# Summarized report

The topic chosen is : game theory.

First of all, what is game theory ? Is it a theory that explains how to win everytime at poker ? No, it is a field of the mathematics, which tries to explain the evolution of a situation (a game) involving people (players) who have a finite range of actions they can set up (strategies) to maximize their payoffs.

I obviously chose this topic because I am fond of maths, but also because which is a discipline of maths, which is very close to the real world, and which aims at predicting and explaining our behaviours. Who never have though about predicting someone else actions ?

The presentation aims to bring people through basic concepts of the theory, and to know what maths could answer to the following question : "In a given situation, do people should play it selfishly to actually maximize their gains ?".

Before starting this walk through game theory we will first give the definition of a game and then, the fundamental concept of equilibrium.

**A game** is composed of :

- a set of $n$ players : $I = \{1, \dots, n\}$
- every player have at disposition a set of strategy $S_i,\ 1 \leq i \leq n$
- to every player we associate a gain function $g_i$, define by :

$$g_i : \begin{cases} S_1 \times \dots \times S_n \to \mathbb{R} \\ (s_1, \dots, s_n) \mapsto g_i(s_1, \dots, s_n) \end{cases}$$

In other words : the gain function gives to a player $i$ (it means : the player marked with the number $i$) the value of his gains for a given profile of strategies. A profile of strategies in a game of two person could be $(s_1^{(1)}, s_1^{(2)})$ : meaning that player 1 and player 2 both chose the strategy number 1.

**Définition : Nash's equilibrium**

A profile of strategies $(s_1^*, \dots, s_n^*)$ is a Nash's equilibrium if every player shouldn't deviate from his strategy $s_i^*$ no matter what are playing the others :

$$\forall i \in \{1, \dots, n\}\ \forall s_i \in S_i \quad u_i(s_i^*, s_1^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_n^*) \geq u_i(s_i, s_1^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_n^*)$$

If we just keep to the definition, a Nash's equilibrium is a profile of strategies, where each strategy is the best for the player who played it, under the hypothesis that each player did not choose his strategy by concerting

the others (selfishly).

For a given game, it could be very intresting to know if such an equilibrium exists, because it could give us the answer to the question seen earlier : "If every one choose their strategy selfishly, what will be the resulting situation ?".

What do have Maths to say about that ? Let see with an academical example : the Prisoner's Dilemma.

We consider Bonnie and Clyde, two ordinary persons. They both are arrested and placed in two different rooms, without any possibility of communication. The prosecutors offers them a bargain. If they both denied the crime, they will both serve 1 year in prison. If only one remains silent and the other denounces him, then the denouncer wil become free and the other will serve 3 years in prison. Finally, if they denounce each other, then both will serve 2 years in prison.

We are in a game's situation : 2 players who can't communicate, both have the same set of strategies : denounce or denie, and the payoffs for both of them are : freedom and serving years in prison. Does any equilibrium exists ? And does it maximize the gain for both players ? This is what we will see in the presentation.