Gardocki_Patrick_HW4_Report

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1. Optimization

a)

$\nabla l(\theta) = l'(\theta)$

$= \sum_{i=1}^{m} \frac{d}{d\theta}(-log(1 + exp(-\theta x^i))) + \frac{d}{d\theta}((y^i - 1)\theta x^i)$

$\nabla l(\theta) = \sum_{i=1}^{m} \left(\frac{x^i exp(-\theta x^i)}{1+exp(-\theta x^i)} + (y^i - 1)x^i \right)$

b)

Initialize: θ, γ, ϵ

While: $|\theta^{t+1} - \theta^t| > \epsilon$

Do: $\theta^{t+1} = \theta^t + \gamma \nabla l(\theta)$

c)

Initialize: $\theta, \gamma, \epsilon, K$

While: $|\theta^{t+1} - \theta^t| > \epsilon$

Do: $\theta^{t+1} = \theta^t + \gamma \sum_{i \subset S_k} \left(\frac{x^i exp(-\theta x^i)}{1+exp(-\theta x^i)} + (y^i - 1)x^i \right)$

d)

Given: $\nabla l(\theta) = \sum_{i=1}^{m} \left(\frac{x^i exp(-\theta x^i)}{1+exp(-\theta x^i)} + (y^i - 1)x^i \right)$

$l''(\theta) = \sum_{i=1}^{m} \frac{d}{d\theta} \left(\frac{-x^i}{1+exp(-\theta x^i)} \right) = \sum_{i=1}^{m} \left(\frac{-{x^i}^2}{(1+exp(-\theta x^i)^2} \right)$

The regression problem is concave because the Hessian matrix is less than 0. There is a global minimum and gradient descent will achieve a unique solution when the gradient is at or near 0.

2. Naive bayes for spam filtering

a)

$P(y=0)=\frac{3}{7};\quad P(y=1)=\frac{4}{7}$

Message: million dollar offer: [0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0]

secret offer today: [1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]

secret is secret: [2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0]

low price for valued customer: [0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0]

play secret sports today: [1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0]

sports is healthy: [0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0]

low price pizza: [0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1]

b)

$l(\theta)=\sum_{i=1}^{m}\sum_{k=1}^{d}x_k^i log\theta$

$log(P(x,y))=log\left(P(y)\prod\theta_{c,k}^{x_k^i}\right)=\sum_{i=1}^{m}\left(log(P(y)+\sum_{k=1}^{n}log\theta_{c,k}^{x_k^i}\right)$

Constraint: $\sum_{k=1}^{m}\theta_{c,k}=1$

$L(P(x,y))=\sum_{i=1}^{m}\left(log(P(y)+\sum_{k=1}^{n}log\theta_{c,k}^{x_k^i}\right)+\lambda\sum_{k=1}^{n}\theta_{c,k}-1$

$\frac{dL}{d\theta}=\sum_{i=1}^{m}(\frac{1^{x_k^i}}{\theta c,k})+\lambda=0$

$\theta_{c,k}=\frac{-\sum_{i=1}^{m}1^{x_k^i}}{\lambda}$

Given Constraint: $\sum_{k=1}^{m}\theta_{c,k}=1=\frac{-\sum_{k=1}^{n}\sum_{i=1}^{m}1^{x_k^i}}{\lambda}$

$\lambda=-\sum_{k=1}^{n}\sum_{i=1}^{m}1^{x_k^i}$

$\theta_{c,k}=\frac{\sum_{i=1}^{m}1^{x_k^i}}{\sum_{k=1}^{n}\sum_{i=1}^{m}1^{x_k^i}}$

$\theta_{0,1}=\frac{3}{4}$

$\theta_{0,7}=\frac{2}{3}$

$\theta_{1,1}=\frac{1}{4}$

$\theta_{1,15}=\frac{1}{1}=1$

c)

"today is secret"

$P(x|y=0)=\prod\theta_{c,k}^{x_k}=\theta_{0,1}\theta_{0,7}\theta_{0,11}=\frac{3}{16}$

$P(x|y=1)=\prod\theta_{c,k}^{x_k}=\theta_{1,1}\theta_{1,7}\theta_{1,11}=\frac{1}{16}$

$P(y=0|x)=\frac{P(x|y=0)P(y=0)}{P(x|y=0)P(y=0)+P(x|y=1)P(y=1)}=\frac{9}{13}$

Since $P(y=0|x)>\frac{1}{2}$, the message is spam.

3. Comparing Classifiers

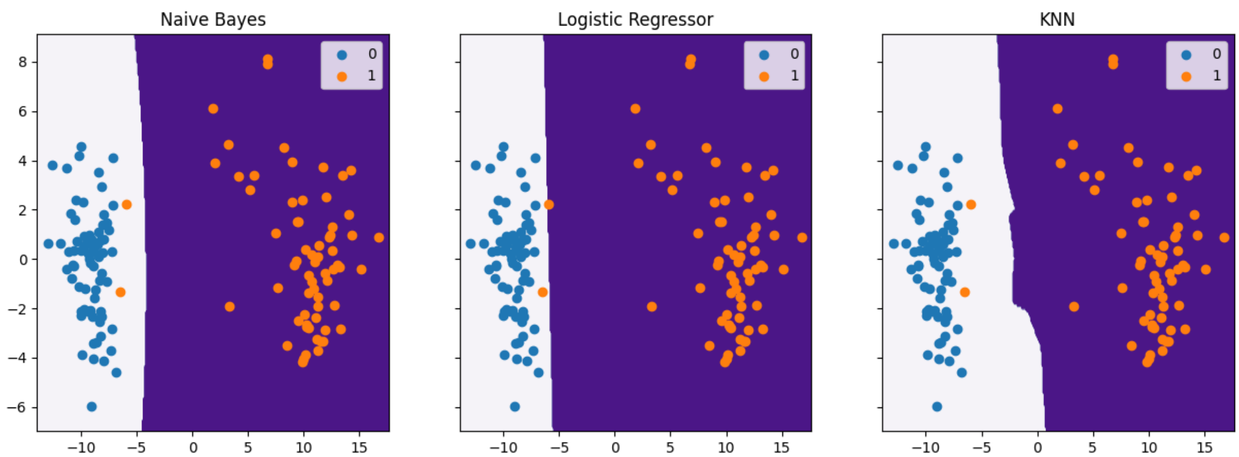
a)

Table 1: Conditional Sample Mean

Classifier	Testing.Accuracy
Naive Bayes	0.9411
Logistic Regression	0.9117
KNN	0.9411

Naive Bayes and KNN performed better than Logistic Regression. This may because the data is not linearly separable, and Logistic Regression is a linear classifier.

b)



Logistic Regression shows a linear decision boundary while Naive Bayes and KNN both show non-linear boundaries.