Math 251 Sections 1,2,10 FAQ 8/31/09

1. Suppose that you open a savings account which earns interest at a 4% annual rate compounded semiannually. Also suppose that your initial deposit is \$1000. After one year what will be the balance in this account, accurate to the exact penny?

ANS.

$$1000(1 + .04/2)^2 = 1000(1 + 2(.02) + (.02)^2) = 1000 + 40 + 0.4 = 1040.40$$

2. Suppose that you open a savings account which earns interest at a 10% annual rate (compounded continuously) with a deposit of \$1000. Also suppose you make continuous payments at the rate of \$500 per year. Write an ODE with initial condition which models the balance $P(t)$ in this account.

ANS. $P' = 0.1P + 500$ $\qquad P(0) = 1000$

3. For the savings account in Problem 2, use a direction field to determine whether it takes longer for $P(t)$ to grow from \$1000. to \$2000 than from \$9000 to \$10000.

ANS. Click here to download Maxima to your computer and plot the direction field with the the plotdf package. By looking at the Direction field it is obvious that the slope of $P(t)$ on the interval where it goes from \$1000. to \$2000 is smaller than the the slope of $P(t)$ any where on the interval where it goes from \$9000. to \$10000. Therefore, it takes more time for $P(t)$ to cross the first interval than it does the second.

4. Solve the IVP which models the balance for the savings account in Problem 2, Exactly how long will it take to accumulate \$10000 in the account.

ANS. The integrating factor for

$$P' - .1P = 500$$

is $e^{-.1t}$. Mutliplying through gives

$$(Pe^{-.1t})' = 500e^{-.1t}$$

Integrating both sides with respect to t gives:

$$Pe^{-.1t} = -5000e^{-.1t} + C$$

Solving for P gives:

$$P = -5000 + Ce^{.1t}$$

Plugging in the initial condition to find C gives:

$$P = -5000 + 6000e^{.1t}$$

We now solve the equation

$$10000 = -5000 + 6000e^{.1t}$$

$$e^{.1t} = 5/2$$

$$t = 10\ln(5/2)$$

That is $t = 9.16$years.

5. Suppose that you take a mortgage of \$50,000 at 6% annual interest rate to buy a house. If you wish to pay off the mortgage 25 years making continuous payments, then what should the annual rate of payment be?

ANS. Let P be the balance at time t years. Then P is modeled by the differential equation

$$P' = .06P + k \qquad P(0) = 50,000$$

Here k will be negative because making an annual payment of k dollars reduces P', the rate of growth of the balance. This is a linear differential equation and the solution to this initial values problem is as follows: The integrating factor for

$$P' - .06P = k$$

is $e^{-.06t}$. Mutliplying through gives

$$(Pe^{-.06t})' = ke^{-.06t}$$

Integrating both sides with respect to t gives:

$$Pe^{-.06t} = \frac{-1}{.06} k e^{-.06t} + C$$

Solving for P gives:

$$P = \frac{-1}{.06} k + Ce^{.06t}$$

Plugging in the initial condition to find C gives:

$$P = \frac{-1}{.06} k + (50000 + \frac{1}{.06} k) e^{.06t}$$

We now plug in $t = 25$ and $P(25) = 0$ and solve for k

$$0 = \frac{-1}{.06} k + (50000 + \frac{1}{.06} k) e^{1.5}$$

Moving everything involving k to the left:

$$\frac{1}{.06}(1 - e^{1.5}) k = 50000 e^{1.5}$$

So

$$k = (.06) 50000 e^{1.5} / (1 - e^{1.5}) = -3861.42$$

Remember, that we expected a negative answer.

6. Suppose that an avalanche starts with a volume of 3x10^6 cubic meters of snow. Also suppose that rate of change of the volume of snow involved is 2 times the volume that is already inolved and that workers on the ski slope are able divert snow from the avalanche at the rate of 4x10^6 cubic meters per hour. Let V the volume of snow in the avalanche at time t. Mathematically model this event by writing down a differential equation and initial condition satisfied by V. By drawing a direction field determine how the workers ability to overcome the avalanche depends on the initial volume.

ANS. A time $t = 0$ the volume is 3x10^6; i.e., $V(0) = 3\text{x}10^6$.
The following information is given about the rate of change of volume: $V' = 2V - 4\text{x}10^6$. Click here to download Maxima to your computer and plot the direction field with the the plotdf package.