1. Find the characteristic equation for the following second order linear homogeneous differential equation with constant coefficients:

$$y'' + y' - 2y = 0$$

ANS.

$$r^2 + r - 2 = (r+2)(r-1)$$

2. Consider the ODE in Problem 1. Suppose the $y = y(t)$ is a solution. What ODE would you expect $y(t+17)$ to satisfy.

ANS. $y(t+17)$ is $y(t)$ shifted to the left 17 units. The ODE in Problems 1 is autonomous. So any solution shifted any amount right or left is also a solution. So $y(t+17)$ satisfies the ODE in Problem 1.

3. What is the general solution to the equation in Problem 1?

ANS. The roots of the characteristic polynomial are $r_1 = -2$ and $r_2 = 1$. So

$$y = c_1 e^{-2t} + c_2 e^t$$

4. Find the solution of the equation in Problem 1 that satisfies the initial conditions:

$$y(0) = 1, \qquad y'(0) = 3$$

ANS. First find $y' = -2c_1 e^{-2t} + c_2 e^t$
Now set $= 0$ in y and in y':

$$c_1 + c_2 = 1$$

$$-2c_1 + c_2 = 3$$

Multiply the first equation by -1 and add it to the second:

$$-3c_1 + 0 = 2$$

So $c_1 = -2/3$ and $c_2 = 5/3$. The solution is

$$y(t) = \frac{1}{3}\left(-2e^{-2t} + 5e^t\right)$$

5. Find the solution of the equation in Problem 1 that satisfies the initial conditions:

$$y(2007) = 1, \qquad y'(2007) = 3$$

ANS. Since we have already solved this problem when $t_0 = 0$ and solutions of this ODE can be shifted horizontally we simply shift the solution in problem 3 to the right by 2007:

$$y(t-2007) = \frac{1}{3}\left(-2e^{-2(t-2007)} + 5e^{t-2007}\right)$$

6. Verify the Superposition Principle for the ODE in Problem 1 and functions $y_5 = e^{5t}$ and $y_6 = e^{6t}$. I.e., verify:

$$L[c_1 y_5 + c_2 y_6] = c_1 L[y_5] + c_2 L[y_6]$$

ANS. We first calculate the left hand side as follows:

$$\begin{array}{rcl}
-2\,(c_1y_5 + c_2y_6 & = & c_1e^{5t} + c_2e^{6t}) \\
+1\,((c_1y_5 + c_2y_6)' & = & 5c_1e^{5t} + 6c_2e^{6t}) \\
+1\,((c_1y_5 + c_2y_6)'' & = & 25c_1e^{5t} + 36c_2e^{6t}) \\
------------------------------ & = & ----------------------- \\
L[c_1y_5 + c_2y_6] & = & 28c_1e^{5t} + 40c_2e^{6t}
\end{array}$$

On the other side we see that:

$$\begin{array}{rcl}
-2\,(y_5 = e^{5t} & & y_6 = e^{6t}) \\
+1\,(y_5' = 5e^{5t} & & y_6' = 6e^{6t}) \\
+1\,(y_5'' = 25e^{5t} & & y_6'' = 36e^{6t}) \\
------------------------------ & = & ----------------------- \\
L[y_5] = 28e^{5t} & & L[y_6] = 40e^{6t} \\
c_1L[y_5] + c_2L[y_6] & = & 28c_1e^{5t} + 40c_2e^{6t}
\end{array}$$

The two answers agree!

7. What can you say about the long time behavior (the limit as $t \to \infty$) of nonzero solutions of the equations

$$y'' + y' - 2y = 0 \qquad y'' + 3y' + 2y = 0 \qquad y'' - 3y' + 2y = 0$$

ANS. The gen'l solution of $y'' + y' - 2y = 0$ is $y = c_1e^{-2t} + c_2e^t$. So some solutions approach 0, others aproach $-\infty$, and others aproach ∞, depending on the values of c_1 and c_2.
The gen'l solution of $y'' + y' - 2y = 0$ is $y = c_1e^{-2t} + c_2e^t$. So some solutions approach 0, others aproach $-\infty$, and others aproach ∞, depending on the values of c_1 and c_2.
The gen'l solution of $y'' + 3' + 2y = 0$ is $y = c_1e^{-t} + c_2e^{-2t}$. So all solutions approach 0, regardless of the values of c_1 and c_2.
The gen'l solution of $y'' - 3y' + 2y = 0$ is $y = c_1e^{2t} + c_2e^t$. So some nonzero solutions approach aproach $-\infty$, and others aproach ∞, depending on the values of c_1 and c_2.