Math 251 Sections 1,2,10 FAQ 8/27/09

1. Pick out the linear ODE's from among the following:

$$yy' = 1 \qquad \sin(t^3) - \cos(t^4)y + 2y' = 0 \qquad 3y' + 4t + 5 - y^2 = 0 \qquad 4t^2y' + e^t y - \ln t = 0 \qquad \sqrt{y'} - y + 1 = 0$$

ANS. To answer this question all the equations must be put into the standard form $y' = f(t, y)$. If $f(t, y)$ does nothing more than multiply y by a function of t and/or add a function of t, then it is linear.

For the second equation we obtain $f(t, y) = \frac{1}{2}(\cos(t^4)y - \sin(t^3))$ which depends on y linearly and for the 4th equation we obtain $f(t, y) = \frac{1}{4t^2}(\ln t - e^t y)$ which also depends on y linearly. For the remaining equation the $f(t, y)$ are as follows: $f(t, y) = 1/y$, $f(t, y) = \frac{1}{3}(y^2 - 4t - 5)$ and $f(t, y) = (1 - y)^2$ and none of these depend on y linearly.

2. Match the descriptions on the right with the differential equations on the left.

i. **a.** $y' = 1 + 2y + 2t + 4ty$	**i.** linear and separable,
iv. **b.** $y' = y^2 + e^t$	**ii.** separable but not linear
iii. **c.** $y' = 2y + e^t$	**iii.** linear but not separable
ii. **d.** $y' = 2e^{t+y}$	**iv.** not separable and not linear

3. What does the product rule say about the derivative of ye^{at}:

$$\frac{d}{dt}\left(ye^{at}\right) = \left(ye^{at}\right)'$$

ANS.

$$\left(ye^{at}\right)' = y'e^{at} + aye^{at}$$

(NOTE: It is better to memorize the product rule as $(fg)' = f'g + fg'$ instead of $(fg)' = fg' + f'g$. Although they are mathematically equivalent, the latter intereferes with the memorization of the quotient rule.)

4. Consider the expression $y' - 2y$. Find a number a such that multiplying $y' - 2y$ by e^{at} gives the following derivative::

$$\left(ye^{at}\right)'$$

The function e^{at} is called an **integrating factor** .

ANS. If we multiply $y' - 2y$ by e^{at}, then we get

$$y'e^{at} - 2ye^{at}$$

Comparing this with the answer from Problem 3 we see that we need the following

$$y'e^{at} - 2ye^{at} = y'e^{at} + aye^{at}$$

So if we choose $a = -2$ everything matches perfectly.

5. Find the general solution of the differential equation $y' + \frac{1}{2}y = e^t$.
ANS. From the above we see that an integrating factor for this differential equation is:

$$e^{\frac{t}{2}}$$

and multiplying both sides of the equation by this gives:

$$(ye^{\frac{t}{2}})' = e^{\frac{3t}{2}}$$

Therefore,

$$ye^{\frac{t}{2}} = \frac{2}{3}e^{\frac{3t}{2}} + C$$

Finally,

$$y = \frac{2}{3}e^t + Ce^{\frac{-t}{2}}$$

6. Solve the following IVP (initial value problem):

$$3y' - y = t, \qquad y(0) = 1$$

ANS. Before we try to solve this differential equation, we make the coefficient of y' equal 1 by dividing through by 3: $y' - \frac{1}{3}y = \frac{1}{3}t$. This equation has the integrating factor:

$$e^{\frac{-t}{3}}$$

and multiplying both sides of the equation by this gives:

$$(ye^{\frac{-t}{3}})' = \frac{1}{3}te^{\frac{-t}{3}}$$

Therefore,

$$ye^{\frac{-t}{3}} = \int \frac{1}{3}te^{\frac{-t}{3}}\,dt$$

We need to use integration by parts to integrate the right hand side; Setting $u = t$ and $dv = \frac{1}{3}e^{\frac{-t}{3}}$ gives $du = dt$ and $v = -e^{\frac{-t}{3}}$, Thus the right hand side is

$$-te^{\frac{-t}{3}} + \int e^{\frac{-t}{3}}\,dt = -te^{\frac{-t}{3}} - 3e^{\frac{-t}{3}} + C$$

We can now solve for the general formula for y:

$$y = -t - 3 + Ce^{\frac{t}{3}}$$

By plugging in $t = 0$ and $y = 1$, we find that the initial condition is satisfied if $C = 4$.