

ISyE 6739 Homework 1

Patrick Gardocki

2023-05-21

2-19

a)

$$A' = 3, 5, 6, 8$$

b)

$$A \cap B = 1, 2$$

c)

$$(A \cap B) \cup C = 1, 2, 3, 4, 5$$

d)

$$(B \cup C)' = 7, 8$$

e)

$$(A \cap B)' \cup C = 1, 3, 4, 5, 6, 7, 8$$

2-81

a)

$$P(A) = \frac{52^8}{62^8} = 0.2448461$$

$$52^8/62^8$$

```
## [1] 0.2448461
```

b)

$$P(B) = \frac{10^8}{62^8} = 4.580011e-7$$

```
10^8/62^8
```

```
## [1] 4.580011e-07
```

c)

$$P(B \geq 1) = 1 - P(A) = 0.7551539$$

```
1 - 52^8/62^8
```

```
## [1] 0.7551539
```

d)

$$P(B = 2) = \frac{\binom{8}{6} \times 10^2 \times 52^6}{62^8} = 0.2535389$$

```
(choose(8,6)*10^2*52^6)/(62^8)
```

```
## [1] 0.2535389
```

2-90

a)

$$P(A) = \frac{\binom{5}{1} \times 36^5}{36^6} = 5/36 \approx 0.138889$$

```
(choose(5,1)*36^5)/36^6
```

```
## [1] 0.1388889
```

b)

$$P(B) = \frac{\binom{5}{1} \times 36^5}{36^6} = 5/36 \approx 0.138889$$

```
(choose(5,1)*36^5)/36^6
```

```
## [1] 0.1388889
```

c)

$$P(A \cap B) = P(A) \cdot P(B) = \frac{25}{36^2} \approx 0.019290$$

```
(5/36) * (5/36)
```

```
## [1] 0.01929012
```

d)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{5}{36} + \frac{5}{36} - \frac{25}{36^2} = 0.258487$$

```
(5/36) + (5/36) - (25/36^2)
```

```
## [1] 0.2584877
```

2-141

a)

$$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A)}{1 - P(B)} = \frac{P(A)}{1 - \frac{10^8}{62^8}}$$
$$P(A) = \frac{52^8}{62^8}; P(B) = \frac{10^8}{62^8}$$
$$\therefore P(A|B') = \frac{\frac{52^8}{62^8}}{1 - \frac{10^8}{62^8}} = 0.24484$$

```
((52^8/62^8)/(1-(10^8/62^8)))
```

```
## [1] 0.2448462
```

b) ???????

$$P(A' \cap B) = P(A'|B)P(B) = P(B) = \frac{10^8}{62^8} = 4.580011e-7$$

B is a subset of A' so $P(A'|B) = 1$

```
10^8/62^8
```

```
## [1] 4.580011e-07
```

c)

Let T be passwords with exactly 2 integers and I be passwords with at least 1 integer.

$$P(T|I) = \frac{P(T \cap I)}{P(I)} = \frac{P(T)}{P(I)} \text{ Since } P(T) \text{ is a subset of } P(I), \frac{P(T \cap I)}{P(I)} = \frac{P(T)}{P(I)}$$

From 2-81 part d, $P(T) = 0.2535$

From 2-81 part c, $P(I) = 0.7551$

$$\therefore P(T|I) = \frac{0.2535}{0.7551} = 0.3357$$

```
0.2535/0.7551
```

```
## [1] 0.3357171
```

2-179

a)

$$P(F|S) = 0.6; P(F|V) = 0.04$$

$$P(F) = P(F|S)P(S) + P(F|V)P(V) = 0.6 \times 0.2 + 0.04 \times 0.8 = 0.152$$

```
0.6*0.2 + 0.8*0.04
```

```
## [1] 0.152
```

b)

$$P(S|F) = \frac{P(F|S)P(S)}{P(F)} = \frac{0.6 \times 0.2}{0.152} = 0.78947$$

```
(0.6*0.2)/0.152
```

```
## [1] 0.7894737
```

c)

$$P(V|F') = \frac{P(F'|V)P(V)}{P(F')} = \frac{0.96 \times 0.8}{1 - 0.152} = 0.90566$$

```
(0.96*0.8)/(1-0.152)
```

```
## [1] 0.9056604
```

Q2

$$\binom{13}{1} \binom{4}{3} \binom{12}{2} \binom{4}{1} \binom{4}{1} = 54,912$$

```
choose(13,1) * choose(4,3)*choose(12,2)*choose(4,1)^2
```

```
## [1] 54912
```