Math 251 Sections 1,2,10 FAQ 10/22/09

1. One of the following system of linear equations has a nontrivial solution (not both x and y equal 0). Which one? Find several nontrivial solutions for it.

$$\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 2 & -1 \\ -6 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

ANS. The determinant of the matrix in the second one is 0, or equivalently the second row is a multiple of the first. Therefore, it has nontrivial solutions (x, y) which satisfy the equation $2x - y = 0$. This equation has infinitely many nontrivial solutions. $(1, 2), (-1, -2), (3, 6)$ are just some examples.

2. Consider the following linear system of ODE's:

$$\begin{aligned} x' &= 1x - 2y \\ y' &= -4x + 3y \end{aligned}$$

Find directions to the trajectories of the solutions to this system at the following points in the phase plane: $(3, 4)$, $(2, 1)$, $(1, 0)$, $(0, 1)$

Check your answers using Professor Mansfield's Phase Portrait applet on the web.

Click

here to check your answer

3. Consider the following 2×2 linear system of ODE's. Verify that the given vector is a solution to this system.

$$\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}\mathbf{x} \qquad \mathbf{x} = e^{2t}\begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

ANS.

If $\mathbf{x} = e^{2t}\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ then $\mathbf{x}' = 2e2t\begin{pmatrix} 4 \\ 2 \end{pmatrix}$.

Also $A\mathbf{x} = e^{2t}\begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix} = e^{2t}\begin{pmatrix} (3)(4) + (-2)(2) \\ (2)(4) + (-2)(2) \end{pmatrix} e^{2t}\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ which coincides with the above expression for $\mathbf{x}'$.

4. Rewrite the following linear system using matrix notation. Find the general solution. Also find the solution satisfying the given initial conditions

$$\begin{aligned} x' &= 5x - y, \qquad x(0) = 2 \\ y' &= 3x + y, \qquad y(0) = -1 \end{aligned}$$

ANS. The given equation is equivalent to

$$\mathbf{x}' = A\mathbf{x} \qquad \mathbf{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

where

$$A = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$$

We have not yet covered the procedure for finding general solutions to 2×2 linear systems of ODE's.