For the equations in problem #1 and 2:
i. determine the equilibrium solutions
ii. determine where the solutions are increasing and where they are decreasing
iii. determine whether the equilibrium solutions are asymptotically stable or unstable. **iv.** determine the values of the y-coordinate of the inflection points of any solution to this ODE

1. $y' = (y+1)(y-2)$

ANS. $f(y) = (y+1)(y-2) = y^2 - y - 2 = 0$. So $f(y) = 0$ when $y = -1$ or $y = 2$. These constant functions are the two equilibrium solutions.
We also see that $y' > 0$ when $2 < y$, $y' < 0$ when $-1 < y < 2$ and that $y' > 0$ when $y < -1$
From this you see that the solution $y = -1$ is asymptotically stable and $y = 2$ is unstable.
For y'' we obain the formula $\frac{df}{dy}y' = (y+1)(y-2)(2y-1)$ and from this we see that all inflection points have y-coordinate equal to $\frac{1}{2}$

2. $y' = (y+1)(2-y)$

ANS. $f(y) = (y+1)(2-y) = -y^2 + y + 2 = 0$. So $f(y) = 0$ when $y = -1$ or $y = 2$. These constant functions are the two equilibrium solutions.
We also see that $y' < 0$ when $2 < y$, $y' > 0$ when $-1 < y < 2$ and that $y' < 0$ when $y < -1$
From this you see that the solution $y = 2$ is asymptotically stable and $y = -2$ is unstable. (Exactly the opposite of the previous example.)
For y'' we obain the formula $\frac{df}{dy}y' = (y+1)(2-y)(-2y+1)$ and from this we see again that all inflection points have y-coordinate equal to $\frac{1}{2}$. However, since in the region $-y < y < 3$ the graph of any solution is decreasing (the opposite of the situation in the previous problem) the concavity of the graph goes from down to up as the y decreases past the value $\frac{1}{2}$.

3. $y' = y(y-1)^2$

ANS. $f(y) = y(y-1)^2 = 0$ when $y = 1$ or $y = 0$. These constant functions are the two equilibrium solutions. To answer the remaining questions let us find an expression for y'' and the arrange information about signs in a table. The formula for y'' is

$$y'' = \frac{df}{dy}(y)f(y)$$

and $\frac{df}{dy} = 3y^2 - 4y + 1 = (3y-1)(y-1)$ changes sign at $y = 1/3, 1$ which means that y'' could change sign at $y = 0, 1/3, 1$.
From this you see that both the equilibrium solution $y = 0$, $y = 1$ are unstable. (According to the textbook the additional adjective **asymptotically semistable** can be attache to the equilibrium solution $y = 1$.) After you try to sketch a few graphs of solutions by hand, use Maxima to verify your sketch.
For y'' we obain the formula $\frac{df}{dy}y' = y(y-1)^3(3y-1)$ and from this we see that all inflection points have y-coordinate equal to $\frac{1}{3}$