Math 251 Sections 1,2,10 FAQ 10/5/09

1. An object whose mass is 1 kgm stretches a spring 1.111111111 meters. The object is pushed up .1 meters above its equilibrium postion and then set into motion with a downward velocity of .3 meters/sec. Determine the displacement $y(t)$ of the object from its equilibrium position at any time $t > 0$.

ANS. Since $mg = kL$ and $g = 10, m = 1, L = 1.11111111$ we have $k = 9$, the differential equation for the displacement $y = y(t)$ is

$$y'' + 9y = 0 \qquad \text{or} \qquad y'' + \omega_0^2 y = 0$$

where $\omega_0 = 9^{1/2} = 3$. The initial conditions satisfied by $y(t)$ are $y(0) = -.1$ and $y'(0) = .3$. We have that $y = c_1 \cos 3t + c_2 \sin 3t$ and also that $c_1 = -.1 = -1/10$, $c_2 = y'(0)/\omega_0 = .3/3 = 1/10$. That is,

$$y = \frac{1}{10}(-\cos 3t + \sin 3t)$$

2. Express the solution to Problem 1 as a single cosine function. What are the amplitude and frequency of the motion?

ANS. The (natural) frequency is $\omega_0 = 3$. This means that the object returns to its initial position every $\pi/3$ seconds. To find the amplitude of the oscillations, we write:

$$y = \frac{1}{10}(-\cos 3t + \sin 3) = R\cos(3t - \delta)$$

and find that $R = \frac{1}{10}\sqrt{(-1)^2 + 1^2} = \sqrt{2}/10$ To find the phase angle we plot $(-1, 1)$ and determine the angle the line joining this point to the origin makes with the positive x-axis. So δ is a second quadrant angle with a reference angle of $\arctan 1$.

3. How many times does the object pass through its equilibrium position? What is the first time that it does so.

ANS. infinite. Since $y(t) = .14\cos(3(t - \pi/4))$, at time $t = 0$ the object is located at $y(0) = .14\cos(-3\pi/4)$, a negative value. The first time the object pass through its equilibrium position $y(t) = 0$ is when $3(t - \pi/4) = -\pi/2$, i.e., $t = \pi/4 - \pi/6 = \pi/12$

4. Sketch a graph of $y(t)$.

ANS. Click here for the second order ODE solver and enter the given differential equation for $y(t)$.

5. If the spring breaks when it is stretched 2 meters from its state with no object attached, with the initial displacement specified in Problem 1 determine the range all possible initial velocities that will not break the spring? (Here we are assuming that Hooke's law remains valid even near the breaking point.)

ANS. With a 1kg object attached to the spring, it is stretched 10/9 meters to equilibrium from its state without any weight attached.
Let v_0 be the initial velocity given the 1kg. We find need to determine v_0 so that the maximum value of $y(t)$ remain below $2 - 10/9 = 8/9$, so that the sum of the $y(t)$ and the stretch to equilibrium remains below 2 meters.
Note that $R = \sqrt{(-0.1)^2 + (v_0/3)^2}$ and that we would like to have $R^2 < 64/81 = .79$. Therefore $v_0^2/9 < .78$; so $v_0^2 < 7.02$ and hence $|v_0| < 2.65$.