

Viterbi Decoding of Complex Channels using Neural Networks

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Outline

First section

Second section

Viterbi Setup

Probability of transmit signal given the received signal

$$Pr(\mathbf{x}|\mathbf{y}) \quad (1)$$

For channels without intersymbol interference

$$\prod_{i=1}^n pr(x_i|y_i) \quad (2)$$

Using Baye's rules

$$Pr(x|y) = \frac{Pr(y|x)Pr(x)}{Pr(y)} \quad (3)$$

With equiprobable transmit symbols

$$Pr(y|x) = Pr(x|y)Pr(y) \quad (4)$$

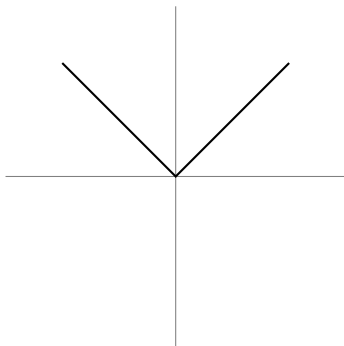
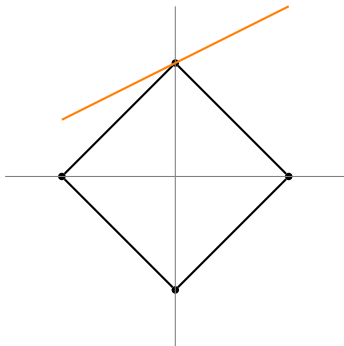
Viterbi Setup Continued

Given a finite channel length and finite symbol constellation, sequence decoding can be formalized as

$$\underset{\mathbf{x}}{\text{maximize}} \quad \prod_{i=1}^N Pr(x_i|y_i) \quad (5)$$

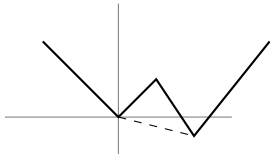
Incorporating Neural Net into Viterbi Decoding

Pictures with tikz



Pictures with tikz

- ▶ convex envelope of (nonconvex) f is the largest convex underestimator g
- ▶ *i.e.*, the best convex lower bound to a function



- ▶ **example:** ℓ_1 is the envelope of **card** (on unit ℓ_∞ ball)
- ▶ **example:** $\|\cdot\|_*$ is the envelope of **rank** (on unit spectral norm ball)
- ▶ various characterizations: *e.g.*, f^{**} or convex hull of epigraph

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Group lasso

(e.g., Yuan & Lin; Meier, van de Geer, Bühlmann; Jacob, Obozinski, Vert)

- ▶ problem:

$$\text{minimize} \quad f(x) + \lambda \sum_{i=1}^N \|x_i\|_2$$

i.e., like lasso, but require groups of variables to be zero or not

- ▶ also called $\ell_{1,2}$ mixed norm regularization

Structured group lasso

(Jacob, Obozinski, Vert; Bach et al.; Zhao, Rocha, Yu; ...)

- ▶ problem:

$$\text{minimize } f(x) + \sum_{i=1}^N \lambda_i \|x_{g_i}\|_2$$

where $g_i \subseteq [n]$ and $\mathcal{G} = \{g_1, \dots, g_N\}$

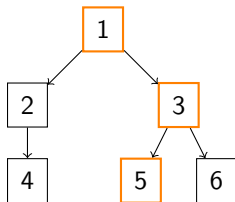
- ▶ like group lasso, but the groups can overlap arbitrarily
- ▶ particular choices of groups can impose 'structured' sparsity
- ▶ e.g., topic models, selecting interaction terms for (graphical) models, tree structure of gene networks, fMRI data
- ▶ generalizes to the **composite absolute penalties family**:

$$r(x) = \|(\|x_{g_1}\|_{p_1}, \dots, \|x_{g_N}\|_{p_N})\|_{p_0}$$

Structured group lasso

(Jacob, Obozinski, Vert; Bach et al.; Zhao, Rocha, Yu; ...)

hierarchical selection:

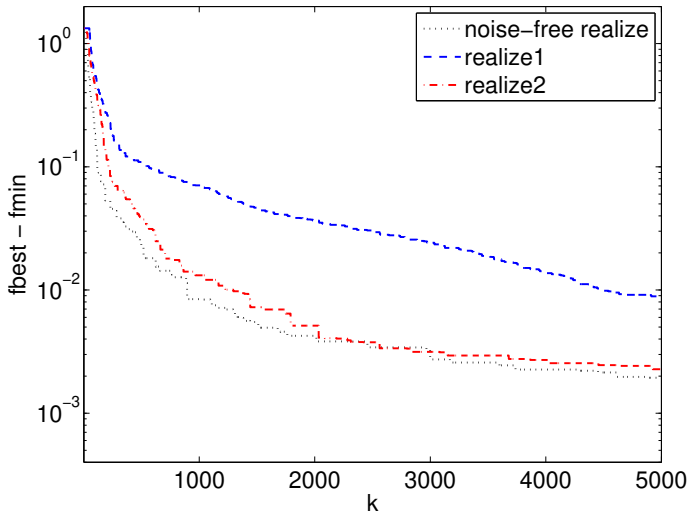


- ▶ $\mathcal{G} = \{\{4\}, \{5\}, \{6\}, \{2, 4\}, \{3, 5, 6\}, \{1, 2, 3, 4, 5, 6\}\}$
- ▶ nonzero variables form a rooted and connected subtree
 - if node is selected, so are its ancestors
 - if node is not selected, neither are its descendants

Sample ADMM implementation: lasso

```
prox_f = @(v,rho) (rho/(1 + rho))*(v - b) + b;  
prox_g = @(v,rho) (max(0, v - 1/rho) - max(0, -v - 1/rho));  
  
AA = A*A';  
L = chol(eye(m) + AA);  
  
for iter = 1:MAX_ITER  
    xx = prox_g(xz - xt, rho);  
    yx = prox_f(yz - yt, rho);  
  
    yz = L \ (L' \ (A*(xx + xt) + AA*(yx + yt)));  
    xz = xx + xt + A'*(yx + yt - yz);  
  
    xt = xt + xx - xz;  
    yt = yt + yx - yz;  
end
```

Figure



Algorithm

if L is not known (usually the case), can use the following line search:

given x^k , λ^{k-1} , and parameter $\beta \in (0, 1)$.

Let $\lambda := \lambda^{k-1}$.

repeat

1. Let $z := \text{prox}_{\lambda g}(x^k - \lambda \nabla f(x^k))$.
2. **break if** $f(z) \leq \hat{f}_\lambda(z, x^k)$.
3. Update $\lambda := \beta \lambda$.

return $\lambda^k := \lambda$, $x^{k+1} := z$.

typical value of β is $1/2$, and

$$\hat{f}_\lambda(x, y) = f(y) + \nabla f(y)^T(x - y) + (1/2\lambda)\|x - y\|_2^2$$