# single\_period\_model

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#### 1 An implementation of the single period Kyle model

Implemented by Paul Friedrich, 2019, ETH Zürich.

#### 2 Initialization

Needed python packages: Keras, TensorFlow, Matplotlib, Numpy, Scipy. If using a CPU, note the comments in the first cell.

```
In [1]: ## Import modules
        import os
        # COMMENT THIS OUT IF USING TENSORFLOW-GPU
        # The line below forces CPU usage. In this file, that is faster than using a GPU,
        # probably due to the relatively small net size and large CPU/GPU handoff performance co
        # due to the requirement of batch size =1.
        os.environ['CUDA_VISIBLE_DEVICES'] = '-1'
        import time
        import tensorflow as tf
        import numpy as np
        import matplotlib.pyplot as plt
        import matplotlib.backends.backend_pdf
        import keras
        import scipy.stats as ss
        from keras.models import Sequential, Model
        from keras.layers import Dense, Input, Lambda, Dropout
        from keras import backend as K
        from keras.callbacks import History
        from keras.utils import plot_model
```

Using TensorFlow backend.

## 3 Configuration

A number of configuration options can be chosen here. Note the comments to see what the possibilities are.

Control which distributions to use for noise trader orders Y and final price Z using the whichdist\_y and whichdist\_z variables. Options are 'normal', 'laplace', 'gumbel', 'gamma', 'gammacent' and 'bimodal'. We suggest leaving y to be normally distributed and only altering Z.

Choices about the parameters are made by passing the appropriate string to the whichconfig variable. Available configurations are given by a tuple ( $\mu_z$ ,  $\sigma_z$ ,  $\sigma_v$ ):

```
'cent': 0, 2, 1.
'non-cent': 0.5, 2, 1.
'non-cent-gamma': 1, √2, 1
```

Control the amount of training epochs for the insider and market maker with the variables epochs\_I and epochs\_MM (default: 3 both).

Control the sample size of both the insider and market maker training data using the variables  $n_I$  and  $n_M$  (default: 5000 both).

Control the amount of maximum training loops with the variable N\_loops (default: 20). Convergence usually occurs between loops 10-15.

Control the type of the initial insider order function which is used for training the market maker model in the first training loop using the variable whichstart. Available options are: - 'linstart' for a start of I\_initial(z) = intercept\_equilibrium + slope\_equilibrium \* z. This represents an insider that has reached the equilibrium. - 'noisestart' for a start of I\_initial(z) = a single sample from  $\mathcal{N}(0,\sigma_y) \sim Y$ . (default) - 'expstart' for a start of I\_initial(z) = intercept\_equilibrium + slope\_equilibrium \* 10 \* ( $e^{0.1z} - 1$ ). This represents an insider that is very similar to the equilibrium but not linear.

Control the size of the transaction cost using the transaction\_cost variable. Good values to use are 0 (no transaction cost, default), 0.5, 1, 1.5.

```
In [3]: #### ADJUST THIS EACH RUN ####
        runnumber = 9999
        transaction_cost = 0
        epochs_I = 3
        epochs_MM = 3
        n_I = 5000
        n_MM = 5000
        N_{loops} = 20
        whichconfig='cent'
        whichstart='noisestart'
        whichactivations='relu'
        whichdist_z='normal'
        whichdist_y='normal'
        ##################################
        whichdist = whichdist_z if whichdist_z == whichdist_y else whichdist_z + whichdist_y
        runname='run'+str(runnumber)
```

```
## Defines mean/stddevs
def get_dist_constants(config_name):
    if config_name=='cent':
        return(0, 2, 1)
    elif config_name=='non-cent':
        return(0.5, 2, 1)
    elif config_name=='non-cent-gamma':
        return(1, np.sqrt(2), 1)
    else: raise NameError('Invalid dist. config chosen')
# Defining constants. mu==mean, sigma==stddev.
mu_z, sigma_z, sigma_y = get_dist_constants(whichconfig)
# Calculate the theoretical constants for linear predictions. For centered gamma: interc
slope_I = sigma_y/sigma_z
intercept_I = 0 if whichdist_z=='gammacent' else -mu_z*sigma_y/sigma_z
intercept_MM = 0 if whichdist_z=='gammacent' else mu_z
slope_MM = sigma_z/(2*sigma_y)
# Initialize insider obj function
def insider_order(x):
    if whichstart=='linstart':
        return(intercept_I+slope_I*x)
    elif whichstart=='noisestart':
        return(np.random.normal(loc=0,scale=sigma_y,size=np.size(x)))
    elif whichstart=='expstart':
        return(intercept_I + slope_I * 10*(np.exp(0.1*x) - 1))
    else:
        raise NameError('Invalid starting order function chosen. Adjust "whichstart".')
# Draws samples
def sample_from_dist(dist, param_1, param_2, sample_size):
    if dist=='normal':
        return(np.array(np.random.normal(loc=param_1, scale=param_2, size=sample_size)))
    elif dist=='laplace':
        return(np.array(np.random.laplace(loc=param_1, scale=param_2/np.sqrt(2), size=sa
    elif dist=='gumbel':
        return(np.array(np.random.gumbel(loc=param_1-param_2*np.sqrt(6)*np.euler_gamma/r
    elif dist=='gamma':
        return(np.array(np.random.gamma(shape=param_2**2 / param_1, scale=param_1**2 / p
    elif dist=='gammacent':
        return(np.array(np.random.gamma(shape=param_2**2 / param_1, scale=param_1**2 / p
    elif dist=='bimodal':
        p_z_mix = 0.5
```

```
mu_z_1 = -2
        mu_z_2 = 2
        sigma_z_1 = 1
        sigma_z_2 = 1
        def mu_mixed(p, mu_1, mu_2):
            return(p*mu_1 + (1-p)*mu_2)
        def sigma_mixed(p, mu_1, mu_2, sigma_1, sigma_2):
            return(np.sqrt(p*(sigma_1**2 + (mu_1-mu_mixed(p, mu_1, mu_2))**2) + (1-p)*(s
        mu_z = mu_mixed(p_z_mix, mu_z_1, mu_z_2)
        sigma_z = sigma_mixed(p_z_mix, mu_z_1, mu_z_2, sigma_z_1, sigma_z_2)
        comps = np.random.choice((0,1),size=sample_size,p=(p_z_mix,1-p_z_mix))
        mus = [(mu_z_1, mu_z_2)[i] \text{ for } i \text{ in } comps]
        sigmas = [(sigma_z_1,sigma_z_2)[i] for i in comps]
        gaussians=np.random.normal(size=sample_size)
        return(np.array(gaussians*sigmas+mus))
   else:
        raise NameError(f'Invalid distribution chosen. Adjust whichdist_z/y.')
# Define plotting function
def model_plotting(data, slope, intercept, name):
   data_sorted = np.sort(data)
   data_min = data_sorted[0]
   data_max = data_sorted[-1]
   data_test = np.linspace(data_min-np.linalg.norm(data_min-data_max)/2, data_max+np.li
   dummy = np.zeros(np.size(data_sorted))
   P_true = data_test*slope + intercept
   if name=='MM':
        P_predicted = MM_model.predict([data_test,np.zeros(1000)])
        plt.plot(data_test, P_true, color='r')
        plt.plot(data_test, P_predicted, linestyle='-',color='g')
        plt.scatter(data_sorted, MM_model.predict([data_sorted, dummy]), marker='|', col
        plt.xlabel('Total Order x+y')
        plt.ylabel('Market Price P(x+y)')
   else:
        P_predicted = insider_model.predict([data_test,np.zeros(1000)])
        plt.plot(data_test, P_true, color='r')
        plt.plot(data_test, P_predicted, linestyle='-',color='g')
        plt.scatter(data_sorted, insider_model.predict([data_sorted, dummy]), marker='|'
        plt.xlabel('End-of-period price z')
        plt.ylabel('Insider order x(z)')
   plt.legend(['Predicted linear behavior','NN output'])
   plt.title(f'{name}, N={i+1}')
```

#### 4 Model training

The model will automatically stop training if a sufficient goodness-of-fit is reached. Alternatively, after 7 iterations, it will ask whether you want to prematurely stop it after every iteration. Answering 'y', 'yes', 's', 'stop' will stop training, for all other inputs the model continues training.

```
In [4]: fig_list = []
        t0 = time.time()
        for i in range(N_loops):
            print(f'Loop {i+1} out of {N_loops}')
            ### Market maker model ###
            # Build market maker sample of (y, z)'s
            y = sample_from_dist(whichdist_y, 0, sigma_y, n_MM)
            z = sample_from_dist(whichdist_z, mu_z, sigma_z, n_MM)
            # Create MM input
            x = insider_order(z).squeeze()
            total\_order = x+y
            # MM model architecture
            in_main = Input(shape=(1,))
            in_aux = Input(shape=(1,))
            layer1_MM = Dense(10, input_shape=(1,),activation=whichactivations)(in_main)
            layer2_MM = Dense(10, activation=whichactivations)(layer1_MM)
            out_main = Dense(1)(layer2_MM)
            # MM loss func
            def MM_loss(z):
                def MM_LS(x_true, x_pred):
                    return(K.square(z-x_pred))
                return (MM_LS)
            # Create and compile model
            MM_model = Model(inputs=[in_main, in_aux],
                             outputs=out_main)
            MM_model.compile(optimizer='adam',
                      loss=MM_loss(MM_model.inputs[1]),
                      metrics=['accuracy'])
            dummy_y = np.zeros((n_MM, 1))
            # Train model
            print('%%% Training MM model %%%')
            MM_model.fit(x=[total_order,z], y=dummy_y, epochs=epochs_MM, batch_size=1, verbose=1
            # Define pricing rule
            def pricing_rule(order):
```

```
return(MM_model([order,K.zeros(1)]))
# Plotting MM
fig_loop = plt.figure(i, figsize=(12,6))
plt.subplot(121)
model_plotting(total_order, slope_MM, intercept_MM, 'MM')
### Insider model ###
# Build insider sample of z's
z_I = sample_from_dist(whichdist_z, mu_z, sigma_z, n_I)
y_I = sample_from_dist(whichdist_y, 0, sigma_y, n_I)
# Insider model architecture
in_main_I = Input(shape=(1,))
in_aux_I = Input(shape=(1,))
layer1_I = Dense(10, activation=whichactivations)(in_main_I)
out_main_I = Dense(1)(layer1_I)
insider_model = Model(inputs=[in_main_I, in_aux_I],
              outputs=out_main_I)
# Insider loss func.
def insider_loss(z, y):
    def I_mean(x_true, x_pred):
       price = pricing_rule(x_pred+y)
        profits = (z-price)*x_pred - transaction_cost*K.abs(x_pred)
        return(-profits)
    return(I_mean)
# Compile Insider Model
insider_model.compile(optimizer='adam',
          loss=insider_loss(insider_model.input[0], insider_model.input[1]), # Call
          metrics=['accuracy'])
dummy_y = np.zeros((n_I,1))
# Train Insider model
print('%%% Training Insider model %%%')
insider_model.fit(x=[z_I,y_I], y=dummy_y, epochs=epochs_I, batch_size=1, verbose=1)
# Define new Insider order function
def insider_order(price):
    return(insider_model.predict([price, np.zeros(np.size(price))]))
# Plotting
plt.subplot(122)
model_plotting(z_I, slope_I, intercept_I, 'Insider')
# Finishing plot
```

```
fig_list.append(fig_loop)
                            plt.show()
                            # Taking time
                            t1 = time.time()
                            run_time = time.strftime('%H:%M:%S', time.gmtime(t1-t0))
                            print(f'Total runtime: {run_time}')
                             # Premature stopping
                             # regression of I
                            reg_I = ss.linregress(z_I, np.squeeze(insider_model.predict([z_I, np.zeros(np.size(z_I, np.zeros(np.zeros(np.size(z_I, np.zeros(np.s
                            stop_I = False
                            if (np.abs(reg_I.slope - slope_I) < 0.075) and (np.abs(reg_I.intercept - intercept_I
                                      stop_I = True
                             # regression of M
                            reg_MM = ss.linregress(total_order, np.squeeze(MM_model.predict([total_order, np.zer
                            stop_MM = False
                            if (np.abs(reg_MM.slope - slope_MM) < 0.075) and (np.abs(reg_MM.intercept - intercept
                                      stop_MM = True
                            # stop condition
                            print(f'MM slope: {reg_MM.slope} ({slope_MM}); MM intercept: {reg_MM.intercept} ({in
                            stop_training=[]
                            if stop_I and stop_MM:
                                      print('%% Sufficient goodness-of-fit is reached. %%%\nWant to stop training? (y
                                      stop_training = input()
                            if stop_training in ['y', 'yes', 's', 'stop']:
                                      break
                             #if (i+1)\%5==0 and transaction_cost!=0:
                            if (i+1)>=7 and transaction_cost!=0:
                                      print(f'%% Want to stop training prematurely? (y/n)')
                                      stop_training = input()
                            if stop_training in ['y', 'yes', 's', 'stop']:
                                      break
                   ## After loop completion
                   # plotting progression
                   pdf = matplotlib.backends.backend_pdf.PdfPages(f'{whichstart}_{runname}_{whichconfig}_{N}
                   for fig in fig_list:
                            pdf.savefig( fig )
                   pdf.close()
Loop 1 out of 20
WARNING:tensorflow:From G:\Programs\Anaconda3\envs\mthesis\lib\site-packages\tensorflow\python\f
```

Instructions for updating:

Colocations handled automatically by placer.

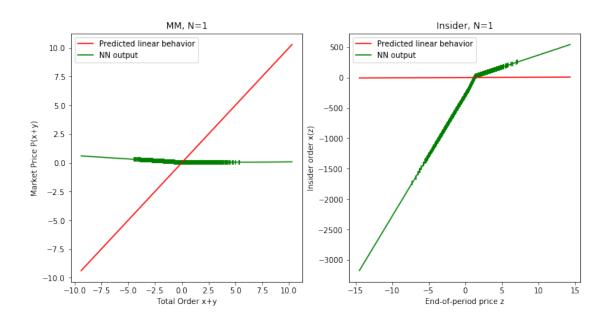
%%% Training MM model %%%

WARNING:tensorflow:From G:\Programs\Anaconda3\envs\mthesis\lib\site-packages\tensorflow\python\o Instructions for updating:

Use tf.cast instead.

Epoch 1/3

```
5000/5000 [=
          ========] - 2s 499us/step - loss: 4.1003 - acc: 0.9996
Epoch 2/3
5000/5000 [==
           =======] - 2s 420us/step - loss: 4.0904 - acc: 0.9998
Epoch 3/3
%%% Training Insider model %%%
Epoch 1/3
Epoch 2/3
Epoch 3/3
```



Total runtime: 00:00:14

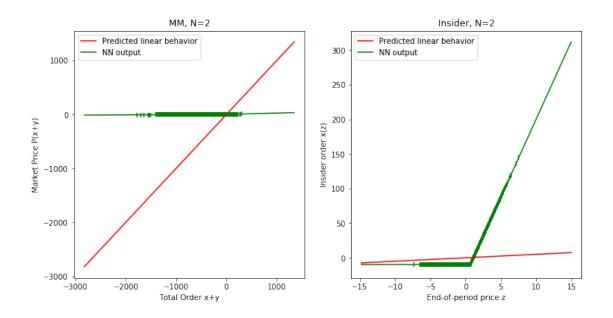
MM slope: -0.02512230316230322 (1.0); MM intercept: 0.07445404407155577 (0) I slope: 166.13658076125094 (0.5); I intercept: -324.1468608242456 (0.0)

Loop 2 out of 20

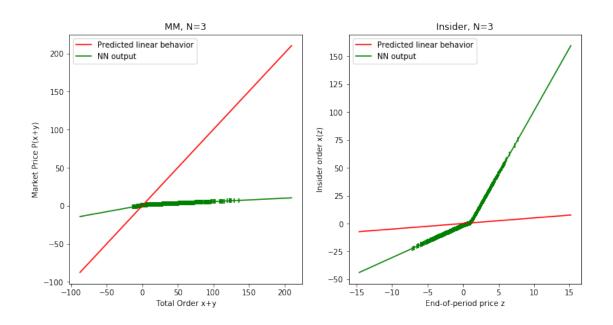
%%% Training MM model %%%

Epoch 1/3

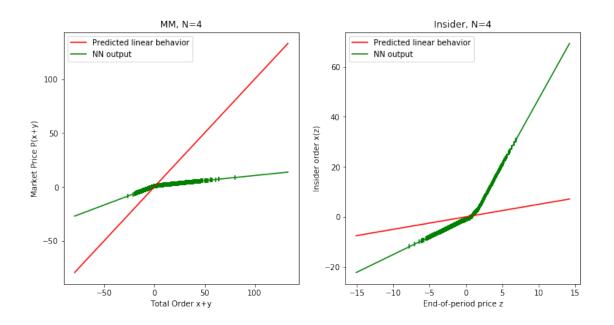
Epoch 2/3



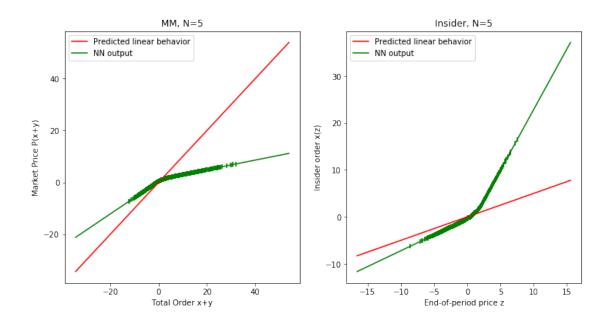
```
Total runtime: 00:00:29
MM slope: 0.005700640897969643 (1.0); MM intercept: 1.856957631576941 (0)
I slope: 8.29994433538599 (0.5); I intercept: 1.9359519034157489 (0.0)
Loop 3 out of 20
%%% Training MM model %%%
Epoch 1/3
Epoch 2/3
5000/5000 [============== ] - 2s 451us/step - loss: 1.3382 - acc: 0.0424
Epoch 3/3
%%% Training Insider model %%%
Epoch 1/3
5000/5000 [==:
                    ========] - 3s 505us/step - loss: -6.9433 - acc: 0.0940
Epoch 2/3
5000/5000 [===
                      =======] - 2s 469us/step - loss: -8.9053 - acc: 0.0310
```



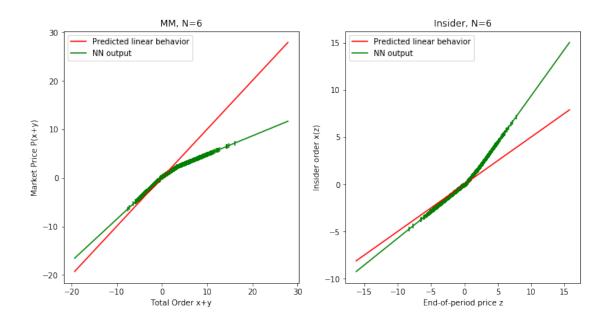
```
Total runtime: 00:00:44
MM slope: 0.07290915849163955 (1.0); MM intercept: -0.1547990845502495 (0)
I slope: 5.200688807600315 (0.5); I intercept: 0.9024175487520765 (0.0)
Loop 4 out of 20
%%% Training MM model %%%
Epoch 1/3
5000/5000 [=============== ] - 3s 504us/step - loss: 0.2663 - acc: 0.2110
Epoch 2/3
5000/5000 [=============== ] - 2s 461us/step - loss: 0.0998 - acc: 0.2044
Epoch 3/3
%%% Training Insider model %%%
Epoch 1/3
Epoch 2/3
5000/5000 [===
                =========] - 2s 476us/step - loss: -4.4582 - acc: 0.0650
Epoch 3/3
```



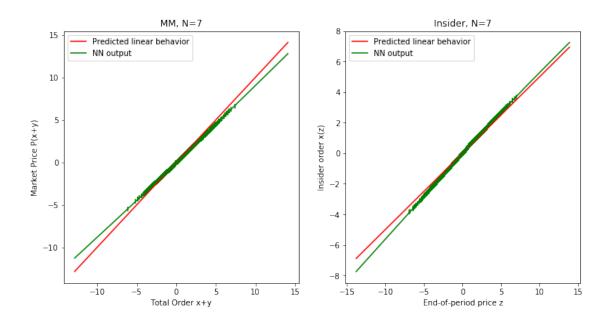
```
Total runtime: 00:00:59
MM slope: 0.16618137375137895 (1.0); MM intercept: -0.15394753180891044 (0)
I slope: 2.501505429244555 (0.5); I intercept: 0.33429489606338086 (0.0)
Loop 5 out of 20
%%% Training MM model %%%
Epoch 1/3
Epoch 2/3
Epoch 3/3
%%% Training Insider model %%%
Epoch 1/3
5000/5000 [===
            =========] - 3s 539us/step - loss: -2.1046 - acc: 0.1718
Epoch 2/3
5000/5000 [===
         Epoch 3/3
```



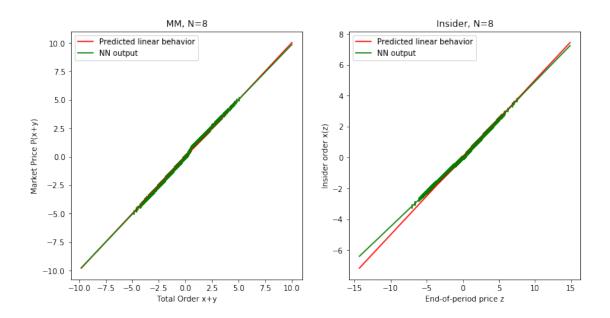
```
Total runtime: 00:01:15
MM slope: 0.34752065706276847 (1.0); MM intercept: -0.18202768140885117 (0)
I slope: 1.1971637201802472 (0.5); I intercept: 0.21985366016376723 (0.0)
Loop 6 out of 20
%%% Training MM model %%%
Epoch 1/3
Epoch 2/3
Epoch 3/3
%%% Training Insider model %%%
Epoch 1/3
5000/5000 [====
              =========] - 3s 561us/step - loss: -1.3284 - acc: 0.2590
Epoch 2/3
5000/5000 [===
           Epoch 3/3
5000/5000 [============== ] - 3s 504us/step - loss: -1.3944 - acc: 0.2900
```



```
Total runtime: 00:01:32
MM slope: 0.6364939280437554 (1.0); MM intercept: -0.17075154153004193 (0)
I slope: 0.7055975257824642 (0.5); I intercept: 0.12784659138110163 (0.0)
Loop 7 out of 20
%%% Training MM model %%%
Epoch 1/3
Epoch 2/3
5000/5000 [============= ] - 3s 512us/step - loss: 1.4234 - acc: 0.2408
Epoch 3/3
%%% Training Insider model %%%
Epoch 1/3
5000/5000 [===
              =========] - 3s 586us/step - loss: -0.9841 - acc: 0.3200
Epoch 2/3
5000/5000 [==:
           Epoch 3/3
```



```
Total runtime: 00:01:49
MM slope: 0.9030329406812784 (1.0); MM intercept: -0.05503504010303997 (0)
I slope: 0.5541292161060504 (0.5); I intercept: -0.03414628410754964 (0.0)
Loop 8 out of 20
%%% Training MM model %%%
Epoch 1/3
Epoch 2/3
5000/5000 [============== ] - 3s 517us/step - loss: 1.7782 - acc: 0.2674
Epoch 3/3
%%% Training Insider model %%%
Epoch 1/3
5000/5000 [======
           Epoch 2/3
5000/5000 [===
           Epoch 3/3
5000/5000 [============== ] - 3s 530us/step - loss: -0.9398 - acc: 0.4092
```



```
Total runtime: 00:02:07

MM slope: 1.0505601185077198 (1.0); MM intercept: -4.031004568208832e-05 (0)

I slope: 0.46887708546869156 (0.5); I intercept: 0.008189797169772632 (0.0)

%%% Sufficient goodness-of-fit is reached. %%%

Want to stop training? (y/n)

y
```

## 5 Plotting final models

Generating and saving the last (and presumably best) model results.

plt.ylabel('Market Price P(x+y)')

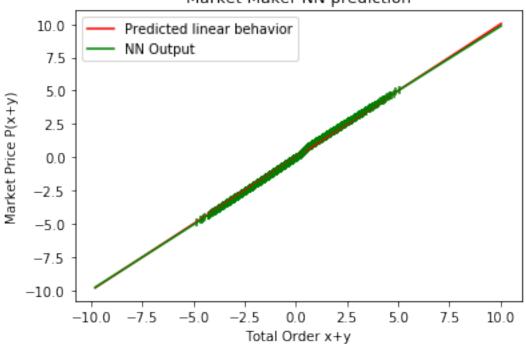
```
plt.legend(['Predicted linear behavior', 'NN Output'])
plt.title('Market Maker NN prediction')

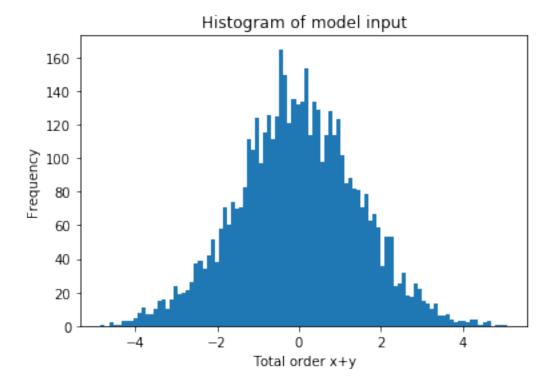
plt.savefig(f'{whichstart}_MM_{runname}_{whichconfig}_{N_loops}_{n_MM}_{epochs_MM}_{n_I})
plt.show()

# Histogram

plt.hist(total_order,bins=100)
plt.title('Histogram of model input')
plt.xlabel('Total order x+y')
plt.ylabel('Frequency')
plt.savefig(f'{whichstart}_MM_{runname}_{whichconfig}_{N_loops}_{n_MM}_{epochs_MM}_{n_I})
plt.show()
```

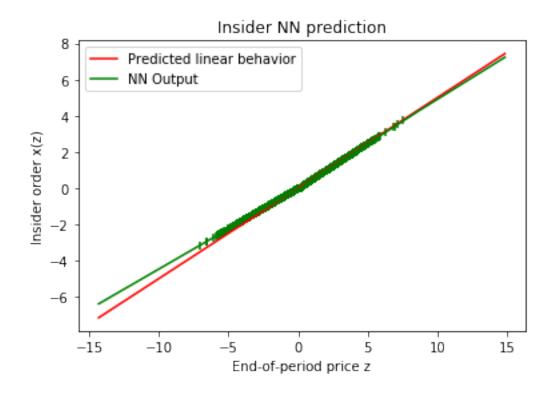


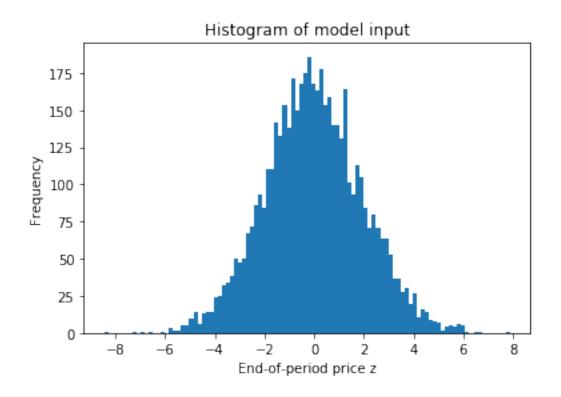




```
In [6]: z_sorted = np.sort(z_I)
        z_sym = z_sorted[np.abs(z_sorted)<3]</pre>
        \#z\_close = z\_sorted[np.abs(z\_sorted-2)<0.5]
        bend = insider_model.predict([z_sym, np.zeros(np.size(z_sym))])
        z_bend_r = z_sym[np.argmax(bend>0.05)] # argmax seeks first 'true'
        z_bend_l = z_sym[np.argmin(bend<-0.05)-1] # argmin seeks first 'false'</pre>
        ## Plot I
        dummy = np.zeros(n_I)
        z_min = z_sorted[0]
        z_{max} = z_{sorted}[-1]
        z_test = np.linspace(z_min-np.linalg.norm(z_min-z_max)/2, z_max+np.linalg.norm(z_min-z_m
        plt.plot(z_test, z_test*slope_I+intercept_I, color='r')
        plt.plot(z_test, insider_model.predict([z_test, np.zeros(1000)]),color='g')
        plt.scatter(z_sorted,insider_model.predict([z_sorted, dummy]), marker='|', color='g')
        if transaction_cost != 0:
            plt.axvline(z_bend_r, color='b', linestyle='--')
            plt.axvline(z_bend_l, color='b', linestyle='--')
            plt.axvline(0, color='grey', linestyle=':')
            plt.xticks(list(plt.xticks()[0]) + [int(z_bend_r*10)/10, int(z_bend_l*10)/10])
            print(f'Approximation: Left bend at: {z_bend_l}, right bend at: {z_bend_r}')
        plt.xlabel('End-of-period price z')
        plt.ylabel('Insider order x(z)')
        plt.legend(['Predicted linear behavior','NN Output'])
```

```
plt.title('Insider NN prediction')
plt.savefig(f'\{ whichstart\}_I_{runname}_{whichconfig}_{N_loops}_{n_MM}_{epochs_MM}_{n_I})
plt.show()
# Plot 0
if transaction_cost != 0:
    plt.plot(z_sym, insider_model.predict([z_sym, np.zeros(np.size(z_sym))]), color='g')
    plt.axvline(z_bend_r, color='b', linestyle='--')
    plt.axvline(z_bend_l, color='b', linestyle='--')
    plt.axvline(0, color='grey', linestyle=':')
    plt.xticks(list(plt.xticks()[0]) + [int(z_bend_r*10)/10, int(z_bend_l*10)/10])
    plt.xlabel('End-of-period price z')
    plt.ylabel('Insider order x(z)')
    plt.title('Location of "bend" in the insider order function')
    plt.savefig(f'{ whichstart}_I_{runname}_{whichconfig}_{N_loops}_{n_MM}_{epochs_MM}_{
    plt.show()
## Histogram
plt.hist(z,bins=100)
plt.title('Histogram of model input')
plt.xlabel('End-of-period price z')
plt.ylabel('Frequency')
plt.savefig(f'{whichstart}_I_{runname}_{whichconfig}_{N_loops}_{n_MM}_{epochs_MM}_{n_I}_
plt.show()
z_min = mu_z-3*sigma_z # approx same as actual z_min
z_{max} = mu_z + 3*sigma_z
z_test = np.linspace(z_min, z_max, 100)
x_test = np.squeeze(insider_model.predict([z_test, dummy])) # squeezing to get (100,1)->
reg = ss.linregress(z_test, x_test)
print(f'slope: {reg.slope}; intercept: {reg.intercept}\nslope_pred: {slope_I}; int_pred:
```





slope: 0.46987225309924896; intercept: 0.04752412810921659

slope\_pred: 0.5; int\_pred: 0.0

R\_sq: 0.999558455556054