Math 251 Sections 1,2,10 FAQ 10/29/09

The linear systems in Problems 1 and 2 have only one eigenvalue. In each problem determine it. Also determine whether there are two eigenvectors which are not multiples of each other. Sketch a phase portrait. Also state the name of the critical point $(0,0)$, and state whether it is unstable, stable or asymptotically stable.

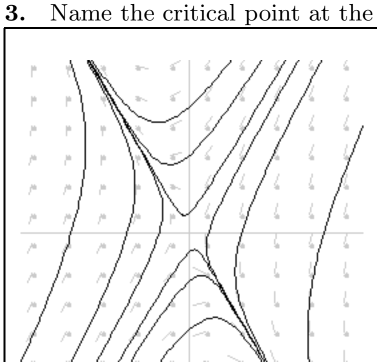
1. $\mathbf{x}' = A\mathbf{x} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \mathbf{x}$

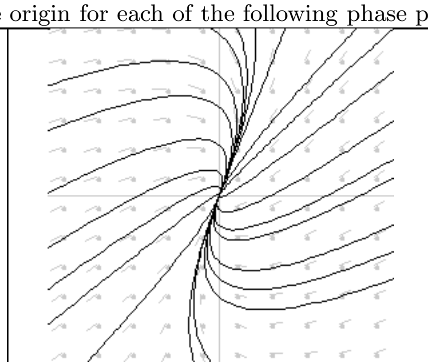
ANS. The characteristic polynomial is $(-2-r)(-2-r) = (r+2)^2$. There is only one eigenvalue $r = -2$. We seek the corresponding eigenvector and find that $A + 2I = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Every nonzero vector is an eigenvector of this matrix. Thus the critical point $(0,0)$ is a proper node and since the eigenvalue is negative it is asymptotically stable.

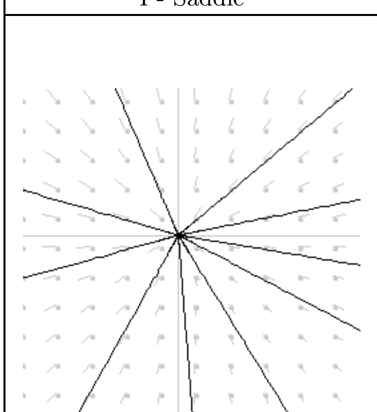
2. $\mathbf{x}' = A\mathbf{x} = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \mathbf{x}$

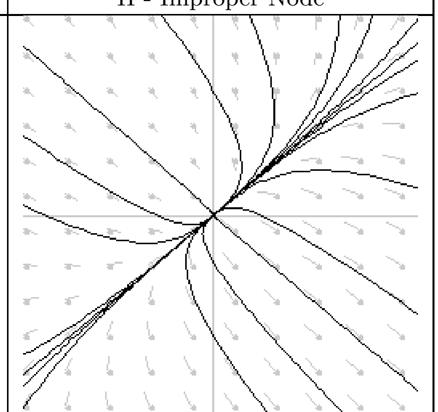
ANS. The characteristic polynomial is $(1-r)(3-r) + 1 = r^2 - 4r + 3 + 1 = (r-2)^2$. There is only one eigenvalue $r = 2$. We seek the corresponding eigenvector and find that $A - 2I = \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}$. An eigenvector of this matrix is $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and every other eigenvector is a multiple of this one. Thus the critical point $(0,0)$ is an improper node and since the eigenvalue is positive it is unstable. To sketch a phase portrait we draw the phase portrait of an unstable node with one eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and then rotate the other eigenvector into this one. There are two possible ways to do this. We choose the one that gives a trajectory with tangent at $(0,1)$ in the direction of $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

3. Name the critical point at the origin for each of the following phase portraits.


I - Saddle


II - Improper Node


III - Proper Node


IV - Node