

# Neural Network Based Decoding over Molecular Communication Channels

Peter Hartig

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## Abstract

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## 0.1 Notation

The following notation conventions are used Expectation Conditional Probability Argmax Vector Indexing

## 0.2 Introduction

Characterizing and obtaining information about communication channels is a fundamental barrier to communication. While optimal and sub-optimal strategies for overcoming this barrier in many contexts have enabled vast and effective communication infrastructure, this barrier still limits communication in others. Molecular Communication channels pose a particularly difficult context in which to overcome this barrier as channel characteristics are often non-linear and may be dependent on the specific transmitted information. In communication contexts, such as wireless, "Pilot" symbol-streams are often used to mitigate the difficulty in obtaining channel information by provide real-time information supporting an underlying channel model. The low symbol rate of Molecular Communication channels often makes such strategies impractical. However, the success of this data-driven technique in wireless channels suggest that perhaps an alternative, data-driven method may be viable in the Molecular Communication context. One potential data-driven method for characterizing these channels is a neural network. Neural networks have shown to be an effective tool in data-driven approximating of probability distributions.

The general communication channel is equivalent to a conditional probability  $P(x|y)$ , in which  $x$  is transmitted information and  $y$  is received information.  $P(x|y)$  takes into account the (potentially random) channel through which the information  $x$  passes, and random noise added prior to receiving  $y$ . The communication problem entails optimizing a form of  $P(x|y)$  over a set of possible, transmitted information  $x$ . In general, sub-optimal solutions do not require perfect knowledge of the distribution  $P(x|y)$  and may be used when  $P(x|y)$  is unknown or impractical to obtain. In this work, a neural network is used to estimate  $P(x|y)$ .

## 0.3 Background

### 0.3.1 MLSE

The form of  $P(x|y)$  used for detection in this work is  $\operatorname{argmin}_x P(y|x)$ . This optimization, known as Maximum Likelihood Sequence Estimation (MLSE), over the set of all possible  $x$  is exponentially complex in the cardinality of  $x$ . Information about the communication channel can, however, reduce the complexity of this problem. In order to illustrate this reduction, the following example is proposed.

Consider the communication channel over which a causal, linear, and time invariant combination of a set of the transmitted information is received.

$$y[k] = \sum_{l=1}^L a[l]x[k-l] \quad (1)$$

In this case,  $P(y|x)$  can be rewritten as  $\prod P(y_i|x_{i-L+1}^i) = \sum \log(P(y_i|x_{i-L+1}^i))$ . The sequence of received symbols  $\mathbf{y}$  can be equivalently represented by the trellis:

TODO IMPORT Trellis picture in which each time-point  $k$  represents a unique set of  $L$  transmitted symbols  $x[k-l] \forall l \in 1..L$ . Due to the finite number of states that channel can be in at a given time, MLSE can be performed using the Viterbi Algorithm over this trellis.

Viterbi Algorithm:

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**given**  $P(y_i|x_{i-L+1}^i) \forall i \in 1..N$ .

Let  $\lambda := \lambda^{k-1}$ .

**for**  $i = 1..N$

**for each state  $s$  at time  $i$**

1. Let  $\text{Path}_s := \operatorname{prox}_{\lambda g}(x^k - \lambda \nabla f(x^k))$ .
2. **break if**  $f(z) \leq \hat{f}_\lambda(z, x^k)$ .
3. Update  $\lambda := \beta \lambda$ .

**return**  $\operatorname{argmin}_s \text{cost}[i], x^{k+1} := z$ .

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For finite state, causal channels, MLSE reduces to the Viterbi Algorithm. Note that while Viterbi algorithm does have exponential complexity in the length of the channel (i.e.  $L$ ), complexity is linear in the length of the sequence.

### 0.3.2 ViterbiNet

As suggested in the introduction, despite the reduction in complexity offered by the Viterbi Algorithm for MLSE, the individual metrics used each step of the algorithm  $P(y_i|x_{i-L+1}^i)$  require knowledge of the channel which may be difficult to obtain. To estimate this distribution using a neural network, Baye's Rule is used.

$$P(y_i|x_{1...L}) = \frac{P(x_{1...L}|y_i)P(y_i)}{P(x_{1...L})} \quad (2)$$

These terms can be interpreted as:

- $P(x_{1...L}|y_i)$  : The probability of being in a channel state given the corresponding received symbol from that time point. In the case of a finite number of states, such a probability can be estimated using a neural network for classification of received signals into channel states.
- $P(y_i)$  : The randomness of the channel. As each received signal represents a state of the system to which noise may be added. A representative mixture-model can be estimated using a set of received signal training data.
- $P(x_{1...L})$  : Assuming the transmitted symbols are equiprobable, this term can be neglected as all states  $x_{1...L}$  will have equal probability.

## 0.4 Simulation Results

### 0.4.1 System Model

Consider the received signal

$$y[k] = \sum_{l=1}^L a[l]x[k-l] + n[k], \quad n[k] \sim \mathcal{N}(0, 1) \quad (3)$$

with  $x[k-l] \in \{-1, +1\}$  and  $n[k] \sim \mathcal{N}(0, 1)$ . The resulting signal to noise ratio (SNR) is  $\frac{E\{x[k]\}}{\sigma^2}$ .

Figure 1: Simulation Channels: LTI Channel

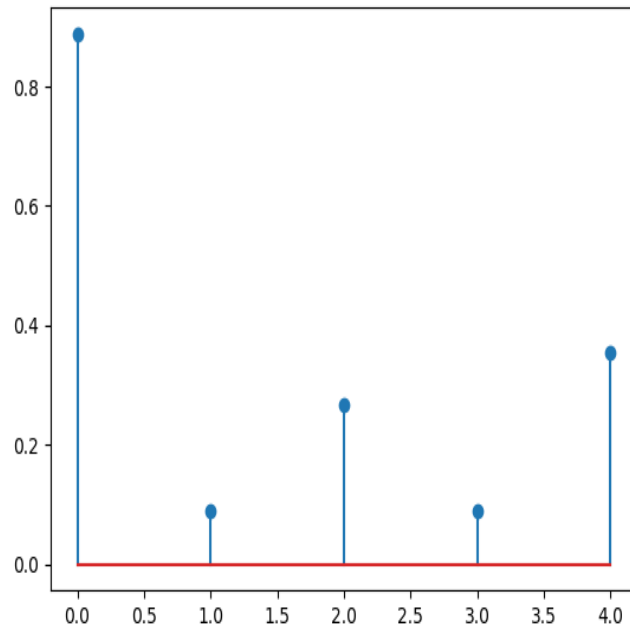
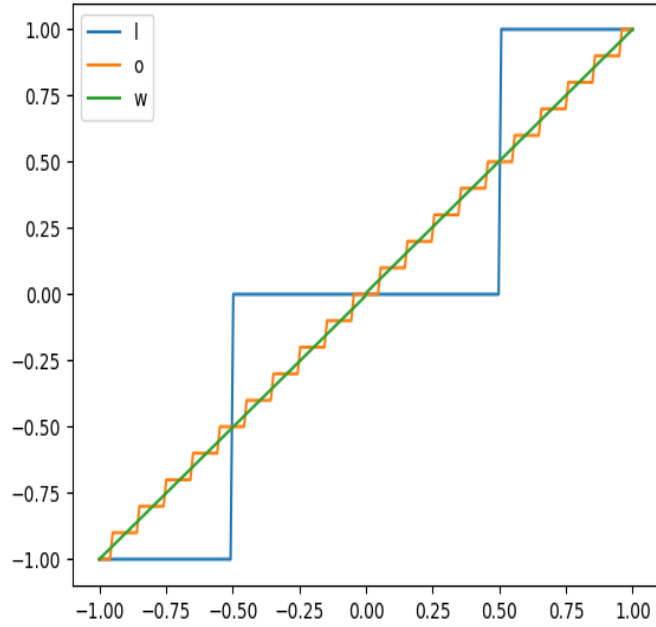


Figure 2: Simulation Channels: Quantizer



Adding quantization at matched filter (prior to noise being added)  
Details of NN architecture and training  
Details of Mixture Model training

## 0.4.2 Results

Proposed Figures



Figure 3: LTI Channel Performance

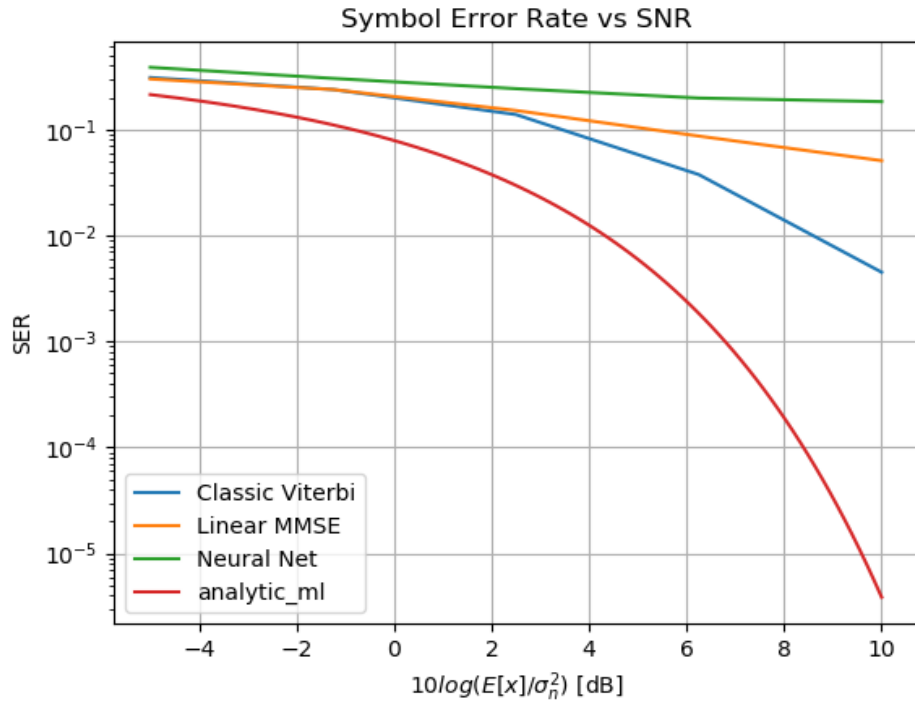


Figure 4: LTI + Quantized Channel Performance

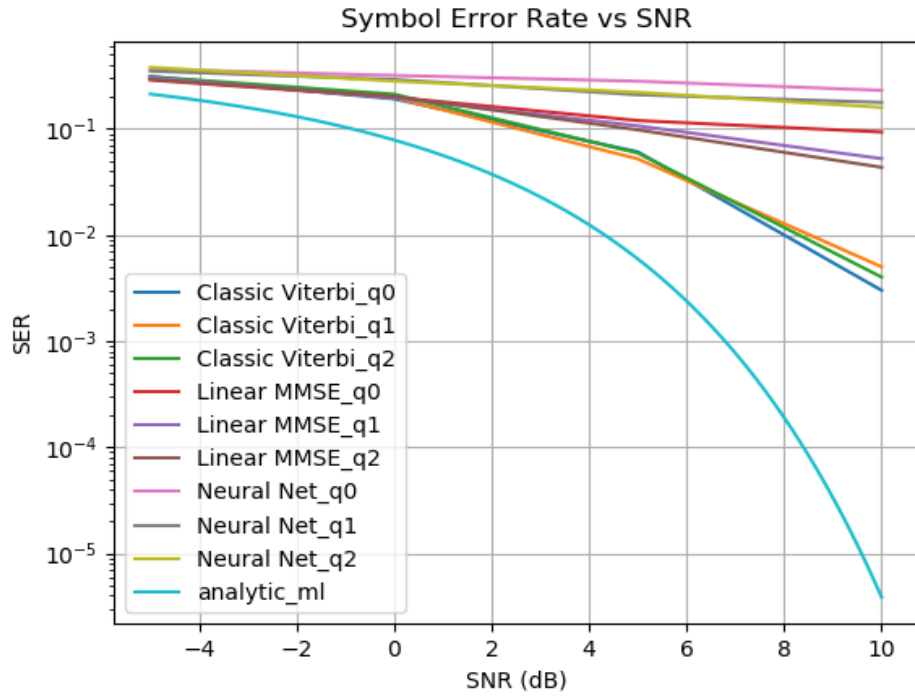
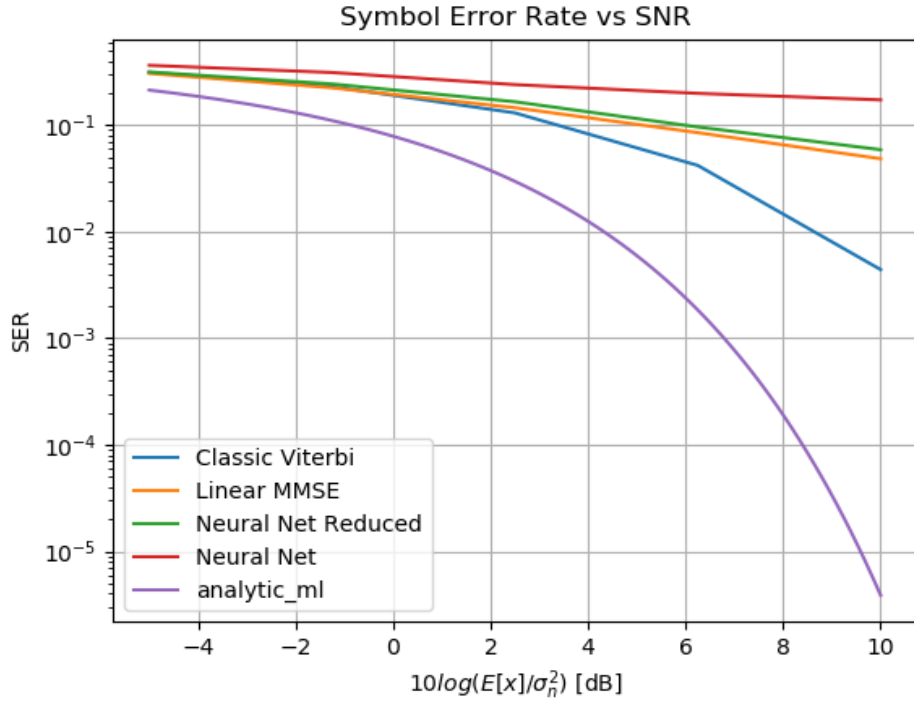
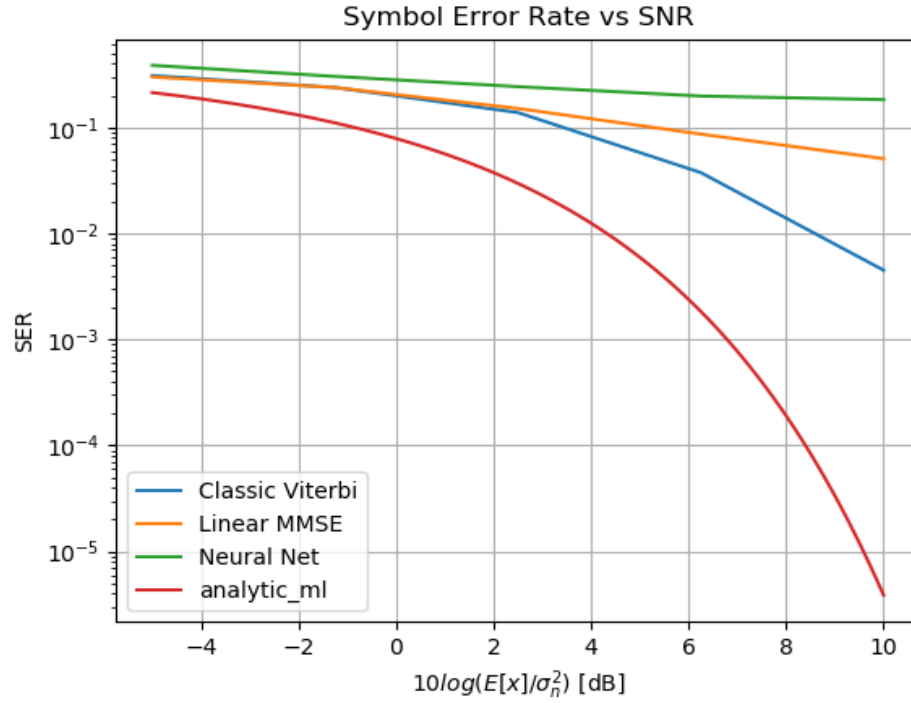


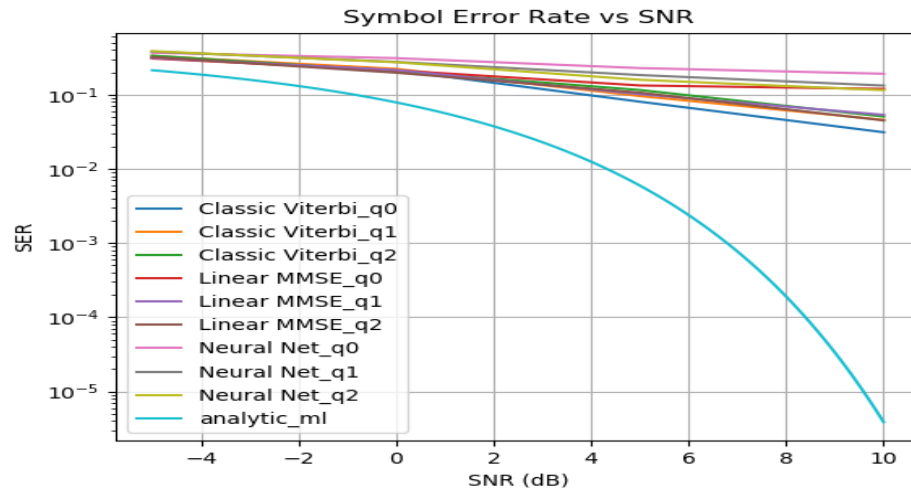
Figure 5: Reduced State ViterbiNet: LTI Channel



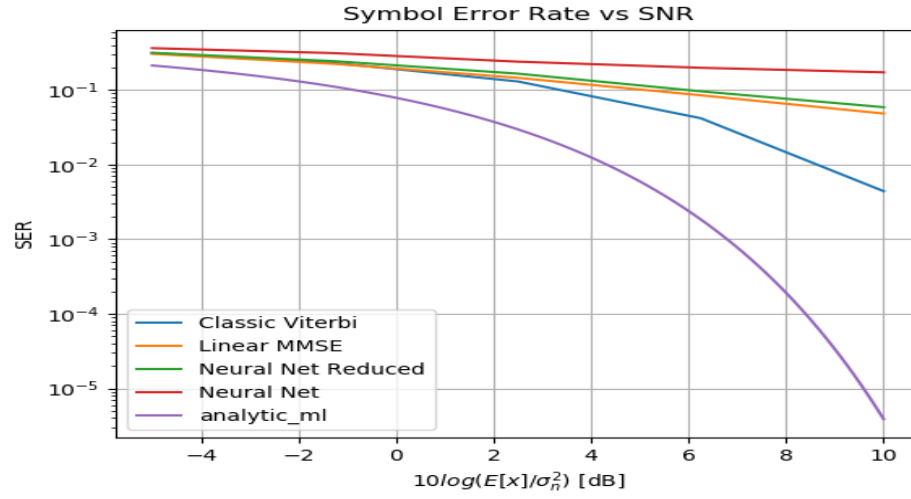
- ViterbiNet Performance compared to MMSE and classic Viterbi LTI Channel



- ViterbiNet Performance compared to MMSE and classic Viterbi non-linear Channel



- Reduced ViterbiNet on LTI Channel



- Reduced ViterbiNet on non-linear Channel

**ViterbiNet**

**Reduced State ViterbiNet**

## 0.5 Conclusion

### 0.5.1 Future Work

Discuss forward backward (APP) algorithms that could be implemented.