

ISyE 6739 Homework 2

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3-109

X = number of passengers that do not show up

X follows a binomial distribution

a)

$$P(X \geq 5) = 1 - P(X \leq 4) = 1 - \sum_{x=0}^4 \binom{125}{x} (0.1)^x (0.9)^{125-x} = 0.9961$$

```
1 - pbinom(4,125,0.1)
```

```
## [1] 0.9961414
```

b)

$$P(X > 5) = 1 - P(X \leq 5) = 1 - \sum_{x=0}^5 \binom{125}{x} (0.1)^x (0.9)^{125-x} = 0.9885$$

```
1 - pbinom(5,125,0.1)
```

```
## [1] 0.9885678
```

3-173

a)

$$\text{ED} = \text{number of emergency department visits per day } \lambda T = 1.8 \quad P(\text{ED} > 5) = 1 - \sum_{x=0}^5 \frac{e^{-1.8} (1.8)^x}{x!} = 0.0103$$

```
1 - ppois(5,1.8)
```

```
## [1] 0.01037804
```

b)

EW = number of emergency department visits per week $\lambda T = 1.8 \times 7 = 12.6$ $P(ED < 5) = \sum_{x=0}^4 \frac{e^{-12.6}(12.6)^x}{x!} = 0.00497$

```
ppois(4,12.6)
```

```
## [1] 0.004979028
```

c)

ET = number of visits in T days $\lambda T = 1.8T$ $P(ET \geq 1) = 1 - P(ET = 0) = 1 - \frac{e^{-1.8T}(1.8T)^0}{0!}$
 $e^{-1.8T} = 0.01 = \ln(e^{-1.8T}) = \ln(0.01) \rightarrow T = 2.558 \text{ days}$

```
log(0.01)/-1.8
```

```
## [1] 2.558428
```

d)

Solve for λ . $P(ED > 5) = 0.1 = 1 - \sum_{x=0}^5 \frac{e^{-\lambda}(\lambda)^x}{x!}$

4-4

$f(x) = \frac{2}{x^3}$ for $x > 1$

a)

$P(X < 2) = \int_1^2 \frac{2}{x^3} dx = \left(\frac{-1}{x^2} \right) \Big|_1^2 = 0.75$

```
integrand <- function(x) {2/x^3}  
integrate(integrand,1,2)
```

```
## 0.75 with absolute error < 8.3e-15
```

b)

$P(X > 5) = \int_5^\infty \frac{2}{x^3} dx = \left(\frac{-1}{x^2} \right) \Big|_5^\infty = 0.04$

```
integrand <- function(x) {2/x^3}  
integrate(integrand,5,Inf)
```

```
## 0.04 with absolute error < 5.8e-07
```

c)

$$P(4 < X < 8) = \int_4^8 \frac{2}{x^3} dx = \left(\frac{-1}{x^2} \right) \Big|_4^8 = 0.0469$$

```
integrand <- function(x) {2/x^3}  
integrate(integrand,4,8)
```

```
## 0.046875 with absolute error < 5.2e-16
```

d)

$$P(X < 4 \text{ or } X > 8) = \int_1^4 \frac{2}{x^3} dx + \int_8^{\infty} \frac{2}{x^3} dx = 0.9531$$

```
integrand <- function(x) {2/x^3}  
a <- integrate(integrand,1,4)  
b <- integrate(integrand,8,Inf)  
a[[1]] + b[[1]]
```

```
## [1] 0.953125
```

e)

Find x , where $P(X < x) = 0.95$.

$$P(X < x) = \int_1^x \frac{2}{x^3} dx = \left(\frac{-1}{x^2} \right) \Big|_1^x = \left(\frac{-1}{x^2} \right) - (-1) = 0.95$$
$$\therefore x = 4.4721$$

4-82

a)