$$\frac{du}{dt} + N(u;\lambda) = 0 \quad , \quad u(t,x) \qquad 1$$

PINNs: - Data-driven discovery of p.d.e.s (system identification)

- Data-driven solution of -11-

e.g. Burger's equation: $N(u;\lambda) = \lambda_1 u \frac{\partial u}{\partial x} + \lambda_2 \frac{\partial^2 u}{\partial x^2}$, $\lambda = (\lambda_1, \lambda_2)$

Given noisy measurements $\{u(t_i,x_i)\}_{i=0}^{K}$ compute:

(a.) $u(t,x)$, $x \in \Omega$, $t \in [0,T]$

(b.) λ

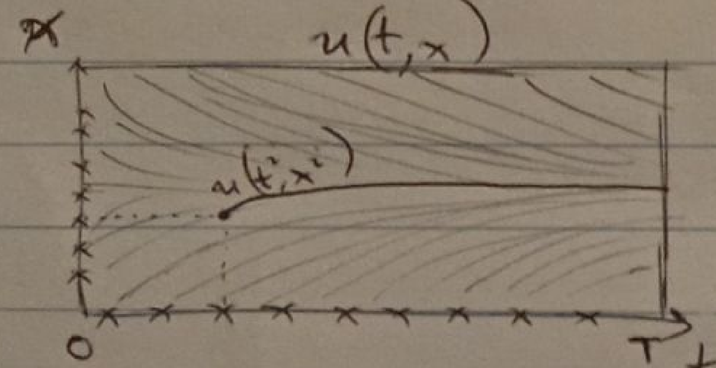
Let $f := \frac{du}{dt} + N(u)$. N is known. If we approximate $u(t,x)$ by an NN, the f is called a "physics-informed NN".

e.g. Burger's equation: $f := \frac{du}{dt} + u\frac{\partial u}{\partial x} - (0.01/\pi) \frac{\partial^2 u}{\partial x^2}$

Let MSE be the training loss function.

$$MSE := MSE_u + MSE_f$$

$$:= \frac{1}{N_u} \sum_{i=1}^{N_u} \| u(t_u^i, x_u^i) - u^i \|_2^2 + \frac{1}{N_f} \sum_{i=1}^{N_f} \left| f(t_f^i, x_f^i) \right|^2$$



$u(t^i, x^i)$ initial condition i

: trajectory computed $u(t,x)$, $t \in [t^i, T]$, $x \in \Omega$