Math 251 Sections 1,2,10 FAQ 10/15/09

1. Consider the unit step function $u(t)$ and the constant function 1.
What is $\mathcal{L}\{u(t)\}$?
What is $\mathcal{L}\{1\}$?
Is it possible for two different functions to have the same Laplace transform?

ANS. It looks like they are the same! It appears that it is possible for two different functions to have the same Laplace transform because the definition of Laplace transform only uses the values of the function on the interval $[0, \infty)$. So working with the Laplace transform of a function we can only recover information about the function's behavior on the interval $[0, \infty)$ and there the functions must be the same (except perhaps at points of discontinuity and there are not too many of those).

2. Sketch a graph of the following function.
Write down a formula that expresses the following function in terms of the unit step function.
Find its Laplace transform.

$$f(t) = \begin{cases} 0, & \text{if} \quad t < 1 \\ 1, & \text{if} \quad 1 \leq t < 2 \\ 2, & \text{if} \quad 2 \leq t \end{cases}$$

ANS.

$$(u(t-1) - u(t-2)) + 2u(t-2) = u(t-1) + u(t-2)$$

$$\mathcal{L}\{f(t)\} = \frac{e^{-s}}{s} + \frac{e^{-2s}}{s}$$

3. Do the same thing for the following "ramp" function:

$$f(t) = \begin{cases} 1, & \text{if} \quad t < 1 \\ 2-t, & \text{if} \quad 1 \leq t < 2 \\ 0, & \text{if} \quad 2 \leq t \end{cases}$$

ANS.

$$f(t) = (1 - u(t-1)) + (2-t)(u(t-1) - u(t-2)$$

For the purpose of taking Laplace transforms we would very much prefer to see someting like a $1-t$, or $t-1$, multiplying the step function $u(t-1)$. This is pretty easy to arrange because: $(2-t) = (1-(t-1))$ So,

$$f(t) = 1 - u(t-1) + (1-(t-1))u(t-1) + (t-2)u(t-2)$$

$$f(t) = 1 - (t-1)(u(t-1) + (t-2)u(t-2)$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s} - \frac{e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}$$

4. Find the following Laplace transforms:
i. $\mathcal{L}\{u(t-\pi/2)\sin(t)\}$

ANS. For the purpose of taking Laplace transforms we would very much prefer to see someting like a $\sin(t-\pi/2)$ or $\cos(t-\pi/2)$ next to $u(t-\pi/2)$. From their graphs we see that indeed $\cos(t-\pi/2) = \sin(t)$. So

$$\mathcal{L}\{u(t-\pi/2)\sin(t)\} = e^{-\pi s/2}\frac{s}{s^2+1}$$

ii. $\mathcal{L}\{u(t-\pi)\cos(t)\}$

ANS. For the purpose of taking Laplace transforms we would very much prefer to see someting like a $\sin(t-\pi)$ or $\cos(t-\pi)$ next to $u(t-\pi)$. From their graphs we see that indeed $\cos(t-\pi) = -\cos(t)$. So,

$$\mathcal{L}\{u(t-\pi)\cos(t)\} = -e^{-\pi s}\frac{s}{s^2+1}$$