1. Suppose that the acceleration of an object of mass 1 kgram is given by $y''(t) = -2\delta(t-2)$. If $y'(0) = 0$, then find $y'(1)$, $y'(3)$. Sketch a graph for $y'(t)$. By examining this graph determine a formula for $y'(t)$ in terms of the unit step function $u(t)$. Also sketch a graph of $y(t)$.
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ANS. $y'(1) = 0$, $y'(3) = -2$. We see that $y'(t) = -2u(t-2)$

2. Find the Laplace transform of $3u(t-1) + 4\delta(t-2)$

ANS.

$$\mathcal{L}\{3u(t-1) + 4\delta(t-2)\} = 3e^{-s}/s + 4e^{-2s}$$

3. Solve the following IVP: $y" + 8y' + 20y = \delta(t-3) \quad y(0) = 1, y'(0) = 1$

ANS. We take the Laplace transform of both sides:
$s(sY-1) - 1 + 8(sY-1) + 20Y = e^{-3s}$
Solving for Y gives:

$$Y = \frac{s+9}{s^2+8s+20} + e^{-3s}\frac{1}{s^2+8s+20}$$

Ignoring the e^{-3s} we find

$$\mathcal{L}\{\frac{1}{2}\mathrm{e}^{-4\mathrm{t}}\sin 2\mathrm{t}\} = \frac{1}{(\mathrm{s}+4)^2+2^2}$$

Therefore by the shift formula

$$\mathcal{L}\{\frac{1}{2}\mathrm{u}(\mathrm{t}-3)\mathrm{e}^{-4(\mathrm{t}-3)}\sin 2(\mathrm{t}-3)\} = \mathrm{e}^{-3\mathrm{s}}\frac{1}{(\mathrm{s}+4)^2+2^2}$$

Finally,

$$\mathcal{L}\{\mathrm{e}^{-4\mathrm{t}}\left(\cos 2\mathrm{t} + \frac{5}{2}\sin 2\mathrm{t}\right)\} = \frac{\mathrm{s}+9}{(\mathrm{s}+4)^2+2^2}$$

Hence the answer to the problem is the sum of the above

$$\mathcal{L}\{\mathrm{e}^{-4\mathrm{t}}\left(\cos 2\mathrm{t} + \frac{5}{2}\sin 2\mathrm{t}\right) + \frac{1}{2}\mathrm{u}(\mathrm{t}-3)\mathrm{e}^{-4(\mathrm{t}-3)}\sin 2(\mathrm{t}-3)\} = \mathrm{Y}$$