HW 4

Patrick Gardocki

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9-20

a)

$\alpha = P(\bar{X} \leq 4.85) + P(\bar{X} > 5.15) = P(\frac{\bar{X}-5}{\frac{0.25}{\sqrt{8}}} \leq \frac{4.85-5}{\frac{0.25}{\sqrt{8}}}) + P(\frac{\bar{X}-5}{\frac{0.25}{\sqrt{8}}} > \frac{5.15-5}{\frac{0.25}{\sqrt{8}}}) = P(Z \leq -1.7) + P(Z > 1.7) = 0.04457 + (1 - 0.95543) = 0.08913$

```
pnorm(-1.7)+(1-pnorm(1.7))
```

```
## [1] 0.08913093
```

b)

Power $= 1 - \beta$
$\beta = P(4.85 \leq \bar{X} \leq 5.15) = P(\frac{4.85-5.1}{\frac{0.25}{\sqrt{8}}} \leq \frac{\bar{X}-5.1}{\frac{0.25}{\sqrt{8}}} \leq \frac{5.15-5.1}{\frac{0.25}{\sqrt{8}}}) = P(-2.83 \leq Z \leq 0.566) = P(Z \leq 0.566) - P(Z \leq -2.83) = 0.71566 - 0.00233 = 0.7119$

```
pnorm(0.566)-(pnorm(-2.83))
```

```
## [1] 0.7119757
```

9-23

a) $\bar{x} = 5.2$

$z_o = \frac{5.2-5}{\frac{0.25}{\sqrt{8}}} = 2.26$
P-value$=2(1 - \Phi(|2.26|)) = 2(1 - 0.988) = 0.0238$

```
2*(1-pnorm(2.26))
```

```
## [1] 0.02382125
```

b) $\bar{x} = 4.7$

$z_o = \frac{4.7-5}{\frac{0.25}{\sqrt{8}}} = 2.26$
P-value$=2(1-\Phi(|-3.39|)) = 2(1-0.999) = 0.00069$

```
2*(1-pnorm(3.39))
```

```
## [1] 0.0006989262
```

c) $\bar{x} = 5.1$

$z_o = \frac{5.1-5}{\frac{0.25}{\sqrt{8}}} = 2.26$
P-value$=2(1-\Phi(|1.131|)) = 2(1-0.870) = 0.258$

```
2*(1-pnorm(1.131))
```

```
## [1] 0.2580551
```

9-43(a)(b)

a)

if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$, reject H_o. $\alpha = 0.05,\ \sigma = 0.9$
$z_o = \frac{2.78-3}{\frac{0.9}{\sqrt{15}}} = -0.95$
$-z_{0.025} = -1.96 < -0.95 \therefore$ fails to reject null hypothesis

b)

$z_{0.025} = 1.96$ $\beta = \Phi(z_{0.025} + \frac{3-3.25}{0.9/\sqrt{15}}) - \Phi(-z_{0.025} + \frac{3-3.25}{0.9/\sqrt{15}}) = \Phi(0.88) - \Phi(-3.04) = 0.80939$

9-64 (a)(b)(e)!!!!!!!!!!!!!!!!!!!!!!!!

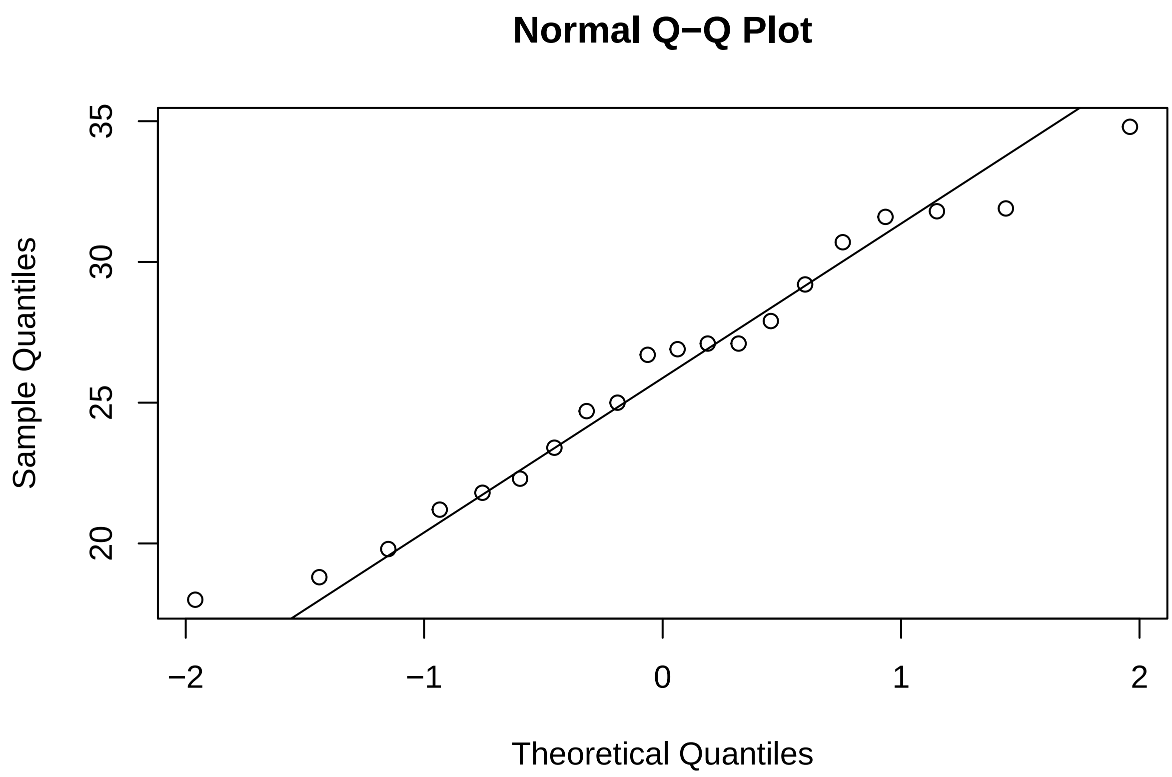
a)

if $t_0 > t_{\alpha,n-1}$, reject H_o. $\alpha = 0.01,\ t_{0.01,19} = 2.539,\ \bar{x} = 26.04,\ s = 4.78$
$t_o = \frac{26.04-25}{\frac{4.78}{\sqrt{20}}} = 0.97$
$t_{0.01,19} = 2.539 > 0.97 \therefore$ fails to reject null hypothesis. Insufficient evidence to show that rainfall is greater than 25 acre-feet.

!!!!!!!!!!!!!!!!!!!!!!!P-value$=2(1-\Phi(|2.539|)) = 8$

b)

```
data = c(18,30.7,19.8,27.1,22.3,18.8,31.8,23.4,21.2,27.9,31.9,27.1,25.0,24.7,26.9,21.8,29.2,34.8,26.7,3
qqnorm(data)
qqline(data)
```



The plot suggests that the data follows a normal distribution.

e)

$26.04 - 2.539(\frac{4.78}{\sqrt{20}}) \leq \mu \rightarrow 23.326 \leq \mu$ Since the lower limit is less than the mean diameter, no evidence that true mean rainfall is greater.

```
qt(.01,19)
```

```
## [1] -2.539483
```

9-98

a)

if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$, reject H_o. $\alpha = 0.05,\ x = 117,\ n = 484$
$z_o = \frac{117-484*0.5}{\sqrt{484*0.5*0.5}} = -11.36$

$-z_{0.025} = -1.96 > -11.36 \therefore$ null hypothesis is rejected and proportion of students planning graduate studies is not 0.5 with $\alpha = 0.05$.

P-value $= 2(1 - \Phi(11.36)) \approx 0$

b)

Using a two-sided confidence interval, if the confidence interval does not contain 0.5, then the proportion of students planning graduate studies is not 0.5 with $\alpha = 0.05$.

$\hat{p} = \frac{117}{484}$
$0.242 - 1.96\sqrt{\frac{0.242*0.758}{484}} \leq p \leq 0.242 + 1.96\sqrt{\frac{0.242*0.758}{484}} = 0.204 \leq p \leq 0.280$

10-4(a)(b)(c)

a)

$H_o: \ \mu_1 - \mu_2 = 0$
$H_1: \ \mu_1 - \mu 2 \neq 0$
$Reject\ H_o\ if\ z_o < -z_{\alpha/2}\ or\ z_o > z_{\alpha/2}$
$\sigma_1 = 0.02, \ \sigma_2 = 0.025, \ \bar{x_1} = 16.015, \ \bar{x_2} = 16.005$
$z_o = \frac{16.015-16.005}{\sqrt{\frac{0.020^2}{10}+\frac{0.025^2}{10}}} = 0.79 \rightarrow -1.96 < 0.702 < 1.96 \therefore$ fail to reject null hypothesis. There is no good proof that the machines fill volumes are different.

P-Value$= 2(1 - \Phi(0.702)) = 0.4826$

```
test1= c(16.03, 16.01, 16.04, 15.96, 16.05, 15.98, 16.05, 16.02, 16.02, 15.99)
test2= c( 16.02, 16.03,15.97, 16.04, 15.96, 16.02,16.01, 16.01, 15.99, 16.00)
mean(test1)
```

```
## [1] 16.015
```

```
mean(test2)
```

```
## [1] 16.005
```

b)

$(16.015-16.005)-1.96\sqrt{\frac{0.020^2}{10}+\frac{0.025^2}{10}} \leq \mu_1 - \mu_2 \leq (16.015-16.005)+1.96\sqrt{\frac{0.020^2}{10}+\frac{0.025^2}{10}} \rightarrow -0.01789 \leq \mu_1 - \mu_2 \leq 0.03789$ Since the interval contains 0, there is no difference between the means with a 95% confidence interval.

c)

$$\beta = \Phi\left(1.96 - \frac{0.04}{\sqrt{\frac{0.020^2}{10}+\frac{0.025^2}{10}}}\right) - \Phi\left(-1.96 - \frac{0.04}{\sqrt{\frac{0.020^2}{10}+\frac{0.025^2}{10}}}\right) = \Phi(1.96-3.95) - \Phi(-1.96-3.95) = 0.02329$$

10-14

a)

$s_p = \sqrt{\frac{(12-1)1.26^2+(16-1)1.99^2}{12+16-2}} = 1.719$
$t_o = \frac{-1.21}{1.719\sqrt{\frac{1}{12}+\frac{1}{16}}} = -1.842$
Degree of Freedom= $n_1 + n_2 - 2 = 12 + 16 - 2 = 26$ P-value: $2(P(t > 1.842)) = 0.077 \rightarrow 2(0.025) < 0.077 < 2(0.05)$ Pooled Standard Deviation: $\sqrt{\frac{1.26^2+1.99^2}{2}} = 1.665$ This is a two sided test because $H_o : \mu_1 - \mu_2 = 0$

b)

Since $2(0.025) < 0.077 < 2(0.05)$, fail to reject the null hypothesis for $\alpha = 0.05\ \textit{and}\ 0.01$.

c)

The sample standard deviations are only slightly different so it can be assumed that the sample variance is also similar.

d)

P-value: $P(t < -1.842) = 0.0326,\ 0.025 < 0.0326 < 0.05 \therefore$ P-value is less than $\alpha = 0.05$, and null hypothesis is rejected.

```
pnorm(-1.8428)
```

```
## [1] 0.03267911
```

10-24

a)

$H_0 : \mu_1 - \mu_2 = 0$
$H_1 : \mu_1 - \mu_2 < 0$
$v = \frac{\left(\frac{12^2}{10}+\frac{22^2}{16}\right)^2}{\frac{(\frac{12^2}{10})^2}{10-1}+\frac{(\frac{22^2}{16})^2}{16-1}} \approx 23$
$t_{0.05,23} = 1.714,\ \bar{x_1} = 290,\ \bar{x_2} = 321,\ s_1 = 12,\ s_2 = 22$
$t_0 = \frac{290-321}{\sqrt{\frac{12^2}{10}+\frac{22^2}{16}}} = -4.64$
Null Hypothesis is rejected if $t_o < -t_{\alpha,v}$. Since -4.64 < -1.714, null hypothesis is rejected and supplier 2 makes gears with higher mean impact strength.

b)

$H_0 : \mu_1 - \mu_2 = 25$
$H_1 : \mu_1 - \mu_{>}25$
$v = \approx 23$

$t_{0.05,23} = 1.714,\ \bar{x_1} = 290,\ \bar{x_2} = 321,\ s_1 = 12,\ s_2 = 22,\ \Delta_0 = 25$
$t_0 = \frac{(321-290)-25}{\sqrt{\frac{12^2}{10}+\frac{22^2}{16}}} = 0.898$

Null Hypothesis is rejected if $t_o > t_{\alpha,v}$. Since $0.898 < 1.714$, fail to reject null hypothesis and no evidence that supplier 2 makes gears with mean impact strength 25 ft-lb higher than supplier 1.

c)

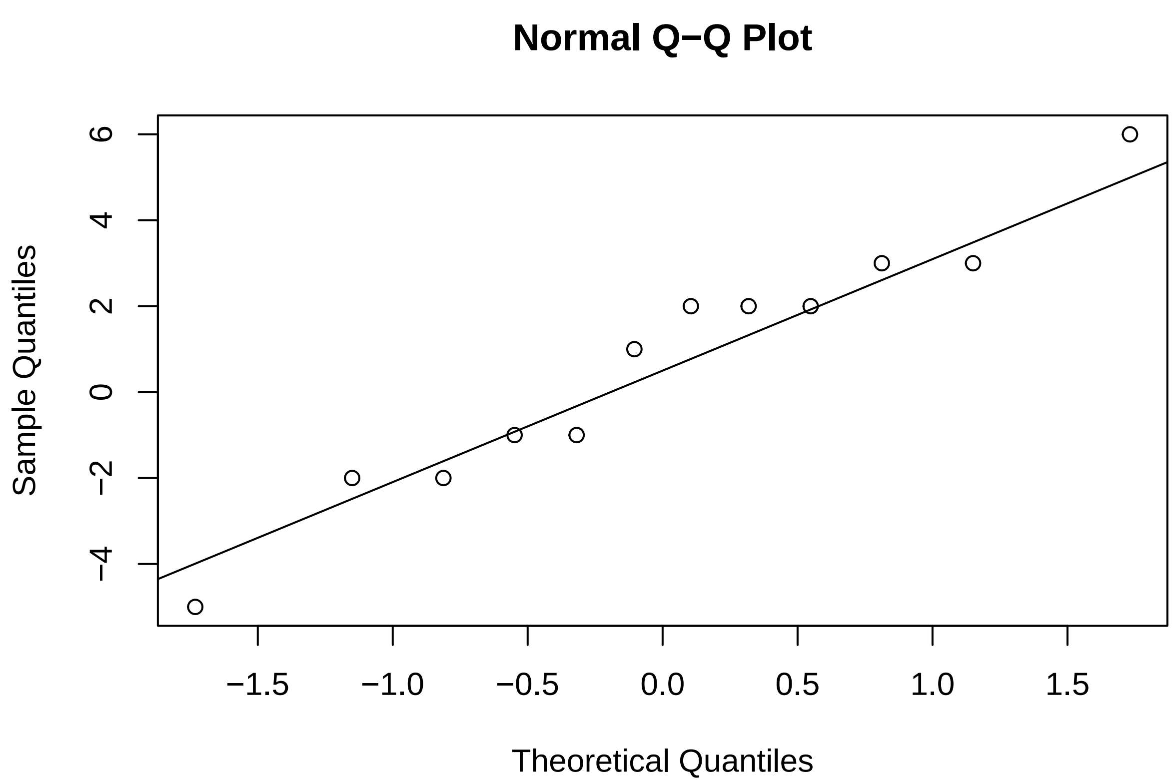
$t_{0.025,25} = 2.0598$
$(321-290) - 2.059\sqrt{\frac{12^2}{10}+\frac{22^2}{16}} \leq \mu_2 - \mu_1 \leq (321-290) + 2.059\sqrt{\frac{12^2}{10}+\frac{22^2}{16}}$
$17.242 \leq \mu_2 - \mu_1 \leq 44.758$

Interval does not contain 0, this means that supplier 2 makes gears with higher impact strength.

10-52

a)

```
data = c(-1,2,2,3,-5,3,6,1,2,-1,-2,-2)
qqnorm(data)
qqline(data)
```



```
mean(data)
```

```
## [1] 0.6666667
```

```
sd(data)
```

```
## [1] 2.964436
```

```
qt(.975,11)
```

```
## [1] 2.200985
```

```
x = c(17,16,21,14,18,24,16,14,21,23,13,18)
y= c(18,14,19,11,23,21,10,13,19,24,15,20)
t.test(x, y, paired = TRUE, alternative = "two.sided")
```

```
##
##  Paired t-test
##
## data:  x and y
## t = 0.77904, df = 11, p-value = 0.4524
## alternative hypothesis: true mean difference is not equal to 0
## 95 percent confidence interval:
##  -1.216846  2.550179
## sample estimates:
## mean difference
##       0.6666667
```

The data seems to follow a normal distribution.

b)

$0.667 - 2.201(\frac{2.964}{\sqrt{12}}) \leq \mu_d \leq 0.667 + 2.201(\frac{2.964}{\sqrt{12}}) \rightarrow -1.216 \leq \mu_d \leq 2.55$

Since interval contains 0, can not determine if either language is preferable.

10-88

a)

$H_o: p_1 = p_2,\ H_1 = p_1 \neq p_2$
$n_1 = 500,\ n_2 = 400,\ x_1 = 385,\ x_2 = 267,\ \hat{p_1} = 0.77,\ \hat{p_2} = 0.6675,\ \hat{p} = \frac{385+267}{500+400} = 0.724$
$z_o = \frac{0.77-0.6675}{\sqrt{0.724(1-0.724)(\frac{1}{500}+\frac{1}{400})}} = 3.42$

H_o rejected if $z_o < -z_{\alpha/2}$ or $z_o > z_{\alpha/2}$, $3.42 > 1.96 \therefore$ null hypothesis is rejected and there is a difference in support between counties.

P-value: $2(1 - P(Z < 3.42)) = 0.000626$

```r
2*(1-pnorm(3.42))
```

```
## [1] 0.0006262114
```

b)

$(0.77-0.6675)-1.96\sqrt{\frac{0.77(1-0.77)}{500}+\frac{0.6675(1-0.6675)}{400}} \leq p_1-p_2 \leq (0.77-0.6675)+1.96\sqrt{\frac{0.77(1-0.77)}{500}+\frac{0.6675(1-0.6675)}{400}} \rightarrow$ $0.0434 \leq p_1-p_2 \leq 0.1616$

Proportion lies between 0.0434 and 0.1616 with 95% confidence. The interval does not contain 0, implying that there is a difference between counties.