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Stimulus Information and Sequential Dependencies in Magnitude Estimation and Cross-Modality Matching

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Descriptive models of magnitude estimation and cross-modality matching derived from two different approaches to psychophysical judgment, the response ratio hypothesis and the fuzzy judgment approach, are compared. The two approaches emphasize different bodies of facts but both attempt to account for sequential dependencies in psychophysical judgments. Both models suggest a hierarchical multiple linear regression model for such data. Some of the predictions of the models are explored in the context of two experiments in which the amount of stimulus information available to subjects in magnitude estimation and cross-modality matching tasks is varied. The fuzzy judgment approach generally does better in explaining the form of such data.

One interesting source of response variability that has been discovered in both magnitude estimation and cross-modality matching, as well as in the more classical methods of judgment (e.g., category judgment), is dependencies of the response to the current stimulus on the values of previous stimuli and responses. The effects in magnitude estimations have now been reported by several investigators (Cross, 1973; Jesteadt, Luce, & Green, 1977; Luce & Green, 1974; Ward, 1971, 1973), and similar effects have been reported in cross-modality matching (Ward, 1975). Although a large number of theoretical approaches to psychophysical judgment have emerged in recent years, only two (to my knowledge) have attempted to deal with these ubiquitous sequential dependencies. The two approaches have different starting points and emphasize different sets of empirical facts.

The response ratio approach (Luce & Green, 1974) grew out of work on a neural timing theory for intensity perception (Luce & Green, 1972). This approach emphasizes the neural coding of stimulus intensity and focuses on reaction time data in detection and discrimination situations and on the basic scaling data. On the other hand, the fuzzy judgment approach (Ward, Note 1, Note 2) emphasizes cognitive processes in scaling judgments, and arises from earlier work on sequential dependencies in absolute identification judgments (e.g., Ward & Lockhead, 1971) and from a different conception of the nature of internal representations of stimuli. The purpose of this article is to compare specific descriptive models of two types of psychophysical judgment derived from the two general approaches, and to test predictions made from these models and their associated metatheories concerning the behavior of the model parameters when the amount of stimulus information available

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to subjects is varied. To this end, I will briefly outline the models and compare them (details of the models can be found in the referenced works), discuss predictions about two experiments, and report the results of the experiments and evaluate the status of each of the two approaches.

The Response Ratio Approach

In one variant of the magnitude estimation task (see Ward, 1971, 1973), the task of the subject is to give a response to the current stimulus so that the ratio of responses to current and immediately previous stimuli reflects the ratio of the intensity of the sensations aroused by those two stimuli. In short, the ratio nature of the task is made explicit. Luce and Green (1974) suggested that this is what subjects attempt to do in all magnitude estimation tasks. That is, their responses are reflections of their attempts to act according to

$$\frac{\mathbf{R}_n}{\mathbf{R}_{n-1}} = C\frac{\mathbf{X}(\mathbf{S}_n)}{\mathbf{X}^*(\mathbf{S}_{n-1})}, \tag{1}$$

where $\mathbf{R}_n$ and $\mathbf{R}_{n-1}$ are random variables representing numerical response on Trials n and $n-1$, C is a constant, and $\mathbf{X}(\mathbf{S}_n)$ and $\mathbf{X}^*(\mathbf{S}_{n-1})$ are random variables representing the internal representations of the stimuli on Trials n and $n-1$. Luce and Green (1974) showed that two such independent internal representations $[\mathbf{X}(\mathbf{S}_n)$ and $\mathbf{X}^*(\mathbf{S}_n)]$ must be generated for each stimulus, and the use of one must destroy it or make it unavailable for further use, or the model is degenerate and uninteresting. This basic model has been generalized by Marley (1976) by making C in Equation 1 also a random variable across trials.

Jesteadt, Luce, and Green (1977) investigated, for magnitude estimations, a multiple linear regression model of the form

$$\log \mathbf{R}_n = \gamma \log \mathbf{I}_n + \sum_{i=1}^{M} \alpha_i \log \mathbf{I}_{n-i} + \sum_{k=1}^{N} \beta_k \log \mathbf{R}_{n-k} + \delta + \boldsymbol{\varepsilon}, \tag{2}$$

where $\mathbf{I}_n$ is a random variable representing the intensity of the stimulus on Trial n. By varying M and n in a series of regression analyses, Jesteadt et al. (1977) concluded that a more limited model, of the form

$$\log \mathbf{R}_n = \gamma \log \mathbf{I}_n + \alpha \log \mathbf{I}_{n-1} + \beta \log \mathbf{R}_{n-1} + \delta + \boldsymbol{\varepsilon} \tag{3}$$

was sufficient to describe the magnitude estimation data of individual subjects. This model follows from the response ratio hypothesis (Equation 1) if we assume that

$$\mathbf{X}(\mathbf{S}_n) = k\mathbf{I}_n{}^m = \mathbf{X}^*(\mathbf{S}_n). \tag{4}$$

Inserting Equation 4 in Equation 1, taking logs of both sides, and solving for $\log \mathbf{R}_n$ yields

$$\log \mathbf{R}_n = m \log \mathbf{I}_n - b \log \mathbf{I}_{n-1} + d \log \mathbf{R}_{n-1} + \log C, \tag{5}$$

where $b = m$ and $d = 1$.[1] Thus, the regression parameters in Equation 3 can be interpreted as estimates of the constants in Equation 5.[2] Although Luce and Green did not explicitly extend the response ratio hypothesis to cross-modality matching, it seems plausible to do so. Identical equations should also describe those data.

The Fuzzy Judgment Approach

On the surface, the fuzzy judgment approach (Ward, Note 1, Note 2) is

[1] The actual equation obtained is

$$\log \mathbf{R}_n = m \log \mathbf{I}_n - m \log \mathbf{I}_{n-1} + \log \mathbf{R}_{n-1} + \log C.$$

The coefficient of $\log \mathbf{I}_{n-1}$ has been changed from m to b and a coefficient $d = 1$ placed in front of $\log \mathbf{R}_{n-1}$ to emphasize the correspondence of this equation to Equation 3 and to facilitate talking about estimates of these parameters from Equation 3. Thus, in Equation 5, $m = b$ by definition.

[2] Of course, $\log \mathbf{R}_{n-1}$ and $\log \mathbf{I}_{n-1}$ are highly correlated and so the problem of multicolinearity arises in this analysis. In such situations, a hierarchical regression model is appropriate, and the order of the variables in the equation depends on the theoretical orientation of the investigator (see Cohen & Cohen, 1975, pp. 116–117, on this point). More appropriate here might be a reversal of the order of $\log \mathbf{R}_{n-1}$ and $\log \mathbf{I}_{n-1}$, since $\mathbf{R}_{n-1}$ is closer in time to $\mathbf{R}_n$ than is $\mathbf{S}_{n-1}$, and should not vary except over trials. However, as will be discussed later, the order of extraction seems to make little difference as long as all factors in Equation 3 are included.

different from the response ratio approach. The first major assumption of the metatheory within which the judgment models are developed is that the perception involved in a psychophysical judgment is categorical. That is, when presented with a stimulus input, the subject's first act is to attempt to categorize it. The subject then proceeds to manipulate the category label, not some representation of the stimulus, to produce the response required. Categorization is conceptualized in the sense of Bruner (1957)—that is, as identification—but the categories are unique to the situation in which the stimuli vary along only a single dimension.

The second major assumption is that both these categories, and the internal representation of the stimulus used in the categorization process, can usefully be conceptualized as "fuzzy subsets" (of the set of possible sensation levels) in the sense of Zadeh (1965). (See also Kaufman, 1975.) A fuzzy subset is a subset in which the elements have grades of membership in the interval (0, 1) inclusive, instead of only 0 or 1 as in ordinary set theory. Hersh and Caramazza (1976) provided an interesting interpretation of natural language concepts referring to quantity (e.g., small, large) as fuzzy subsets over a stimulus ensemble. Oden and Massaro (1978) used fuzzy logic to model subjects' matching of perceived speech sounds to prototypes of phonemes. An alternative way of conceptualizing this is that rather than representing a stimulus (or a category prototype) as a single value of a random variable, as has been done classically, the subject represents it as a sample distribution of random variable values (perhaps taken from several parallel neural channels) and proceeds to work with this distribution. Physiologically, the sample distribution could be composed of a group of reciprocals of interarrival times between neural pulses (as in a neural timing theory) or a group of "pulse densities" sampled over some time interval.

What follows is a brief outline of the stages of processing assumed by the models developed for magnitude estimation and cross-modality matching. On each trial of the judgment task, a stimulus is presented to the subject. In Stage 1, this stimulus energy is transduced into central nervous system activity by the appropriate sensory system according to the operating characteristics of the several transducer stages. The central nervous system activity is then converted by automatic processes into an internal representation of the stimulus, which is held in short-term memory. The internal representation is assumed to be a fuzzy subset of the set of possible sensation levels (or, as mentioned previously, a sample distribution of random variable values). In addition, it is assumed that because of the excitatory-center inhibitory-surround nature of such neural representations (see, e.g., Levine & Grossberg, 1976), the center of the internal representation of the stimulus on Trial n is moved away from (contrasted with) its center on Trial $n - 1$.

In Stage 2, the internal representation of the stimulus is categorized into one of N categories on an ordinal category scale. This is done by means of a process similar to the type of memory scanning studied by Sternberg (1969) and others. Each category is assumed to have a prototype (see Rosch, 1975) that is a fuzzy subset (or sample distribution) similar to the internal representation of the stimulus. The scale consists of the set of prototypes, which are implicitly rank ordered. These prototypes (or procedures for generating them) are assumed to be stored in long-term memory (see Ward & Lockhead, 1971), and are produced when needed. The subject is assumed to use a series of (probably subliminal) decisions to choose a single category label for the internal representation of the stimulus from among those whose prototypes are sufficiently similar to the internal representation of the stimulus. These include a heuristic procedure[3] to decide on one single category even when

[3] A heuristic is a procedure that usually produces an answer to a problem for whose solution an algorithm cannot be used. Here, the heuristic used resembles the representativeness heuristic of Kahneman and Tversky (1973).

there is no obvious candidate. Basically, the heuristic attempts to minimize the distance between the identity of the current stimulus and that of the previous stimulus, perhaps because it has been observed that in random sequences, small distances occur much more often than do large ones (see Ward & Lockhead, 1971). Its effect is to cause the present category response, on the average, to be assimilated to the previous category response, and this effect is passed on to the actual responses by the next two stages.

In Stage 3, there occurs a simple one-to-one mapping of the category chosen as the identity of the stimulus on the stimulus continuum category scale (e.g., a long line) to the corresponding category on a similar category scale for the response continuum (e.g., a large number or a long duration).

In Stage 4, a representative of the response continuum is produced that is a close match to the prototype of the response continuum category indicated by the mapping of Stage 3. At this point, the models for magnitude estimation and cross-modality matching, which have so far been identical, must diverge, since the response continua are different for the different scaling techniques. However, they both assume the attempt to produce a response that matches the prototype of the response category. Of course, in cross-modality matching, the response continuum is also subject to a transducer-operating characteristic transformation, and in magnitude estimation the number continuum is involved. If both continua have power function operating characteristics, then we have the typical Stevensonian situation. If both are log-transformed, then we have a model similar to that of MacKay (1963). More generally, the model falls into the class generated by Staddon's (1978) differential equations.

One aspect of the model presented briefly above deserves further comment. This is the problem of the relationship between the amount of information available in the stimulus and the fuzziness of its internal representation. A plausible assumption (at a metatheoretical level) is that the less clear, precise, and informative the stimulus, the fuzzier its internal representation. It seems reasonable to suppose that a subject would be aware of roughly how precise a representation it is possible to construct from the information at hand. When the internal representation is very fuzzy, judgment strategies might be altered to accommodate to this fact. A subject might, for example, use fewer categories on the input and output category scales or shorten the range of responses, or do both. Also, several structural mechanisms in the model should be affected by the amount of information in the stimulus. The position of the center of the internal representation in the range of sensations should vary more from trial to trial when stimulus information is low, and a fuzzier (more spread out) internal representation should increase the neural contrast effect. Thus, a change in the amount of information available from a stimulus could have widespread effects throughout the judgment process. As a result, it is difficult to make exact quantitative predictions of outcomes of experiments that manipulate this quantity. However, a variety of qualitative predictions from computer simulations are possible and are discussed later.

The process model outlined above is compatible with a number of different mathematizations. Although presented informally here, the models of cross-modality matching and magnitude estimation have been formalized in computer programs. Thus, at least qualitative predictions can be made, and the logic of the models can be checked. In any specific process model, of course, a particular mathematization can be introduced—and this has been done in the present case to facilitate comparison with the work of Jesteadt, Luce, and Green (1977). Although the models are compatible with either of the two contenders for operating characteristics of sensory transducers, the log transform has been assumed for both stimulus and response continua. (See Lipetz, 1969, for convincing arguments against universal application of the power function option.) In this mathematization, the output of

Stage 1 is written as

$$\mathbf{X}'(\mathbf{S}_n) = k_1' \log \mathbf{I}(\mathbf{S}_n) - k_2' \log \mathbf{I}(\mathbf{S}_{n-1}) + c_1, \quad (6)$$

where $\mathbf{S}_n$ is the nominal stimulus on Trial n, $\mathbf{X}'(\mathbf{S}_n)$ is a random variable representing the location of the center of the fuzzy internal representation of the stimulus on Trial n, $\mathbf{I}(\mathbf{S}_n)$ is the physical intensity of the stimulus on Trial n, and k_1', k_2', and c are constants. Equation 6 expresses $\mathbf{X}(\mathbf{S}_n)$ (the original location of the center of the internal representation of $\mathbf{S}_n$) as modified by the neural sharpening process, $\mathbf{X}'(\mathbf{S}_n)$. The output of Stage 1 becomes the input to Stage 2, in which $\mathbf{Z}_I(\mathbf{S}_n)$, the category label assigned to $\mathbf{S}_n$ on the input category scale (now assumed to be an interval scale), is produced. When Stage 2 is completed, after its various heuristics have been applied,

$$\mathbf{Z}_I(\mathbf{S}_n) = k_3'\mathbf{X}'(\mathbf{S}_n) + k_4'\mathbf{Z}_I(\mathbf{S}_{n-1}) + c_2. \quad (7)$$

Stage 3 simply performs

$$\mathbf{Z}_0(\mathbf{S}_n) = \mathbf{Z}_I(\mathbf{S}_n), \quad (8)$$

where $\mathbf{Z}_0(\mathbf{S}_n)$ is the output scale label corresponding to $\mathbf{Z}_I(\mathbf{S}_n)$. In Stage 4, a response, $\mathbf{I}(\mathbf{R}_n)$, is produced on the output continuum according to the appropriate algorithm. In all cases, $\mathbf{X}(\mathbf{R}_n)$, the center of the fuzzy internal representation of $\mathbf{I}(\mathbf{R}_n)$, is determined from the first generated internal representation on the output continuum that sufficiently matches the prototype of $\mathbf{Z}_0(\mathbf{S}_n)$. Assuming that $\mathbf{X}(\mathbf{R}_n) = k_5 \log \mathbf{I}(\mathbf{R}_n) + c_3$, the output of Stage 4 can be shown to be (Ward, Note 2):

$$\mathbf{I}(\mathbf{R}_n) = (\exp c)\mathbf{I}(\mathbf{S}_n)^m \times \mathbf{I}(\mathbf{R}_{n-1})^d \mathbf{I}(\mathbf{S}_{n-1})^{-b}, \quad (9)$$

where c, m, d, and b are constants that are functions of the constants in the earlier equations. The logarithmic form of Equation 9 is

$$\log[\mathbf{I}(\mathbf{R}_n)] = m \log[\mathbf{I}(\mathbf{S}_n)] - b \log[\mathbf{I}(\mathbf{S}_{n-1})] + d \log[\mathbf{I}(\mathbf{R}_{n-1})] + c, \quad (10)$$

which closely resembles Equation 5. However, in Equation 5, $d = 1$, and $m = b$, whereas here $0 < d < 1$ and $m > b$ except where $m = b = 0$. (See Ward, Note 2.) Thus, the two models (Equations 5 and 10) make different predictions about the magnitude of sequential dependencies in psychophysical judgment as expressed by b and d. The regression model of Equation 3 is also suggested by Equation 10, and can be used to estimate m, d and b.

Rationale of the Experiments

The multiple regression analyses performed on magnitude estimation data by Jesteadt et al. (1977) showed that on average, $0 < d < 1(\bar{d} = .382)$ and $m > b$ ($\bar{m} = .273$, $\bar{b} = .052$).[4] These results agree closely with my own reanalyses of the data of Ward (1973) ($\bar{d} = .598$, $\bar{m} = .502$, $\bar{b} = .111$). Thus, although these estimated values agree with those expected under the fuzzy judgment approach, they show that the response ratio hypothesis cannot hold strictly. Jesteadt et al. argued that there are a variety of factors that tend to attenuate estimates of d. They also argued that the response ratio strategy may not always be used; other response strategies not involving $\mathbf{R}_{n-1}$ and $\mathbf{S}_{n-1}$ could be used on some trials and further attenuate estimates of d. Finally, they observed that the magnitude of d also depends on the separation between $\mathbf{S}_n$ and $\mathbf{S}_{n-1}$, although these more local estimates of d almost never approach the predicted value of 1 (see Luce & Green, 1978, for data of one unusual subject). Further experiments are needed to explore the usefulness of the response ratio approach and the fuzzy judgment approach as explanations of the observed sequential dependencies.

A possible starting place is a consideration of the role of stimulus uncertainty in the judgment process. According to the fuzzy judgment approach, stimulus un-

[4] Such regression analyses are somewhat vitiated by the finding of Jesteadt et al. (1977) that the magnitude of d depends on the separation between $\mathbf{S}_n$ and $\mathbf{S}_{n-1}$. However, the overall regression analyses should continue to provide a useful summary of sequential dependencies in a form related to the psychophysical power law.

certainty may be related to several aspects of the internal representation of the stimulus and decision processes regarding that stimulus. This is certainly a factor that could vary across different experimental scaling situations; its relationship to such things as stimulus range, presentation time, viewing or listening conditions, variability in the stimulus delivery apparatus (see Cain, 1977), and so forth, suggests that it may play a role in explaining different experimenters' different results from what is ostensibly the same method.

Early studies of absolute identification judgments focused on manipulations of the amount of information in the stimulus as measured by information theory. One of these studies (McGill, 1957) showed that the dependency of $\mathbf{R}_n$ on $\mathbf{R}_{n-1}$ was affected by such a manipulation of stimulus information. In a more recent study of sequential dependencies in absolute judgments, Ward and Lockhead (1971) showed that such a manipulation increased the assimilative dependency of $\mathbf{R}_n$ on $\mathbf{R}_{n-1}$. Given the similarity between the various forms of psychophysical judgment, it seems reasonable to test the effects of such manipulations on magnitude estimation and cross-modality matching judgments.

To ascertain the predictions of a rigorous version of the fuzzy judgment approach for such a manipulation, several computer simulations of the models outlined above were run, simulations in which parameter values were manipulated singly and in groups in the ways asserted by the metatheory for the effects of manipulations of stimulus information. The results of these simulations are summarized in Table 1. These simulations were of the cross-modality matching model, but results from the magnitude estimation model would be identical in form, since the only difference would be in the output stage, and the two output stages are mathematically equivalent. The entries in Table 1 are the average least-square multiple regression estimates of the parameters of Equation 3 made from the data produced by five 1,000-trial runs of the simulation model in each condition. The baseline condition is one that closely mimics data collected in previous experiments. The most important aspects of

Table 1
Regression Analyses (Equation 3) of Simulated Cross-modality Matching Data

Condition	Average regression parameter estimates			Average variance accounted for		
	γ	β	α	S_n	R_{n-1}	S_{n-1}
1. Baseline	.570	.183	−.062	.926	.007	.0008
2. Increase neural contrast	.607	.061	−.058	.935	.001	.001
3. Increase fuzziness of internal representation	.360	.377	−.041	.759	.085	.002
4. Conditions 2 and 3	.400	.377	−.094	.827	.033	.008
5. Decrease number of categories used from 10 to 6 and make categories fuzzier and more overlapping	.521	.081	−.042	.917	.00015	.00056
6. Same as Condition 5 but with only four categories	.437	.085	−.024	.895	.002	.00004
7. Increase variance of distribution of center of internal representations	.365	.192	−.029	.773	.017	.001
8. Conditions 4, 5, and 7	.287	.404	−.062	.540	.083	.012
9. Conditions 4, 6, and 7 and reduce maximum response range	.315	.069	−.035	.774	.0004	.002
10. Condition 9 but always same stimulus	—	.541	—	—	.294	—

Table 1 are that increases in fuzziness of internal representations alone (which are assumed to result from decreases in stimulus information) cause increases in β (the estimate of the effect of $\mathbf{R}_{n-1}$ on $\mathbf{R}_n$—d in Equations 5 and 10) and decreases in γ (m—the effect of $\mathbf{S}_n$ on $\mathbf{R}_n$). Conditions 3 and 4 demonstrate this. Even changes in the (implicitly ordinal) category scale used (Conditions 5 and 6) do not cancel out the effect of a large increase in fuzziness (Condition 8), although alone they tend to decrease the effects of previous stimuli and responses. A combination of a smaller fuzziness increase, a large neural contrast increase, a reduction in the number of categories used, and a reduction of the maximum response range (Condition 9) also leads to a result with a lower β than the baseline. Notice also that only increasing neural contrast alone increases γ (the effect of $\mathbf{S}_n$ on $\mathbf{R}_n$—the "true" exponent); all other manipulations decrease γ. Finally, α (the estimate of b) is always negative, and seems generally to be (absolutely) decreased by the various manipulations. It is always (absolutely) about an order of magnitude smaller than γ.

To summarize, then, the fuzzy judgment model asserts that decreasing stimulus information would be expected to decrease γ, increase β, and (absolutely) decrease α, although β may also be decreased under some conditions.

It is difficult to wrest predictions of this type from the response ratio approach, since stimulus information and response uncertainty are not explicitly considered there. Jesteadt et al. (1977) argued that the response ratio decision rule may be used only when both the current and previous stimuli are within a band of attention that tends to be centered around the previous stimulus location. An increase in stimulus uncertainty should increase the variability of the internal representation of the stimulus. However, it is not clear just what effect this would have on the probability of both current and previous signals being within the attention band. It could either increase, decrease, or leave unchanged the use of the response ratio response strategy, depending on a number of factors not discussed by Jesteadt et al. At any rate, if use of the response ratio rule changed in frequency, the changes in d and b in Equation 5 should occur together, with d approaching 1 and b approaching m as use increases, and d and b approaching 0 as use decreases. The effects of decreases in stimulus information on m are also difficult to predict, since m is a transducer operating characteristic in the response ratio approach. In the strict Stevensonian view, central tendency and variability of responses are unrelated and the exponent of the psychophysical power function depends only on central tendency. Taking this view seriously would predict minimal effects of changes in stimulus clarity and such on m. Any other prediction would necessitate specification of some functional relationship between central tendency and variability—a specification not easily derivable from the response ratio approach.

To test some of the above predictions, two experiments were run in which the amount of stimulus information available was manipulated by varying the duration and contrast of the visual stimuli. Subjects were asked to make magnitude estimation or cross-modality matching judgments of the separation between pairs of small dots. These stimuli were used because of the requirement to manipulate stimulus clarity and because it was thought to be desirable to replicate some of the effects in a non-auditory modality in order to increase their generality. Amount of stimulus information was conceived similarly to the concept of information or uncertainty in information theory. (See Garner, 1962.) Such manipulations have been shown to affect the amount of information transmission for a variety of similar stimuli. (See, e.g., Lockhead, 1966.) Information transmission was measured, in order to check on the manipulation, by requiring absolute identification judgments in the various conditions after the magnitude estimation or cross-modality matching judgments were completed. An additional condition, in which the same stimulus was always presented under degraded conditions (see McGill, 1957), contained no stimulus information at all in this sense. This condition gave an estimate of the maximum dependency of $\mathbf{R}_n$ on $\mathbf{R}_{n-1}$ that can be expected in these scaling methods. Comparison of these data with those from the experiment by Ward and Lockhead (1971), in which there were no stimuli at all, should allow an assessment of the degree of involvement of purely response processes (such as heuristics) in the sequential dependencies.

Experiment 1: Magnitude Estimation

Method

Subjects. Eight volunteer subjects, of both sexes and between 18 and 30 years of age, were paid $2.50 an hr. All subjects had normal or corrected-to-normal visual acuity.

Stimuli and apparatus. The 10 stimuli were pairs of small black dots printed on white cards, front-lighted in a two-channel Gerbrands tachistoscope. Separation between dot pairs ranged from .5 cm to 5 cm in .5-cm steps. These separations were the values of the stimulus continuum to be judged (i.e., interdot distance). In the high-information condition, the tachistoscopic field was brightly lit and each stimulus was exposed for 500 msec. In the low-information condition and the no-information condition, the tachistoscopic field was dimly lit and exposure duration was only 130 msec. In the high- and low-information conditions, a randomly chosen dot pair of the 10 different dot-pair stimuli was presented on each trial. In the no-information condition, however, the 2.5-cm separation dot pair was presented on every trial, so that there was no variation in the nominal stimulus at all. At the onset of each trial, subjects were fixating on a tiny red dot in a dim field, slightly to one side of the location of the stimulus dots. The stimulus presentation was foveal in all cases.

Procedure. All subjects performed in all three conditions of the experiment, in counterbalanced orders, in three separate 1-hr sessions. In each condition, subjects made about 300 magnitude estimations (about 30 per stimulus) of the distance between the two dots. The magnitude estimation technique was the one used by Ward (1971, 1973) and by Curtis (1970) and others, in which on each trial the previous stimulus serves as the standard. The task of the subject is to calculate the response to the current stimulus so that the ratio of current and previous responses reflects the judged ratio of the sensation values of the current and previous stimuli. This is a literal expression of the ratio nature of the magnitude estimation task. Results in the task do not differ in form from those in the more usual magnitude estimation task (cf. Cross, 1973; Luce & Green, 1974). In the low- and no-information conditions, subjects sometimes complained that the stimulus was not clearly visible, as would be expected from the conditions of low contrast and short exposure duration. Complainers were told that judgments made in such circumstances often still did reflect sensation magnitude, albeit perhaps "subliminally," and that they were not to despair but to continue to try to follow instructions.

To objectively assess the amount of stimulus information that could be transmitted in the high- and low-information conditions, all subjects performed 100 trials of absolute identification judgments without feedback in each of those conditions. This was done in an additional 1-hour session after all magnitude estimations had been completed.

Table 2
Results of the Absolute Judgment Experiments

Information condition	n	Average individual information transmission[a]	Average % correct
	Experiment 1—magnitude estimation[b]		
High	8	2.002	46.12
Low	8	.905	17.00
	Experiment 2—cross-modality matching[c]		
High	11	2.330	48.76
Medium	11	2.219	43.16
Low	7	1.761	43.51

[a] See Garner and Hake (1951).
[b] High- and low-information conditions differ significantly (by t test) in both information transmission and percentage correct (α = .01).
[c] High- and medium-information conditions do not differ significantly in information transmission at α = .05; both differ from the low-information condition at α = .01. None of the three conditions differ in percentage correct at α = .05.

Results and Discussion

Absolute judgments. Table 2 shows the results of the absolute identification judgments done to check the manipulation of amount of available stimulus information. The manipulation was effective, since the average information transmissions in the two conditions are significantly different. These average information transmissions may be somewhat inflated since they are based on only 100 trials per subject per condition, so they should not be considered to be accurate estimates of actual information available; such estimates merely establish the effectiveness of the manipulation. The manipulation of stimulus information also had the expected effect on the variability of responses to individual stimuli in the magnitude estimation phase of the experiment. The average response variance in the high-information condition was significantly different from that in the low-information condition for each of the 10 stimuli ($\alpha = .01$).

Multiple regression analyses. Table 3 displays a summary of the results of a multiple regression analysis of the magnitude estimation data. In this analysis, Equation 3 was the regression equation whose parameters were estimated. This is the regression equation recommended by Jesteadt et al. (1977). The order of entry of $\mathbf{I}_{n-1}$ and $\mathbf{R}_{n-1}$ into the equation makes no difference for these data. Duplicating the complete analysis sequence of Jesteadt et al. (1977) on the present data gave similar results for individual subjects in the high-information condition, which is closest to their own situation, in that most subjects showed statistically significant effects only for the present stimulus and the immediately previous stimulus and response. Similar analyses on all subjects' data combined showed that higher order effects, although small, were statistically reliable. In addition, sign tests on the estimated coefficients from the individual subjects' data indicated that the direction of higher order effects was consistent up to several trials back in the sequence. However, since such effects were quite small, they were ignored for the purposes of this article.

It can be seen in Table 3 that the results for the high-information condition are very similar to those for magnitude estimations and category judgments of loudnesses (cf. Jesteadt et al., 1977). The estimate of γ (m in Equations 5 and 10), .931, is very close to the typical power function exponent for line length of about 1.0. The estimate of β (d) is somewhat smaller than for Jesteadt et al.'s data, and that of α (b) is somewhat larger, but they are of the same order of magnitude and have the same signs. These data confirm once more, and for visual stimuli, that for conglomerate data, $0 < d < 1$ and $m > b$ as predicted by the fuzzy judgment model. (The data of the individual subjects are highly similar

Table 3
Regression Analyses (Equation 3) of Magnitude Estimation and Cross-Modality Matching Data

Information condition	n	Average regression parameter estimates			Average variance accounted for		
		γ	β	α	S_n	R_{n-1}	S_{n-1}
Experiment 1—magnitude estimation							
High	8	.931	.143	−.109	.809	.008	.011
Low	8	.191	.422	.011	.088	.273	.019
No	8	—	.416	—	—	.305	—
Experiment 2—cross-modality matching							
High	22	.539	.269	−.126	.758	.010	.015
Medium	7	.381	.175	−.078	.489	.009	.008
No	12	—	.340	—	—	.148	—

to the average data, as was the case for the data of Jesteadt et al., and so only the averages are reported here.)

Comparing the parameter estimates for the low- and high-information conditions reveals the effects of the manipulation of stimulus information. The estimated γ decreased dramatically from .931 to .191; the estimated α increased to near 0, and the estimated β increased to .422. The variance accounted for generally reflects these changes, with that of $\mathbf{R}_{n-1}$ increasing to 27.3% from .8%, that of $\mathbf{S}_n$ decreasing dramatically, and that of $\mathbf{S}_{n-1}$ increasing slightly. In Table 1 it can be seen that this is one of the patterns predicted by the fuzzy judgment model (Conditions 3 or 4). Thus, this result is consistent with that model. However, it is inconsistent with the response ratio hypothesis. Although β is closer to 1, α should have become absolutely larger (more negative); instead, it approached 0 rather than γ. Also, the decrease in γ would not be expected under the operating-characteristic viewpoint.

The no-information condition, of course, only provides an estimate of β, since $\mathbf{S}_n$ and $\mathbf{S}_{n-1}$ did not vary. The value of .416 is not significantly different from the value of .422 for the low-information condition, indicating that this may be near an upper limit for β under these conditions. The variance accounted for does increase marginally in this condition, but is nowhere near 1.0. The remainder of the variance (69.5%) in this condition could be caused by subjects' number production biases (see Baird & Noma, 1975), which are quite noticeable in magnitude estimation. The data for this condition, then, follow closely the predictions of the fuzzy judgment model (see Condition 10 in Table 1). The response ratio hypothesis would also predict such a result.

Experiment 2: Cross-Modality Matching

Method

Subjects. Twelve volunteer subjects, of both sexes and between 18 and 38 years of age, were paid $2.50 an hr. All subjects had normal or corrected-to-normal visual acuity.

Stimuli and apparatus. Stimulus presentation and response timing were controlled by a PDP 11/10 computer system. Stimuli were pairs of dots (single points) presented on a Tektronix 611 oscilloscope screen. The 10 separations between the dots were 1.08 cm, 2.16 cm., 3.24 cm., 4.31 cm., 6.47 cm., 8.63 cm., 10.78 cm., 12.94 cm., 15.10 cm., and 17.26 cm. Viewing distance was about 1 m. In the high-information condition, stimuli were present for 1 sec, centered on the screen, from .5 to 1.5 sec after a fixation "X" disappeared from the screen. Room illumination was low and the dots were clearly visible. The medium-information condition was identical to the high-information condition except that stimulus duration was only 100 msec. In the low-information condition, stimuli were presented for only 1 msec and moved randomly about the center of the screen from trial to trial. In addition, room illumination was high so that the stimulus dots were more difficult to see. The no-information condition was identical to the low-information condition except that the 6.47-cm stimulus was presented on every trial. Cross-modality matching responses were the elapsed time between two key presses on a keyboard located in front of the subject (timed by the computer). Thus, durations were generated to match the perceived distances between dot pairs on the oscilloscope screen. Absolute identification (without feedback) responses were the numbers 1 through 0 (for 10) typed on the keyboard. The computer also recorded response latencies in all conditions.

Procedure. All subjects performed in all four conditions of the cross-modality matching experiment, in counterbalanced orders, in separate 1-hr sessions. However, because of a programming error, some data were lost and the numbers of subjects reduced to 11, 11, and 7 for the high-, medium-, and low-information conditions, respectively. In each session, subjects made about 500 cross-modality matches (about 50 per stimulus). Subjects were told to produce a duration on the keyboard that subjectively matched the distance between the pair of dots presented on each trial. If subjects complained of poor visibility of the stimuli (as they sometimes did), they were asked to continue to try to do their best to follow instructions.

As in Experiment 1, after completing the cross-modality matching conditions, all subjects made absolute identification judgments under the high-, medium-, and low-information conditions, in order to objectively assess available stimulus information. Subjects made about 10 judgments per stimulus (100 trials in all), without feedback in each condition, in an additional 1-hr session.

Results and Discussion

Absolute judgments. Table 2 shows the results of the absolute identification judgments done to check the manipulation of amount of available stimulus information. This manipulation was not as successful as that in Experiment 1, since the high- and medium-information conditions differed on neither information transmission nor percentage of correct responses. Thus, henceforth in this article, the data from these two conditions are collapsed and reported as the high-information condition. Also, although both high- and medium-information conditions differed significantly from the low-information condition in information transmission, information transmission in the low-information condition was not really very low. Thus, henceforth, this condition is referred to as the medium-information condition. Average cross-modality matching response variances in the new high- and medium-information conditions mirrored the information transmission results in that they were significantly larger for each stimulus in the medium information condition (α = .05).

Multiple regression analyses. Table 3 displays the average results of a multiple regression analysis (Equation 3) for the cross-modality matching data. Once again, the data of individual subjects were highly similar to the average data and so are not presented. Also, again, the complete series of multiple regression analyses performed by Jesteadt et al. (1977) gave similar results when performed on the present data. The major dependencies of $\mathbf{R}_n$ were on $\mathbf{S}_n$, $\mathbf{S}_{n-1}$ and $\mathbf{R}_{n-1}$, and so only the results of analyses using Equation 3 are presented.

The high-information condition data depart somewhat from expectation. The average estimated γ = .539 is not very close to the expected exponent of about 1.0. (Exponents for both line length and duration are about 1, and 1/1 = 1.) The average values of both β and α are absolutely larger than for the magnitude estimation data, although very similar to those obtained in the previous study of sequential dependencies in cross-modality matching (β = .269, α = −.070; reanalysis of data of Ward, 1975). The variance-accounted-for results indicate that, in line with the low value for γ, $\mathbf{S}_n$ accounts for only 75.8% of the variance in $\mathbf{R}_n$. Since $\mathbf{R}_{n-1}$ and $\mathbf{S}_{n-1}$ add only 1% and 1.5% each, there must be a large additional source of variance. This is undoubtedly the variance inherent in producing duration responses; intramodal duration matches are highly variable and the variance increases with duration matched. (See Ward, 1975.)

The medium-information result is also somewhat unexpected. Although both estimated γ and α decreased in absolute magnitude (as expected from the majority of the simulation runs of the fuzzy judgment model), the value of β also decreased compared with the high-information condition. Only Conditions 5, 6, and 9 gave this result in the simulations, indicating that from the fuzzy judgment point of view, the manipulation must have resulted in a decrease in the number of categories used that was not wholly compensated for by the increase in fuzziness of the internal representations of the stimuli. Why this happened in cross-modality matching but not in magnitude estimation is still a mystery, since there were several differences other than response mode between the two experiments. At any rate, the fuzzy judgment model also can account for these results, although they would not be expected under most conditions. From the point of view of the response ratio hypothesis, as extended to cross-modality matching, these data would reflect an overall decrease in the use of the response ratio strategy with decreasing stimulus information, although, again, the decrease in γ is not dealt with.

The no-information condition gives essentially identical results in cross-modality matching to those in magnitude estimation, except that the value of β (.340) is somewhat lower, as is the average variance accounted for (.148). Perhaps the larger, necessary, response variance caused by the inability to repeat responses exactly or to produce exactly the desired response, is responsible for this lower estimate of a maximum for β in cross-modality matching. As in the magnitude estimation study, this condition cannot in principle differentiate between the two approaches.

Response latencies. A result of interest from the point of view of the fuzzy judgment model is that of the response latencies in the high- and medium-information conditions. That model would predict longer latencies in the medium-information condition, since the biasing heuristics would have to be used more there because of increased average size of the "indifference set." This prediction is confirmed by the data, with average response latency in the high-information condition of 577 msec, whereas that in the medium information condition was 903 msec. This difference is significant at $\alpha = .01$ by t test.

Second-Order Dependencies

Jesteadt et al. (1977) reported an analysis of their data for what I will term second-order dependencies. They found that the magnitude of the sequential dependency of $\mathbf{R}_n$ on $\mathbf{R}_{n-1}$ itself depended on the separation between $\mathbf{S}_n$ and $\mathbf{S}_{n-1}$. When $(\mathbf{S}_n - \mathbf{S}_{n-1})$ was small, the effect of $\mathbf{R}_{n-1}$ on $\mathbf{R}_n$ was relatively large, although not very near 1 in any case; the effect of $\mathbf{R}_{n-1}$ on $\mathbf{R}_n$ decreased with increasing $(\mathbf{S}_n - \mathbf{S}_{n-1})$. This result is not at all expected under the response ratio hypothesis. Jesteadt et al. (1977), therefore, argued that subjects may base $\mathbf{R}_n$ on $\mathbf{S}_n$ alone when one or both signals fall outside the attention band (more likely when $[\mathbf{S}_n - \mathbf{S}_{n-1}]$ was large), and only use the response ratio strategy when both $\mathbf{S}_n$ and $\mathbf{S}_{n-1}$ fall within the band (more likely when $[\mathbf{S}_n - \mathbf{S}_{n-1}]$ was small if subjects tended to center the attention band around $\mathbf{S}_{n-1}$). It seems important to examine the present data to see whether the result is predicted by the fuzzy judgment models.

Jesteadt et al. (1977) used the following regression equation to obtain estimates of β, the amount of dependency of $\mathbf{R}_n$ on $\mathbf{R}_{n-1}$:

$$\log (\mathbf{R}_n/k\mathbf{I}_n{}^{\gamma}) = \beta \log (\mathbf{R}_{n-1}/k\mathbf{I}^{\gamma}{}_{n-1}) + \delta + \varepsilon, \quad (11)$$

for various ratios $\mathbf{I}_{n-1}/\mathbf{I}_n$. In Equation 11,

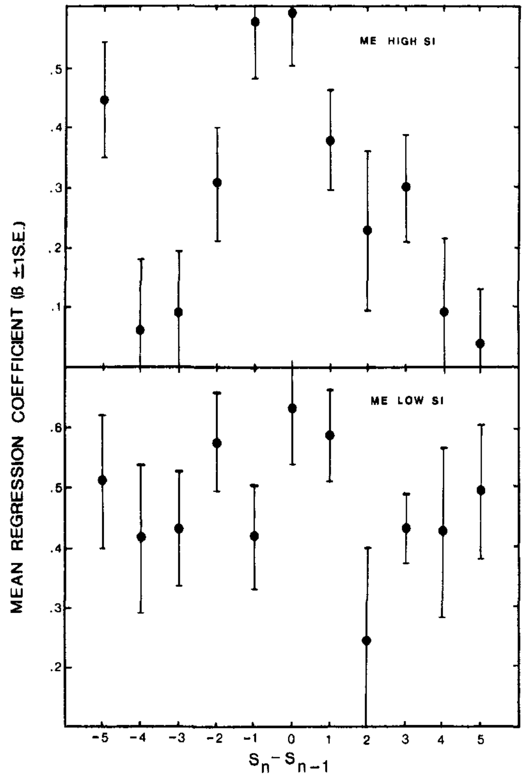

Figure 1. Average regression coefficients ($\beta \pm SE$) from Equation 11 for the data of the magnitude estimation experiment. (High SI = high stimulus information condition; low SI = low stimulus information condition. Note that $[\mathbf{S}_n - \mathbf{S}_{n-1}]$ is in "steps," not physical units. Each part of the figure comprises coefficients estimated from eight 300-trial runs.)

k and γ are estimated from standard power function fits to the data of individual subjects (or single runs of the simulation model). This regression analysis was done for all of the present magnitude estimation and cross-modality matching data except the no-information conditions, and for the data from ten 1,000-trial runs of the simulation model of fuzzy cross-modality matching under each of two conditions (baseline and 9). The general form of the simulation data would be the same for magnitude estimation except that the differences between conditions might be different if different conditions were run. The results of all of these analyses are summarized in Figures 1, 2, and 3. In these figures, only the average estimated βs

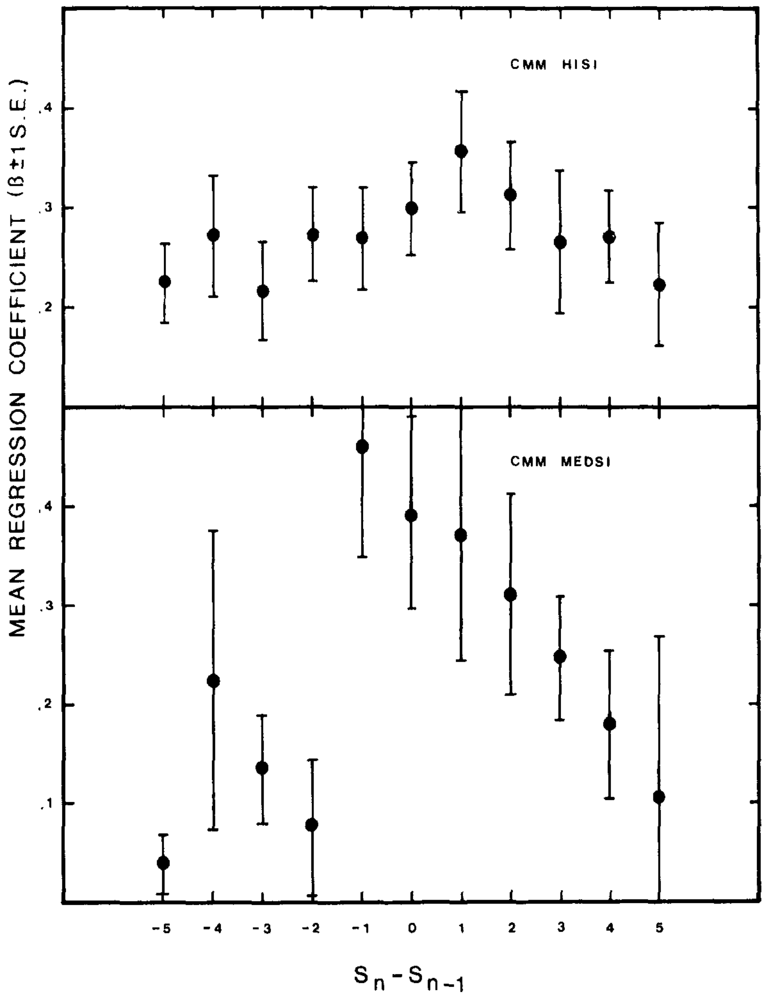

Figure 2. Average regression coefficients ($\beta \pm SE$) from Equation 11 for the data of the cross-modality matching experiment. (HISI = high stimulus information condition; MEDSI = medium stimulus information condition. Note that $[S_n - S_{n-1}]$ is in "steps," not physical units. The high stimulus information figure includes two 500-trial runs for each of 11 subjects. The medium stimulus information figure is based on one 500-trial run from each of 7 subjects.)

for separations of up to five steps between S_n and S_{n-1} are displayed. (β values for larger separations were based on too few responses to be stable.) Note also that the abscissa of each figure is in "step" units, not the physical stimulus units.

The pattern of results in Figure 1 closely resembles that found by Jesteadt et al. (1977) in their magnitude estimation data, although the pattern here is more variable than theirs because the points are based on smaller numbers of trials. Also, the variability precludes making any firm conclusions about the effects, if any, of the manipulation of stimulus information on the decrease of β with increasing $(S_n - S_{n-1})$, although the figure for the low-information condition does look flatter. The cross-modality matching pattern (Figure 2) is clearer, and represents another striking similarity between cross-modality matching and magnitude estimation. The pattern is also unequivocal for the simulation data (Figure 3), even to the point of reproducing the gradual flattening of the curves at large values of $(S_n - S_{n-1})$.

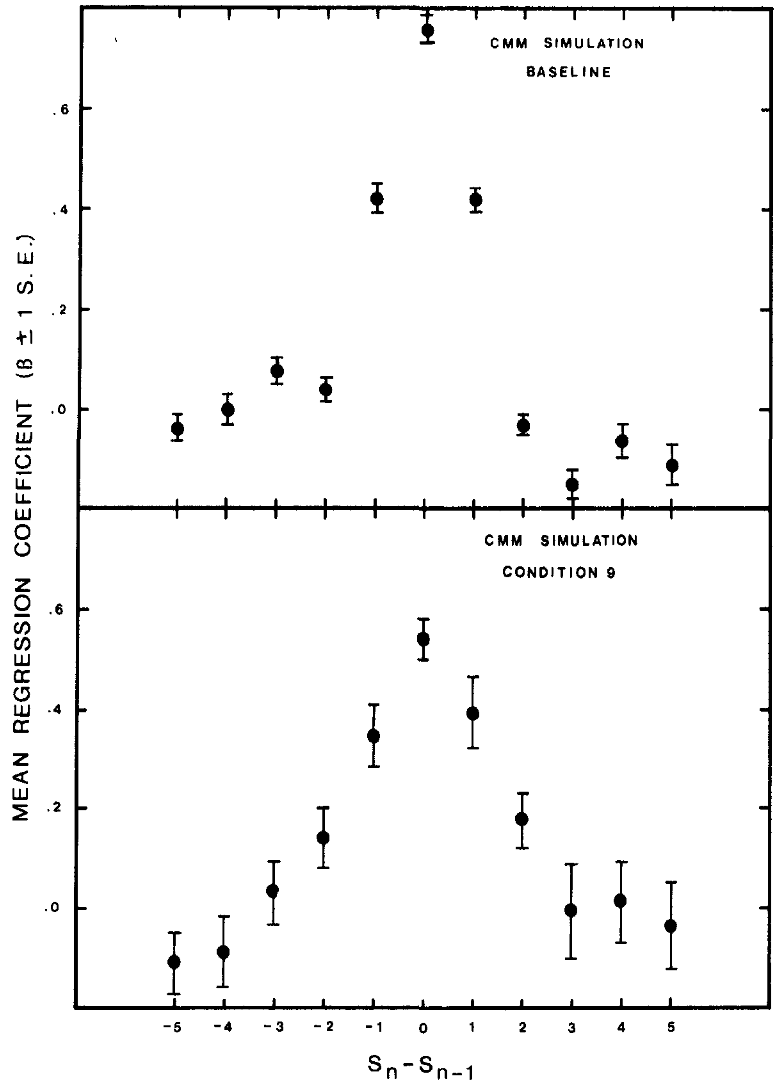

Figure 3. Average regression coefficients ($\beta \pm SE$) from Equation 11, based on the data from 10 1,000-trial runs of the computer simulation model in each of two conditions (Baseline and 9 in Table 1). (Note that $[S_n - S_{n-1}]$ is in "steps," not physical units.)

Since no special changes were made in the model to accommodate these findings, the presence of the inverted-V pattern in the simulated data is a rather striking confirmation of the fuzzy judgment model. The same pattern, with some minor variations, has also appeared in simulated data under a selection of other parameter values. So far, changes in parameter values have moved the inverted V up or down but have not appreciably changed its shape.

The natural question as to which aspect of the fuzzy judgment model is responsible for this pattern is difficult to answer, for the model as implemented on the computer is complex. However, the major factor is probably the fact that the larger $(S_n - S_{n-1})$, the more likely it is that S_n is near one end or the other of the stimulus scale. For such extreme stimuli, the set of category labels among which the (simulated) subject is uncertain is usually smaller than for more central stimuli, since there are no category prototypes beyond the end of the category scale for the internal representation of $\mathbf{S}_n$ to overlap with. The smaller uncertainty set will lead to more frequent classification of $\mathbf{S}_n$ into a category without the use of the biasing heuristic, and a smaller bias when the heuristic is used. Since the heuristic is the source of the relationship between $\mathbf{R}_n$ and $\mathbf{R}_{n-1}$ in this approach, the smaller and less frequent heuristic-caused bias would result in a weaker effect of $\mathbf{R}_{n-1}$ on $\mathbf{R}_n$ (β in the present analyses). The same result is predicted by the model for both cross-modality matching and magnitude estimation, since the identical biasing factor operates prior to the different final stages of judgment.

Conclusions

This article has investigated descriptive models of magnitude estimation and cross-modality matching derived from two different approaches to a theory of psychophysical scaling judgments. Although the descriptive models are similar in form, they differ in their predictions about the parameter values to be expected under various conditions. A pair of experiments in magnitude estimation and cross-modality matching in which stimulus information was manipulated allowed a further exploration of the usefulness of the two approaches. The response ratio approach seems to be inferior to the fuzzy judgment approach as a process model of the generation of the data available to date for the following reasons: (a) A strict view of the response ratio hypothesis has $d = 1$ and $m = b$ in Equation 5, whereas a simple mathematization of the fuzzy judgment approach has $0 < d < 1$ and $m > b$ in Equation 10; in all of the data for magnitude estimation and cross-modality matching available to date, the predictions of the fuzzy judgment approach are confirmed, whereas those of the response ratio approach are disconfirmed. (b) The manipulation of stimulus information causes changes in sequential dependencies in magnitude estimation and cross-modality matching that are all predicted by a computer simulation of the fuzzy judgment approach, whereas the response ratio approach has difficulty with decreases in the standard power function exponent and with different directions of change in the coefficients of $\mathbf{S}_{n-1}$ and $\mathbf{R}_{n-1}$. (c) The no-information conditions show the presence of sequential dependencies even when no stimulus information (in the information transmission sense) is available, and similar response contingencies occur even when there is no stimulus at all (see Ward & Lockhead, 1971); such data suggest a biasing heuristic to be responsible for the response dependencies—indeed, the no-information condition data are predicted by the fuzzy judgment approach. (d) The ubiquitous second-order dependencies discovered by Jesteadt et al. (1977) are explained naturally by the simulation model derived from the fuzzy judgment approach; the response ratio hypothesis does not predict such dependencies and can only be made consistent with them by resorting to ad hoc arguments about special conditions under which it may not be used (when at least one of $\mathbf{S}_n$ or $\mathbf{S}_{n-1}$ is not in the attention band).

Although the fuzzy judgment models and the response ratio hypothesis make some different predictions, the approaches from which they were derived are not totally incompatible. If the response ratio hypothesis is disconfirmed, there is still a need for a hypothesis regarding the judgment process in the neural timing theory approach. Something like the judgment process specified by the fuzzy judgment models might be very useful here. A successful merger of the two approaches would certainly have more usefulness and generality than either approach alone.

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