ME6406 Machine Vision

Professor Kok-Meng Lee
Georgia Institute of Technology
George W. Woodruff School of Mechanical Engineering
Atlanta, GA 30332-0405
Email: kokmeng.lee@me.gatech.edu

Part 2D:
Detection of dominant points on plane curve

http://kmlee.gatech.edu/me6406

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Course Outline

- Introduction and low-level processing
  - Physics of digital images, histogram equalization, segmentation, edge detection, linear filters.
- **Model-based Vision**
  - **Hough transform, pattern representation, matching**
- Geometric methods
  - Camera model, calibration, pose estimation
- Neural network for machine vision
  - Basics, training algorithms, and applications
- Color images and selected topics
  - Physics, perception, processing and applications

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Contents

- **Detection of dominant points on plane curve**
  - Curvature
  - Scale-space filtering
  - Scale space description of corner

In Canvas ("Reading materials" folder)

A. Rattarangsi and T. R. Chin, "Scale-based Detection of corners and planar curves," *IEEE Transactions on Pattern Analysis and Machine Intelligence,* Vol. 14, no 4m April 1992.

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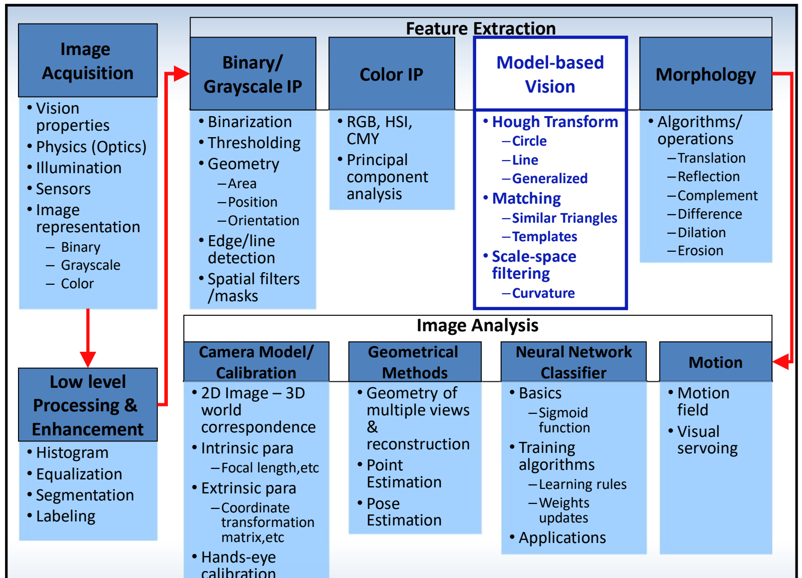

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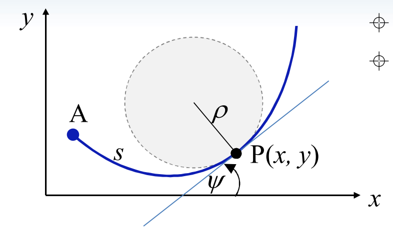
Detection of dominant points on plane curve

- Curvature
- Scale-space filtering
- Scale space description of corner

Curvature

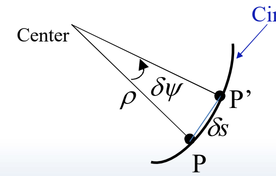


- A is a fixed point.
- $P(x, y)$ is a variable point.
- Let s is the arc-length AP, and ψ is the angle between the x-axis and the tangent at P.

$$\tan\psi = \frac{dy}{dx}$$

Curvature is the rate of turning of the tangent with respect to the arc:

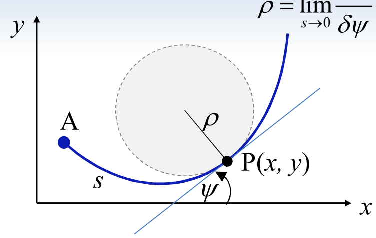
$$\text{Curvature } K = \frac{d\psi}{ds}$$



$$\text{Radius of curvature, } \rho = \lim_{s\to 0}\frac{\delta s}{\delta\psi} = \frac{ds}{d\psi} = \frac{1}{K}$$

Curvature

$$\rho = \lim_{s\to 0}\frac{\delta s}{\delta\psi} = \frac{ds}{d\psi} = \frac{1}{K}$$



$$\tan\psi = \frac{dy}{dx}$$

Differentiating with respect to s,

$$\underbrace{\sec^2\psi\frac{d\psi}{ds}}_{1/\rho} = \frac{d}{ds}\left(\frac{dy}{dx}\right) = \underbrace{\frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{ds}}_{\frac{d^2y}{dx^2}\cos\psi}$$

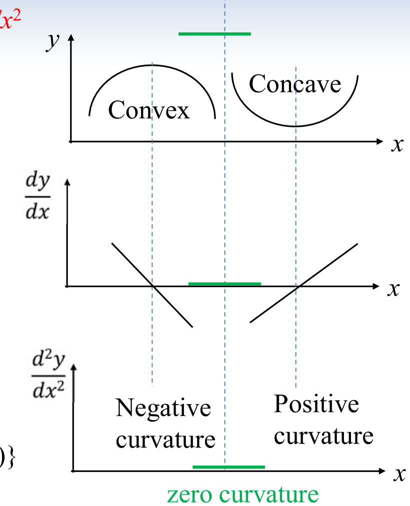
$$\sec^2\psi\frac{1}{\rho} = \frac{d^2y}{dx^2}\cos\psi$$

$$\Rightarrow \rho = \frac{\sec^3\psi}{\frac{d^2y}{dx^2}} = \frac{\left(1+\tan^2\psi\right)^{3/2}}{\frac{d^2y}{dx^2}}$$

$$K = \frac{1}{\rho} = \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

The sign of K depends on d^2y/dx^2

$$K = \frac{1}{\rho} = \frac{\frac{d^2y}{dx^2}}{\underbrace{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}_{\text{Always positive}}}$$



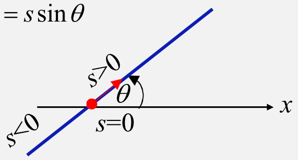
In 2D images, $K = \{x(s), y(s)\}$

In parametric form, $x(s)$ and $y(s)$

Examples

$x(s) = s\cos\theta$
$y(s) = s\sin\theta$

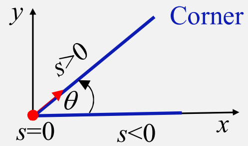
Straight line



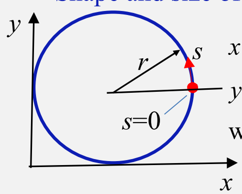
$$x(s) = \begin{cases} -s & s < 0 \\ s\cos\theta & s \ge 0 \end{cases}$$

$$y(s) = \begin{cases} 0 & s < 0 \\ s\sin\theta & s \ge 0 \end{cases}$$

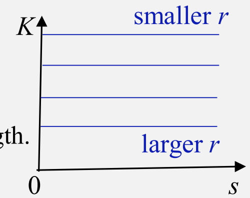
Corner



Shape and size of circle.



$x(s) = r\cos(2\pi s / N)$
$y(s) = r\sin(2\pi s / N)$
where N is the path length.



In parametric form,

$x(s)$; $\dot{x} = \dfrac{dx}{ds}$ and $\ddot{x} = \dfrac{d^2x}{ds^2}$

Similarly,

$y(s)$; $\dot{y} = \dfrac{dy}{ds}$ and $\ddot{y} = \dfrac{d^2y}{ds^2}$

Thus,

$y' = \dfrac{\dot{y}}{\dot{x}}$ where $y' = \dfrac{dy(s)}{dx(s)}$

$$y'' = \frac{d^2y}{dx^2} = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\dot{x}^3}$$

Recall $$K(s) = \frac{y''}{\left[1 + (y')^2\right]^{3/2}} = \frac{\dfrac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\dot{x}^3}}{\left[1 + \left(\dfrac{\dot{y}}{\dot{x}}\right)^2\right]^{3/2}}$$

$$K(s) = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\left(\dot{x}^2 + \dot{y}^2\right)^{3/2}}$$

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Scale-space filtering method

Let $g_\sigma(s) = \dfrac{1}{\sigma\sqrt{2\pi}} \exp\left(\dfrac{-s^2}{2\sigma^2}\right)$ be the Gausspan filter.

Smoothing the curve with $g_\sigma(s)$is the same as convolving with $x(s)$ and $y(s)$. Mathematically,

$$X(s,\sigma) = x * g_\sigma = \int_{-\infty}^{\infty} x(s-u) g_\sigma(u)du$$

$$Y(s,\sigma) = y * g_\sigma = \int_{-\infty}^{\infty} y(s-u) g_\sigma(u)du$$

X(s, σ) and Y(s, σ) are the x and y coordinates of the smoothed curve, respectively; and * is the convolution operator.

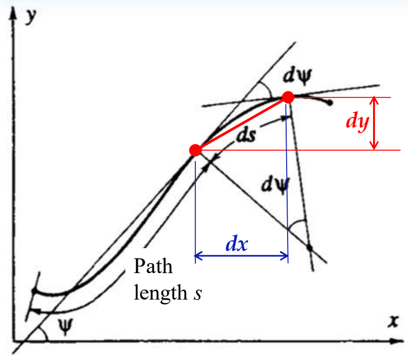
Recall curvature,

$$K[x(s), y(s)] = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\left(\dot{x}^2 + \dot{y}^2\right)^{3/2}}$$

$$K_\sigma[X(s,\sigma), Y(s,\sigma)] = \frac{\dot{X}\ddot{Y} - \dot{Y}\ddot{X}}{\left(\dot{X}^2 - \dot{Y}^2\right)^{3/2}}$$

where $\dot{X} = \dfrac{\partial X}{\partial s}, \dot{Y} = \dfrac{\partial Y}{\partial s}, \ddot{X} = \dfrac{\partial^2 X}{\partial s^2}, \ddot{Y} = \dfrac{\partial^2 Y}{\partial s^2}$

Scale-space filtering method (cont.)



$$\frac{dx}{ds}=\cos\psi;\quad \frac{dy}{ds}=\sin\psi$$

$$\dot{x}^2+\dot{y}^2=\cos^2\psi+\sin^2\psi=1$$

Recall curvature,

$$K\left[x(s),y(s)\right]=\frac{\dot{x}\ddot{y}-\dot{y}\ddot{x}}{\left(\dot{x}^2+\dot{y}^2\right)^{3/2}}=\dot{x}\ddot{y}-\dot{y}\ddot{x}$$

$$K\left(s,\sigma\right)=\dot{X}(s,\sigma)\ddot{Y}(s,\sigma)-\dot{Y}(s,\sigma)\ddot{X}(s,\sigma)$$

Scale-space filtering method (cont.)

Let $F(s,\ \sigma)$ represents the function, $X(s,\ \sigma)$ or $Y(s,\ \sigma)$; and

$F(s,\ 0)$ is the unsmooth coordinates, $x(s)$ and $y(s)$, respectively.

$$F(s,\sigma)=\int_{-\infty}^{\infty} f(s-u)g_\sigma(u)du=\int_{-\infty}^{\infty} g_\sigma(s-u)f(u)du$$

$$f*g_\sigma=g_\sigma*f$$

$$\frac{dF(s,\sigma)}{ds}=\frac{d}{ds}(f*g_\sigma)=\int_{-\infty}^{\infty}\frac{df(s-u)}{ds}g_\sigma(u)du=\int_{-\infty}^{\infty}\frac{dg_\sigma(s-u)}{ds}f(u)du$$

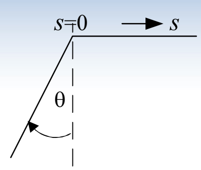
$$\dot{F}(s,\sigma)=\frac{d\left(F(s,0)*g_\sigma\right)}{ds}=\frac{dF(s,0)}{ds}*g_\sigma$$

Discrete approximation:

$$F(s,\sigma)=f*g_\sigma=\sum_{u=-5\sigma}^{5\sigma} f(s-u)g_\sigma(u)$$

$$\frac{dF(s,\sigma)}{ds}\approx F(j+1,\sigma)-F(j-1,\sigma);\quad \frac{d^2F(s,\sigma)}{ds^2}\approx F(j+1,\sigma)-2F(j)+F(j-1,\sigma)$$

Example 1: Compute $\dot{X}(s,\sigma)$ and $\dot{Y}(s,\sigma)$



$$x(s)=\begin{cases}\boxed{s\sin\theta \quad s<0}\\ \boxed{s \quad s\ge 0}\end{cases}\qquad y(s)=\begin{cases}s\cos\theta & s<0\\ 0 & s\ge 0\end{cases}$$

$$\dot{X}(s,\sigma)=\dot{x}*g_\sigma=\int_{-\infty}^{\infty}\dot{x}(s-u)g_\sigma(u)du$$

$$=\boxed{\int_{-\infty}^{s}\dot{x}(s-u)g_\sigma(u)du}+\boxed{\int_{s}^{\infty}\dot{x}(s-u)g_\sigma(u)du}$$

$$=\int_{-\infty}^{s}g_\sigma(u)du+\int_{s}^{\infty}\sin\theta g_\sigma(u)du$$

$$=G_\sigma+(1-G_\sigma)\sin\theta$$

$$=(1-\sin\theta)G_\sigma+\sin\theta$$

Notes:
1st integral
$u\le s$ and hence $t=s-u\ge 0$
$\dot{x}(t)=ds/ds=1$

while in the 2nd integral
$u\ge s$ and hence $t=s-u\le 0$
$\dot{x}(t)=d\left(s\sin\theta\right)/ds=\sin\theta$

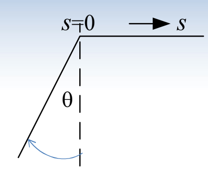
Similarly,

$$\dot{Y}(s,\sigma)=\dot{y}*g_\sigma=\int_{s}^{\infty}\cos\theta g_\sigma(u)du=(1-G_\sigma)\cos\theta$$

$$G_\sigma=\int_{-\infty}^{s}g_\sigma(u)du$$

where $g_\sigma(u)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-u^2}{2\sigma^2}}$

Example 2: Compute $X(s,\sigma)$ and $Y(s,\sigma)$



$$x(s)=\begin{cases}s\sin\theta & s<0\\ s & s\ge 0\end{cases}\qquad y(s)=\begin{cases}s\cos\theta & s<0\\ 0 & s\ge 0\end{cases}$$

$$X(s,\sigma)=x*g_\sigma=\int_{-\infty}^{\infty}x(\underbrace{s-u}_{t})g_\sigma(u)du$$

$$=\int_{-\infty}^{s}\underbrace{(s-u)}_{u\le s,\,t=s-u\ge 0}g_\sigma(u)du+\int_{s}^{\infty}\underbrace{(s-u)\sin\theta}_{u>s,\,t=s-u<0}g_\sigma(u)du$$

$$=s\underbrace{\int_{-\infty}^{s}g_\sigma(u)du}_{G_\sigma}+s\sin\theta\underbrace{\int_{s}^{\infty}g_\sigma(u)du}_{1-G_\sigma}-\int_{-\infty}^{s}ug_\sigma(u)du-\sin\theta\int_{s}^{\infty}ug_\sigma(u)du$$

Recall: $G_\sigma=\int_{-\infty}^{s}g_\sigma(u)du$ where $g_\sigma=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-s^2}{2\sigma^2}}$

$$X(s,\sigma)=sG_\sigma+s\sin\theta\left(1-G_\sigma\right)-\boxed{\int_{-\infty}^{s}ug_\sigma(u)du}-\sin\theta\boxed{\int_{s}^{\infty}ug_\sigma(u)du}$$

Example 2 (cont.): Let $\tau = \frac{-u^2}{2\sigma^2} \Rightarrow du = -\frac{\sigma^2 d\tau}{u}$

$$-\int u g_\sigma(u)du = -\int u\left[\frac{e^\tau}{\sigma\sqrt{2\pi}}\right]\left(-\frac{\sigma^2 d\tau}{u}\right) = \frac{\sigma^2}{\sigma\sqrt{2\pi}}\int e^\tau d\tau = \sigma^2 g_\sigma(u)$$

Since $g_\sigma(u \to \pm\infty) = \left[\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-u^2}{2\sigma^2}}\right]_{u\to\pm\infty} = 0$

$$\Rightarrow -\int_{-\infty}^{s} u g_\sigma(u)du = \sigma^2 g_\sigma(u)\Big|_{-\infty}^{s} = \sigma^2 g_\sigma(s)$$

and $-\int_{s}^{\infty} u g_\sigma(u)du == \sigma^2 g_\sigma(u)\Big|_{-\infty}^{s} = -\sigma^2 g_\sigma(s)$

$$X(s,\sigma) = sG_\sigma + s\left(1-G_\sigma\right)\sin\theta - \underbrace{\int_{-\infty}^{s} u g_\sigma(u)du}_{\sigma^2 g_\sigma(s)} - \underbrace{\sin\theta\int_{s}^{\infty} u g_\sigma(u)du}_{-\sigma^2 \sin\theta g_\sigma(s)}$$

$$= sG_\sigma + s\left(1-G_\sigma\right)\sin\theta + \sigma^2\left(1-\sin\theta\right)g_\sigma(s)$$

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Example 2 (cont.):

Similarly, $Y(s,\sigma) = y * g_\sigma = \int_{-\infty}^{\infty} y(s-u)g_\sigma(u)du$

$$= \int_{s}^{\infty}(s-u)\cos\theta g_\sigma(u)du$$

$$= \cos\theta\left[s\int_{s}^{\infty} g_\sigma(u)du - \int_{s}^{\infty} u g_\sigma(u)du\right]$$

$$= \cos\theta\left[s(1-G_\sigma) - \sigma^2 g_\sigma(s)\right]$$

$$\boxed{\begin{aligned} &X(s,\sigma) = s\left[\left(1-\sin\theta\right)G_\sigma + \sin\theta\right] + \sigma^2\left(1-\sin\theta\right)g_\sigma(s) \\ &Y(s,\sigma) = \cos\theta\left[s(1-G_\sigma) - \sigma^2 g_\sigma(s)\right] \end{aligned}}$$

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Example 3: $\dot{X}(s,\sigma)$ and $\dot{Y}(s,\sigma)$ can be derived from the derivative of $X(s,\sigma)$ and $Y(s,\sigma)$.

$$\boxed{\begin{aligned} &X(s,\sigma) = s\left[\left(1-\sin\theta\right)G_\sigma + \sin\theta\right] + \sigma^2\left(1-\sin\theta\right)g_\sigma(s) \\ &Y(s,\sigma) = \cos\theta\left[s(1-G_\sigma) - \sigma^2 g_\sigma(s)\right] \end{aligned}}$$

$$\dot{X}(s,\sigma) = \left[\left(1-\sin\theta\right)G_\sigma + \sin\theta\right] + s\frac{d}{ds}\left[\left(1-\sin\theta\right)G_\sigma + \sin\theta\right] + \sigma^2\left(1-\sin\theta\right)\frac{dg_\sigma(s)}{ds}$$

$$\dot{Y}(s,\sigma) = \cos\theta\left[(1-G_\sigma) + s\frac{\partial}{\partial s}(1-G_\sigma) - \sigma^2\frac{dg_\sigma(s)}{ds}\right]$$

where $\frac{dg_\sigma(s)}{ds} = \frac{1}{\sigma\sqrt{2\pi}}\frac{de^{-s^2/(2\sigma^2)}}{ds} = \frac{1}{\sigma\sqrt{2\pi}}\left[e^{-s^2/(2\sigma^2)}\left(\frac{-\cancel{2}s}{\cancel{2}\sigma^2}\right)\right] = -\frac{s g_\sigma(s)}{\sigma^2}$

$$\dot{X}(s,\sigma) = (1-\sin\theta)G_\sigma + \sin\theta + \underbrace{s(1-\sin\theta)g_\sigma - s(1-\sin\theta)g_\sigma}_{\text{cancel each other}} = (1-\sin\theta)G_\sigma + \sin\theta$$

$$\dot{Y}(s,\sigma) = \cos\theta\left[(1-G_\sigma) + \underbrace{s(-g_\sigma) - \sigma^2\left(-\frac{sg_\sigma}{\sigma^2}\right)}_{\text{cancel each other}}\right] = (1-G_\sigma)\cos\theta$$

$\boxed{\dot{X}(s,\sigma) = (1-\sin\theta)G_\sigma + \sin\theta \quad \text{and} \quad \dot{Y}(s,\sigma) = (1-G_\sigma)\cos\theta}$ Same as Example 1

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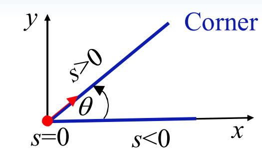
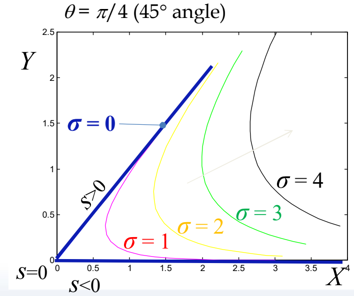
Scale-space filtering method (cont.)

$$x(s) = \begin{cases} -s & s < 0 \\ s\cos\theta & s \geq 0 \end{cases}$$

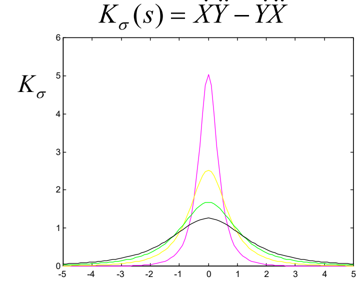
$$y(s) = \begin{cases} 0 & s < 0 \\ s\sin\theta & s \geq 0 \end{cases}$$



$\theta = \pi/4$ (45° angle)



$K_\sigma(s) = \dot{X}\ddot{Y} - \dot{Y}\ddot{X}$



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Curvature optimization for locating maxima

$$K(s,\sigma) = \dot X(s,\sigma)\ddot Y(s,\sigma) - \dot Y(s,\sigma)\ddot X(s,\sigma)$$

- Given the scale parameter σ, solve for all the locations that have maximum absolute curvature $|K(s, \sigma)|$, which are the positive maxima (concave) or negative maxima (convex).
- Apply curvature optimization (treating σ as a given constant),
  - For $K(s, \sigma) > 0$, solve for the solutions $\{s_i\}$ in

$$\frac{d}{ds}K\left[X(s,\sigma),Y(s,\sigma)\right] = 0 \text{ subject to } \frac{d^2}{ds^2}K\left[X(s,\sigma),Y(s,\sigma)\right] < 0$$

  - For $K(s, \sigma) < 0$, solve for the solutions $\{s_i\}$ in

$$\frac{d}{ds}K\left[X(s,\sigma),Y(s,\sigma)\right] = 0 \text{ subject to } \frac{d^2}{ds^2}K\left[X(s,\sigma),Y(s,\sigma)\right] > 0$$

Scale space curvature optimization

$$\boxed{K(s,\sigma) = \dot X(s,\sigma)\ddot Y(s,\sigma) - \dot Y(s,\sigma)\ddot X(s,\sigma)}$$

$$\frac{dK}{ds} = \frac{d\left(\dot X\ddot Y - \dot Y\ddot X\right)}{ds} = \dot X\dddot Y + \underbrace{\ddot X\ddot Y - (\ddot X\ddot Y}_{\text{cancel out}} + \dot Y\dddot X) = 0$$

$$\boxed{\frac{\partial K}{\partial s} = \dot X(s,\sigma)\dddot Y(s,\sigma) - \dot Y(s,\sigma)\dddot X(s,\sigma) = 0}$$

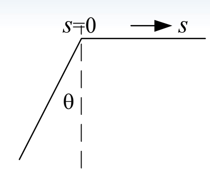
Recall: $F(s, \sigma)$ represents the function, $X(s, \sigma)$ or $Y(s, \sigma)$; and $F(s, 0)$ is the unsmooth coordinates, $x(s)$ and $y(s)$, respectively.

$$\dddot F(s,\sigma) = \frac{d^3}{ds^3}\left[F(s,0)*g_\sigma(s)\right] = \ddot F(s,0)*\dot g_\sigma(s) = \dot F(s,0)*\ddot g_\sigma(s)$$

where $\frac{dg_\sigma(s)}{ds} = -\frac{s\,g_\sigma(s)}{\sigma^2}$

Scale space description of corner

$$\boxed{K_\sigma(X,Y) = \dot X\ddot Y - \dot Y\ddot X}$$



Scale-space curvature optimization

$$\partial K(X,Y)/\partial s = 0$$

$$\dot X(s,\sigma)\dddot Y(s,\sigma) - \dot Y(s,\sigma)\dddot X(s,\sigma) = 0$$

$\dot X(s,\sigma) = (1-\sin\theta)G_\sigma + \sin\theta$	$\dot Y(s,\sigma) = (1-G_\sigma)\cos\theta$
$\ddot X(s,\sigma) = (1-\sin\theta)g_\sigma$	$\ddot Y(s,\sigma) = -g_\sigma\cos\theta$
$\dddot X(s,\sigma) = (1-\sin\theta)\dot g_\sigma$	$\dddot Y(s,\sigma) = -\dot g_\sigma\cos\theta$

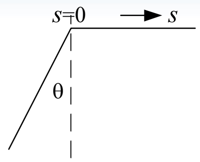
$$\left[(1-\sin\theta)G_\sigma + \sin\theta\right]\left(-\dot g_\sigma\cos\theta\right) - \left[(1-G_\sigma)\cos\theta\right](1-\sin\theta)\dot g_\sigma = 0$$

$$\underbrace{[-\dot g_\sigma\cos\theta(1-\sin\theta)G_\sigma - \dot g_\sigma\cos\theta\sin\theta + \dot g_\sigma\cos\theta(1-\sin\theta)G_\sigma + \dot g_\sigma\cos\theta\sin\theta]}_{\text{cancel each other}} - \dot g_\sigma\cos\theta = 0$$

$$\boxed{-\dot g_\sigma\cos\theta = 0}$$

Scale space description of corner

$$-\dot{g}_\sigma \cos\theta = 0$$



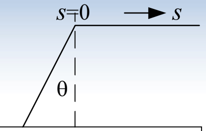
- At **0 = 90°, it becomes a straight line.**

  In this case, any solution s will satisfy **(*)** but not the constraint of $\dfrac{d^2K(s,\sigma)}{ds^2} > 0$

  Hence, there is no absolute maximum.

- In the case of **0 > –90°** but not equal to **90°,** $\quad \dot{g}_\sigma = 0$
  - Therefore, the only solution is at $s = 0$ independent of the corner angle **0, and the scale parameter σ.**
  - This produces a vertical line in scale space, that is, the absolute maxima occur at the same contour location independent of smoothing.

Scale space description of corner



Alternatively, $\ddot{F} = \int_{-\infty}^{\infty} \dot{f}(s-u)\ddot{g}_\sigma(u)du$

$$\ddot{X}(s,\sigma) = \dot{x} * \ddot{g}_\sigma = \int_{-\infty}^{\infty} \dot{x}(s-u)\ddot{g}_\sigma(u)du = \int_{-\infty}^{s} \ddot{g}_\sigma(u)du + \int_{s}^{\infty} \sin\theta\, \ddot{g}_\sigma(u)du$$

$$= \underbrace{\int_{-\infty}^{s} \ddot{g}_\sigma(u)du}_{\dot{g}_\sigma(s)} + \sin\theta \underbrace{\left(\int_{-\infty}^{\infty} \ddot{g}_\sigma(u)du - \int_{-\infty}^{s} \ddot{g}_\sigma(u)du\right)}_{0-\dot{g}_\sigma} \quad \text{where} \quad \underbrace{\int_{-\infty}^{\infty} \ddot{g}_\sigma(u)du}_{\frac{d^2}{ds^2}\int_{-\infty}^{\infty} g_\sigma(u)du} = 0$$

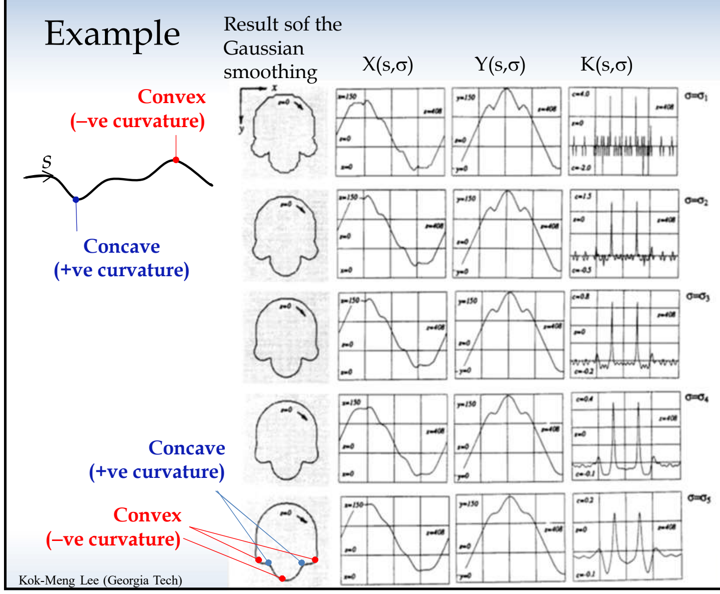
$$= (1-\sin\theta)\dot{g}_\sigma$$

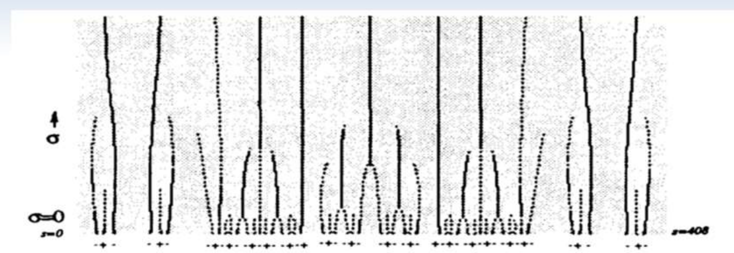
$$\ddot{Y}(s,\sigma) = \dot{y} * \ddot{g}_\sigma = \int_{-\infty}^{\infty} \dot{y}(s-u)\ddot{g}_\sigma(u)du = \int_{s}^{\infty} \cos\theta\, \ddot{g}_\sigma(u)du$$

$$= \cos\theta\left[\int_{-\infty}^{\infty} \ddot{g}_\sigma(u)du - \int_{-\infty}^{s} \ddot{g}_\sigma(u)du\right]$$

$$= -\dot{g}_\sigma \cos\theta$$

Example





Example

Scale space map of maxima of absolute curvature. The horizontal axis is the arc length of the curve at $\sigma = 0$. ***The vertical axis is the Gaussian*** **function parameter** ***D determining the degree of smoothing. The line pattern*** **in the map represents the locations of the local maxima of the curvature. A +sign indicates downward concavity, and a - sign indicates upward concavity.**

The curvature method

1. Find the outside boundary
   bwboundaries(~BW, ‘noholes’)
2. Gaussian smoothing
   conv.m
3. Calculate curvature

$$K(s) = \dot{X}(s)\ddot{Y}(s) - \ddot{Y}(s)\dot{X}(s)$$
$$(s) \cong diff(X),\ \dot{Y}(s) \cong diff(Y)$$
$$(s) \cong diff(\dot{X}),\ \ddot{Y}(s) \cong diff(\dot{Y})$$

4. Find peaks
   findpeaks.m

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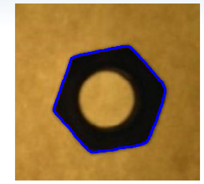
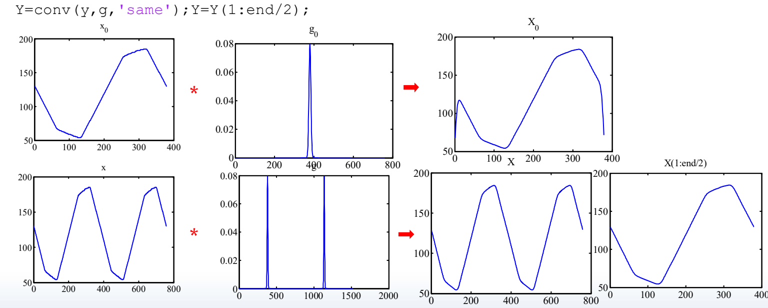
Gaussian smoothing

```
  x=[x0;x0];y=[y0;y0];
% x, y are the xy position of edges
  n=length(x);
  s=linspace(-n/2,n/2,n);
  sigma=5;
  g0=1/sqrt(2*pi*sigma^2)*exp(-s.^2/(2*sigma^2));
  g=[g0 g0]';
%convolve x(s), y(s) with Gaussian
  X=conv(x,g,'same');X=X(1:end/2);
% same: Returns the central part of the convolution
of the same size as x
  Y=conv(y,g,'same');Y=Y(1:end/2);
```





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