INSTANTANEOUS Power Spectrum – a powerful tool for ML

Short-term Fourier Transform and its magnitude-squared, "Spectrogram", are well known tools in signal processing. This article re-introduces an important variation - "Instantaneous" Fourier Transform and Kalman Time Frequency Distribution ("KTFD"); KTFD is an estimate of the power spectrum at an instant of time! This may raise some conceptual concerns which I will address below. In any case, some recent work I was doing in digital twins showed me (again) the amazing power of KTFD and how it could be a very valuable tool in Machine Learning.

Back in 2017, I published an article called, "Importance of Feature Extraction in IoT", https://www.linkedin.com/pulse/importance-feature-extraction-iot-pg-madhavan/ , where I discussed a "new" signal processing tool for time series. Since IoT generates a lot of time series, an additional powerful method in any Data Scientist's toolkit will come in handy sooner or later. This work is based on a much earlier published research - PG Madhavan & WJ Williams, Kalman Filtering and Time Frequency Distribution of Random Signals, SPIE Proceedings, SPIE Vol. 2846, pp. 164-173, 1996.

Time frequency distributions have long history with significant research activities in the 1980s and 90s. What TFD does is to take a time series (amplitudes on time axis) and transform it into a distribution on the time-frequency plane; the amplitude of the distribution is an estimate of the power spectral density at each time-frequency location.

What is a frequency spectrum at a point in time?! For frequencies to "exist", you need a certain extend of consecutive time samples. This is the idea of short-term Fourier Transform. Take a smaller and smaller window of points around the time point of interest and calculate FFT. As the window gets smaller, the frequency estimates "spread out" due to the windowing effect. So, there is a time resolution - frequency resolution trade-off.

The research article cited above brought state-space model and Kalman filtering into the estimation process. The article explains the full theory and describes various simulations; the comparison with other TFD methods (such as Spectrogram) shows that KTFD is far superior.

The best way to see what KTFD can do for you is to show an example.

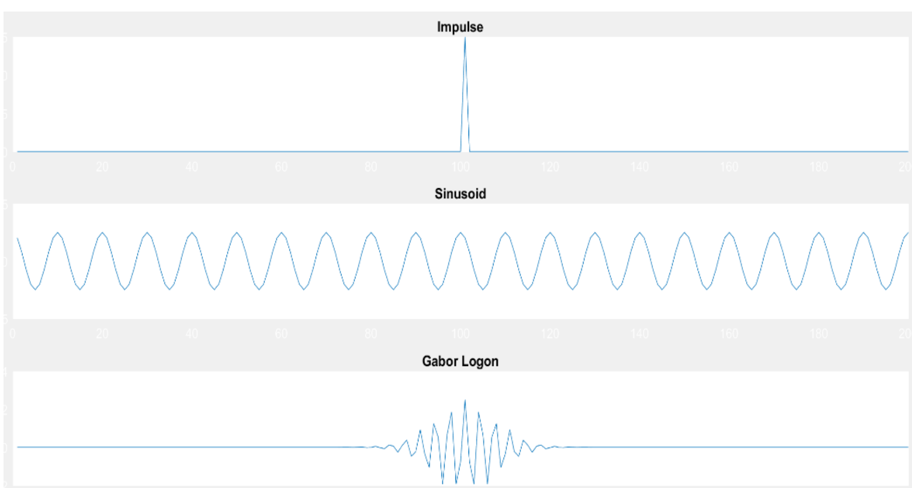

Figure 1. Time series (3) that are added together for KTFD estimation

Signal at the top in figure 1 is an impulse, middle is a sinusoid and bottom is called a “Gabor logon”. Gabor logon has a special place in TFD literature (you can see the definition in my research publication cited above). These 3 signals are combined at different time and frequency locations and its KTFD is estimated. The results are shown in figure 2.

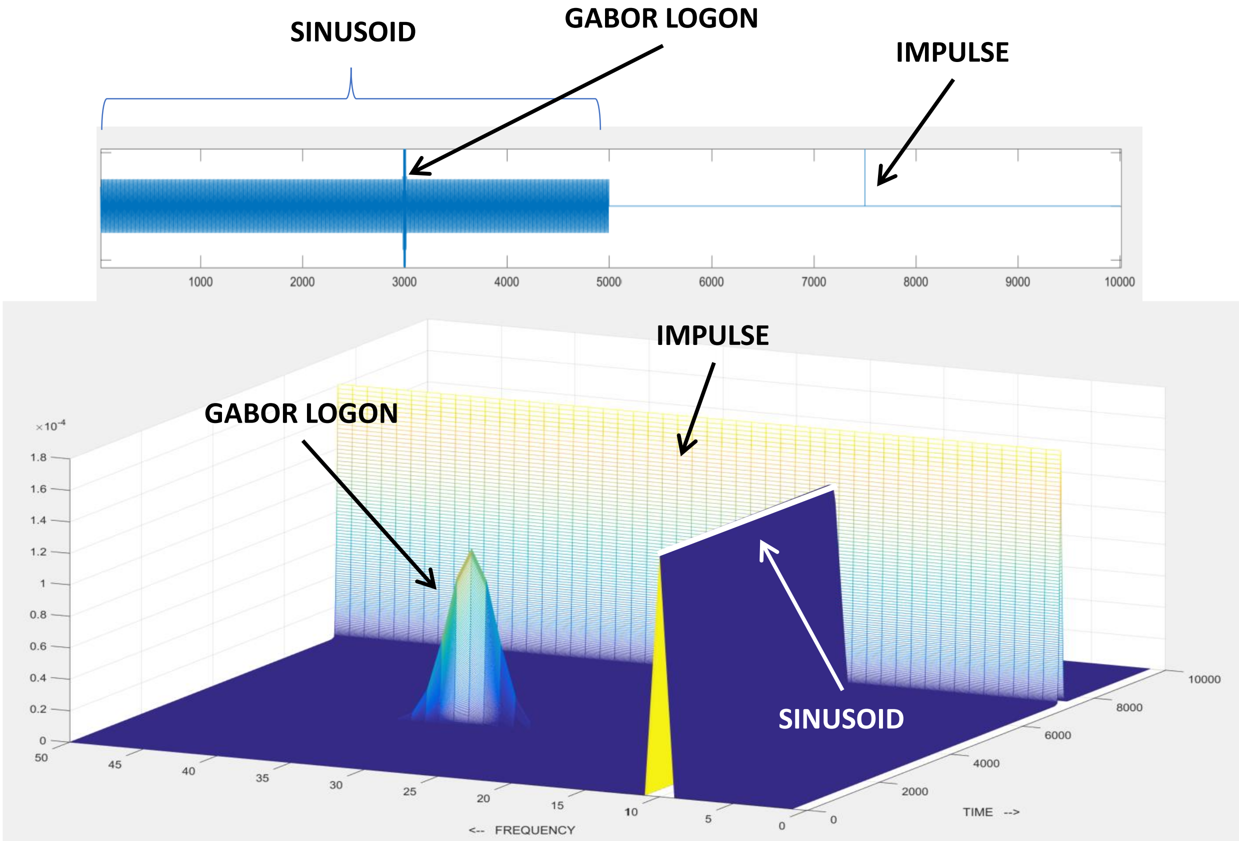

Figure 2. Kalman TFD with time series on the top.

Sinusoid spectrum is a knife-edge at its frequency fraction running along the time direction (dark BLUE). Impulse spectrum is across ALL frequencies at its time location. Gabor logon has a Gaussian bell shape at the time and frequency location it was placed.

For example, Gabor logon peak is located at {t, f} = (3000, 30}. If we consider a frequency “slice” at t=2000, going left along the frequency axis, we will see a non-zero power spectrum value at {2000, 10} corresponding to the sinusoid. So we are indeed obtaining the INSTANTANEOUS power spectral density at a single instant of time! Comparison to Cramer spectral representation theorem and related technical discussions can be followed in my research publication from 1997.

Of course, if we collapse the 3-D picture onto the frequency axis, we get the traditional power spectral density estimate. Since sinusoid is located at a specific frequency (‘10’ in this case which is 1/10 th of the sampling frequency), it shows up very clearly in figure 2. However, in a test case, the observer will be hard-pressed to identify the Gabor logon at ‘30’ and will have even more difficulty seeing an impulse!

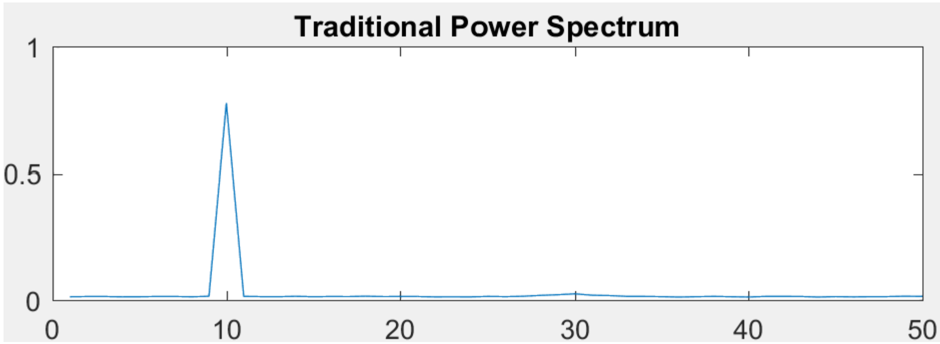

Figure 3. Traditional power spectrum of data in figure 2.

One of the well-known advantages of TFD in general is its characteristic excellent noise performance. It is demonstrated for KTFD below by adding 0db SNR white noise to the pure signals in figure 1.

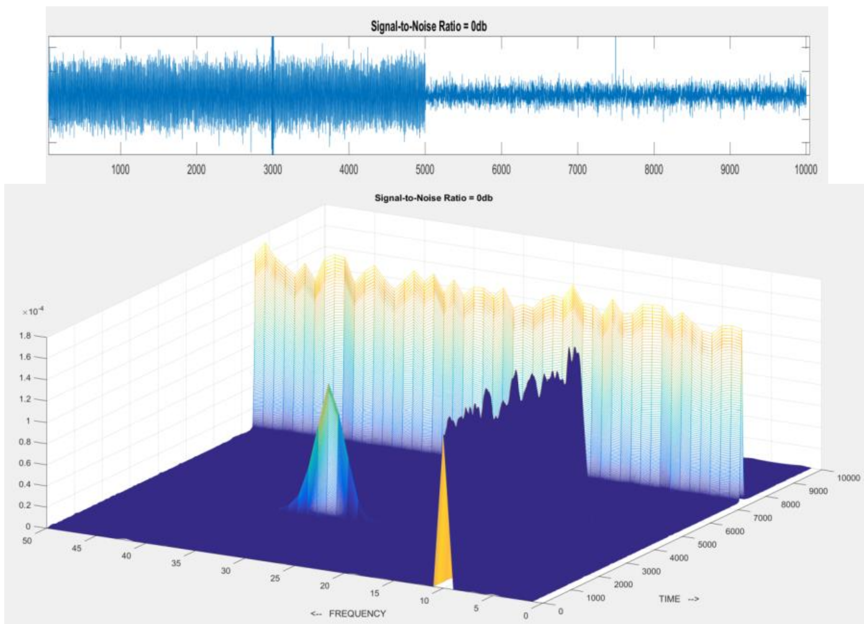

Figure 4. KTFD in the presence of 0db SNR white noise

In figure 4, the top portion shows the noisy time series. There is no hope of detecting the sinusoid at all from the time series, let alone get a good power spectrum estimate by traditional methods. Identifying the impulse and Gabor logon may also be difficult since they appear similar to some outlier noise. In the TFD case, noise energy is distributed evenly (since the noise is white) on the time-frequency plane and its “height” is thus small compared to the signal with restricted time and or frequency support.

However, figure 4 bottom panel shows that the KTFD clearly shows all 3 signal components and pretty accurate spectral estimates even though with higher variance than what we see in the pure data case in figure 2.

Kalman-TFD in ML:

I see two major uses for KTFD in ML.

1. Feature Engineering: Instead of working with the time series itself or the traditional power spectrum (as in figure3), if you preprocess your IoT time series data channel with KTFD, the kind of features that can be detected and then used in ML will be highly information bearing which enhances the chance of faster learning, higher accuracy and more robustness over time.

2. Deep Learning: KTFD output can be projected down to the time-frequency plane to get a "matrix' 2-D view. Also, one can use a 2-D image of the 3-D output from a fixed point of view as input to deep learning (DL) systems. DL will then be able to learn more subtle features from the image than the eye can see (or from pre-processing).

To facilitate ML applications by anyone, I have placed KTFD Matlab function on GitHub including the original research article for those who want to go deeper into the theory.

1. KTFD.m

2. PG Madhavan & WJ Williams, Kalman Filtering and Time Frequency Distribution of Random Signals, SPIE Proceedings, SPIE Vol. 2846, pp. 164-173, 1996.

Here is the GitHub link.

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#Kalman #powerspectrum #ktfd #ML #timeseries #signalprocessing #iot