

ME6406 HW 1

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1. Pin-hole Optics

1.a

Assume: $R_1 = \frac{dp}{2}$; $R_2 = \frac{dL}{2}$; $S = \frac{s}{dp} = \frac{2s}{R_1}$

Area of Segment for any given circle

$$\delta O = \pi R^2 \quad \delta a = \pi R^2 - \delta a_i \rightarrow \frac{\delta a}{\delta O} = 1 - \frac{\delta a_i}{\delta O}$$

$$\delta a_i = \pi R^2 \left(\frac{2\alpha}{2\pi} \right) - sh = R^2 \alpha - s \sqrt{R^2 - s^2}$$

$$\text{If: } S = \frac{s}{R} \rightarrow \alpha = \cos^{-1} S$$

$$\frac{\delta a}{\delta O} = 1 - (\cos^{-1} S - S \sqrt{1 - S^2})$$

Relationship of two projections by given distance between intersection points

$$R_1^2 - s_1^2 = R_2^2 - s_2^2 \rightarrow s_2^2 = R_2^2 - R_1^2 + s_1^2 \rightarrow s_2 = R_1 \sqrt{\frac{1}{\rho}^2 + S_1^2 - 1}$$

$$S_2 = \frac{s_2}{R_2} = \frac{2s_2}{d_L} = -\frac{R_1 \sqrt{\frac{1}{\rho}^2 + S_1^2 - 1}}{R_2} = \rho \sqrt{\frac{1}{\rho}^2 + S_1^2 - 1}$$

Finally, $\frac{\delta a}{\delta O} = (\frac{\delta a}{\delta O})_{PPC} + (\frac{\delta a}{\delta O})_{Cl}$

$$\frac{\delta a}{\delta O} = 2 - \frac{1}{\pi} (\cos^{-1} S - S \sqrt{1 - S^2} + \cos^{-1} S_2 - S_2 \sqrt{1 - S_2^2})$$

$$\frac{\delta a}{\delta O} = 2 - \frac{1}{\pi} (\cos^{-1} S - S \sqrt{1 - S^2} + \cos^{-1} (\rho \sqrt{\frac{1}{\rho}^2 + S_1^2 - 1}) - (\rho \sqrt{\frac{1}{\rho}^2 + S_1^2 - 1}) \sqrt{1 - (\rho \sqrt{\frac{1}{\rho}^2 + S_1^2 - 1})^2})$$

1.b

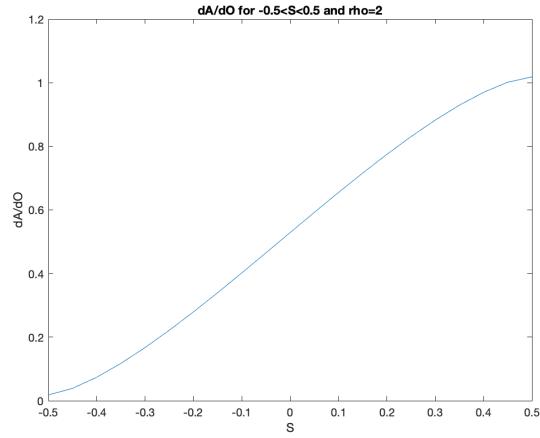


Figure 1: $\frac{dA}{dO}$ as function of S

2. Histogram Equalization

2.a

66	66	60	53	50	50	51	55
68	68	60	53	48	49	53	57
69	68	60	51	46	48	54	58
71	68	58	48	44	47	54	58
73	68	57	47	43	47	55	58
78	69	57	49	46	51	54	56

Figure 2: Sub-Region Before Filtering

202	202	191	106	74	74	90	133
228	228	191	106	53	64	106	154
239	228	191	90	21	53	122	175
244	228	175	53	11	37	122	175
250	228	154	37	5	37	133	175
255	239	154	64	21	90	122	138

Figure 3: Sub-Region After Filtering

Gray-level	# of Pixels	CDF	qk	round(qk)
43	1	1	5.3125	5
44	1	2	10.6250	11
45	0	2	10.6250	11
46	2	4	21.2500	21
47	3	7	37.1875	37
48	3	10	53.1250	53
49	2	12	63.7500	64
50	2	14	74.3750	74
51	3	17	90.3125	90
52	0	17	90.3125	90
53	3	20	106.2500	106
54	3	23	122.1875	122
55	2	25	132.8125	133
56	1	26	138.1250	138
57	3	29	154.0625	154
58	4	33	175.3125	175
59	0	33	175.3125	175
60	3	36	191.2500	191
61	0	36	191.2500	191
62	0	36	191.2500	191
63	0	36	191.2500	191
64	0	36	191.2500	191
65	0	36	191.2500	191
66	2	38	201.8750	202
67	0	38	201.8750	202
68	5	43	228.4375	228
69	2	45	239.0625	239
70	0	45	239.0625	239
71	1	46	244.3750	244
72	0	46	244.3750	244
73	1	47	249.6875	250
74	0	47	249.6875	250
75	0	47	249.6875	250
76	0	47	249.6875	250
77	0	47	249.6875	250
78	1	48	255	255

Figure 4: Histogram Equalization Table

2.b

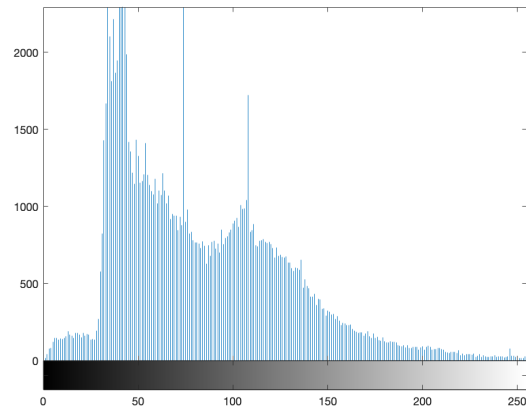


Figure 5: Original Histogram

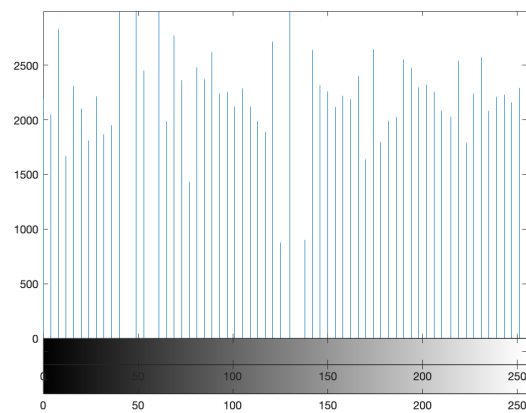


Figure 6: Filtered Histogram

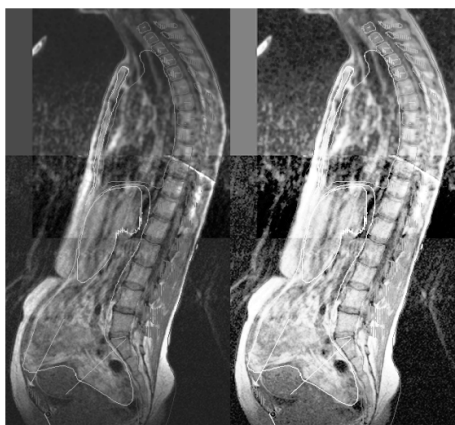


Figure 7: Original - Filtered

3. Filtering Masks

3.a

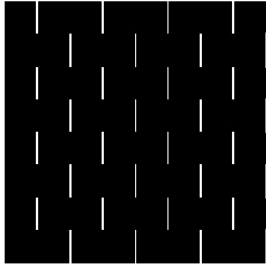
The gradient magnitude is 18.601 and the direction is 53.746°.

66	66	60	53	50	50	51	55
68	68	60	53	48	49	53	57
69	68	60	51	46	48	54	58
71	68	58	48	44	47	54	58
73	68	57	47	43	47	55	58
78	69	57	49	46	51	54	56

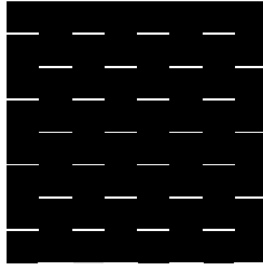
Figure 8: Direction of Gradient at pixel (3,6)

3.b

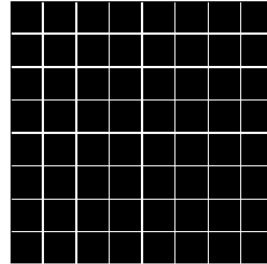
Gradient in X direction



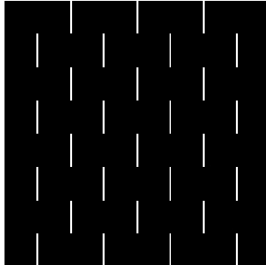
Gradient in Y direction



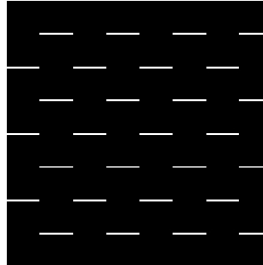
Magnitude of Gradient



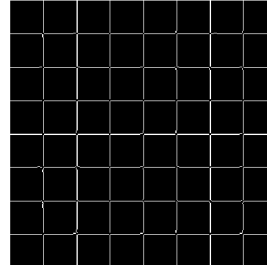
Gradient in X direction (MATLAB imgradientxy)



Gradient in Y direction (MATLAB imgradientxy)



Edge Detection (MATLAB edge)



No differences between Matlab packages and own implementations are observed.

3.c

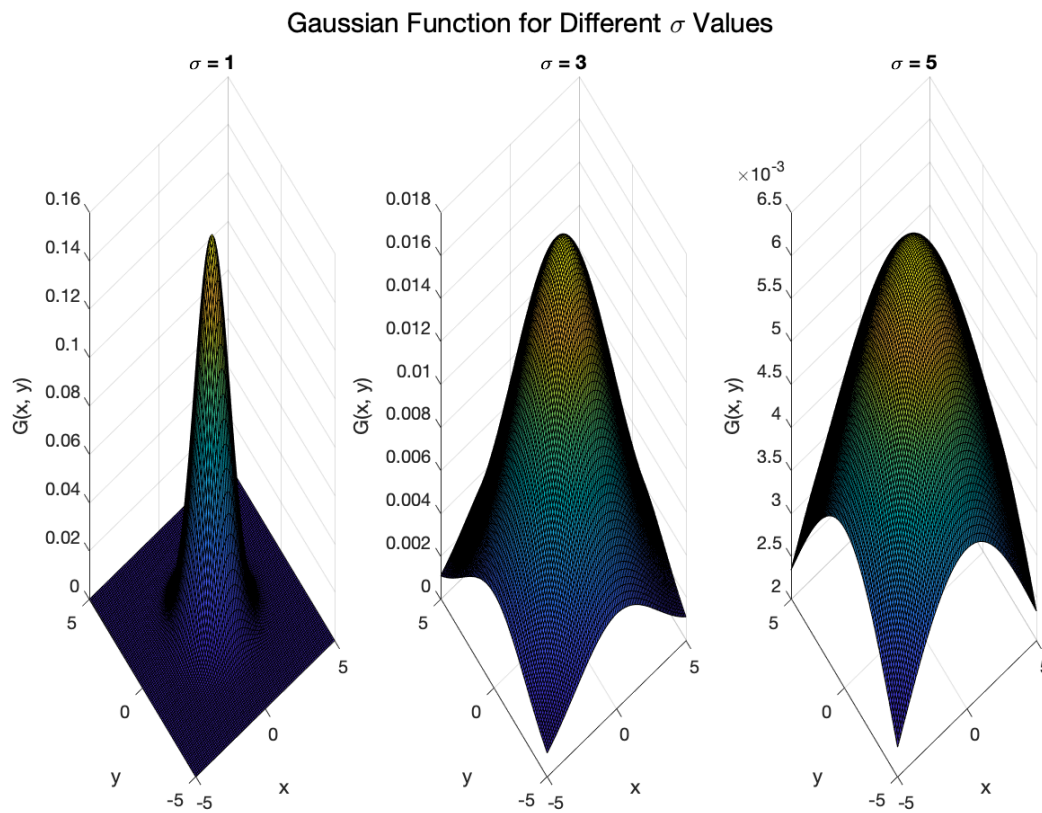


Figure 9: Gaussian Functions for various sigma

3.d

Gaussian Smoothing with Different σ Values

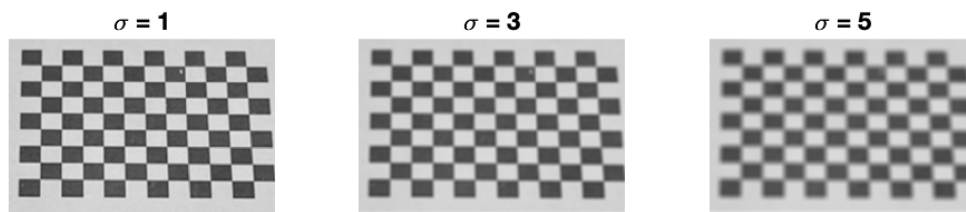
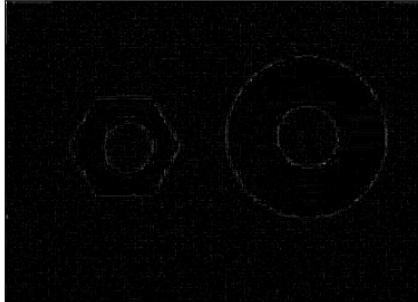


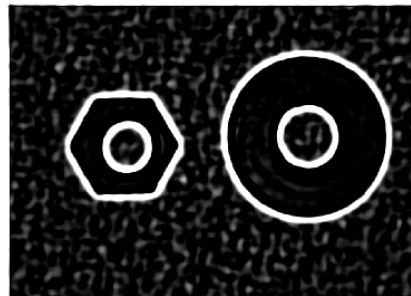
Figure 10: Effects of different sigma for Gaussian Filtering

3.e

Edge Detection ($\sigma = 1$)



Edge Detection ($\sigma = 10$)



3.f

3.f.i

3.f.ii

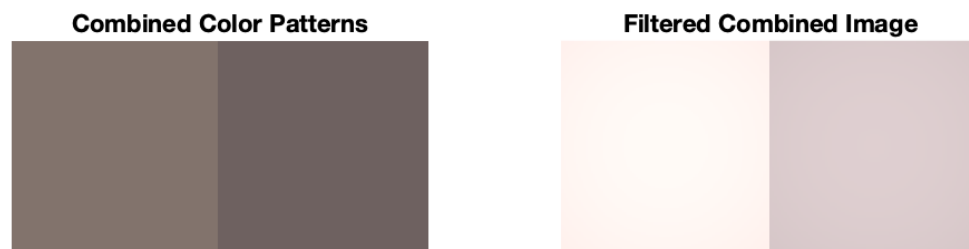


Figure 11: Color Patterns Before and After Filtering

3.f.iii

Before Filtering: $d = 29.46$

After Filtering: $d = 74.23$

4. Low-level Information Processing

4.a

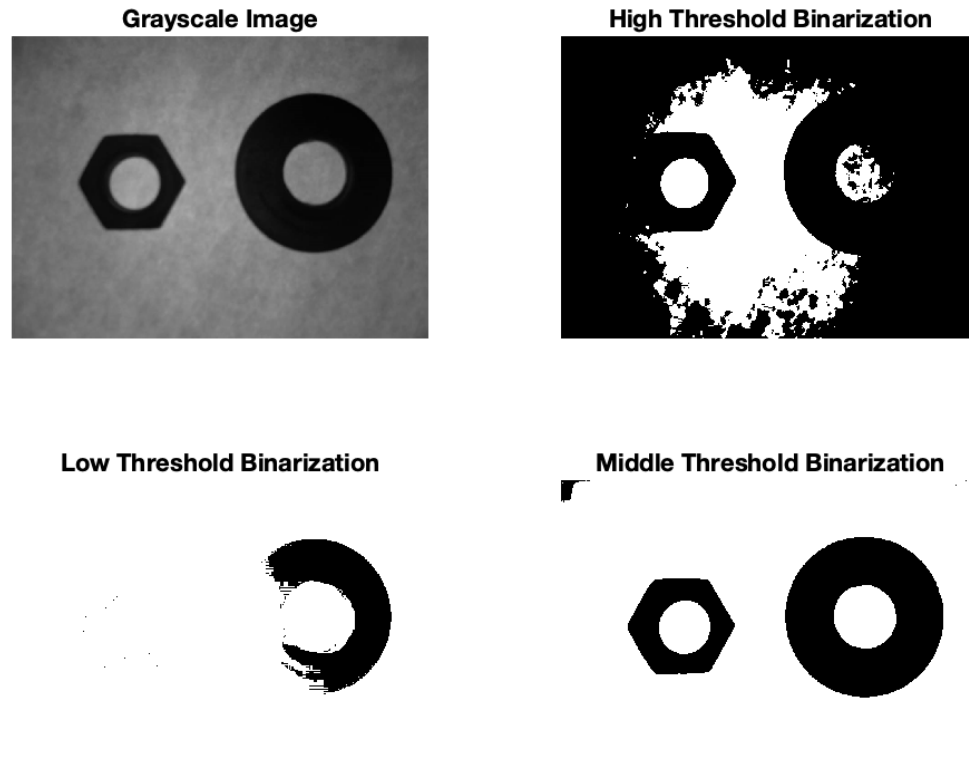


Figure 12: Binarized Image at various Thresholds

4.b

Nut Area: 10331 Nut Centroid: [272.15, 324.19]

Shell Area: 15072 Shell Centroid: [668,05, 301.27]

4.c

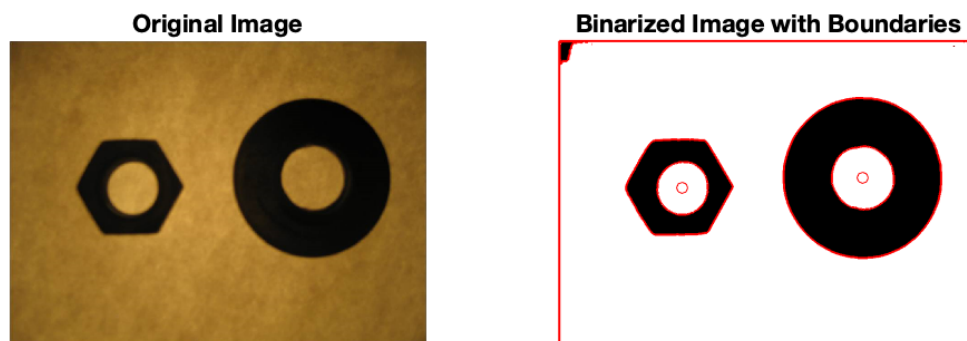


Figure 13: Boundaries of Nut and Shell