Math 251 Sections 1,2,10 FAQ 10/12/09

1. Find the Laplace transforms of the following functions: $e^{3t}\cos 4t$ $e^{-4t}\sin 3t$

ANS. Recall that

$$\mathcal{L}\{\cos 4t\} = \frac{s}{s^2+4^2}$$

By the shiftformula

$$\mathcal{L}\{e^{3t}\cos 4t\} = \frac{s-3}{(s-3)^2+4^2}$$

Recall that

$$\mathcal{L}\{\sin 3t\} = \frac{3}{s^2+3^2}$$

By the shift formula

$$\mathcal{L}\{e^{-4t}\sin 3t\} = \frac{3}{(s+4)^2+3^2}$$

2. Find the inverse Laplace transforms of the following functions: $\dfrac{1}{s^2+2s+10}$ $\dfrac{s+3}{s^2+2s+10}$

ANS. Since the denominator does not factor we complete the square:

$$\frac{1}{s^2+2s+10} = \frac{1}{(s+1)^2+3^2}$$

We also recall the formula:

$$\mathcal{L}\{\frac{1}{3}\sin 3t\} = \frac{1}{s^2+3^2}$$

So invoking the shift formula:

$$\mathcal{L}\{\frac{1}{3}e^{-t}\sin 3t\} = \frac{1}{(s+1)^2+3^2}$$

To find the inverse Laplace transform of the second function we again complete the sqaure and apply the shift formula to

$$\mathcal{L}\{\cos 3t\} = \frac{s}{s^2+3^2}$$

in order to obtain:

$$\mathcal{L}\{e^{-t}\cos 3t\} = \frac{s+1}{(s+1)^2+3^2}$$

But the shift formula does not give us what we want. So we rewrite

$$\frac{s+3}{(s+1)^2+3^2} = \frac{s+1}{(s+1)^2+3^2} + \frac{2}{(s+1)^2+3^2}$$

And now we easily get:

$$\mathcal{L}\{e^{-t}\cos 3t + \frac{2}{3}e^{-t}\sin 3t\} = \frac{s+1}{(s+1)^2+3^2} + \frac{2}{(s+1)^2+3^2}$$

3. Use Laplace transforms to solve the following IVP: $y'' + 14y' + 50y = 0 \quad y(0) = 1, \quad y'(0) = 0$

ANS. Let $\mathcal{L}\{\mathrm{y(t)}\} = \mathrm{Y(s)}$. We start taking Laplace transforms of both sides:

$$s(sY - 1) - 0 + 14(sY - 1) + 50Y = 0$$

$$(s^2 + 14s + 50)Y = s + 14$$

Now solving for Y

$$\begin{aligned} Y &= \frac{s+14}{s^2+14s+50} \\ &= \frac{s+7}{(s+7)^2+1} + \frac{7}{(s+7)^2+1} \\ &= \mathcal{L}\{e^{-7t}(\cos t + 7\sin t)\} \end{aligned}$$

That is,

$$y(t) = e^{-7t}(\cos t + 7\sin t)$$

4. Use Laplace transforms to solve the following IVP: $y'' + 14y' + 48y = 0 \quad y(0) = 1, \quad y'(0) = 0$

ANS. Let $\mathcal{L}\{\mathrm{y(t)}\} = \mathrm{Y(s)}$. We start taking Laplace transforms of both sides:

$$s(sY - 1) - 0 + 14(sY - 1) + 48Y = 0$$

$$(s^2 + 14s + 48)Y = s + 14$$

Now solving for Y

$$\begin{aligned} Y &= \frac{s+14}{s^2+14s+48} = \frac{s+14}{(s+6)(s+8)} \\ &= \frac{4}{s+6} - \frac{3}{s+8} \\ &= \mathcal{L}\{4e^{-6t} - 3e^{-8t}\} \end{aligned}$$

That is,

$$y(t) = 4e^{-6t} - 3e^{-8t}$$