Estimation of Channel Distribution Functions using a Neural Network

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The channel state perspective

The optimization framework

Incorporating a Neural Network

Extension of ViterbiNet: Reduced

Simulation Results

Conclusion

Outline

The Channel State

- Observations are made of some channel in a point-to-point communication system.
- For each observation, this channel takes on a state $s[k] \in \mathcal{S}$.
- The true state $s[k]$ is hidden by the addition of noise to an observation $y[k]$.

Sampling Channel State

Over many observations, the sequence $\mathbf{y}$ corresponds to a sequence of channel states $\mathbf{s} \in S^N$

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For a channel represented by an LTI system, the state is determined entirely by the transmitted information $\mathbf{x}$.

Estimating the True Channel State

Goal:

We attempt to estimate the true, hidden, sequence of channel states, $\mathbf{s}$, based the sequence of samples $\mathbf{y}$.

Note

We assume that we known how many states the channel $|S|$

Outline

MAP Sequence Detection

$$\underset{\mathbf{s} \in \mathcal{S}^N}{\text{maximize}}\ p(\mathbf{s}|\mathbf{y}).$$

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this simplifies to

$$\underset{\mathbf{s}\in S^N}{\text{maximize}}\; \prod_{k=0}^{N-1} p(y[k]|s[k])p(\mathbf{s}).$$

MAP Sequence Detection

For the LTI channel

$$p(\mathbf{s})$$
$$=$$
$$p(s[N]|s[N-1]\ldots s[0])p(s[N-1]|s[N-2]\ldots s[0])\ldots p(s[1]|s[0])p(s[0])$$

describes the consistency of transmitted symbols implied by the state sequence. The channel states of the LTI channel satisfy the Markov property

$$p(s[N]|s[N-1]\ldots s[0]) = p(s[N]|s[N-1]]).$$

Example with LTI channel - Continued

With these assumptions,

$$\underset{\mathbf{s}\in S^N}{\text{maximize}} \prod_{k=0}^{N-1} p(y[k]|s[k])p(\mathbf{s})$$

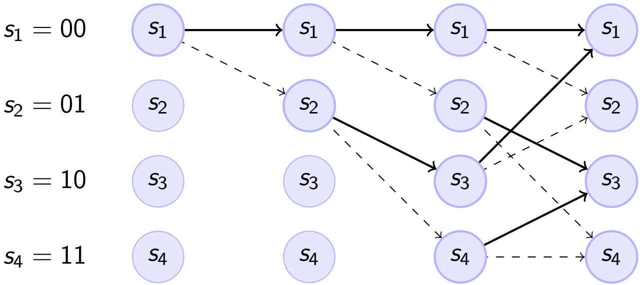
is equivalent to

$$\underset{\mathbf{s}\in S^N}{\text{minimize}} \sum_{k=0}^{N-1} -log(p(y[k]|s[k])p(s[k]|s[k-1])).$$

For the LTI channel $p(s[k]|s[k-1])$ is 0 if states contradict transmission sequence, otherwise this term is constant.

Viterbi Algorithm

$$\underset{\mathbf{s}\in S^N}{\text{minimize}} \sum_{k=0}^{N-1} -log(p(y[k]|s[k])p(s[k]|s[k-1])).$$



Example with channel impulse response length 2 and constellation size 2.

Outline

Decomposing Terms in the Viterbi Algorithm

The individual terms in

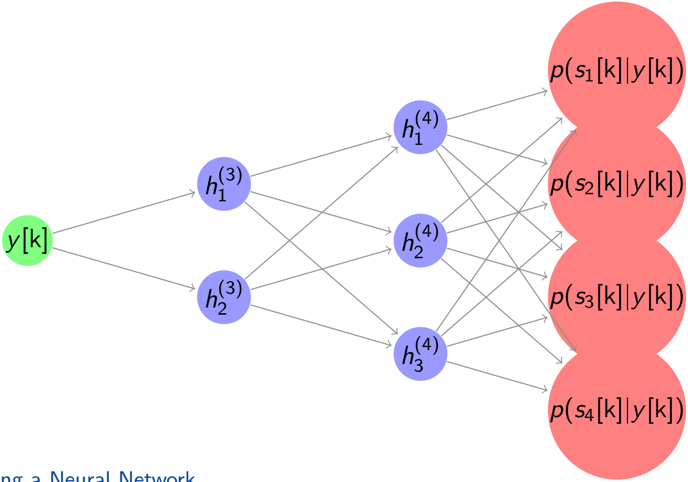
$$\underset{\mathbf{s} \in S^N}{\text{minimize}} \sum_{k=0}^{N-1} -log(p(y[k]|s[k])p(s[k]|s[k-1])).$$

can be rewritten

$$p(y[k]|s[k])p(s[k]|s[k-1]) = \frac{p(s[k]|y[k])p(y[k])}{p(s[k])} p(s[k]|s[k-1]).$$

Decomposing Terms in the Viterbi Algorithm

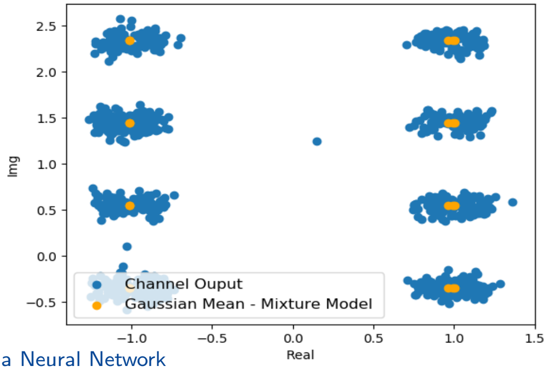
$$\frac{\color{red}{p(s[k]|y[k])}p(y[k])}{p(s[k])}$$



Decomposing Terms in the Viterbi Algorithm

$$\frac{p(s[k]|y[k])\color{red}{p(y[k])}}{p(s[k])}$$

$$p(y[k]) = \sum_{s_i \in \mathcal{S}} p(y[k], s_i)$$



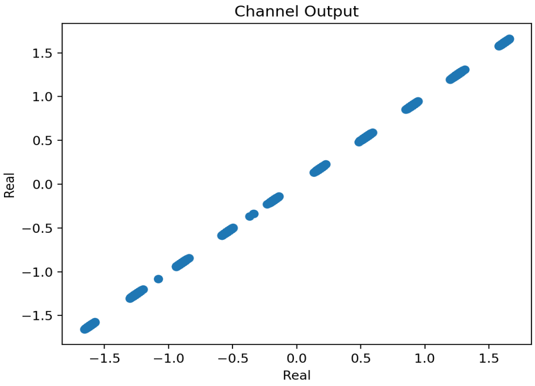
Decomposing Terms in the Viterbi Algorithm

$$\frac{p(s[k]|y[k])p(y[k])}{p(s[k])}$$

Outline

State Redundancy

Include picture of channel taps



Exploiting State Redundancy

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2. For states with ambiguous channel input, use majority decision
3. *Choosing too few states will degrade performance

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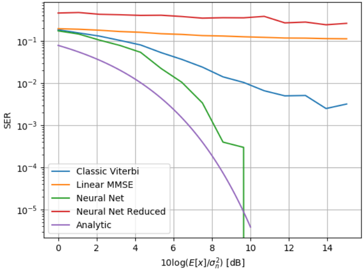
Simulation System

- BPSK with AWGN at receiver with SNR given by

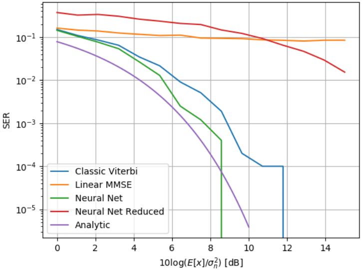
$$\mathrm{SNR} = \frac{E\{|x[k]|^2\}}{E\{|n[k]|^2\}}$$

Detection Performance: LTI Channel

$\text{CIR} = [0.9, 0.7, 0.3, 0.5, 0.1]$

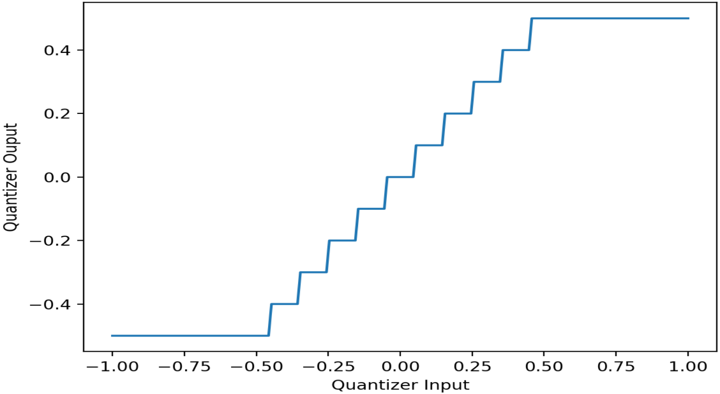


$\text{CIR} = [.9, 0, .0, .4, .7]$



*Reduced states uses 8 states in both figures above

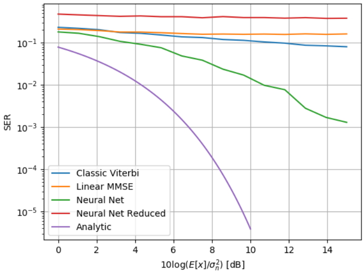
Detection Performance: Quantizer



Easy implementation by rounding down to chosen decimal place (one in the following) after adding noise.

Detection Performance: LTI Channel with Quantization

CIR $= [0.9, 0.7, 0.3, 0.5, 0.1]$



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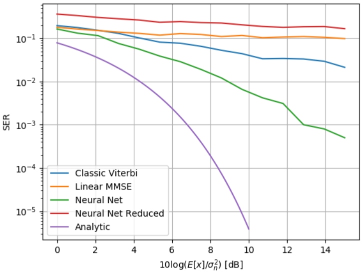

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*Note that the "Classic" Viterbi is no longer using ideal metric here

Outline

Other Notes

- Can be applied to other algorithms (BCJR).
- Generate training data for molecular communications channel and test on real data.

Thank You.

Questions or Comments?