Math 251 Sections 1,2,10 FAQ 9/11/09

In the following first determine whether or not the given ODE is exact. If it is, then find the general solution using the method for solving exact differential equations. If not, then perhaps it is possible to solve using another method?

1. $(t^2-1)y' + 2ty = 0$

ANS. We seek a solution of the form $F(t,y) = C = \mathit{constant}$. This means that $F_t = 2ty$, $F_y = t^2 - 1$. We first check F_{ty} and F_{yt} to see that they are the same. They are both equal to $2t$; so the equation is exact. Consequently $F = t^2y + h(y)$. We also must have $F_y = t^2 - 1$. Therefore $\frac{dh}{dy} = -1$. We have found that $F(t,y) = t^2y - y$. In other words, the general solution is $t^2y - y = C$, or $y = C/(t^2-1)$.

2.

$$(e^t \cos y + 2\cos t)y' + e^t \sin y - 2y \sin t = 0$$

ANS. First rewrite the equation:

$$e^t \sin y - 2y\sin t + (e^t \cos y + 2\cos t)y'$$

We seek a solution of the form $F(t,y) = C$ =constant. This means that $F_t = e^t \sin y - 2y \sin t$, $F_y = e^t \cos y + 2\cos t$. We first check F_{ty} and F_{yt} to see that they are the same. They are both equal to $e^t \cos y - 2\sin t$ This equation is exact. Consequently $F = e^t \sin y + 2y\cos t + h(y)$. We also must have

$$F_y = e^t \cos y + 2\cos t + \frac{dh}{dy} = e^t \cos y + 2\cos t$$

. Therefore $\frac{dh}{dy} = 0$. So we have found the general solution is $e^t \sin y + 2y \cos t = C$.

3.

$$(9t^2 + y - 1)dt - (4y - t)dy = 0$$

ANS. We first put this ODE in the form that exact equations are usually written:

$$9t^2 + y - 1 + (t - 4y)y' = 0$$

and then seek a solution of the form $F(t,y) = C$ =constant. This means that $F_t = 9t^2 + y - 1$, $F_y = t - 4y$. We first check F_{ty} and F_{yt} to see that they are the same. They are both equal to 1. This equation is exact. Consequently $F = 3t^3 + ty - t + h(y)$. We also must have $F_y = t + \dfrac{dh}{dy} = t - 4y$. Therefore $\frac{dh}{dy} = -4y$ and hence $h(y) = -2y^2$. So we have found that the general solution is $3t^3 + ty - t - 2y^2 = C$.

4.
$(2x + 3y)dy + (2y + 3x)dx = 0$

ANS. We first put this ODE in the form that exact equations are usually written. Since we have been using t as the independent variable, let's also replace x by t:

$$(3t + 2y)dy + (3y + 2t)dt = 0$$

We divide through by the symbol dt to obtain:

$$(2t + 3y)y' + (2y + 3t) = 0$$

and then we rearrange into the standard form for exact ODE's:

$$2y + 3t + (2t + 3y)y' = 0$$

We finally can check whether it is exact or not by assuming that $F_t = 3t + 2y$ and $F_y = 2t + 3y$. Indeed $F_{ty} = 2 = F_{yt}$. Therefore, we may seek $F = \dfrac{3}{2}t^2 + 2ty + h(y)$ and then compare $F_y = 2t + \frac{dh}{dy}$ with $2t + 3y$ to see that $\frac{dh}{dy} = 3y$ and hence h should be chosen to be $\frac{3}{2}y^2$. Finally, the general solution of this ODE is: $\frac{3}{2}(t^2 + y^2) + 2yt = C$

5. Solve the following IVP

$$e^t + 2t\cos y - (t^2\sin y + e^y)y' = 0 \qquad y(1) = 0$$

ANS. We seek a solution of the form $F(t, y) = C$ =constant. This means that $F_t = e^t + 2t\cos y$, $F_y = -t^2\sin y - e^y$. We first check F_{ty} and F_{yt} to see that they are the same. They are the same: $-2t\sin y$. So the equation is exact. Consequently, $F = e^t + t^2\cos y + h(y)$. We also must have $F_y = -t^2\sin y - e^y$. Therefore $\frac{dh}{dy} = -e^y$. We have found the general solution

$$e^t + t^2\cos y - e^y = C$$

and setting $t = 1$ and $y = 0$ gives $C = e$. The solution to the IVP is $e^t + t^2\cos y - e^y = e$