HW3

Patrick Gardocki

2023-06-09

6-30

```
octane = read.table("6-30.txt",header = TRUE)

octane=octane$Rating

o = sort(octane)

quantile(o, c(0.25,0.5,0.75),type=6)
```

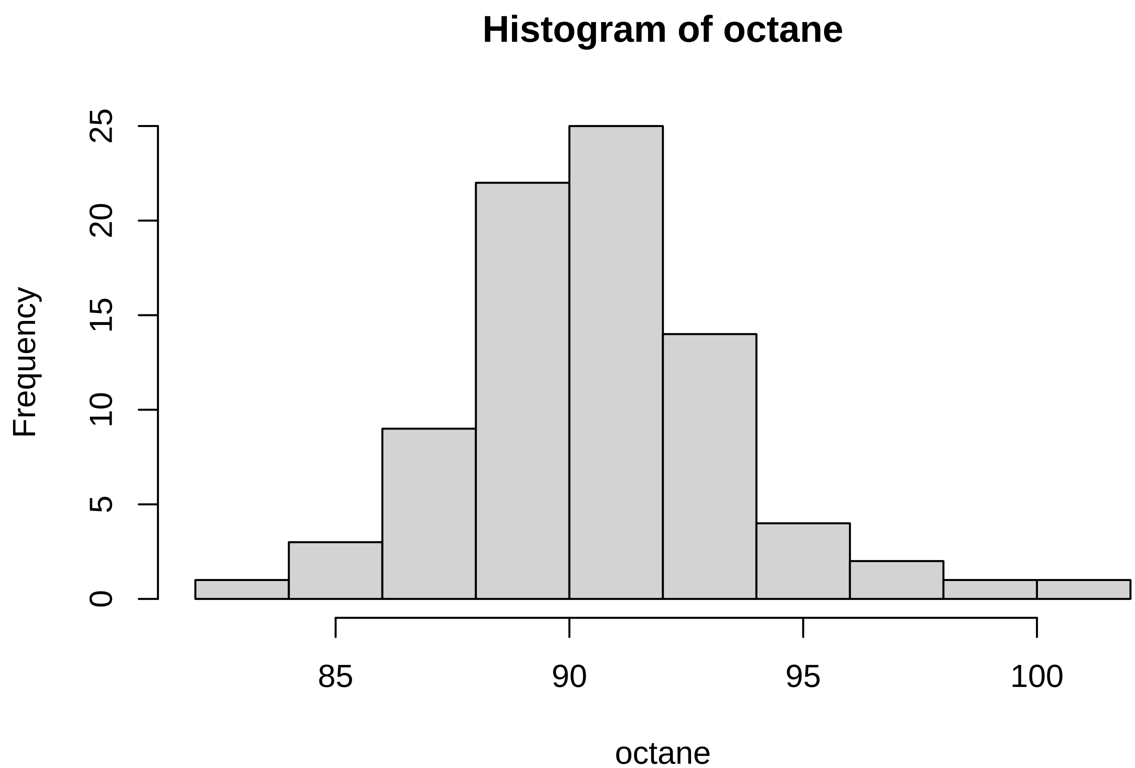
```
##    25%    50%    75%
## 88.575 90.400 92.200
```

```
stem(octane, scale=2)
```

```
##
##   The decimal point is at the |
##
##    83 | 4
##    84 | 33
##    85 | 3
##    86 | 777
##    87 | 456789
##    88 | 23334556679
##    89 | 0233678899
##    90 | 0111344456789
##    91 | 0001112256688
##    92 | 22236777
##    93 | 023347
##    94 | 2247
##    95 |
##    96 | 15
##    97 |
##    98 | 8
##    99 |
##   100 | 3
```

6-46

```
octane = read.table("6-30.txt",header = TRUE)

octane=octane$Rating

obj = hist(octane,breaks=8)
```




```
length(octane)
```

```
## [1] 82
```

```
library(knitr)
library(kableExtra)
df = data.frame(Frequency = obj$counts)

n=length(obj$breaks)
for(i in 1:(n-2)){
  df$Class[i]=paste(obj$breaks[i],"$\\le x <$", obj$breaks[i+1])
}

df$Class[n-1] = paste(obj$breaks[n-1],"$\\le x \\le$", obj$breaks[n])
```

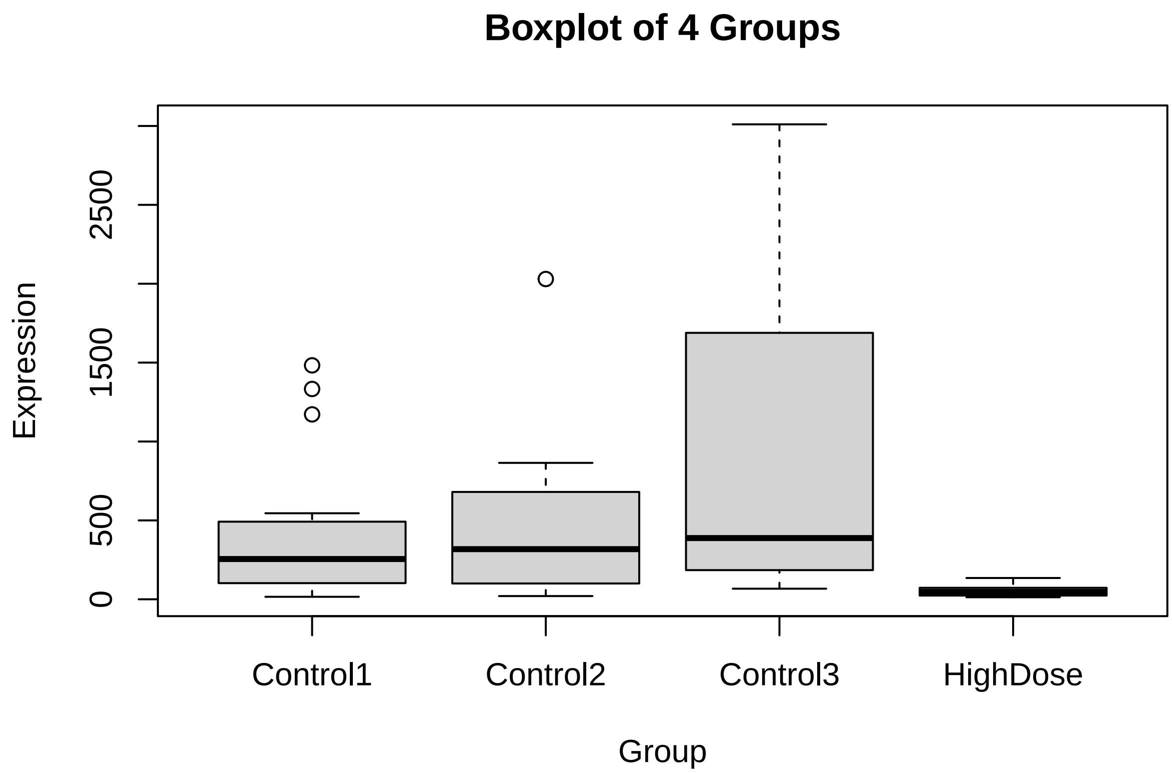
Table 1: Frecquency Distribution Table

Class	Frequency	Relative Frequency	Cumulative Frequency	Cumulative Relative Frequency
$82 \leq x < 84$	1	0.0121951	1	0.0121951219512195
$84 \leq x < 86$	3	0.0365854	4	0.0487804878048781
$86 \leq x < 88$	9	0.1097561	13	0.158536585365854
$88 \leq x < 90$	22	0.2682927	35	0.426829268292683
$90 \leq x < 92$	25	0.3048780	60	0.731707317073171
$92 \leq x < 94$	14	0.1707317	74	0.902439024390244
$94 \leq x < 96$	4	0.0487805	78	0.951219512195122
$96 \leq x < 98$	2	0.0243902	80	0.975609756097561
$98 \leq x < 100$	1	0.0121951	81	0.98780487804878
$100 \leq x \leq 102$	1	0.0121951	82	1

```
df$"Relative Frequency"= obj$density*2
for(i in 1:(n-1)){
  df$"Cumulative Frequency"[i]=paste(cumsum(obj$counts)[i])
  df$"Cumulative Relative Frequency"[i]=paste(cumsum(obj$density*2)[i])
}

kable(df[c(2,1,3,4,5)], "latex", align="c", caption="Frecquency Distribution Table", escape = FALSE)
```

6-81

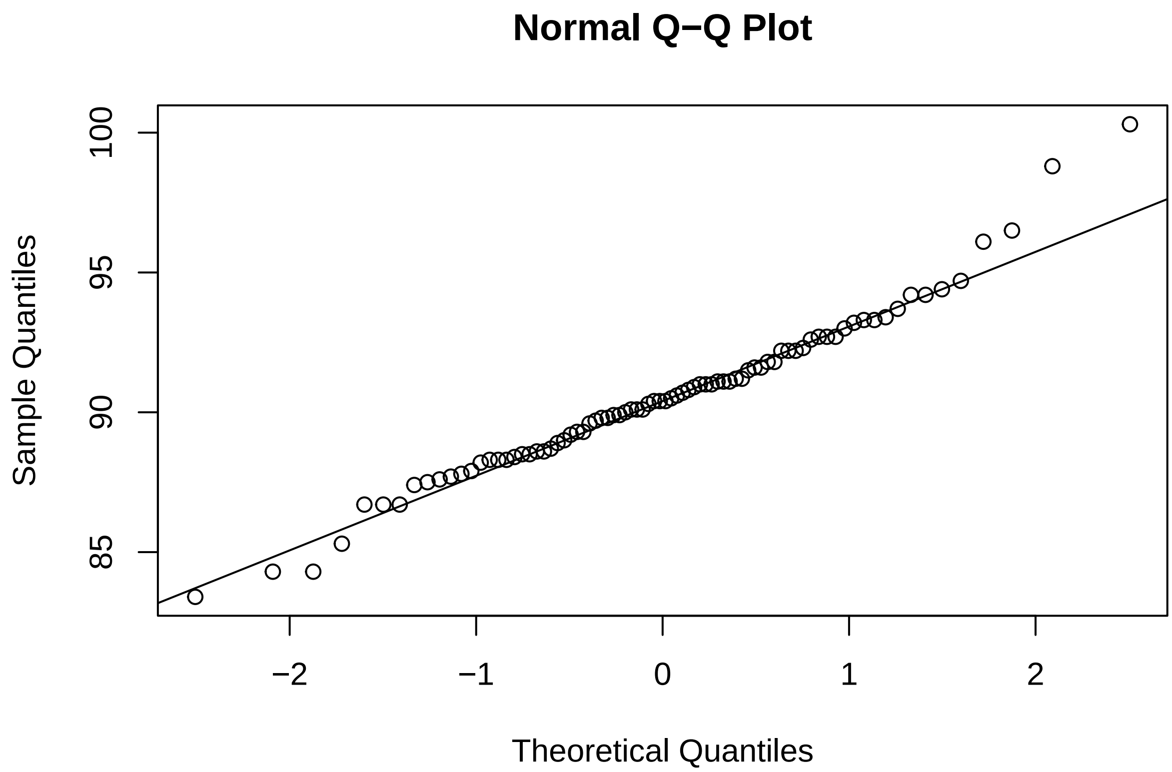
```
Expr = read.table("6-81.txt",header = TRUE)

attach(Expr)

boxplot(Expression~Group, main="Boxplot of 4 Groups",
   xlab="Group", ylab="Expression")
```



The control groups seem to have the same median value but drastically different variances. The ‘high dose’ group has a mush smaller variance and slightly lower median expression as well.

6-98

```
octane = read.table("6-30.txt",header = TRUE)

octane=octane$Rating
qqnorm(octane)
qqline(octane)
```



The Normal Probability plot suggests that the data is normally distributed but seems to break down at both ends of the data range.

7-12

$\mu_X = 8.2\,minutes,\ n = 49,\ \sigma_X = 1.5\ minutes,\ \sigma_{\bar{x}} = \frac{\sigma_X}{\sqrt{n}} = 0.2143$ Under Central Limit Theorem, $\bar{X}$ is approx. normally distributed.

(a)

$P(\bar{X} < 10) = P(Z < \frac{10-\mu}{\sigma_{\bar{X}}}) = P(Z < 8.4) = 1$

(b)

$P(5 < \bar{X} < 10) = P(\frac{5-\mu}{\sigma_{\bar{X}}} < Z < \frac{10-\mu}{\sigma_{\bar{X}}}) = P(Z < 8.4) - P(Z < -14.932) = 1 - 0 = 1$

(c)

$P(\bar{X} < 6) = P(Z < \frac{6-\mu}{\sigma_{\bar{X}}}) = P(Z < -10.27) = 0$

7-37

a)

if $E(\hat{\Theta}) = \Theta$ then unbiased estimator
$E(\hat{X}_1 - \hat{X}_2) = E(\hat{X}_1) - E(\hat{X}_2) = \mu_1 - \mu_2 \therefore$ unbiased

b)

$$\sigma_{\bar{X}} = \sqrt{V(\hat{\Theta})} = \sqrt{V(\hat{X}_1 - \hat{X}_2)} = \sqrt{V(\hat{X}_1) + V(\hat{X}_2) + 2COV(\hat{X}_1, \hat{X}_2)} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Estimating the standard error could be done by replacing the variance, σ with sample standard deviation, S.

c)

$S_P^2 = \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$
$E(S_P^2) = E(\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}) = \frac{1}{n_1+n_2-2}\left((n_1-1)E(S_1^2) + (n_2-1)E(S_2^2)\right) = \frac{1}{n_1+n_2-2}\left((n_1-1)*\sigma_1^2 + (n_2-1)*\sigma_2^2\right) = \sigma^2 * \frac{n_1+n_2-2}{n_1+n_2-2} = \sigma^2$

7-44

$f(x) = p(1-p)^{x-1}$

$L(p) = \prod_{i=1}^{n} p(1-p)^{x_i-1} = p^n(1-p)^{\sum_{i=1}^{n} x_i - n}$

$l(p) = nln(p) + (\sum_{i=1}^{n} x_i - n)ln(1-p)$

$\frac{\partial l}{\partial u} = 0 = \frac{n}{p} - \frac{\sum_{i=1}^{n} x_i - n}{1-p} = \frac{n(1-p) - p\sum_{i=1}^{n} x_i - n}{p(1-p)} = n - p\sum_{i=1}^{n} x_i \therefore$
$\hat{p} = \frac{n}{\sum_{i=1}^{n} x_i}$

8-1

a)

$z_0 = 2.14 \rightarrow P(Z < 2.14) = 0.9838 \therefore \alpha = 2 \times (1 - 0.9838) \rightarrow CI = 100 \times (1-\alpha) = 96.76\%$

b)

$z_0 = 2.49 \rightarrow P(Z < 2.49) = 0.9838 \therefore \alpha = 2 \times (1 - 0.9963) \rightarrow CI = 100 \times (1-\alpha) = 98.72\%$

c)

$z_0 = 1.85 \to P(Z < 1.85) = 0.9678 \therefore \alpha = 2 \times (1 - 0.9678) \to CI = 100 \times (1 - \alpha) = 93.56\%$

d)

$z_0 = 2.00 \to P(Z < 2.00) = 0.9772 \therefore \alpha = (1 - 0.9772) \to CI = 100 \times (1 - \alpha) = 97.72\%$

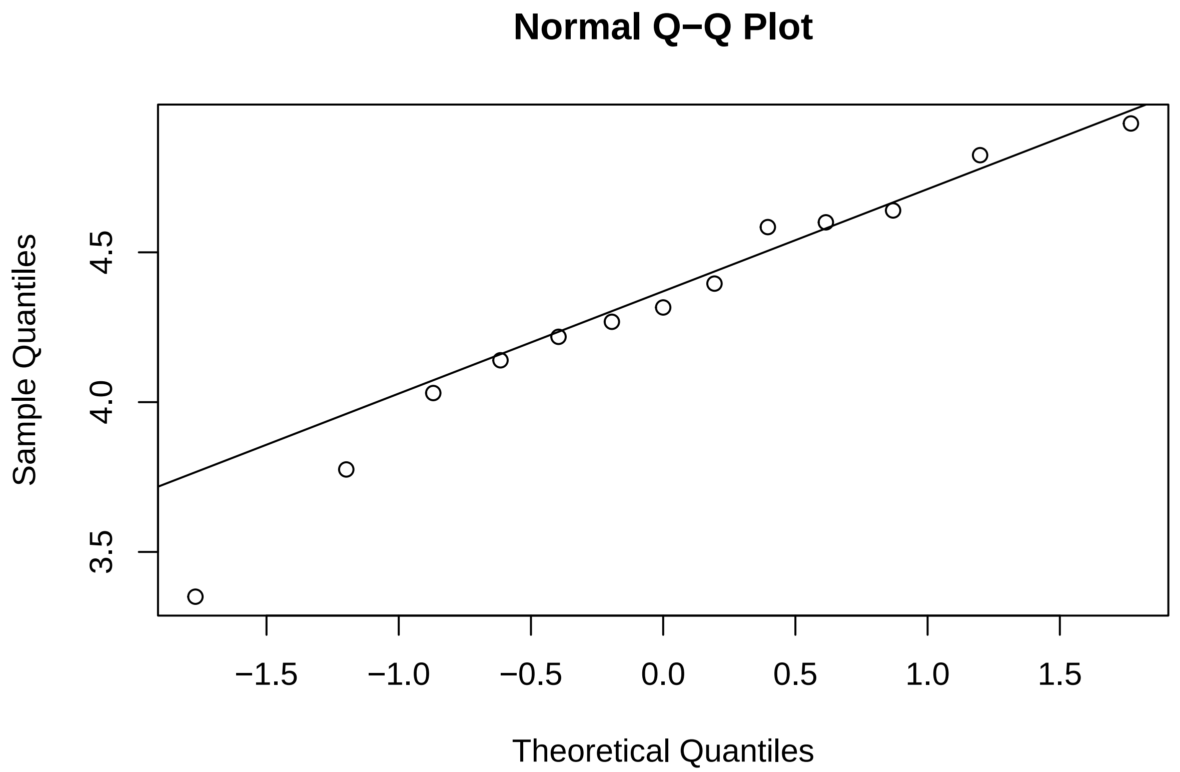
e)

$z_0 = 1.96 \to P(Z < 1.96) = 0.9750 \therefore \alpha = (1 - 0.9750) \to CI = 100 \times (1 - \alpha) = 97.50\%$

8-41

a)

```
speed = c(3.775302, 3.350679, 4.217981, 4.030324, 4.639692, 4.139665, 4.395575, 4.824257, 4.268119, 4.58
qqnorm(speed)
qqline(speed)
```



```r
library(nortest)
ad.test(speed)
```

```
##
##  Anderson-Darling normality test
##
## data:  speed
## A = 0.23339, p-value = 0.7448
```

Based on the plot, the data seems to follow a Normal distribution.

b)

$\bar{x} - t_{0.025,12}\left(\frac{s}{\sqrt{n}}\right) \leq \mu \leq \bar{x} + t_{0.025,12}\left(\frac{s}{\sqrt{n}}\right)$
$\mu = 4.313;\ s = 0.4328;\ n = 13;\ t_{0.025,12} = 2.179$
$4.313 \pm 2.179\left(\frac{0.4328}{\sqrt{13}}\right) \rightarrow 4.051 \leq \mu \leq 4.575$

```r
mean(speed)
```

```
## [1] 4.313222
```

```r
sd(speed)
```

```
## [1] 0.4328017
```

```r
length(speed)
```

```
## [1] 13
```

```r
qt(.025,12)
```

```
## [1] -2.178813
```

```r
qt(.05,12)
```

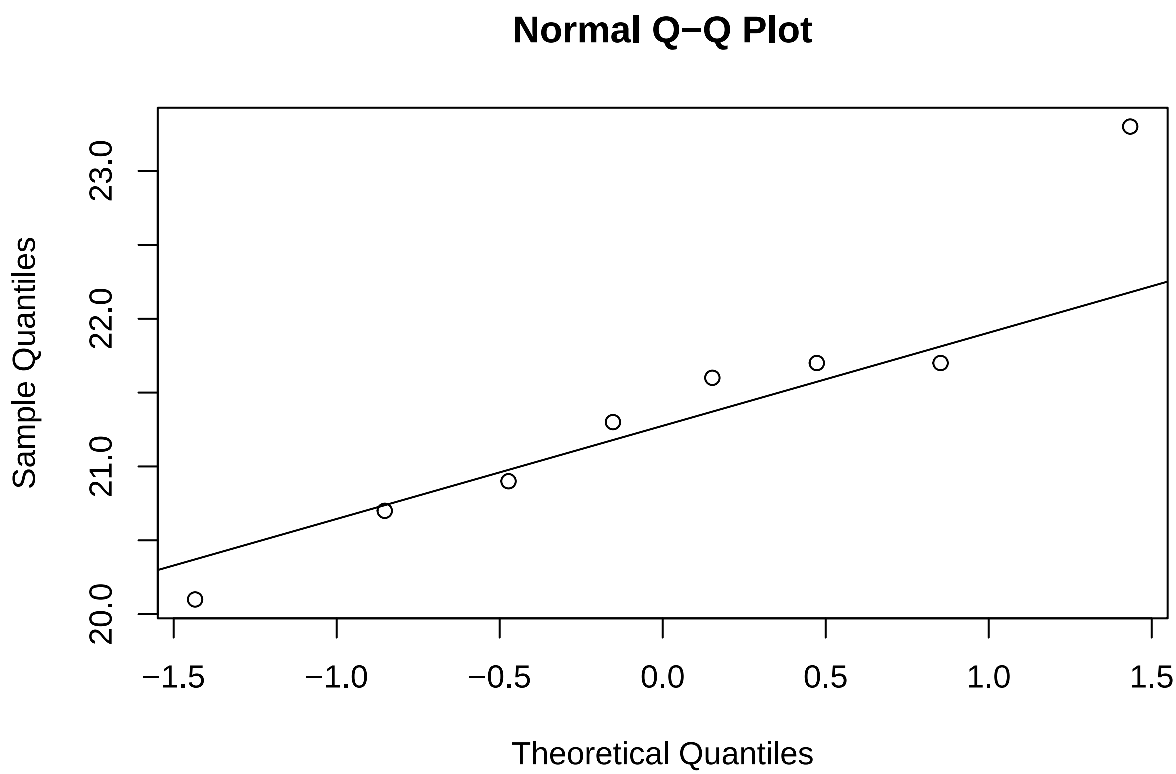
```
## [1] -1.782288
```

c)

$\bar{x} - t_{0.05,12}\left(\frac{s}{\sqrt{n}}\right) \leq \mu$
$\mu = 4.313;\ s = 0.4328;\ n = 13;\ t_{0.05,12} = 1.782$
$4.313 - 1.782\left(\frac{0.4328}{\sqrt{13}}\right) \rightarrow 4.099 \leq \mu$

8-53

```r
temp = c(23.3, 21.7, 21.6, 21.7, 21.3, 20.7, 20.9, 20.1)
qqnorm(temp)
qqline(temp)
```




```r
library(nortest)
ad.test(temp)
```

```
##
##  Anderson-Darling normality test
##
## data:  temp
## A = 0.35247, p-value = 0.3666
```

```r
sd(temp)
```

```
## [1] 0.9463275
```

```r
length(temp)
```

```
## [1] 8
```

$n = 8;\ s = 0.9463$
$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,7} = 16.012;\ \chi^2_{0.975,7} = 1.689$
$\frac{7(0.9463)^2}{16.012} \leq \sigma^2 \leq \frac{7(0.9463)^2}{1.689} = 0.392 \leq \sigma^2 \leq 3.709 \rightarrow 0.626 \leq \sigma \leq 1.926$
Based on the plot, the data seems to follow a Normal distribution.

Q2

a)

for μ_1: $E(\frac{\frac{\sum X_i}{n}+\frac{\sum Y_j}{m}}{2}) = \frac{1}{2}\left(E(\frac{\sum X_i}{n}) + E(\frac{\sum Y_j}{m})\right) = \frac{1}{2}\left(\frac{1}{n}E(\sum X_i) + \frac{1}{m}E(\sum Y_j)\right) = \frac{1}{2}(\mu+\mu) = \mu \therefore$ unbiased
for μ_2: $E(\frac{\sum X_i + \sum Y_j}{n+m}) = \frac{1}{n+m}\left(E(\sum X_i) + E(\sum Y_j)\right) = \frac{1}{n+m}\left(n\mu + m\mu\right) = \frac{n+m}{n+m}(\mu) = \mu \therefore$ unbiased

b)

for μ_1: $Var(\frac{\frac{\sum X_i}{n}+\frac{\sum Y_j}{m}}{2}) = \frac{1}{2}\left(\frac{1}{n^2}Var(\sum X_i) + \frac{1}{m^2}Var(\sum Y_j)\right) = \frac{1}{2}\left(\frac{1}{n^2}n\sigma^2 + \frac{1}{m^2}m\sigma^2\right) = \frac{1}{2}(\frac{\sigma^2}{n} + \frac{\sigma^2}{m}) = \frac{1}{2}(\frac{m\sigma^2}{nm} + \frac{n\sigma^2}{nm}) = \frac{\sigma^2}{2nm}(n+m)$
for μ_2: $Var(\frac{\sum X_i + \sum Y_j}{n+m}) = \frac{1}{n+m}(Var(\sum X_i) + Var(\sum Y_j)) = \frac{1}{n+m}(n\sigma^2 + m\sigma^2) = \sigma^2$

c)

Relative Efficiency: $\frac{MSE(\hat{\mu}_1)}{MSE(\hat{\mu}_2)}$
$MSE(\hat{\mu}_1) = Var(\hat{\mu}_1) - Bias^2 \therefore Relative\ Efficiency = \frac{Var(\hat{\mu}_1)}{Var(\hat{\mu}_2)} = \frac{\frac{\sigma^2(n+m)}{2nm}}{\sigma^2} = \frac{n+m}{2nm}$

d)

Based on relative efficiency being less than 1, $\hat{\mu}_1$ is the better estimator.