ISyE 6739 Homework 2

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3-109

X = number of passengers that do not show up

X follows a binomial distribution

a)

$P(X \geq 5) = 1 - P(X \leq 4) = 1 - \sum_{x=0}^{4} \binom{125}{x}(0.1)^x(0.9)^{125-x} = 0.9961$

```
1 - pbinom(4,125,0.1)
```

```
## [1] 0.9961414
```

b)

$P(X > 5) = 1 - P(X \leq 5) = 1 - \sum_{x=0}^{5} \binom{125}{x}(0.1)^x(0.9)^{125-x} = 0.9885$

```
1 - pbinom(5,125,0.1)
```

```
## [1] 0.9885678
```

3-173

a)

ED = number of emergency department visits per day $\lambda T = 1.8$ $P(ED > 5) = 1 - \sum_{x=0}^{5} \frac{e^{-1.8}(1.8)^x}{x!} = 0.0103$

```
1 - ppois(5,1.8)
```

```
## [1] 0.01037804
```

b)

EW = number of emergency department visits per week $\lambda T = 1.8 \times 7 = 12.6$ $P(ED < 5) = \sum_{x=0}^{4} \frac{e^{-12.6}(12.6)^x}{x!} = 0.00497$

```
ppois(4,12.6)
```

```
## [1] 0.004979028
```

c)

ET = number of visits in T days $\lambda T = 1.8T$ $P(ET \geq 1) = 1 - P(ET = 0) = 1 - \frac{e^{-1.8T}(1.8T)^0}{0!}$ $e^{-1.8T} = 0.01 = ln(e^{-1.8T}) = ln(0.01) \to T = 2.558\ days$

```
log(0.01)/-1.8
```

```
## [1] 2.558428
```

d)

Solve for λ. $P(ED > 5) = 0.1 = 1 - \sum_{x=0}^{5} \frac{e^{-\lambda}(\lambda)^x}{x!}$

4-4

$f(x) = \frac{2}{x^3}\ for\ x > 1$

a)

$P(X < 2) = \int_1^2 \frac{2}{x^3} dx = \left(\frac{-1}{x^2}\right)|_1^2 = 0.75$

```
integrand <- function(x) {2/x^3}
integrate(integrand,1,2)
```

```
## 0.75 with absolute error < 8.3e-15
```

b)

$P(X > 5) = \int_5^\infty \frac{2}{x^3} dx = \left(\frac{-1}{x^2}\right)|_5^\infty = 0.04$

```
integrand <- function(x) {2/x^3}
integrate(integrand,5,Inf)
```

```
## 0.04 with absolute error < 5.8e-07
```

c)

$P(4 < X < 8) = \int_4^8 \frac{2}{x^3} dx = \left(\frac{-1}{x^2}\right)|_4^8 = 0.0469$

```
integrand <- function(x) {2/x^3}
integrate(integrand,4,8)
```

```
## 0.046875 with absolute error < 5.2e-16
```

d)

$P(X < 4 \, or \, X > 8) = \int_1^4 \frac{2}{x^3} dx + \int_8^\infty \frac{2}{x^3} dx = 0.9531$

```
integrand <- function(x) {2/x^3}
a <- integrate(integrand,1,4)
b <- integrate(integrand,8,Inf)
a[[1]] + b[[1]]
```

```
## [1] 0.953125
```

e)

Find x, where $P(X < x) = 0.95$.
$P(X < x) = \int_1^x \frac{2}{x^3} dx = \left(\frac{-1}{x^2}\right)|_1^x = \left(\frac{-1}{x^2}\right) - (-1) = 0.95$
$\therefore x = 4.4721$

4-82

Let W be required water in million gallons. Let S be storage capacity of 350 million gallons. $\mu = 310 \, million$; $\sigma = 45 \, million$
$X \sim N(310, 45^2)$ ## a) $P(W > S) = 1 - P(X \leq S) = 1 - \Phi(\frac{350-310}{45}) = 0.1870$

```
pnorm(310,350,45)
```

```
## [1] 0.1870314
```

b) ???????????????

$P(W > w) = P(Z > \frac{w-310}{45}) = 1 - \Phi(\frac{w}{45}) = 0.01$
$\Phi(\frac{w-310}{45}) = 0.99 \rightarrow \frac{w-310}{45} = 2.33$(From Cumulative Standard Normal Table)
$x = 414.85 \, million \, gallons$

c) ??????????????

$P(W < w) = P(Z < \frac{w-310}{45}) = \Phi(\frac{w-310}{45}) = 0.05$
$\Phi(\frac{w-310}{45}) = 0.05 \rightarrow \frac{w-310}{45} = -1.64$ (From Cumulative Standard Normal Table)
$x = 236.2\, million\ gallons$

d)

Find μ, $X \sim N(\mu, 45^2$
$P(W > S) = 1 - P(X \leq S) = 1 - \Phi(\frac{350-\mu}{45}) = 0.01$
$\Phi(\frac{350-\mu}{45}) = 0.99 \rightarrow \mu = 350 - 2.33 \times 45 = 245.15$
Given 1.4 million people, mean daily demand per person = 175.107 gallons

4-117

a)

b)

c)

d)

5-6

Q2