Math 251 Sections 1, 2,10 FAQ 8/24/09

1. Which among the following equations is (are) not differential equations?

$$y + \sin t = \frac{dy}{dt} \qquad 1 = 2y' \qquad y^2 + y' = 1 \qquad y + \frac{1}{y} = t \qquad y + \frac{de^2}{dt} = t \qquad y'' + \frac{1}{y} = t \qquad t^2 y''' + \frac{1}{y} y'' - 1 = 0$$

Is (are) any of the above differential equation(s) autonomous? What are the orders of the DE's listed above?

ANS. The fifth equation does does NOT involve a derivative of the unknown $y = y(t)$ function, since $\frac{de^2}{dt} = 0$, and hence not a derivative of y. Also the fourth equation is not a DE, because it involves no derivative of y with respect to t. All the other eqns are DE's. The second and third equations are autonomous. The last DE is third order and next to last is second order. All other DE's are first order.

2. Consider the differential equation $y' - \cos(y) - t = 0$. What is the slope of the tangent line to the graph of the solution of this differential equation at the point $(3/2, \pi)$?

ANS. To find the slope of the tangent line to the graph of the solution plug in the given point and solve for y'. That is,

$$y' - \cos(\pi) - 3/2 = 0 \text{ or } y' = 1/2$$

NOTE: If t and the y are reversed when plugging the point $(3/2, \pi)$ into the equation an the correct answer is produced!

3. Sketch a direction field for the differential equation

$$y' = 4 - 2y$$

ANS. Plot a bunch of arrows along the y-axis and then, using the fact that this is an autonomous DE, translate each one several times to the right. To check your sketch, click here to download Maxima to your computer and plot the direction field with the instructions load(plotdf); plotdf $(4 - 2 * y)$;

4. What is (are) the equilibrium solution(s) of the above differential equation?

ANS. There is one critical point 2 and hence one equilibrium solution $y = 2$.

5. Sketch the solution y of the differential equation in problem 3 using a value y_0 for $y(0)$ that is larger than the equilibrium solution found in Problem 4. Do the same for a value y_0 which is less than the equilibrium solution found in Problem 4.

ANS. In your Maxima plot click the mouse button at $(0, 3)$ and then click the mouse button at $(0, 1)$.

6. Redo Problems 3, 4 and 5 for the DE

$$y' = 2y - 4$$

ANS. Plot a bunch of arrows along the y-axis and then, using the fact that this is an autonomous DE, translate each one several times to the right. Use Maxima to verify the correctness of your sketch.

7. During the four summer months water evaporates from a lake at a rate that is equal to 2 times the volume of water contained in the lake at that time. (Volume is measured in units of millions of liters and evaporation rate is measure in units of million liters per month). Also assume there are 4 million liters of rain falling into the lake each month. How many million liters should be in the lake at the beginning of the summer in order to ensure that

ANS. Let $V = V(t)$ denote the volume of water in the lake at time t where t denotes the number of months from the beginning of the summer. The translation to mathematics of the first sentence in this problem is

$$V' = -2V$$

The minus sign is needed to reflect that the volume of water decreases due to evaporation. The second sentence states that the rate of change in the volume of water is altered due to rain falling into the lake. Therefore, the actual rate of change is given by the following formula

$$V' = 4 - 2V$$

(Note that there is no reason to put a t next to the 4 because at this point we are trying to write a formula for the rate of change of volume NOT the actual volume.) Since we have solved this DE graphically in problem Problem 3, we easily see that the answers to the last three questions are the same: 2 million liters, i.e., the equilibrium solution of this DE.