Additional Bayes' Rule Problems

1. A friend who works in a big city owns two cars, one small and one large. Three-quarters of the time he drives the small car to work, and one-quarter of the time he drives the large car. If he takes the small car, he usually has little trouble parking, and so is at work on time with probability 0.9. If he takes the large car, he is at work on time with probability 0.6. Given that he was on time on a particular morning, what is the probability that he drove the small car?

2. At a certain gas station 40% of the customers request regular gas, 35% request unleaded gas, and 25% request premium gas. Of those customers requesting regular gas, only 30% fill their tanks. Of those customers requesting unleaded gas, 60% fill their tanks, while of those requesting premium, 50% fill their tanks. If the next customer fills the tank, what is the probability that regular gas is requested?

3. Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose that a light aircraft has disappeared. If it has an emergency locator, what is the probability that it will be discovered?

☞ **See solutions on next page.**

Solutions

For each of the following problems, draw a tree diagram, if necessary, to find the numerator and denominator of the requested conditional probability. Otherwise, carry out the formula, as illustrated, using the multiplication rule successively.

1. Let S = event friend drives small car
   and L = event friend drives large car
   and T = event friend is at work on time

   Given: $P(S) = 0.75$
   $P(L) = 0.25$
   $P(T|S) = 0.90$
   $P(T|L) = 0.60$

   So, $P(S|T) = P(T \text{ and } S)/P(S)$
   $= \{P(S) \times P(T|S)\}/P((T \text{ and } S) \text{ OR } (T \text{ and } L))$
   $= \{P(S) \times P(T|S)\}/\{P(T \text{ and } S) + P(T \text{ and } L)\}$
   $= \{P(S) \times P(T|S)\}/\{P(S) \times P(T|S) + P(L) \times P(T|L)\}$
   $= (0.75\times0.90)/\{0.75\times0.90 + 0.25\times0.60\}$
   $= 0.675/(0.675 + 0.15)$
   $= 0.675/0.825$
   $= 0.818$

2. Let R = event customer requests regular gas
   and U = event customer requests unleaded gas
   and P = event customer requests premium gas
   and F = event customer fills tank

   Given: $P(R) = 0.40$ $P(F|R) = 0.30$
   $P(U) = 0.35$ $P(F|U) = 0.60$
   $P(P) = 0.25$ $P(F|P) = 0.50$

   So, $P(R|F) = P(R \text{ and } F)/P(F)$
   $= P(R \text{ and } F)/P((R \text{ and } F) \text{ OR } (U \text{ and } F) \text{ OR } (P \text{ and } F))$
   $= P(R \text{ and } F)/\{P(R \text{ and } F) + P(U \text{ and } F) + P(P \text{ and } F)\}$
   $= \{P(R) \times P(F|R)\}/\{(P(R) \times P(F|R)) + (P(U) \times P(F|U)) + (P(P) \times P(F|P))\}$
   $= (0.40 \times 0.30)/((0.40\times0.30) + (0.35\times0.60) + (0.25\times0.50))$
   $= 0.12/(0.12 + 0.21 + 0.125)$
   $= 0.12/0.455$
   $= 0.264$

3. Let D = event light aircraft is discovered if and when it disappears when in flight and E = event aircraft has an emergency locator

Given: $P(D) = 0.70 \Rightarrow P(D') = 1 - P(D) = 1 - 0.70 = 0.30$
$P(E|D) = 0.60$
$P(E'|D') = 0.90 \Rightarrow P(E|D') = 1 - P(E'|D') = 1 - 0.90 = 0.10$

Then, $P(D|E)$

$$= P(D \text{ and } E)/P(E)$$
$$= P(D \text{ and } E)/P((E \text{ and } D) \text{ OR } (E \text{ and } D'))$$
$$= P(D \text{ and } E)/\{P(E \text{ and } D) + P(E \text{ and } D')\}$$
$$= \{P(D) \times P(E|D)\}/\{(P(D) \times P(E|D)) + (P(D') \times P(E|D'))\}$$
$$= (0.70 \times 0.60)/((0.70\times0.60) + (0.30 \times 0.10))$$
$$= 0.42/(0.42+0.03)$$
$$= 0.42/0.45$$
$$= 0.93$$