# ISyE 6739 Homework 2

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## 3-109

X = number of passengers that do not show up

X follows a binomial distribution

### a)

$P(X \geq 5) = 1 - P(X \leq 4) = 1 - \sum_{x=0}^{4} \binom{125}{x} (0.1)^x (0.9)^{125-x} = 0.9961$

```
1 - pbinom(4,125,0.1)
```

```
## [1] 0.9961414
```

### b)

$P(X > 5) = 1 - P(X \leq 5) = 1 - \sum_{x=0}^{5} \binom{125}{x} (0.1)^x (0.9)^{125-x} = 0.9885$

```
1 - pbinom(5,125,0.1)
```

```
## [1] 0.9885678
```

## 3-173

### a)

ED = number of emergency department visits per day $\lambda T = 1.8$ $P(ED > 5) = 1 - \sum_{x=0}^{5} \frac{e^{-1.8}(1.8)^x}{x!} = 0.0103$

```
1 - ppois(5,1.8)
```

```
## [1] 0.01037804
```

### b)

EW = number of emergency department visits per week $\lambda T = 1.8 \times 7 = 12.6$ $P(ED < 5) = \sum_{x=0}^{4} \frac{e^{-12.6}(12.6)^x}{x!} = 0.00497$

```
ppois(4,12.6)
```

```
## [1] 0.004979028
```

### c)

ET = number of visits in T days $\lambda T = 1.8T$ $P(ET \geq 1) = 1 - P(ET = 0) = 1 - \frac{e^{-1.8T}(1.8T)^0}{0!}$
$e^{-1.8T} = 0.01 = ln(e^{-1.8T}) = ln(0.01) \rightarrow T = 2.558\ days$

```
log(0.01)/-1.8
```

```
## [1] 2.558428
```

### d)

Solve for $\lambda$. $P(ED > 5) = 0.1 = 1 - \sum_{x=0}^{5} \frac{e^{-\lambda}(\lambda)^x}{x!}$

## 4-4

$f(x) = \frac{2}{x^3}\ for\ x > 1$

### a)

$P(X < 2) = \int_1^2 \frac{2}{x^3} dx = \left(\frac{-1}{x^2}\right)|_1^2 = 0.75$

```
integrand <- function(x) {2/x^3}
integrate(integrand,1,2)
```

```
## 0.75 with absolute error < 8.3e-15
```

### b)

$P(X > 5) = \int_5^\infty \frac{2}{x^3} dx = \left(\frac{-1}{x^2}\right)|_5^\infty = 0.04$

```
integrand <- function(x) {2/x^3}
integrate(integrand,5,Inf)
```

```
## 0.04 with absolute error < 5.8e-07
```

## c)

$P(4 < X < 8) = \int\limits_{4}^{8} \frac{2}{x^3} dx = \left(\frac{-1}{x^2}\right) |_4^8 = 0.0469$

```
integrand <- function(x) {2/x^3}
integrate(integrand,4,8)
```

```
## 0.046875 with absolute error < 5.2e-16
```

## d)

$P(X < 4\, or\, X > 8) = \int\limits_{1}^{4} \frac{2}{x^3} dx + \int\limits_{8}^{\infty} \frac{2}{x^3} dx = 0.9531$

```
integrand <- function(x) {2/x^3}
a <- integrate(integrand,1,4)
b <- integrate(integrand,8,Inf)
a[[1]] + b[[1]]
```

```
## [1] 0.953125
```

## e)

Find x, where $P(X < x) = 0.95$.
$P(X < x) = \int\limits_{1}^{x} \frac{2}{x^3} dx = \left(\frac{-1}{x^2}\right) |_1^x = \left(\frac{-1}{x^2}\right) - (-1) = 0.95$
$\therefore x = 4.4721$

# 4-82

Let W be required water in million gallons. Let S be storage capacity of 350 million gallons. $\mu = 310\, million$; $\sigma = 45\, million$
$X \sim N(310, 45^2)$ ## a) $P(W > S) = 1 - P(X \leq S) = 1 - \Phi(\frac{350-310}{45}) = 0.1870$

```
pnorm(310,350,45)
```

```
## [1] 0.1870314
```

## b) ??????????????

$P(W > w) = P(Z > \frac{w-310}{45}) = 1 - \Phi(\frac{w}{45}) = 0.01$
$\Phi(\frac{w-310}{45}) = 0.99 \rightarrow \frac{w-310}{45} = 2.33$(From Cumulative Standard Normal Table)
$x = 414.85\, million\, gallons$

## c) ?????????????

$P(W < w) = P(Z < \frac{w-310}{45}) = \Phi(\frac{w-310}{45}) = 0.05$
$\Phi(\frac{w-310}{45}) = 0.05 \rightarrow \frac{w-310}{45} = -1.64$ (From Cumulative Standard Normal Table)
$x = 236.2\ million\ gallons$

## d)

Find $\mu$, $X \sim N(\mu, 45^2$
$P(W > S) = 1 - P(X \leq S) = 1 - \Phi(\frac{350-\mu}{45}) = 0.01$
$\Phi(\frac{350-\mu}{45}) = 0.99 \rightarrow \mu = 350 - 2.33 \times 45 = 245.15$
Given 1.4 million people, mean daily demand per person = 175.107 gallons

# 4-117

$\lambda = \frac{1}{E(X)} = \frac{1}{15}$ X = time until first call ## a) $P(X > 30) = \int\limits_{30}^{\infty} \frac{1}{15} e^{\frac{-x}{15}} dx = e^{-2} = 0.1353$

```
integrand <- function(x) {(1/15)*exp(-x/15)}
integrate(integrand,30,Inf)
```

```
## 0.1353353 with absolute error < 3.6e-05
```

## b)

$P(X \leq 10) = P(X > 10)$ Since expect 0 calls in first 10 minutes $P(X > 10) = \int\limits_{10}^{\infty} \frac{1}{15} e^{\frac{-x}{15}} dx = e^{-2/3} = 0.5134$

```
integrand <- function(x) {(1/15)*exp(-x/15)}
integrate(integrand,10,Inf)
```

```
## 0.5134171 with absolute error < 8e-07
```

## c)

$P(5 < X < 10) = \int\limits_{5}^{10} \frac{1}{15} e^{\frac{-x}{15}} dx = e^{-1/3} - e^{-2/3} = 0.2031$

```
integrand <- function(x) {(1/15)*exp(-x/15)}
integrate(integrand,5,10)
```

```
## 0.2031142 with absolute error < 2.3e-15
```

## d)

$P(X < x) = 0.90 \rightarrow P(X < x) = \int\limits_{0}^{x} \frac{1}{15} e^{\frac{-x}{15}} dx = 1 - e^{-x/15} = 0.90 \rightarrow ln(0.10) \times -15$
$\therefore x = 34.5387\ minutes$

```r
log(0.1)*-15
```

```
## [1] 34.53878
```

## 5-6

### a)

The table below shows the PMF table for $x, y \in \{0, 1, 2, 3, 4\}$

```r
df = data.frame()
library(kableExtra)
library(knitr)
for (i in 0:4){
  c0 = choose(60,4-i)*choose(30,0)*choose(10,i)/choose(100,4)
  c1 = choose(60,4-1-i)*choose(30,1)*choose(10,i)/choose(100,4)
  c2 = choose(60,4-2-i)*choose(30,2)*choose(10,i)/choose(100,4)
  c3 = choose(60,4-3-i)*choose(30,3)*choose(10,i)/choose(100,4)
  c4 = choose(60,4-4-i)*choose(30,4)*choose(10,i)/choose(100,4)
  out = c(c0,c1,c2,c3,c4)
  df = rbind(df,out)
}

colnames(df)<-c("x=0","x=1","x=2","x=3","x=4")
row.names(df) <- c("y=0","y=1","y=2","y=3","y=4")

kable(df,"latex",align="c", caption="PMF",label="5-6a")%>%
    kable_styling(latex_options = "hold_position")
```

Table 1: PMF

| | x=0 | x=1 | x=2 | x=3 | x=4 |
|---|---|---|---|---|---|
| y=0 | 0.1243578 | 0.2618059 | 0.1963545 | 0.0621234 | 0.0069889 |
| y=1 | 0.0872686 | 0.1354169 | 0.0665608 | 0.0103539 | 0.0000000 |
| y=2 | 0.0203125 | 0.0206568 | 0.0049921 | 0.0000000 | 0.0000000 |
| y=3 | 0.0018362 | 0.0009181 | 0.0000000 | 0.0000000 | 0.0000000 |
| y=4 | 0.0000536 | 0.0000000 | 0.0000000 | 0.0000000 | 0.0000000 |

### b)

$f_x(x)$ is the sum of the columns in the PMF table.
$f_x(0) = 0.2338$ ; $f_x(1) = 0.4187$ ; $f_x() = 0.2679$ ; $f_x(3) = 0.0724$ ; $f_x(4) = 0.0069$

```r
colSums(df[ ], na.rm=TRUE)
```

```
##         x=0         x=1         x=2         x=3         x=4
## 0.233828714 0.418797697 0.267907350 0.072477351 0.006988887
```

### c)

$E(X) = \sum_0^4 x_i * f(x_i) = 1.2001$

```
0*0.2338 + 1*0.4188 + 2*0.2679 + 3*0.0725 + 4*0.0070
```

```
## [1] 1.2001
```

### d)

$f_{Y|3}(y) = \frac{f_{xy}(3,y)}{f_x(3)}$
From 5-6 part b, using when x=3, and$f_x(3) = 0.0724$, the PMF

```
df1 = data.frame(y=c("0","1","2","3","4"),f=c(0.857,0.143,0,0,0))
names(df1) <- c("y","$f_{Y|3}(y)$")

kable(df1,"latex",align="c", caption="PMF",label="5-6a",escape = FALSE)%>%
    kable_styling(latex_options = "hold_position")
```

Table 2: PMF

| y | $f_{Y\|3}(y)$ |
|---|---|
| 0 | 0.857 |
| 1 | 0.143 |
| 2 | 0.000 |
| 3 | 0.000 |
| 4 | 0.000 |

### e)

$E(Y|X = 3) = \sum_0^4 y_i * f_{Y|3}(y)_i = 0 * 0.857 + 1 * 0.143 = 0.143$

### f)

$V(Y|X = 3) = \sum_0^4 (y_i^2 * f_{Y|3}(y)_i) - E(Y|X = 3)^2 = 0^2 * 0.857 + 1^2 * 0.143 - 0.143^2 = 0.123$

### g)

X and Y are not independent.
$f_{xy}(x, y) \neq f_x(x) \times f_y(y) \ for \ all \ x \ and \ y$
For example, $f_{xy}(1, 1) = 0.1354$ ; $f_x(1) = 0.4187$ ; $f_y(1) = 0.2994$
$0.1354 \neq 0.4187 \times 0.2994$

## Q2

### a)

$P(X) = \binom{3}{x} \times 0.98^x \times 0.02^{3-x}$

```
df = data.frame()
library(kableExtra)
library(knitr)
c0 = dbinom(0:3,3,0.98)
for (x in 1:4){

  out = c(c0[x])
  df = rbind(df,out)
}

names(df) <- c("P(x)")
row.names(df) <- c("$f(0)$","$f(1)$","$f(2)$","$f(3)$")

kable(df,"latex",align="c", caption="PMF",label="Q2",escape = FALSE)%>%
    kable_styling(latex_options = "hold_position")
```

Table 3: PMF

| | P(x) |
|---|---|
| $f(0)$ | 0.000008 |
| $f(1)$ | 0.001176 |
| $f(2)$ | 0.057624 |
| $f(3)$ | 0.941192 |

### b)

```
c0 = pbinom(0:3,3,.98)
df = data.frame()
library(kableExtra)
library(knitr)

for (x in 1:4){

  out = c(c0[x])
  df = rbind(df,out)
}

names(df) <- c("P(x)")
row.names(df) <- c("$F(0)$","$F(1)$","$F(2)$","$F(3)$")

kable(df,"latex",align="c", caption="CDF",label="Q2",escape = FALSE)%>%
    kable_styling(latex_options = "hold_position")
```

$F(x) = 0 \; for \; x \leq 0$
$F(x) = 1 \; for \; x \geq 3$

Table 4: CDF

| | P(x) |
|---|---|
| $F(0)$ | 0.000008 |
| $F(1)$ | 0.001184 |
| $F(2)$ | 0.058808 |
| $F(3)$ | 1.000000 |

### c)

$$E(X) = \sum_{x=0}^{3} x * f(x) = 2.94$$

$$V(X) = \sum_{x=0}^{3} x^2 * f(x) - E(X)^2 = 0.0588$$

```
0*.000008 + 1*0.001176 + 2*0.057624 + 3*.941192
```

```
## [1] 2.94
```

```
0^2*.000008 + 1^2*0.001176 + 2^2*0.057624 + 3^2*.941192 - 2.94^2
```

```
## [1] 0.0588
```