and Q_0 and P_0[13] are increasing, it must be that Q also increases in equilibrium.

It can also be seen that in equilibrium an increase in α will reduce the share of the population working in routine jobs λ_r. The increase of $\overline{\varphi}$ and $\underline{\varphi}$ meant that the average productivity of the routine worker and manager increases. If the new total output of the Heterogeneous Good was $Q = \alpha Q'$[14] where Q' was the output in the previous equilibrium before the change in α, it would have to be that to adapt for the new higher productivity of routine and cognitive workers the share of workers in the routine sector was smaller. But because there is an increase in $\overline{\varphi}$ it must be that $Q < \alpha Q'$ even if $Q > Q'$, further accentuating the fall of workers in the routine labour.

Therefore if Q and Q_0 are gross complements an increase in α leads to:

- Increase in $\underline{\varphi}$

13 $P_0 = w_r\underline{\varphi}$ and both w_r and $\underline{\varphi}$ are increasing
14 Assume that $\alpha = 1$ before the change to $\alpha > 1$

- Increase in $\overline{\varphi}$
- Increase in Q_0
- Increase in Q
- Decrease in λ_r

PROOF OF LEMMA 3:

Using implicit differentiation and some rearraning I obtain the implicit derivative of $\overline{\varphi}(\underline{\varphi})$ over σ.

$$\frac{\partial \overline{\varphi}}{\partial \sigma} = \frac{L_r\left(2\sigma - \frac{\sigma^2}{\sigma-1}\right) - (\sigma-1)\varphi^{*(2-\sigma)}\mathbf{A}}{\mathbf{B}(2-\sigma)(\sigma-1)\varphi^{*(1-\sigma)} - \overline{\varphi}g(\overline{\varphi})(\sigma^2+\sigma-1)}$$

Where:

$$\begin{aligned} \mathbf{A} &= \int_{\underline{\varphi}}^{\infty} \varphi^{\sigma-1} g(\varphi)\log(\varphi)d\varphi \\ \mathbf{B} &= \int_{\underline{\varphi}}^{\infty} \varphi^{\sigma-1} g(\varphi)d\varphi \end{aligned}$$

That is, $\mathbf{A}$ is the derivative of $\mathbf{B}$ over σ. The bottom of the derivative is negative. The top is also negative (and therefore the derivative positive) unless:

$$\mathbf{B} > \mathbf{A}\frac{\sigma(\sigma-1)}{\sigma-2}$$

First the expression which contains σ has a minimum at $\sigma = 2 + \sqrt(2)$ which reaches a value of $3 + 2\sqrt(2)$[15]. Therefore for this to be true it must be that most of the weight of the distribution is in values very close to 1.

PROOF OF COROLLARY 2:

Using Proposition 3, and the fact that:

15 close to 6

$$
\begin{aligned}
\frac{\partial L_r}{\partial \sigma} &= \frac{\partial L_r}{\partial \overline{\varphi}} \varphi^{*'} \\
\frac{\partial L_r}{\partial \sigma} &= \overline{\varphi} g(\overline{\varphi}) \varphi^{*'}
\end{aligned}
$$