Math 251 Sections 1,2,10 FAQ 10/8/09

1. An object whose mass is 1 kgm stretches a spring 1.111111111 meters. The object is connected to a damper with damping constant $\gamma = 10$ and an external force equal to $3\cos 3t$ is also applied. The object is pushed up .1 meters above its equilibrium postion and then set into motion with a downward velocity of .3 meters/sec. Then determine the displacement $y(t)$ at any time $t \geq 0$.

ANS. With damping and external force, the IVP that we must solve is

$$y'' + 10y' + 9y = 3\cos 3t \qquad y(0) = -.1 \qquad y'(0) = .3$$

The characteristic polynomial is $r^2 + 10r + 9$. So the general solution to the associated homogeneous equation is $y_h = c_1e^{-t} + c_2e^{-9t}$. We now seek a solution y_p to the given equation by complexifying it:

$$y'' + 10y' + 9y = 3e^{3it}$$

We expect that a solution will be of the form $y_c = Ae^{3it}$. We plug this in to the above to obtain

$$(-9A + 30iA + 9A)e^{3it} = 3e^{3it}$$

Therefore $A = 3/30i = -i/10$. We now take the real part of

$$\frac{-i}{10}(\cos 3t + i\sin 3t)$$

to obtain a particular solution of the original equation:

$$y_p = \frac{1}{10}\sin 3t = .1\sin 3t$$

So the general solution is $y_h + y_p$ and the only thing that remains is to determine the c_1 and c_2:

$$y = c_1e^{-t} + c_2e^{-9t} + .1\sin 3t \qquad y' = -c_1e^{-t} - 9c_2e^{-9t} + .3\cos 3t$$

Plugging $t = 0$ into these equation gives:

$$c_1 + c_2 = -.1 \qquad -c_1 - 9c_2 + .3 = .3$$

which gives $c_2 = 1/80$ and $c_1 = -9/80$.

2. What is the transient part of the solution to Problem 1 and what is the steady state part of the solution to Problem 1?

ANS. The transient solution is $(1/80)(-9e^{-t} + e^{-9t})$ and the steady state solution is $y_p = .1\sin 3t$.

3. If the damping device is not removed from the system in Problem 1 then explain why the external force of $\cos\omega t$ will not produce resonance for any value of ω.

ANS. Because when there is damping then the real part of the roots of the characteristic polynomial cannot be zero but the real part of $i\omega$ is zero.

4. If damping is removed from the system in Problem 1 then determine which value of ω in the external force of $\cos\omega t$ will produce resonance.

ANS. Resonance occurs when the natural frequency coincides with the frequency of the the external force. In Problem 3 the natural frequency is 3 when the damping device is removed.