Georgia Institute of Technology
George W. Woodruff School of Mechanical Engineering
ME6406 Machine Vision (Fall 2023)
Assignment #3: (ME6406Q Due Wednesday, **November 4th** , 2023**, 23:59pm EDT**)

All programs should be written using MATLAB. Solutions must be consolidated into a **single pdf file** (including all results and an explanation of results) and a **zip file** (including all m-files or mlx-files used for the results). Solutions must be submitted electronically through **Canvas**. Late solutions will be penalized at a 10% deduction from the homework score and will NOT be accepted 24 hours after the due date. Without a signed honor code, your exams and assignments will NOT be graded.

Problem 1: Camera Model and Calibration

a) Camera Model. Write a program (CameraModel.m) to transform the 3D world coordinates ($X_wY_wZ_w$) of the 20 calibration points (represented by '*' in Fig. 1 and Table 1) to the 2D image coordinates (u_dv_d) using Tsai's camera model Steps 1 to 3, which account for radial lens distortion. Use [$\mathbf{R}_x$(135°)], $\mathbf{T}$=[3 3.5 7.5]^T, f=1.3, k_1=0.01 to illustrate your solutions, you may assume variables/parameters are in consistent physical units. Determine and show these 20 feature points in the u_dv_d plane. Save the (X_w, Y_w), (u_u, v_u) and (u_d, v_d) values in camera_calibration_data.mat for b).

b) Camera Calibration. Write a program (CameraCalibration.m) to calibrate compute f, [**R**], **T**. Given the above data in camera_calibration_data.mat. Compute f, [**R**], **T**, k_1.

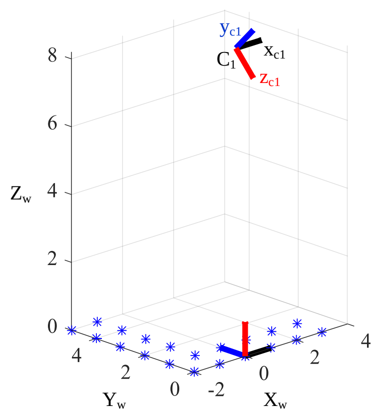

Fig. 1 Camera model and calibration

Table 1 Camera calibration points

X_w	−2	−1	0	1	2
Y_w	0	0	0	0	0
Z_w	0	0	0	0	0
X_w	3	−2	−1	0	1
Y_w	0	1	1	1	1
Z_w	0	0	0	0	0
X_w	2	3	−1	−2	−1
Y_w	1	1	2	2	3
Z_w	0	0	0	0	0
X_w	−2	−1	−2	−1	−2
Y_w	3	4	4	5	5
Z_w	0	0	0	0	0

Problem 2: Morphology

a) Let A denote the set shown in Fig. 2(a). Refer to the structuring elements shown (the black dots denote the origin). *Hand-sketch* the result obtained from each of the following morphological operations:

1) (A ⊖ B4) ⊕ B2
2) (A ⊖ B1) ⊕ B3
3) (A ⊕ B1) ⊕ B3
4) (A ⊕ B3) ⊖ B2

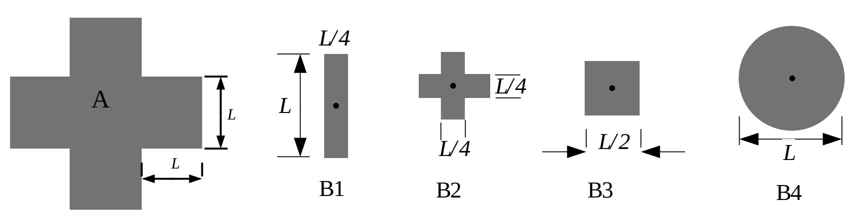

Fig. 2(a)

b) Use the following steps (1. A⊕B, 2. A⊖B, and 3. A⊕B−(A⊖B), and the structure element B in Fig. 2(b) to perform a morphological filtering on the head-CT image A (Fig. 2c). Show the corresponding images obtained from each of the operations.

Suggested MATLAB function:
imdilate.m, *imerode.m*

1	1	1
1	1	1
1	1	1

Fig. 2(b)

Fig. 2(c)

Problem 3: Robot Eye-on-Hand Calibration

Figure 3(a) shows the setup for performing an eye-on-hand calibration where a stationary planar calibration board is viewed at 3 different locations by a camera mounted on a robot gripper. Figure 3(b) shows the images in three camera image planes. The transformation matrices from *CW* to *Ci* can be determined by the camera calibration ([**H**ci] where *i*=1, 2, 3). The rigid body transformations of the robot gripper from Stations 1 to 2 and 2 to 3 are given by the robot controller, which are denoted respectively by [**H**g12] and [**H**g23]. Write a MATLAB program to perform an eye-on-hand calibration; use [**H**c1], [**H**c2], and [**H**c3] data in '*robot_hand_eye_data.mat*' to illustrate your solutions:

1) Compute ([**R**c12], **T**c12) and ([**R**c23], **T**c23).
2) Obtain the equivalent angle-axis representation (n, θ) for each of the rotation matrixes:

   [**R**c12], [**R**c23], [**R**g12] and [**R**g23].
3) Compute P_{c12}, P_{c23}, P_{g12} and P_{g23}. Check your solutions by computing [$\mathbf{R}_{g12}$] and [$\mathbf{R}_{g23}$] using Equations (8) and (10) in [2] and comparing with those given in the data file '*robot_hand_eye_data.mat*'.
4) Use the procedure in [2] to compute $\mathbf{P}_{cg}$, [$\mathbf{R}_{cg}$] and $\mathbf{T}_{cg}$.

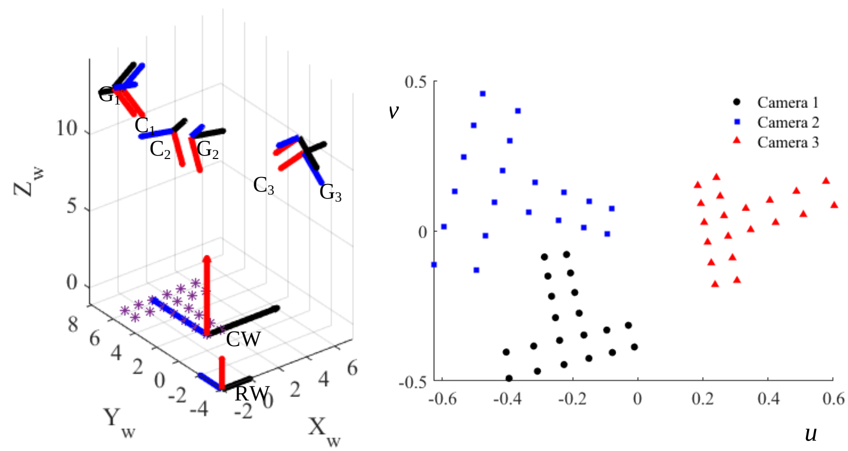

Fig. 3(a) Fig. 3(b)

Problem 4: Ellipse-Circle Correspondence

A circle captured by a camera (with focal length *f*=0.825cm) in the image plane has the following general ellipse equation: $Au^2 + 2Buv + Cv^2 + 2Du + 2Ev + F = 0$. The coefficients are given in file 'coef2023.mat', and the circle radius r = 7.5cm. Determine the following parameters:

1) The center of the circle with respect to the camera frame.
2) The plane equation (with respect to the camera frame) that contains the circle.
3) With no additional information, multiple solutions are possible. Find all valid solutions.

Reference:

[1] Tsai, R. "A Versatile Camera Calibration Technique for High-accuracy 3D Machine Vision Metrology using Off-the-shelf TV Cameras and Lenses," *IEEE Trans. on Robotics and Automation*, Vol. 3, No.4, pp. 323- 344, 1987.

[2] Tsai, R.Y. and R.K. Lenz, "A New Technique for Fully Autonomous and Efficient 3D Robotics Hand/Eye Calibration," *IEEE Trans. on Robotics and Automation*, Vol. 5, No. 3, 1989.

[3] Qiang Ji, Mauro Costa, Robert Haralick, and Linda Shapiro, "An Integrated Linear Technique for Pose Estimation from Different Features," *International Journal of Pattern Recognition and Artificial Intelligence*, Vol. 13, No. 5, 1999.