Math 251 Sections 1,2,10 FAQ 9/16/09

1. Determine the largest interval on which the following initial value problem has a unique solution: $t(t-5)y'' + 2ty' + 3y = 4(t-5), \quad y(3) = 1, \quad y'(3) = 2$

ANS. Rewrite the equation so that the coefficient of y'' is 1: $y'' + \frac{2t}{t(t-5)}y' + \frac{3}{t(t-5)}y = \frac{4}{t}$. Note the coefficients p, q, and g are discontinuous at $t = 0$ and $t = 5$. The largest open interval containig $t = 3$ not containing the above discontinuities is $(0, 5)$.

2. Consider the differential equation $y'' + py' + qy = g$ where p,q, and g, are continuous on the interval $I = (-1, 1)$. Explain why e^t and $1 + t$ cannot both be solutions.

ANS. The solution to IVP's are unique on I. Note the e^t and $1 + t$ satisfy the same value at $t = 0$ and their derivatives are also equal at $t = 0$. So both of these functions cannot be solutions to the differential equation.

3. Find the Wronskian $W(y_1, y_2)(t)$ where $y_1 = e^{2t}$ and $y_2 = te^{2t}$.

ANS.

$$W(y_1, y_2)(t) = \begin{vmatrix} e^{2t} & te^{2t} \\ 2e^{2t} & (2t+1)e^{2t} \end{vmatrix} = e^{4t}$$

4. Pick a pair of functions from the following collection of functions that might be a fundamental set of solutions for some homogeneous differential equation.

$$0, \quad e^{2t}, \quad 2e^{2t}, \quad e^{t-2}, \quad e^{2t-2}$$

ANS. Note that $e^{2t-2} = e^{-2}e^{2t}$. Therefore the second, third and fifth function are multiples of each other. Also, 0 can never be part of a fundamental set. So, e^{t-2} together with one of the the second, third and fifth functions form a fundamental set.

5. Find the general solution for the ODE $y'' - y = 0$.

ANS. The characteristic polynomial is $r^2 - 1$ which has roots $r_1 = 1$ and $r_2 = -1$. Thus e^t and e^{-t} are solutions. Since they are not clearly one is not a constant multiple of the other $y = c_1e^t + c_2e^{-t}$ is the general solution.

6. Consider the functions $\cosh t = (e^t + e^{-t})/2$ and $\sinh t = (e^t - e^{-t})/2$. Show that these functions also are solution of the ODE in Problem 5 and that $y = c_1 \cosh t + c_2 \sinh t$ also a general solution for it.

ANS. The superposition principle says that $\cosh t$ and $\sinh t$ are solutions. Observe that whereas the range of $\sinh t$ is $(-\infty, \infty)$, the range of $\cosh t$ is $[1, \infty)$. Therefore, one is not a constant multiple of the other. So we see that $y = c_1 \cosh t + c_2 \sinh t$ is also a general solution for this ODE. The advantage of using $\cosh t$ and $\sinh t$ instead of e^t and e^{-t} as a fundamental pair is in the fact that the $\cosh t$ has 1 and 0 for its value and that of its derivative at $t = 0$ whereas for $\sinh t$ it is exactly the reverse. Therefore if one is trying to solve the IVP $y(0) = \alpha$ and $y'(0) = \beta$ with this fundamental pair, then one just needs to take $c_1 = \alpha$ and $c_2 = \beta$. For other linear homogeneous ODE's the same simplifying strategy may be applied.