MATH 6701 Homework #1

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7.1.44

Given: $y = -x^2 + 3x$

Let $x = t$, $y = -t^2 + 3t$

For point $(0, 0)$, $t = 0$ $\vec{r}(t) = t\mathbf{i} + (-t^2 + 3t)\mathbf{j}$

Tangent vector: $\frac{dr}{dt} = \mathbf{i} + (-2t + 3)\mathbf{j}$

Unit Tangent Vector at point $(0, 0)$:

$\frac{\frac{dr}{dt}}{\|\frac{dr}{dt}\|} = \frac{1}{\sqrt{1+9}}(\mathbf{i} + 3\mathbf{j}) = \frac{\sqrt{10}}{10}(\mathbf{i} + 3\mathbf{j})$

7.2.26

$\|P_1, P_2\| = \sqrt{1^2 + 2^2 + 4^2} = \sqrt{21}$

$\|P_1, P_3\| = \sqrt{3^2 + 2^2 + (2\sqrt{2})^2} = \sqrt{21}$

$\|P_2, P_3\| = \sqrt{2^2 + 0^2 + (2\sqrt{2} - 4)^2} = \sqrt{28 - 16\sqrt{2}}$

$\|P_1, P_2\| = \|P_1, P_3\| \neq \|P_2, P_3\|$

$\therefore$ *isosceles triangle*

7.3.45

Given: $\|F\| = 20,\ \theta = 60°,\ \|d\| = 100$

$W = \|F\|\|d\|cos\theta = 20(100)cos(60°) = 1000$ ft-lb

7.3.53

First solve 7.3.52 *Prove* $n = a\mathbf{i} + b\mathbf{j}$ *is* $\perp$ *to* $ax + by + c = 0$

$P_1(x_1, y_1)$ *and* $P_2(x_2, y_2)$ are points on the line. $\vec{P_1P_2} = (x_2 - x_1, y_2 - y_1)$ If $n \cdot \vec{P_1P_2} = 0$ Then perpendicular.
$(a, b) \cdot (x_2 - x_1, y_2 - y_1) = (ax_2 + by_2) - (ax_1 + by_1) = -c - (-c) = 0$
$\therefore$ n is perpendicular to the line

Prove $d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$d = \|\vec{P_1P_2}\| cos\theta = \frac{|n \cdot \vec{P_1P_2}|}{\|n\|} = \frac{(ax_2+by_2)-(ax_1+by_1)}{\sqrt{a^2+b^2}} = \frac{|c-(ax_1+by_1)|}{\sqrt{a^2+b^2}} = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

7.4.52

$$b \times c = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & 1 \\ 1 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 4 & 1 \\ 1 & 5 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} \mathbf{k} = (20 - 1)\mathbf{i} - (5 - 1)\mathbf{j} + (1 - 4)\mathbf{k} = 19\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$$

$v = |a \cdot (b \times c)| = (3\mathbf{i} + \mathbf{j} + \mathbf{k}) \cdot 19(\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}) = |57 - 4 - 3| = 50$ *cu. units*

7.5.67

Let $z = t$ $4x - 2y - z = 1;\ x + y + 2z = 1 \rightarrow 4x - 2y = 1 + t;\ x + y = 1 - 2t$
Solve system of equations. $x = \frac{1}{2} - \frac{1}{2}t;\ y = \frac{1}{2} - \frac{3}{2}t;\ z = t$

7.6.24

$p_1(x) = x + 1;\ p_2(x) = x - 1$

a)

To be linearly independent: $c_1p_1 + c_2p_2 = 0$

$c_1(x + 1) + c_2(x - 1) = (c_1 + c_2)x + (c_1 - c_2) = 0 \rightarrow\ c_1 + c_2 = 0;\ c_1 - c_2 = 0 \therefore\ c_1 = 0;\ c_2 = 0$

b)

$p(x) = 5x + 2$

$c_1 + c_2 = 5;\ c_1 - c_2 = 2 \rightarrow\ c_1 = \frac{7}{2}, c_2 = \frac{3}{2} \therefore p(x) = \frac{7}{2}p_1(x) + \frac{3}{2}p_2(x)$

7.6.35 (you may assume that Problem 9 gave a vector space)

Find Basis, a linearly independent set of vectors that can linearly combine to form vector space:

Taking the following set of matrices: $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix};\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix};\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix};\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

These are linearly independent and can represent $M_{22} \therefore$ a basis.

The dimension is 4 given the 4 matrices used to represent M_{22}

Review Chapter 7.39

Find line through $P = (7, 3, -5)$ that is parallel to $\frac{(x-3)}{4} = \frac{(y+4)}{-2} = \frac{(z-9}{6}$

$\mathbf{d} = (4, -2, 6) \therefore \parallel \ line\ is\ \frac{(x-7)}{4} = \frac{(y-3)}{-2} = \frac{(z+5}{6}$

Review Chapter 7.51

Checking subspace: P_n contains $p_1\ and\ p_2$.

Vector addition: If $\frac{d^2 p_1}{dx^2} = 0,\ \frac{d^2 p_2}{dx^2} = 0 \rightarrow \frac{d^2 p_1}{dx^2} + \frac{d^2 p_2}{dx^2} = 0$

Scalar Multiplication: Given c as a scalar, $\frac{d^2 cp}{dx^2} = c \cdot \frac{d^2 p}{dx^2} = c \cdot 0 = 0$

$\therefore$ the set of polynomials in P_n satisfying $\frac{d^2 p}{dx^2} = 0$ is a supspace of P_n

Find a basis: Assume a monomial x^k, Find $\frac{d^2}{dx^2}(x^k) = 0$
$\frac{d^2}{dx^2}(x^k) = k(k-1)x^{k-2} = 0,\ When\ k = 1, x \therefore$ Basis is {1,x}