HW4

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3.6: 28

$x^2y'' - 3xy' + 4y = 0;\ y(1) = 5;\ y'1) = 2$

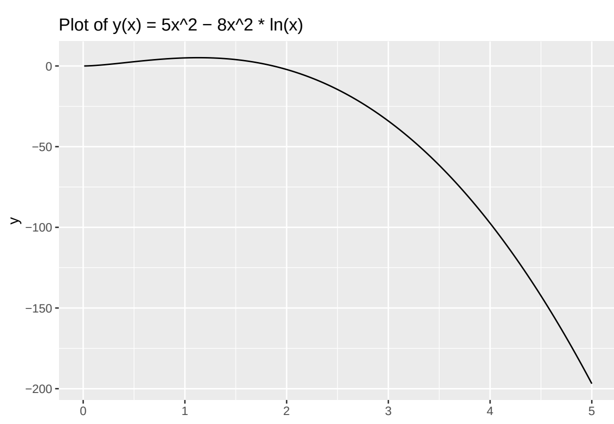
$y'(x) = mx^{m-1};\ y''(x) = m(m-1)x^{m-2}$

$(m(m-1) - 3m + 4)x^m = 0$
$(m^2 - 4m + 4)x^m = 0$
$(m-2)^2 = 0$
$\therefore m_1 = m_2 = 2$
$y_c(x) = c_1x^2 + c_2x^2lnx$

$y'(x) = 2c_1x + 2c_2xlnx + c_2x$
$y(1) = 5 = c_1$
$y'(1) = 2 = 2c_1 + c_2$

$c_1 = 5$
$c_2 = -8$

$y(x) = 5x^2 - 8x^2lnx$



3.8: 41

$\frac{d^2x}{dt^2} + 4x = -5sin2t + 3cos2t;\ x(0) = -;\ x'(0) = 1$

$m^2 + 4 = 0 \rightarrow m_1 = -2i,\ m_2 = 2i$

$x_c(t) = c_1cos2t + c_2sin2t$

$x_p(t) = Atcos2t + Btsin2t$
$x_p'(t) = (A + 2Bt)cos2t + (-2A + Bt)sin2t$
$x_p''(t) = (-4At + 4B)cos2t + (-4A - 4Bt)sin2t$

$4Bcos2t - 4Asin2t = 3cos2t - 5sin2t$
$4B = 3 \rightarrow B = \frac{3}{4}$
$-4A = -5 \rightarrow A = \frac{5}{4}$

$x_p(t) = \frac{5}{4}tcos2t + \frac{3}{4}tsin2t$

$x(t) = c_1cos2t + c_2sin2t + \frac{5}{4}tcos2t + \frac{3}{4}tsin2t$

Given: $x(0) = -;\ x'(0) = 1$

$x(t) = -cos2t - \frac{1}{8}sin2t + \frac{5}{4}tcos2t + \frac{3}{4}tsin2t$

3.9: 12

$y'' + \lambda y = 0;\ y'(0) = 0;\ y(\pi/4) = 0$

For $\lambda = 0$, $y(x) = c_1x + c_2,\ y'(0) = y(\pi/4) = 0 \rightarrow c_1 = c_2 = 0$

$y(x) = 0$ For $\lambda = 0$, no nontrivial solutions, and is not eigenvalue.

For $\lambda < 0$, $\lambda = -\alpha^2;\ y'' - \alpha^2y = 0;\ m^2 - \alpha^2 = 0 \rightarrow m_1 = -\alpha,\ m_2 = \alpha$

$y(x) = c_1cosh\alpha x + c_2sinh\alpha \rightarrow c_1 = c_2 = 0 \rightarrow y(x) = 0$

For $\lambda < 0$, no nontrivial solutions, and is not eigenvalue.

For $\lambda > 0$, $\lambda = \alpha^2;\ y'' + \alpha^2y = 0;\ m^2 + \alpha^2 = 0 \rightarrow m_1 = -i\alpha,\ m_2 = i\alpha$

$c_2 = 0;\ c_1cos\frac{\alpha\pi}{4} = 0$

For $c_1 \neq 0$, nontrivial solutions. $cos\frac{\alpha\pi}{4} = 0 \rightarrow \alpha = 2 + 4n$

$\lambda_n = (2 + 4n)^2$

$y_n(x) = sin(2 + 4n)x$

5.1: 8

$\frac{1-x}{2+x} = \frac{3-2-x}{2-x} = -1 + \frac{3}{2+x} = -1 + (\frac{3}{2})(\frac{1}{1+\frac{x}{2}}) = \frac{1}{2} - \frac{3x}{4} + \frac{3x^2}{8} - \frac{3x^3}{16} + ...$

$|\frac{x}{2}| < 1 \rightarrow$ Interval of Convergence $(-2, 2)$

5.1: 19

5.2: 6

5.2: 19

10.2: 22

10.2: 38

10.4: 14 by using variation of parameters using the matrix exponential formula