

Assignment #1: Extended due date **Saturday, September 23rd, 2023 (23:59 EDT)**

All programs should be written using MATLAB. Solutions must be consolidated into a **single pdf file** (including all results and an explanation of results) and a **zip file** (including all m-files used for the results). Solutions must be submitted electronically through **Canvas**. Late solutions will be penalized at a 10% deduction from the homework score and will NOT be accepted 24 hours after the due date. **Without a signed honor code, your exams and assignments will NOT be graded.**

1. Pin-hole optics

Consider a pinhole projection circle (PPC with diameter d_P) scanning along the diagonal line ($C_P C_L$) to the circular landmark as shown in Fig. 1, where s ($-d_P/2 \leq s \leq +d_P/2$) is the distance from the intersecting line to the PPC center C_P .

- Derive** the mathematical expression for $\delta a / \delta O$ as a function of $S (= s/d_P)$, where δa is the overlapping area and δO is the pin-hole projection area, in terms of the ratio $\rho (= d_L/d_P)$.
- Plot** $\delta a / \delta O$ for the range, $-0.5 \leq S \leq 0.5$ and $\rho = 2$.
Discuss the effect of ρ on $\delta a / \delta O$ by considering two cases, $0 < \rho < 1$, and $\rho \geq 1$.

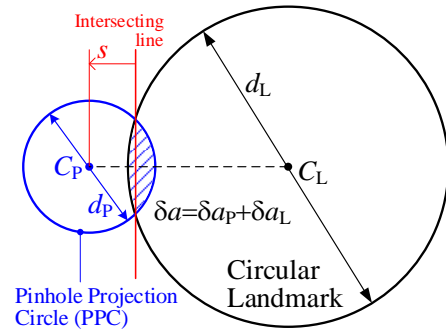


Fig. 1. Pin-hole model of a landmark

2. Histogram Equalization (HE)

Fig. 2(a) shows an 8-bit gray-level MRI image (Posture_MRI.png) of a human and a subregion in a matrix (Fig. 2b). For your convenience, the data are also given in the 'sub_region_matrix.xlsx' file.

- Compute the HE of the region matrix by completing Table 1. Show the sub-region matrixes before and after the HE.

(MATLAB *histeq* command is NOT allowed in 2a).

Table 1

Gray-level	# of pixels	CDF	q_k	$\text{round}(q_k)$
43	1	1	5.313	5
:	:	:	:	:
78	2	48	255	255



Fig. 2

(a) Posture_MRI.png

(b) subregion

	66	66	60	53	50	50	51	55
	68	68	60	53	48	49	53	57
	69	68	60	51	46	48	54	58
	71	68	58	48	44	47	54	58
X	73	68	57	47	43	47	55	58
	78	69	57	49	46	51	54	56

- Perform the HE on an image by writing a Matlab script. Demonstrate your work as follows:
 - Read in and display the 'Posture_MRI.png'.
 - Compare by displaying the original and processed images and their histograms.
You may use the Matlab functions: *imshow*, *imhist* or *hist*, *histeq* for Part 2b).

3. Filtering masks

Sobel operator (Parts 3a, b)

- Use a 3x3 Sobel operator to calculate the *magnitude* and *direction* of the gradient at pixel (3, 5) in Fig. 2(b). Indicate the direction of the gradient on the pixel. (Note: Sobel operator is coordinate dependent. Be sure to use consistent coordinate systems on the sub-regions.)

- b) Write a Matlab script to compute the gradient of an image. For illustration, use the Sobel operator on the image “checker.png” shown in Fig. 3(a). Display the gradient images (G_x , G_y , \mathbf{G}). You are encouraged to compare with that computed using Matlab functions *edge* and *imgradientxy*; explain if any differences.

Gaussian operator (Parts 3c to f)

- c) Plot the Gaussian function in the range $(-5 \leq x \leq 5, -5 \leq y \leq 5)$ for $\sigma = 1, 3, 5$:

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(\frac{-(x^2 + y^2)}{2\sigma^2}\right)$$

Suggested Matlab functions: *meshgrid.m*, *surf.m*

- d) Use a Gaussian filtering mask to smooth the image ‘Texture.png’ (Fig. 3b); specifically, compare the smoothing effects of different σ ($= 1, 3$, and 5) on the image.

Note: Use the proper mask size for a specified σ to smooth an image. *Suggested Matlab functions: imfilter.m*

- e) Perform an edge detection using the DoG function (difference of two Gaussians on the image “nut_and_shell.png” (Fig.4), demonstrating your work with $\sigma_c = 1$ and $\sigma_s = 10$.

- f) An interesting observation in a human visual system (HVS) suggests our HVS uses DoG functions (Part 3e) to discriminate closely similar colors and detect an edge. Consider an image consisting of two 100^2 -pixel RGB color patterns with [Red, Green, Blue] values:

Color Pattern I (left): [130, 115, 108] and **Color Pattern II (right):** [110, 97, 96].

- i) Compute the 3 transformed components [h_1 , h_2 , h_3] to create a new color image using the equations on the right, where h_i (with $i=1, 2$, and 3) represent the transformed values:
- $$h_1(x, y) = DoG * R + G_{\sigma_s} * [2R + G - B]$$
- $$h_2(x, y) = DoG * G + G_{\sigma_s} * [R + 2G - B]$$
- $$h_3(x, y) = DoG * B + G_{\sigma_s} * [R + G]$$

- ii) Plot the color patterns before and after the color filtering;

- iii) Computing the norm d between the two color vectors before and after the filtering:

$$d = \sqrt{(R_I - R_{II})^2 + (G_I - G_{II})^2 + (B_I - B_{II})^2}$$

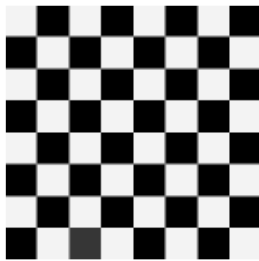


Figure 3(a)
checker.png

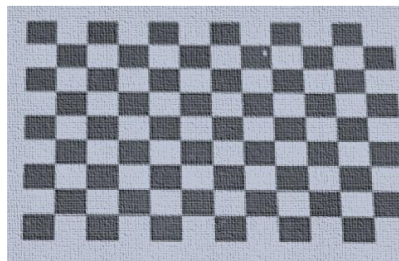


Figure 3(b)
Texture.png



Figure 4.
nut_and_shell.png

4. Low-level information processing

This problem shows the process of obtaining some low-level information from an image.

- a) Read in and convert the image ‘nut_and_shell.png’ into a gray-scale image. Binarize the image using 3 different thresholds. Preferably one over- and one under-estimate and the other in between. (Hint: Use the histogram to get an idea of what thresholds to pick).
- b) Obtain the area and centroid of the nut and shell in the image with the appropriate threshold.
- c) Find all the boundaries of the nut and shell.

Suggested Matlab functions: rgb2gray.m, im2bw.m, bwlabel.m, regionprops.m, bwboundaries.m