ME6406 Machine Vision

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Part 2 Model-based Vision
B. Template matching

http://kmlee.gatech.edu/me6406

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Course Outline

- Introduction and low-level processing
  - Physics of digital images, histogram equalization, segmentation, edge detection, linear filters.
- **Model-based Vision**
  - **Hough transform, pattern representation, matching**
- Geometric methods
  - Camera model, calibration, pose estimation
- Neural network for machine vision
  - Basics, training algorithms, and applications
- Color images and selected topics
  - Physics, perception, processing and applications

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Contents

- **Hierarchical Feature Extraction**
- **Hough Transform (HT)**
  - Lines (without gradient)
  - Foot normal (line detection using gradient)
  - Circles and ellipses
  - Generalized HT

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Hierarchical Feature Extraction

- **Most features are extracted by combining a small set of primitive features (edges, corners, regions)**
  - Grouping: which edges/corners/curves form a group? (Perceptual organization at the intermediate-level of vision)
  - Model Fitting: what structure best describes the group?
- **Consider a slightly simpler problem… From Edges to curves?**
  - Given local edge elements, can we organize these into more 'complete' structures, such as straight lines?
  - General idea:
    - Find an alternative space in which lines map to points.
    - Each edge element 'votes' for the straight line which it may be a part of.
    - Points receiving a high number of votes might correspond to actual straight lines in the image.
  - The idea behind the **Hough transform** is that a change in representation converts a point grouping problem into a peak detection problem.

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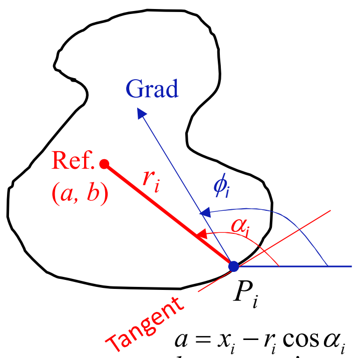
Generalizing the HT

The HT can be extended to finding curves that do not have a simple analytic form. For illustration, assume that scaling and rotation have been fixed.



$$a = x_i - r_i \cos\alpha_i$$
$$b = y_i - r_i \sin\alpha_i$$

1. Pick a reference point (a, b)
2. For $i = 1, \dots, n$:
   a) Draw segment to P_i on the boundary.
   b) Measure its length r_i, and its orientation α_i.
   c) Write the ref coordinates (a, b) as a function of r_i and a_i
   d) Record the gradient orientation ϕ_i at P_i.
3. Build a table with the data, indexed by ϕ_i .

Generalized HT Algorithm for objects with no analytical form

R-Table (pre-stored template):	
ϕ_1	$(r_1^1,\alpha_1^1),(r_2^1,\alpha_2^1),\dots(r_{n1}^1,\alpha_{n1}^1)$
ϕ_2	$(r_1^2,\alpha_1^2),(r_2^2,\alpha_2^2),\dots(r_{n2}^2,\alpha_{n2}^2)$
…	
ϕ_m	$(r_1^m,\alpha_1^m),(r_2^2,\alpha_2^2),\dots(r_{nm}^m,\alpha_{nm}^m)$

1. Form a R-Table
2. Initialize Accumulator (x, y).
3. From each pixel point, do the following:
   a) Compute ϕ
   b) Calculate possible centers
   c) Increment Accumulator.
4. Increment the point in parameter space .
5. Local maxima in the accumulator array now correspond to collinear points in the object shape.

Find a **rotated, scaled** and **translated** version of the curve:

1. Form a H accumulator array (x, y, S, θ) of possible reference points (a, b), scaling factor S and rotation angle θ.
2. For each edge (x, y) in the image:
   1. Compute $\phi(x,y)$
   2. For each (r, a) corresponding to $\phi(x,y)$ do:

      For each S and θ:
      $$a = x_i + r(\phi) S \cos\left[\alpha(\phi) + \theta\right]$$
      $$b = y_i + r(\phi) S \sin\left[\alpha(\phi) + \theta\right]$$
      $$H(a, b, S, \theta) \mathrel{++}$$
3. Find maxima of H.

Summary of Hough Transform (HT)

- HT is a “voting” scheme: points vote for a set of parameters describing a line or curve.
- The more votes for a particular set: the more evidence that the corresponding curve is present in the image.
- Can detect MULTIPLE curves in one shot.
- Computational cost increases with the number of parameters describing the curve.

Template matching (http://kmlee.gatech.edu/me6406/)

4. Lee, K-M. and S. Janakiraman, "A Model-based Vision Algorithm for Real-Time Flexible Part-feeding and Assembly," Paper number: MS 92-211. *SME Applied Machine Vision Conf.,* June 1-4, 1992, Atlanta, GA.

In Canvas ("Reading materials" folder)

A. Rattarangsi and T. R. Chin, "Scale-based Detection of corners and planar curves," *IEEE Transactions on Pattern Analysis and Machine Intelligence,* Vol. 14, no 4m April 1992.

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Image Acquisition
- Vision properties
- Physics (Optics)
- Illumination
- Sensors
- Image representation
  - Binary
  - Grayscale
  - Color

Low level Processing & Enhancement
- Histogram
- Equalization
- Segmentation
- Labeling

Feature Extraction

Binary/ Grayscale IP
- Binarization
- Thresholding
- Geometry
  - Area
  - Position
  - Orientation
- Edge/line detection
- Spatial filters /masks

Color IP
- RGB, HSI, CMY
- Principal component analysis

Model-based Vision
- Hough Transform
  - Circle
  - Line
  - Generalized
- Matching
  - Similar Triangles
  - Templates
- Scale-space filtering
  - Curvature

Morphology
- Algorithms/ operations
  - Translation
  - Reflection
  - Complement
  - Difference
  - Dilation
  - Erosion

Image Analysis

Camera Model/ Calibration
- 2D Image – 3D world correspondence
- Intrinsic para
  - Focal length,etc
- Extrinsic para
  - Coordinate transformation matrix,etc
- Hands-eye calibration

Geometrical Methods
- Geometry of multiple views & reconstruction
- Point Estimation
- Pose Estimation

Neural Network Classifier
- Basics
  - Sigmoid function
- Training algorithms
  - Learning rules
  - Weights updates
- Applications

Motion
- Motion field
- Visual servoing

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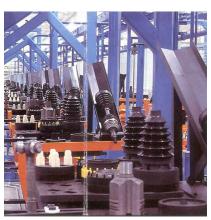
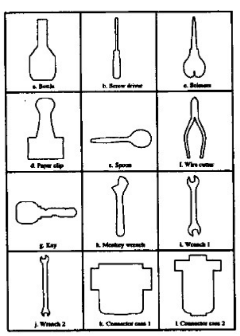
Template matching an introduction

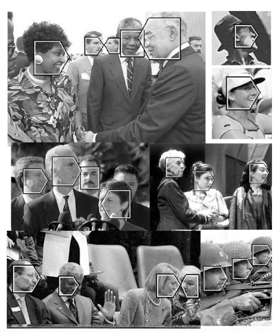
❑ **Typical tasks:**
- Model objects
- Identify objects
- Locate objects



❑ **Pattern Recognition**



❑ **Face Recognition**



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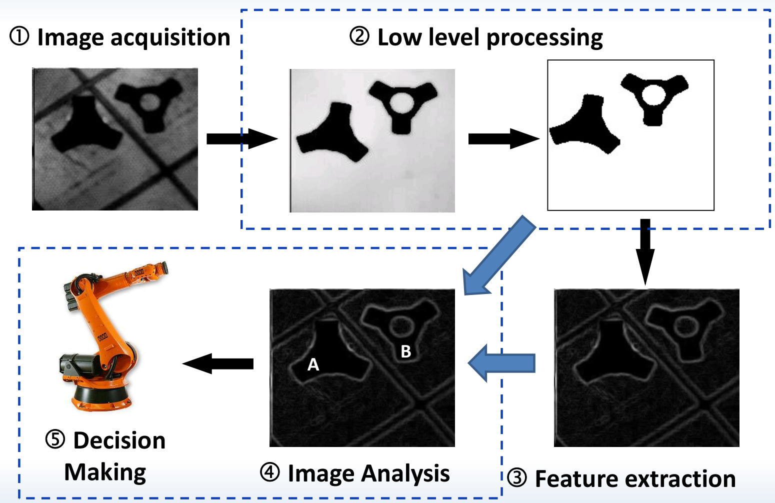
Machine Vision System for Part presentation



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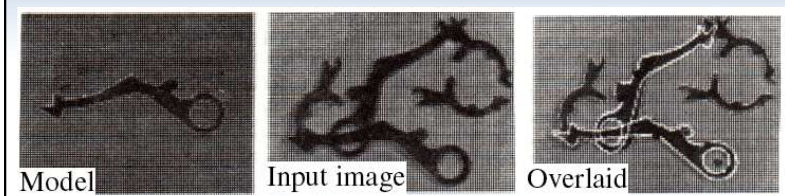

Image of the object to be identified.

Image of the targeted object along with some other stuff.

Satisfactory correspondences align many model edge points with image edge points.

Feature points are detected, and then correspondences between group (for example, Hough transform), are searched. Each correspondence given rise to an affine transformation from the model to the image.

Figure from "Object recognition using alignment," D.P. Huttenlocher and S. Ullman, Proc. Int. Conf. Computer Vision, 1986,

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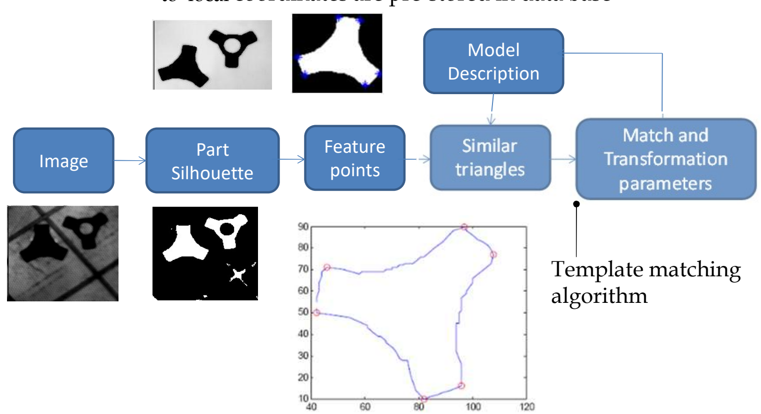
Template matching

Feature point position and orientation with respect to local coordinates are pre-stored in data base



Template matching algorithm

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Feature representation (many methods)

- Assumptions
  - Object on flat surface.
  - Two-dimensional images
  - Orientation can be determined from the outline or feature points.
- A one-dimensional functional representation of a boundary
- Real time need (feature points)
  - Feature points are characteristics of the objects.
  - Easy to find, unique to each object, and robust.
  - Sufficient information for fast recognition and accurate computation.
- Object representation (signatures)
  - Method of r-θ plot, also called ρ-θ signature.
  - Median length
  - Curvature

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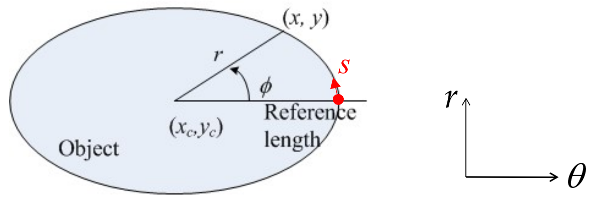
Feature representation

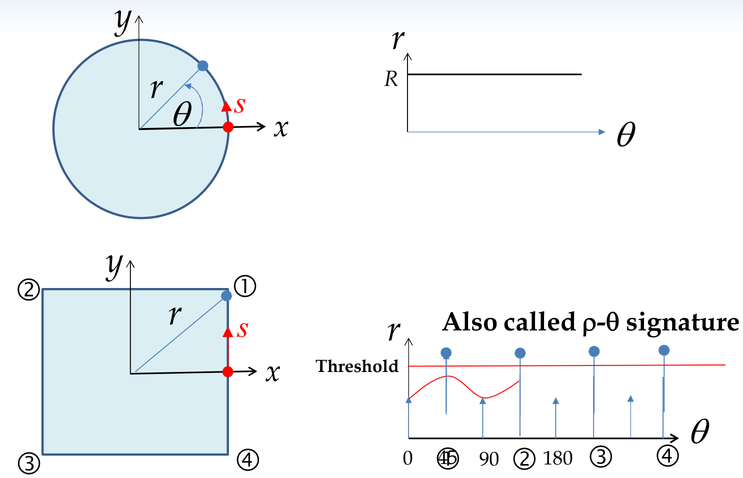
1. **Method of r-θ plot** : The basic idea is to reduce the boundary (edge points) representation to a one 0dimenasional feature which is easier to describe.



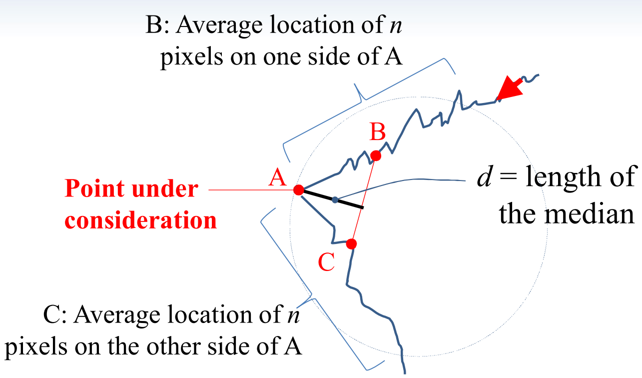
❑ General steps

- Find centroid to serve as the origin.
- Major axis (object orientation) as the reference length.
- Go along the edge curve (path length, s).
- At each edge point, compute and plot on the magnitude r and angle θ.

1. Method of r-θ plot (Examples)

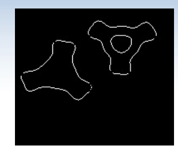


2. Method of median length (Use as a relative curve)

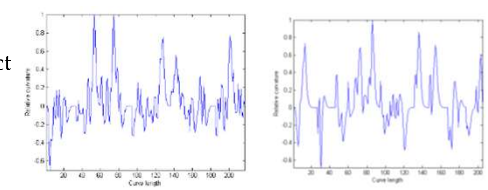


❑ A is a critical point if
d > threshold distance, and if
d is maximum in a neighborhood of '2n' pixels.

2. Method of median length (Example)

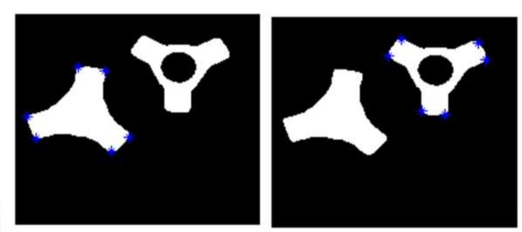


Left object



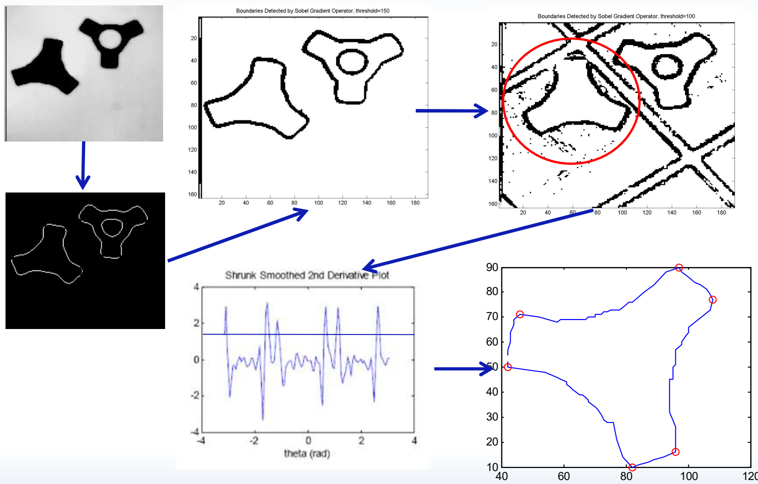
Right Object

Detected corners in left object



Detected corners in right object

2. Method of median length (Example)



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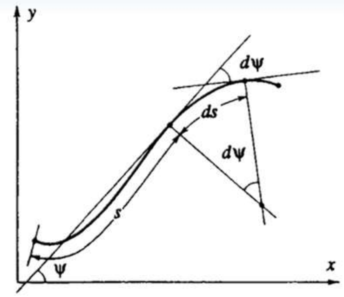
3. Method of Curvature



$$g_\sigma(s)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(\frac{-s^2}{2\sigma^2}\right)$$

$$X(s,\sigma)=x*g_\sigma=\int_{-\infty}^{\infty}x(s-u)g_\sigma(u)du$$

$$Y(s,\sigma)=y*g_\sigma=\int_{-\infty}^{\infty}y(s-u)g_\sigma(u)du$$

$$\boxed{K_\sigma(X,Y)=\dot{X}\ddot{Y}-\dot{Y}\ddot{X}}$$

$$\boxed{\text{Curvature, } K(x,y)=\frac{\partial\psi}{\partial s}=\dot{x}\ddot{y}-\dot{y}\ddot{x}}$$

$$\dot{x}=\frac{\partial x}{\partial s}\quad \dot{y}=\frac{\partial y}{\partial s}\quad \ddot{x}=\frac{\partial^2 x}{\partial s^2}\quad \ddot{y}=\frac{\partial^2 y}{\partial s^2}$$

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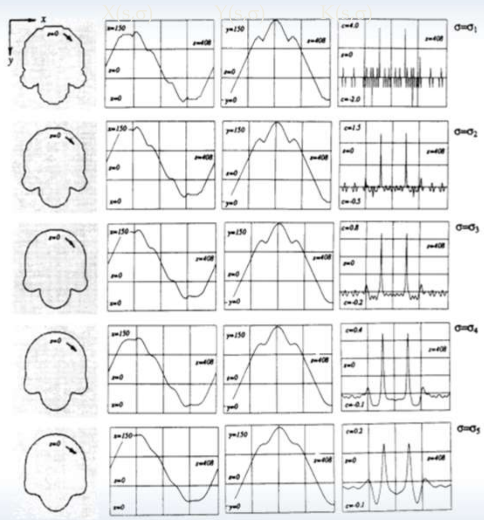
3. Method of Curvature (Example)

Results of
the Gaussian
smoothing



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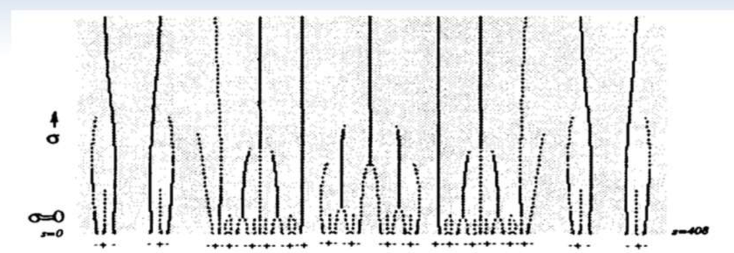

Example

Scale space map of maxima of absolute curvature. The horizontal axis is the arc length of the curve at ***σ = 0. The vertical axis is the Gaussian*** **function parameter** ***D determining the degree of smoothing. The line pattern*** **in the map represents the locations of the local maxima of the curvature. A +sign indicates downward concavity, and a - sign indicates upward concavity.**

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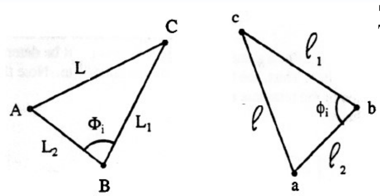
Templating matching (identification and location)

- Practical imaging problems
  - Shadows and highlights
  - Partially overlapped parts
  - Missing objects.
  - Oversized object (difficult to image the whole object without sacrificing details)
  - For part presentation, both identification, location and orientation are needed.

Templating matching (identification and location)

- The precompiled description of the object is referred to here as a template.
- Use multiple of three feature points (corners and locations of high curves) that form triangles as a basis for a template.
- Approach
  1. Search for similar triangles in the image.
  2. Determine the transformation parameters that allow the template to be mapped onto the silhouette (object in the image).
  3. Verify the remaining points in the image by using the template description.
  4. Determine the location and orientation from the best match.

Templating matching (identification)



Similar triangles:
Two constraints must be made.

$$\left|\frac{L_i}{L}-\frac{\ell_i}{l}\right|<\varepsilon_i \quad i=1,2$$

Computation

$$\left|\Phi_i-\phi_i\right|<\varepsilon_\phi$$

$$\frac{l}{L}=\frac{\ell_i}{L_i}=k=\text{scaling factor}$$ ← Refer to image and template

$$\rho(>0)=\frac{|\delta|}{\ell}=\frac{|\delta_i|}{\ell_i}=\text{Error bound (tolerance)}$$

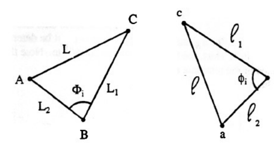
Refer two feature points of the object in the image (due to fabrication tolerances, for example)

$$\ell_i=kL_i\pm\delta_i$$
$$\ell=kL\pm\delta$$

Given the dimensional tolerance, find ε in terms of ρ.

Templating matching (identification)

Given the dimensional tolerance, find ε in terms of ρ.



$$\left|\frac{L_i}{L}-\frac{\ell_i}{l}\right|<\varepsilon_i \quad (1)$$
$$\frac{l}{L}=\frac{\ell_i}{L_i}=k \quad (2)$$
$$\rho=\frac{|\delta|}{\ell}=\frac{|\delta_i|}{\ell_i} \quad (3)$$

$$\underbrace{\left|\frac{L_i}{L}-\frac{kL_i\pm\delta_i}{kL\pm\delta}\right|}<\varepsilon_i$$

$$\frac{L_i(kL\pm\delta)-L(kL_i\pm\delta_i)}{L(kL\pm\delta)}$$

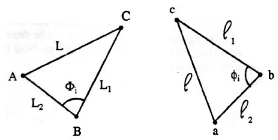
$$\boxed{\cancel{L_i kL}\pm L_i\delta-\cancel{LkL_i}\pm L\delta_i}$$

$$\boxed{kL} \quad \boxed{kL_i}$$

$$\frac{\pm L_i\delta\pm L\delta_i}{L(kL\pm\delta)}=\frac{\pm L_i(\rho\ell)\pm L(\rho\ell_i)}{L(kL\pm\rho\ell)}$$

Templating matching

Given the dimensional tolerance, find ε in terms of ρ.



$$\left|\frac{L_i}{L}-\frac{kL_i\pm\delta_i}{kL\pm\delta}\right|<\varepsilon_i$$

$$\left|\frac{L_i}{L}-\frac{\ell_i}{l}\right|<\varepsilon_i \quad (1)$$

$$\frac{2\rho}{1\pm\rho}\left(\frac{\ell_i}{\ell}\right)=\frac{2\rho}{1\pm\rho}\left(\frac{L_i}{L}\right)<\varepsilon_i$$

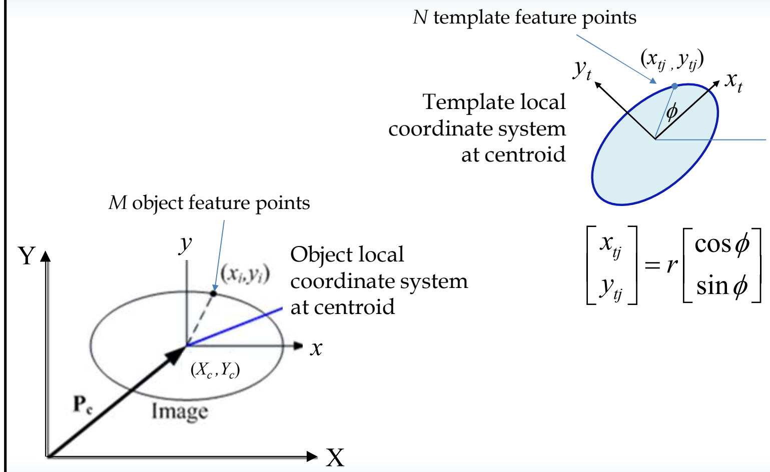
$$\frac{l}{L}=\frac{\ell_i}{L_i}=k \quad (2)$$

$$0<\rho<1$$

$$\boxed{\left(\varepsilon_i\right)_{\max}=\frac{2\rho}{1-\rho}\left(\frac{\ell_i}{\ell}\right)}$$

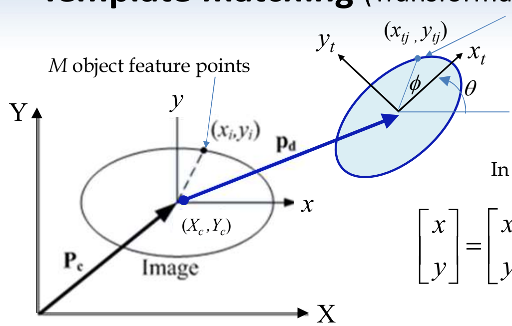
$$\rho=\frac{|\delta|}{\ell}=\frac{|\delta_i|}{\ell_i} \quad (3)$$

Template matching (Transformation parameters)



$$\begin{bmatrix} x_{tj} \\ y_{tj} \end{bmatrix}=r\begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

Template matching (Transformation parameters)



$$\begin{bmatrix} x_{tj} \\ y_{tj} \end{bmatrix}=r\begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

In object coordinate system,

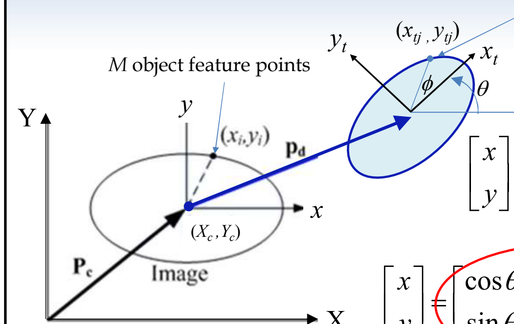
$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} x_d \\ y_d \end{bmatrix}+r\begin{bmatrix} \cos(\phi+\theta) \\ \sin(\phi+\theta) \end{bmatrix}$$

$$\mathbf{P}_c+\mathbf{p}_d=\begin{bmatrix} X_c+x_d \\ Y_c+y_d \end{bmatrix}$$

$$r\cos(\phi+\theta)=\underbrace{r\cos\phi}_{x_{tj}}\cos\theta-\underbrace{r\sin\phi}_{y_{tj}}\sin\theta$$

$$r\sin(\phi+\theta)=\underbrace{r\sin\phi}_{x_{tj}}\cos\theta+\underbrace{r\cos\phi}_{y_{tj}}\sin\theta$$

Template matching (Transformation parameters)



$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} x_d \\ y_d \end{bmatrix}+r\begin{bmatrix} \cos(\phi+\theta) \\ \sin(\phi+\theta) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x_{tj} \\ y_{tj} \end{bmatrix}+\begin{bmatrix} x_d \\ y_d \end{bmatrix}$$

In Image coordinate system and accounting the scaling factor k

$$\begin{bmatrix} X_{tj} \\ Y_{tj} \end{bmatrix}=k\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x_{tj} \\ y_{tj} \end{bmatrix}+\begin{bmatrix} X_c+x_d \\ Y_c+y_d \end{bmatrix}$$

Template matching (Transformation parameters)

If the scaled template identically matches the object, and (x_{tj}, y_{tj}) corresponds to (x_i, y_i)

$$\begin{bmatrix} X_i \\ Y_i \end{bmatrix} = \begin{bmatrix} X_{tj} \\ Y_{tj} \end{bmatrix} = k \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_{tj} \\ y_{tj} \end{bmatrix} + \begin{bmatrix} X_c + x_d \\ Y_c + y_d \end{bmatrix}$$

$$X_i = \underbrace{k\cos\theta}_{q_1} x_{tj} - \underbrace{k\sin\theta}_{q_2} y_{tj} + \underbrace{X_c + x_d}_{q_3}$$

$$Y_i = \underbrace{k\sin\theta}_{q_2} x_{tj} + \underbrace{k\cos\theta}_{q_1} y_{tj} + \underbrace{Y_c + y_d}_{q_4}$$

$$\mathbf{Q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} k\cos\theta \\ k\sin\theta \\ X_c + x_d \\ Y_c + y_d \end{bmatrix}$$

$$k = \sqrt{q_1^2 + q_2^2}, \quad \theta = \tan^{-1}(q_2 / q_1)$$

$$x_d = q_3 - X_c, \quad y_d = q_4 - Y_c$$

Template matching (Transformation parameters)

$$\mathbf{Q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} k\cos\theta \\ k\sin\theta \\ X_c + x_d \\ Y_c + y_d \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} x_{tj} & -y_{tj} & 1 & 0 \\ y_{tj} & x_{tj} & 0 & 1 \end{bmatrix}}_{[\mathbf{A}]} \underbrace{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}}_{\mathbf{Q}} = \underbrace{\begin{bmatrix} X_i \\ Y_i \end{bmatrix}}_{\mathbf{R}}$$

$$k = \sqrt{q_1^2 + q_2^2}$$

$$\theta = \tan^{-1}(q_2 / q_1)$$

$$x_d = q_3 - X_c$$

$$y_d = q_4 - Y_c$$

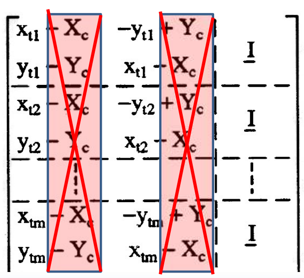
$$[\mathbf{A}]\mathbf{Q} = \mathbf{R} \qquad \mathbf{Q} = \left[\mathbf{A}^T\mathbf{A}\right]^{-1}\mathbf{A}^T\mathbf{R}$$

$$\begin{bmatrix} X_{tj} \\ Y_{tj} \end{bmatrix} = k \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_{tj} \\ y_{tj} \end{bmatrix} + \begin{bmatrix} X_c + x_d \\ Y_c + y_d \end{bmatrix}$$

$$E_M = \sum_{j=1}^{n} \sqrt{\left(X_{tj} - X_i\right)^2 + \left(Y_{tj} - Y_i\right)^2}$$

$$\underbrace{\begin{bmatrix} x_{tj} & -y_{tj} & 1 & 0 \\ y_{tj} & x_{tj} & 0 & 1 \end{bmatrix}}_{[\mathbf{A}]} \underbrace{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}}_{\mathbf{Q}} = \underbrace{\begin{bmatrix} X_i \\ Y_i \end{bmatrix}}_{\mathbf{R}}$$

A =



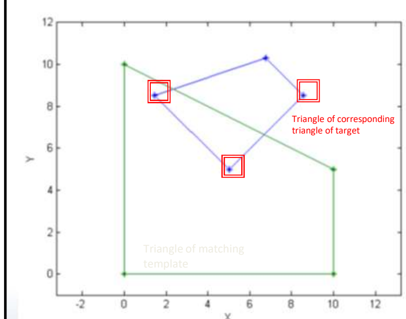
4. Lee, K-M. and S. Janakiraman, "A Model-based Vision Algorithm for Real-Time Flexible Part-feeding and Assembly," Paper number: MS 92-211. *SME Applied Machine Vision Conf.*, June 1-4, 1992, Atlanta, GA.

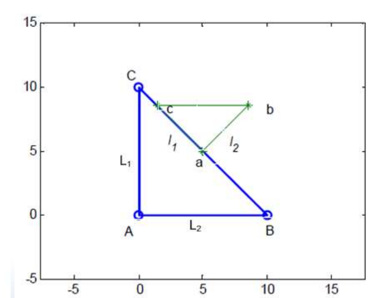
(8)

Template matching example

	Template		Target Object	
	x-coord	y-coord	x-coord	y-coord
Feature 1	0	0	8.5355	8.5355
Feature 2	10	0	5	5
Feature 3	10	5	1.4645	8.5355
Feature 4	0	10		

***k*=0.5, *θ*=45°, x_d=y_d=5**



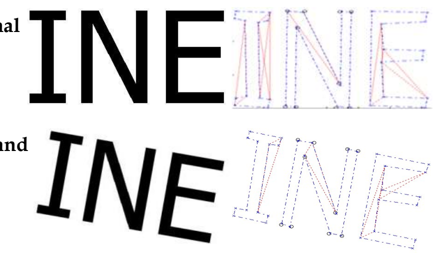


Template matching example

Template Image **Pattern Result**

Original

Scaled and Rotated



	x_d	y_d	Scale, k	Θ	E_M
	87	4	0.9925	0.5443°	21
	85	15	15	0.7404	10.7°

$$E_M = \sum_{j=1}^{n} \sqrt{\left(X_{tj} - X_i\right)^2 + \left(Y_{tj} - Y_i\right)^2}$$