Quadratic Formula Derivation

$y = ax^2 + bx + c$

Now complete the square:

$y = a\left(x^2 + \frac{b}{a}x\right) + c$

$y = a\left[\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a^2}\right] + c$

$y = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] + c$

$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$

Now multiply by 4a:

$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$

Now set y to zero and solve for x:

$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0$

$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} + c$

Now multiply by 4a:

$4a^2\left(x + \frac{b}{2a}\right)^2 = b^2 - 4ac$

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$

$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$

$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} \Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$