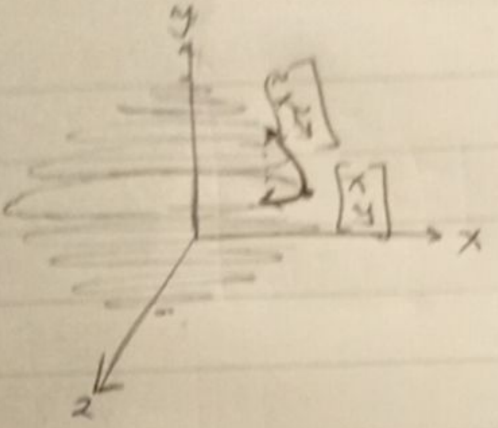
Rotation in 3D



- Rotation around z-axis (i.e. on the x-y plane) corresponds to <u>Rotation on 2D</u> (x-y)

$$\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} = \begin{bmatrix}\cos\vartheta & -\sin\vartheta & 0\\ \sin\vartheta & \cos\vartheta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = D_z(\vartheta)\,\underline{x}$$

- Rotation around y-axis (ie. on the x-z plane) corresponds to <u>Rotation on (x-z)</u>

$$= \begin{bmatrix}\cos\vartheta & 0 & -\sin\vartheta\\ 0 & 1 & 0\\ \sin\vartheta & 0 & \cos\vartheta\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

$$\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} = \begin{bmatrix}\cos\vartheta & -\sin\vartheta & 0\\ 0 & 1 & 0\\ \sin\vartheta & \cos\vartheta & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = R_y(\vartheta)\,\underline{x}$$

- Rotation around x-axis (ie. on the y-z plane) corresponds to <u>Rotation on y-z</u>

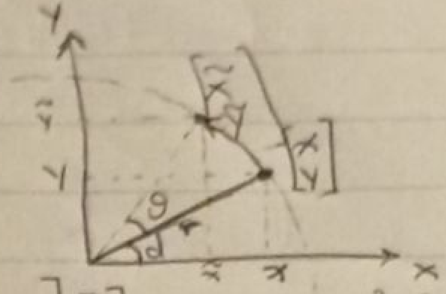
$$\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\vartheta & -\sin\vartheta\\ 0 & \sin\vartheta & \cos\vartheta\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

$$\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ \cos\vartheta & -\sin\vartheta & 0\\ \sin\vartheta & \cos\vartheta & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = R_x(\vartheta)\,\underline{x}$$

Note: Any movement on 3D (sphere) translates to cascading three rotations

$$\underline{\tilde{x}} = R_x(\vartheta_x)R_y(\vartheta_y)R_z(\vartheta_z)\,\underline{x} = R\,\underline{x} = \begin{bmatrix}r_{11} & r_{12} & r_{13}\\ r_{21} & r_{22} & r_{23}\\ r_{31} & r_{32} & r_{33}\end{bmatrix}\underline{x}$$

Counter-Clockwise Rotation in 2D



$x^2+y^2=r^2$

$$\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} = \begin{bmatrix}\alpha_1 & \alpha_2\\ \alpha_3 & \alpha_4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$$

$$\tilde{x} = R(\theta)x$$

?

x moves $\tilde{x}-x$, y more $y-\tilde{y}$

$$\cos\alpha = \frac{x}{r} \Rightarrow x = r\cos\alpha$$

$$\sin\alpha = \frac{y}{r} \Rightarrow y = r\sin\alpha$$

$$\cos\theta = \frac{\tilde{x}}{r} \Rightarrow \tilde{x} = r\cos(\theta-\alpha)$$

$$\sin\theta = \frac{\tilde{y}}{r} \Rightarrow \tilde{y} = r\sin(\theta-\alpha)$$

$$\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} = r\begin{bmatrix}\cos(\theta-\alpha)\\ \sin(\theta-\alpha)\end{bmatrix} \circledast \Longrightarrow$$

$\circledast$

- $\cos(\theta-\alpha) = \cos\theta\,\underline{\cos\alpha} - \sin\theta\cdot\underline{\sin\alpha} = \cos\theta\,\frac{x}{r} - \sin\theta\,\frac{y}{r}$
- $\sin(\theta-\alpha) = \sin\theta\,\underline{\cos\alpha} - \cos\theta\,\underline{\sin\alpha} = \sin\theta\,\frac{x}{r} - \cos\theta\,\frac{y}{r}$

$$\overset{\circledast}{\Longrightarrow} \begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} = \begin{bmatrix}x\cos\theta - y\sin\theta\\ x\sin\theta - y\cos\theta\end{bmatrix} = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & -\cos\theta\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = R(\theta)x$$

Note: indipendent of radius r. Radius iformation is embedded into vector $\begin{bmatrix}x\\ y\end{bmatrix}$ and $\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix}$