Math 251 Sections 1,2,10 FAQ 10/7/09

1. An object whose mass is 1 kg stretches a spring 1 meter to equilibrium. A damper with damping constant 6Ns/m is attached to the object. The object is pushed up .3 meters above its equilibrium postion and then set into motion with a upward velocity of .5 meters/sec. Determine the displacement $y(t)$ of the object from its equilibrium position at any time $t > 0$.

ANS. Since $mg = kL$ and $g = 10, m = 1, L = 1$ we have $k = 10$, the differential equation for the displacement $y = y(t)$ is

$$y'' + 6y' + 10y = 0$$

The initial conditions satisfied by $y(t)$ are $y(0) = -.3$ and $y'(0) = -.5$. We have that $y = e^{-3t}(c_1 \cos t + c_2 \sin t)$ and $y' = -3y + e^{-3t}(-c_1 \sin t + c_2 \cos t)$ and
also that $c_1 = -.3 = -3/10$, $-3(-.3) + c_2 = -.5$. $c_2 = -\frac{14}{10}$

$$y = \frac{e^{-3t}}{10}(-3\cos t - 14 \sin t)$$

2. How many times does the object cross its equilibrium position. What is the time interval between successive crossings.

ANS. This object crosses its equilibrium position inifinitely many times. The time between crossings is $\frac{1}{2}2\pi = \pi$

3. If the damper in the above spring mass system is a variable γ, then what value produces critical damping.

ANS. The differential equation for the displacement $y = y(t)$ when a damper (shock absorber) is added is:

$$y'' + \gamma y' + 10y = 0$$

The characteristic polynomial is

$$r^2 + \gamma r + 10 = 0$$

We have critical damping exactly when the characteristic polynomial is making the transition from to real roots to complex roots; that is, when it has double roots. This of course happens when the discriminant is equal to 0.

$$\gamma^2 = 4(1)(10) = 40 \qquad \text{or} \qquad \gamma = \sqrt{40} = 2\sqrt{10}$$

Click on this to use the second order ODE solver to solve this problem graphically or check the answer graphically.

4. As γ varies, what are the possibilities for the total number of crossings of the equilibrium position.

ANS. If γ is below critical damping then infinitely many. At crtical damping and higher one or zero crossings.

5. Suppose the shock absorbers on a car are weak. Should heavy objects be added or remove from the trunk in order to improve its chances of passing the state safety inspection without changing the shock aborbers.

ANS. If the damping constant of the shock absorbers is above critical damping the car passes the state safety inspection. The value of critical damping is reduced if heavy objects are removed making it more likely that the car will pass inspection.