

1 Game and System Setup

This game considers a wireless communications network which includes a both macro cell and femto cell users.

1.1 Players

Individual femto cells are the players in this game. Here each player $f \in \{1...F\}$ is considered to have a number of antennas T_f with which to transmit to K_f femto cell users.

In this setting femto cells are assumed to be spaced far apart in distance permitting the simplification that femto cell f cause no interference to the users of femto cell $j \in \{1...F\} \setminus f$

Femto base stations with multiple antennas can beamform their transmission with transformation $\mathbf{U}_f \in \mathbb{C}_{T_f \times L_f}$.

Femto base station f has power constraint $\sum_{i=1}^{L_f} p_{f,i} \leq P_f^{Total}$.

Femto base stations are assumed to have a utility function $U_f()$ based upon the quality of service provided to its users.

Macro Cell user $m \in \{1...M\}$ experiences receiver interference to due transmission by femto cell base stations. These macro cell users have limits to the amount of interference they will tolerate $\sum_{f=1}^F \mathbf{p}_f^t \tilde{\mathbf{h}}_m^T \mathbf{U}_f \leq I_m^{Threshold}$.

Femto base station f is assumed to have full channel information (\mathbf{H}_f) for all user with which it transmits. The received signal for user j of femto base station f experiences the inner product of $\mathbf{h}_{fj}^H \mathbf{u}_{fj}$

Femto cell f is assumed to have full channel information (\tilde{H}_f) for all user with which it interferes .

Users of femto cell f have SINR $\gamma_{f,i} = \frac{p_{f,i} |\mathbf{h}_{fj}^H \mathbf{u}_{fj}|^2}{\sigma_n^2 + \sum_{\tilde{f} \neq f} \sum_{u=1}^{K_{\tilde{f}}} + \sum_{\tilde{k} \neq i} } + i \in \{1...K_f\}$
with AWGN $\sim \mathcal{N}(0, \sigma_n^2)$

Assuming sufficient base station spacing this becomes $\gamma_{f,i} = \frac{p_{f,i}|\mathbf{h}_{\mathbf{f}}^H \mathbf{u}_{\mathbf{f}}|^2}{\sigma_n^2 + \sum_{k \neq i}} +$
 $i \in \{1 \dots K_f\}$

This further simplifies assuming that users use a zero-forcing beam-former
 $\gamma_{f,i} = \frac{p_{f,i}|\mathbf{h}_{\mathbf{f}}^H \mathbf{u}_{\mathbf{f}}|^2}{\sigma_n^2}$

1.2 Scenarios

1.2.1 Case: $T_f \geq M + K_f$

Base Stations could potentially zero-beam-form towards all macro users. However, as base stations have power constraints, it may be beneficial to cause certain amounts of interference.

1.2.2 Case: $K_f \leq T_f < M + K_f$

Base Stations can send unique signals to all users but does not have sufficient DOF to zero-beam-form for all macro users.

1.3 Optimization Problem

Each player f attempts to maximize utility function $U_f()$ while playing a strategy that falls in the region constrained by the interference limits imposed by the macro cell users.

The player optimization problem can be written as:

$$\begin{aligned} & \underset{\mathbf{U}_f, \mathbf{P}_f}{\operatorname{argmax}} U_f() \\ \text{s.t. } & \sum_{f=1}^F \mathbf{P}_f^t \tilde{\mathbf{h}}_{\mathbf{m}}^T \mathbf{U}_f \leq I_m^{\text{Threshold}} \end{aligned}$$

2 Solving