Math 251 Sections 1,2,10 FAQ 10/09/09

1. What is the definition of the Laplace transforms of $f(t)$?
What is the Laplace transform of e^{2t}, e^{3t}, e^{5t}
Which of the following formula(s) are(is) true.

a. $\mathcal{L}\{7e^{2t}+8e^{3t}\} = 7\mathcal{L}\{e^{2t}\}+8\mathcal{L}\{e^{3t}\}$
b. $\mathcal{L}\{7e^{2t}8e^{3t}\} = 56\dfrac{1}{s-2}\dfrac{1}{s-3}$
c. $\mathcal{L}\{e^{2t}\sin 3t\} = \dfrac{3}{(s-2)(s^2+3^2)}$
d. $\mathcal{L}\{2\cos^2 t\} = \frac{1}{s}+\dfrac{s}{(s^2+2^2)}$
e. $\mathcal{L}\{2\sin t\cos t\} = 2\dfrac{1}{s^2+1}\dfrac{s}{(s^2+1)}$

ANS. If we set $\mathcal{L}\{\mathrm{f(t)}\} = \mathrm{F(s)}$, then

$$F(s) = \int_0^\infty e^{-st}f(t)\,dt$$

$\mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}$ Set $a = 2, /; 3/; 5$
a. is "the Superposition Principle" for Laplace transforms, TRUE.
b. is obviously FALSE.
c. is FALSE. And that will become obvious in a few days.
d. is TRUE because of a very basic trig identity.
e. is FALSE because of the double angle formula for sine.

2. Find the inverse Laplace transforms of the following functions: (i.e. find a function of t whose Laplace transform is the given function of s)
$\dfrac{1}{s+3}$, $\dfrac{1}{s-3i}$, $\dfrac{s}{s^2+16}$, $\dfrac{3}{s^2+36}$, $\dfrac{2}{s^2+16}$, $\dfrac{3}{s^2-9}$, $\dfrac{s}{s^2-49}$

ANS. $\mathcal{L}\{\mathrm{e}^{-3t}\} = \dfrac{1}{\mathrm{s}+3}$
$\mathcal{L}\{\cos 4t\} = \dfrac{\mathrm{s}}{\mathrm{s}^2+16}$
$\frac{1}{2}\mathcal{L}\{\sin 6t\} = \frac{1}{2}\dfrac{6}{\mathrm{s}^2+36}$
$\mathcal{L}\{\cosh 3t\} = \dfrac{\mathrm{s}}{\mathrm{s}^2-9}$
$\frac{1}{2}\mathcal{L}\{\sin 4t\} = \dfrac{2}{\mathrm{s}^2+16}$
$\mathcal{L}\{\frac{1}{2}(\mathrm{e}^{3\mathrm{t}}-\mathrm{e}^{-3\mathrm{t}})\} = 3\dfrac{3}{\mathrm{s}^2-9}$
$\mathcal{L}\{\frac{1}{2}(\mathrm{e}^{7\mathrm{t}}+\mathrm{e}^{-7\mathrm{t}})\} = \dfrac{\mathrm{s}}{\mathrm{s}^2-49}$

3. Which of the following functions $f(t)$ have Laplace transform $F(s)$ that exists for sufficiently large s? (DO NOT try to find their Laplace transforms.) $f(t)=t^3$ $f(t)=\sin t^3$ $f(t)=\tan t^3$ $f(t)=e^{t^{1/3}}$ $f(t)=e^{t^3}$

ANS. $f(t)=t^3$: Using calculus it is easy to see that $t^3/e^t < 27/e^3$ if $t>3$. Therefore $|f(t)|$ is eventually bounded by an exponential and consequently has a Laplace transform.
$f(t)=\sin t^3$: The absolute value of this function is bounded by 1 which is bounded by e^t for $t>0$.
$f(t)=\tan t^3$: This is not piecewise continuous. The Laplace transform does not exist.
$f(t)=e^{t^{1/3}}$: Here the absolute value of the function is bounded by is bounded by e^t for $t>1$.
$f(t)=e^{t^3}$: For any constant a the ratio $e^{t^3}/e^{at}\to\infty$ as $t\to\infty$. Therefore this function cannot be eventually bounded by a constant times an exponential function.

4. Use Laplace transforms to solve the following IVP:
$y'+4y=e^{4t}$ $y(0)=3$

ANS. Let $\mathcal{L}\{\mathrm{y(t)}\} = \mathrm{Y(s)}$. Taking Laplace transforms of both sides of the given equation gives: $sY-3+4Y=\frac{1}{s-4}$

Solving for Y gives

$$Y = \frac{3}{s+4} + \frac{1}{(s+4)(s-4)} = \frac{3}{s+4} + \frac{1/8}{s-4} - \frac{1/8}{s+4} = \mathcal{L}\{3\mathrm{e}^{-4\mathrm{t}} + \frac{1}{8}\mathrm{e}^{4\mathrm{t}} - \frac{1}{8}\mathrm{e}^{-4\mathrm{t}}\} = \mathcal{L}\{\frac{23}{8}\mathrm{e}^{-4\mathrm{t}} + \frac{1}{8}\mathrm{e}^{4\mathrm{t}}\}$$