The Standard Model Higgs boson as the inflaton

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Abstract

We argue that the Higgs boson of the Standard Model can lead to inflation and produce cosmological perturbations in accordance with observations. An essential requirement is the non-minimal coupling of the Higgs scalar field to gravity; no new particle besides already present in the electroweak theory is required.

Key words: Inflation, Higgs field, Standard Model, Variable Planck mass, Non-minimal coupling PACS: 98.80.Cq, 14.80.Bn

1. Introduction

The fact that our universe is almost flat, homogeneous and isotropic is often considered as a strong indication that the Standard Model (SM) of elementary particles is not complete. Indeed, these puzzles. together with the problem of generation of (almost) scale invariant spectrum of perturbations, necessary for structure formation, are most elegantly solved by inflation [1, 2, 3, 4, 5, 6]. The majority of present models of inflation require an introduction of an additional scalar—the "inflaton". This hypothetical particle may appear in a natural or not so natural way in different extensions of the SM, involving Grand Unified Theories (GUTs), supersymmetry, string theory, extra dimensions, etc. Inflaton properties are constrained by the observations of fluctuations of the Cosmic Microwave Background (CMB) and the matter distribution in the universe. Though the mass and the interaction of the inflaton with matter fields are not fixed, the well known considerations prefer a heavy scalar field with a mass $\sim 10^{13}\,{\rm GeV}$ and extremely small self-interacting quartic coupling constant $\lambda \sim 10^{-13}$ [7]. This value of the mass is close to the GUT scale, which is often considered as an argument in favour of existence of new physics between the electroweak and Planck scales.

The aim of the present Letter is to demonstrate that the SM itself can give rise to inflation. The spectral index and the amplitude of tensor perturbations can be predicted and be used to distinguish this possibility from other models for inflation; these parameters for the SM fall within the 1σ confidence contours of the WMAP-3 observations [8].

To explain our main idea, consider Lagrangian of the SM non-minimally coupled to gravity,

$$L_{\rm tot} = L_{\rm SM} - \frac{M^2}{2} R - \xi H^{\dagger} H R , \qquad (1)$$

where $L_{\rm SM}$ is the SM part, M is some mass parameter, R is the scalar curvature, H is the Higgs field, and ξ is an unknown constant to be fixed later. ¹ The third term in (1) is in fact required by the renormalization properties of the scalar field in a curved space-time background [9]. If $\xi = 0$, the coupling of the Higgs field to gravity is said to be "minimal". Then M can be identified with Planck scale M_P related to the Newton's constant as

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¹ In our notations the conformal coupling is $\xi = -1/6$.

 $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18} \,\text{GeV}$. This model has "good" particle physics phenomenology but gives "bad" inflation since the self-coupling of the Higgs field is too large and matter fluctuations are many orders of magnitude larger than those observed. Another extreme is to put M to zero and consider the "induced" gravity [10, 11, 12, 13, 14], in which the electroweak symmetry breaking generates the Planck mass [15, 16, 17]. This happens if $\sqrt{\xi} \sim 1/(\sqrt{G_N} M_W) \sim 10^{17}$, where $M_W \sim$ 100 GeV is the electroweak scale. This model may give "good" inflation [12, 13, 14, 18, 19, 20] even if the scalar self-coupling is of the order of one, but most probably fails to describe particle physics experiments. Indeed, the Higgs field in this case almost completely decouples from other fields of the SM² [15, 16, 17], which corresponds formally to the infinite Higgs mass m_H . This is in conflict with the precision tests of the electroweak theory which tell that m_H must be below 285 GeV [21] or even 200 GeV [22] if less conservative point of view is taken.

These arguments indicate that there may exist some intermediate choice of M and ξ which is "good" for particle physics and for inflation at the same time. Indeed, if the parameter ξ is sufficiently small, $\sqrt{\xi} \ll 10^{17}$, we are very far from the regime of induced gravity and the low energy limit of the theory (1) is just the SM with the usual Higgs boson. At the same time, if ξ is sufficiently large, $\xi \gg 1$, the scalar field behaviour, relevant for chaotic inflation scenario [7], drastically changes, and successful inflation becomes possible. We should note, that models of chaotic inflation with both nonzero M and ξ were considered in literature [12, 14, 19, 20, 23, 24, 25], but in the context of either GUT or with an additional inflaton having nothing to do with the Higgs field of the Standard Model.

The Letter is organised as follows. We start from discussion of inflation in the model, and use the slow-roll approximation to find the perturbation spectra parameters. Then we will argue in Section 3 that quantum corrections are unlikely to spoil the classical analysis we used in Section 2. We conclude in Section 4.

2. Inflation and CMB fluctuations

Let us consider the scalar sector of the Standard Model, coupled to gravity in a non-minimal way. We will use the unitary gauge $H = h/\sqrt{2}$ and neglect all gauge interactions for the time being, they will be dis-

cussed later in Section 3. Then the Lagrangian has the form:

$$S_{J} = \int d^{4}x \sqrt{-g} \left\{ -\frac{M^{2} + \xi h^{2}}{2} R + \frac{\partial_{\mu} h \partial^{\mu} h}{2} - \frac{\lambda}{4} (h^{2} - v^{2})^{2} \right\}.$$
 (2)

This Lagrangian has been studied in detail in many papers on inflation [14, 19, 20, 24], we will reproduce here the main results of [14, 19]. To simplify the formulae, we will consider only ξ in the region $1 \ll \sqrt{\xi} \ll 10^{17}$, in which $M \simeq M_P$ with very good accuracy.

It is possible to get rid of the non-minimal coupling to gravity by making the conformal transformation from the Jordan frame to the Einstein frame

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \;, \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2} \;.$$
 (3)

This transformation leads to a non-minimal kinetic term for the Higgs field. So, it is convenient to make the change to the new scalar field χ with

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2 / M_P^2}{\Omega^4}} \ . \tag{4}$$

Finally, the action in the Einstein frame is

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2} \hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - U(\chi) \right\}, (5)$$

where \hat{R} is calculated using the metric $\hat{g}_{\mu\nu}$ and the potential is

$$U(\chi) = \frac{1}{\Omega(\chi)^4} \frac{\lambda}{4} \left(h(\chi)^2 - v^2 \right)^2 . \tag{6}$$

For small field values $h \simeq \chi$ and $\Omega^2 \simeq 1$, so the potential for the field χ is the same as that for the initial Higgs field. However, for large values of $h \gg M_P/\sqrt{\xi}$ (or $\chi \gg \sqrt{6}M_P$) the situation changes a lot. In this limit

$$h \simeq \frac{M_P}{\sqrt{\xi}} \exp\left(\frac{\chi}{\sqrt{6}M_P}\right) \,.$$
 (7)

This means that the potential for the Higgs field is exponentially flat and has the form

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2} \left(1 + \exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right) \right)^{-2} . \tag{8}$$

The full effective potential in the Einstein frame is presented in Fig. 1. It is the flatness of the potential at $\chi \gg M_P$ which makes the successful (chaotic) inflation possible.

This can be seen most easily by rewriting the Lagrangian (1), given in the Jordan frame, to the Einstein frame, see also below.

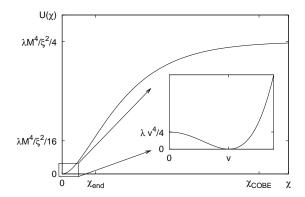


Fig. 1. Effective potential in the Einstein frame.

Analysis of the inflation in the Einstein frame 3 can be performed in standard way using the slow-roll approximation. The slow roll parameters (in notations of [28]) can be expressed analytically as functions of the field $h(\chi)$ using (4) and (6) (in the limit of $h^2\gg M_P^2/\xi\gg v^2$),

$$\epsilon = \frac{M_P^2}{2} \left(\frac{dU/d\chi}{U}\right)^2 \simeq \frac{4M_P^4}{3\xi^2 h^4} \,, \tag{9}$$

$$\eta = M_P^2 \frac{d^2 U/d\chi^2}{U} \simeq -\frac{4M_P^2}{3\xi h^2} ,$$
(10)

$$\zeta^2 = M_P^4 \frac{(d^3 U/d\chi^3) dU/d\chi}{U^2} \simeq \frac{16 M_P^4}{9\xi^2 h^4} \,. \tag{11}$$

Slow roll ends when $\epsilon \simeq 1$, so the field value at the end of inflation is $h_{\rm end} \simeq (4/3)^{1/4} M_P/\sqrt{\xi} \simeq 1.07 M_P/\sqrt{\xi}$. The number of e-foldings for the change of the field h from h_0 to $h_{\rm end}$ is given by

$$N = \int_{h_{\text{end}}}^{h_0} \frac{1}{M_P^2} \frac{U}{dU/dh} \left(\frac{d\chi}{dh}\right)^2 dh \simeq \frac{6}{8} \frac{h_0^2 - h_{\text{end}}^2}{M_P^2/\xi} . (12)$$

We see that for all values of $\sqrt{\xi} \lll 10^{17}$ the scale of the Standard Model v does not enter in the formulae, so the inflationary physics is independent on it. Since interactions of the Higgs boson with the particles of the SM after the end of inflation are strong, the reheating happens right after the slow-roll, and $T_{\rm reh} \simeq (\frac{2\lambda}{\pi^2 g^*})^{1/4} M_P/\sqrt{\xi} \simeq 2\times 10^{15}\,{\rm GeV}$, where $g^*=106.75$ is the number of degrees of freedom of the SM. So, the number of e-foldings for the the COBE scale entering the horizon $N_{\rm COBE} \simeq 62$ (see [28]) and $h_{\rm COBE} \simeq 9.4 M_P/\sqrt{\xi}$. Inserting (12) into the COBE normalization $U/\epsilon = (0.027 M_P)^4$ we find the required value for ξ

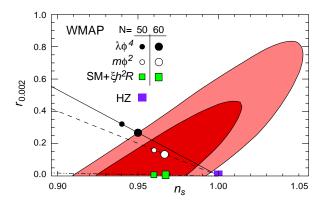


Fig. 2. The allowed WMAP region for inflationary parameters (r,n). The green boxes are our predictions supposing 50 and 60 e-foldings of inflation. Black and white dots are predictions of usual chaotic inflation with $\lambda\phi^4$ and $m^2\phi^2$ potentials, HZ is the Harrison-Zeldovich spectrum.

$$\xi \simeq \sqrt{\frac{\lambda}{3}} \frac{N_{\text{COBE}}}{0.027^2} \simeq 49000 \sqrt{\lambda} = 49000 \frac{m_H}{\sqrt{2}v} \ .$$
 (13)

Note, that if one could deduce ξ from some fundamental theory this relation would provide a connection between the Higgs mass and the amplitude of primordial perturbations. The spectral index $n=1-6\epsilon+2\eta$ calculated for N=60 (corresponding to the scale $k=0.002/{\rm Mpc}$) is $n\simeq 1-8(4N+9)/(4N+3)^2\simeq 0.97$. The tensor to scalar perturbation ratio [8] is $r=16\epsilon\simeq 192/(4N+3)^2\simeq 0.0033$. The predicted values are well within one sigma of the current WMAP measurements [8], see Fig. 2.

3. Radiative corrections

An essential point for inflation is the flatness of the scalar potential in the region of the field values $h \sim 10 M_P/\sqrt{\xi}$, what corresponds to the Einstein frame field $\chi \sim 6 M_P$. It is important that radiative corrections do not spoil this property. Of course, any discussion of quantum corrections is flawed by the non-renormalizable character of gravity, so the arguments we present below are not rigorous.

There are two qualitatively different type of corrections one can think about. The first one is related to the quantum gravity contribution. It is conceivable to think [29] that these terms are proportional to the energy density of the field χ rather than its value and are of the order of magnitude $U(\chi)/M_P^4 \sim \lambda/\xi^2$. They are small at large ξ required by observations. Moreover, adding non-renormalizable operators h^{4+2n}/M_P^{2n} to the Lagrangian (2) also does not change the flatness of the

³ The same results can be obtained in the Jordan frame [26, 27].

potential in the inflationary region. 4

Other type of corrections is induced by the fields of the Standard Model coupled to the Higgs field. In one loop approximation these contributions have the structure.

$$\Delta U \sim \frac{m^4(\chi)}{64\pi^2} \log \frac{m^2(\chi)}{\mu^2} , \qquad (14)$$

where $m(\chi)$ is the mass of the particle (vector boson, fermion, or the Higgs field itself) in the background of field χ , and μ is the normalization point. Note that the terms of the type $m^2(\chi)M_P^2$ (related to quadratic divergences) do not appear in scale-invariant subtraction schemes that are based, for example, on dimensional regularisation (see a relevant discussion in [30, 31, 32, 33]). The masses of the SM fields can be readily computed [14] and have the form

$$m_{\psi,A}(\chi) = \frac{m(v)}{v} \frac{h(\chi)}{\Omega(\chi)} , \quad m_H^2(\chi) = \frac{d^2 U}{d\chi^2}$$
 (15)

for fermions, vector bosons and the Higgs (inflaton) field. It is crucial that for large χ these masses approach different constants (i.e. the one-loop contribution is as flat as the tree potential) and that (14) is suppressed by the gauge or Yukawa couplings in comparison with the tree term. In other words, one-loop radiative corrections do not spoil the flatness of the potential as well. This argument is identical to the one given in [14].

Another important correction is connected with running of the non-minimal coupling ξ to gravity. The corresponding renormalization group equation is [34, 35]

$$\mu \frac{d\xi}{d\mu} = \left(\xi + \frac{1}{6}\right) \frac{\left(12\lambda + 12y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}{g'}^2\right)}{16\pi^2} , (16)$$

where $y_t=m_t/v$ is the top Yukawa coupling, g and g' are SU(2) and U(1) couplings of the Standard Model and μ is the characteristic scale. The renormalization of ξ from $\mu \sim M_W$ to the Planck scale is considerable, $\xi(M_P)\approx 2\xi(M_W)$. At the same time, the change of ξ in the inflationary region is small, $\delta\xi/\xi\approx 0.2$. Thus, the logarithmic running of ξ does not change the behaviour of the potential required for inflation.

There is also the induced one-loop pure gravitational term of the form $\xi^2 R^2/64\pi^2$. During the inflationary epoch it is smaller than the tree term $M_P^2 R$ by the Higgs self-coupling $\lambda/64\pi^2$ and does not change the conclusion.

4. Conclusions

In this Letter we argued that inflation can be a natural consequence of the Standard Model, rather than an indication of its weakness. The price to pay is very modest—a non-minimal coupling of the Higgs field to gravity. An interesting consequence of this hypothesis is that the amplitude of scalar perturbations is proportional to the square of the Higgs mass (at fixed ξ), revealing a non-trivial connection between electroweak symmetry breaking and the structure of the universe. The specific prediction of the inflationary parameters (spectral index and tensor-to-scalar ratio) can distinguish it from other models (based, e.g. on inflaton with quadratic potential), provided these parameters are determined with better accuracy.

The inflation mechanism we discussed has in fact a general character and can be used in many extensions of the SM. Thus, the ν MSM of [36, 37] (SM plus three light fermionic singlets) can explain simultaneously neutrino masses, dark matter, baryon asymmetry of the universe and inflation without introducing any additional particles (the ν MSM with the inflaton was considered in [30]). This provides an extra argument in favour of absence of a new energy scale between the electroweak and Planck scales, advocated in [32].

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 $[\]overline{^4}$ Actually, in the Jordan frame, we expect that higher-dimensional operators are suppressed by the effective Planck scale $M_P^2 + \xi h^2$.

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