Game Theoretic Solutions to Multiple Antenna Power Control in Heterogeneous Networks

Peter Hartig

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Abstract

This work investigates resource allocation strategies for a communication system with uncoordinated users and MIMO capable base stations. Following a game theoretic solution, it is shown that unlike the SISO case investigated in [GCA15], the general MIMO problem does not admit the N-Concave game framework. Refinements of the general problem are then made to use the N-Concave game framework. These refinements require pre-selection of transmission beamformers prior to the allocation of power and therefore the beamformer design is also considered to increase the achievable player utility. These results are verified and further analyzed in simulations.

Contents

Chapter 1

Introduction

Modern communication systems often incorporate numerous, uncoordinated users competing for a limited resource. In this work, a game theoretic approach considers a wireless communication network with uncoordinated macrocell users and femtocell base stations. The goal of this approach is to find Nash Equilibrium in which femtocell base stations optimize individual utility while ensuring interference tolerances are not violated for macrocell users. Previous work has shown solutions for systems in which femtocell base stations transmit over a single input, single output (SISO) channel; therefore the multiple input, multiple ouput (MIMO) case is investigated here.

This work focuses on achieving Nash Equilibrium (NE) using distributed solutions. In the communications context, distribution of system optimization is preferable in order to minimize the amount of information passing overhead required by central optimization.

1.1 Notation

The following notation is used throughout the remainder. $E[x]$ is the expected value of a random variable x. Vectors are denoted by bold font, lower-case letters ($\mathbf{x}$) and are assumed to be column vectors. Matrices are denoted by bold font, upper-case letters ($\mathbf{X}$). The transpose and hermetian of $\mathbf{X}$ are denoted by $\mathbf{X}^T$ and $\mathbf{X}^H$ respectively. $\mathbf{I}$ denotes the identity matrix.

1.2 Relevant Tools/Theory

It is useful to first review relevant game theoretic tools. Together, these tools provide a series of steps to follow in solving for Nash Equilibrium via distributed solutions.

1. The Concave N-Person game: A framework for proving existence and uniqueness of Nash Equilibrium.

   - **Definition:** A game in which individual players maximize a concave utility function over a convex strategy set resulting from problem constraints (potentially player coupled).
   - **Existence of Nash Equilibrium:** Pure Strategy NE exist for Concave N-Person games [Ros64, Thm1].
   - **Uniqueness of Nash Equilibrium:** To prove uniqueness of NE for Concave N-Person games, the Normalized Nash Equilibrium is introduced. First, take a weighted sum of the player utility functions $U_{\mathrm{f}}(b_{\mathrm{f}})$.

     $$\sigma(\mathbf{b},\mathbf{r}) = \Sigma_{\mathrm{f}=1}^{\mathrm{F}} r_{\mathrm{f}} U_{\mathrm{f}}(b_{\mathrm{f}}),\ \mathbf{r} = (r_1...r_{\mathrm{F}}),\ \mathbf{u} = (b_1...b_{\mathrm{F}}),\ r_{\mathrm{f}} \geq 0.$$

     The gradient of $\sigma(\mathbf{b},\mathbf{r})$ is

     $$g(b,r) = \begin{bmatrix} r_1 \nabla U_1(b_1) \\ r_2 \nabla U_2(b_2) \\ \vdots \\ r_F \nabla U_{\mathrm{F}}(b_{\mathrm{F}}) \end{bmatrix}. \tag{1.1}$$

     The matrix valued function $G(b,r)$ is defined as the Jacobian of $g(b,r)$.
     Negative definiteness of the matrix $[G(b,r)+G^T(b,r)]$ is a sufficient condition for diagonally strict concavity of $\sigma(\mathbf{b},\mathbf{r})$ which is a sufficient condition for uniqueness of a NNE [Ros64, Thm4].

2. The Potential Function: An equivalent but central optimization problem representing a game.

   - **Definition:** A function $\Psi(b_{\mathrm{f}})$ which satisfies:

     $$\frac{\partial \Psi(\mathbf{b})}{\partial b_{\mathrm{f}}} = \frac{\partial U_f(b_{\mathrm{f}})}{\partial b_{\mathrm{f}}},\ \mathbf{b} = [b_1, b_2...b_{\mathrm{F}}] \tag{1.2}$$

     for a game with player utility functions $U_f(b_{\mathrm{f}})$. In words, $\Psi(\mathbf{b})$ is a function whose gradient with respect to the strategy b_{f} of a player, is equal to the gradient of that player's utility function with respect to b_{f}.
   - **Result 1:** Global optima of a potential function are Nash Equilibrium [MS96].

- **Result 2:** For a Concave N-Person games admitting a potential function, the potential function is concave, allowing for use of convex optimization tools to find a NE.

3. Distributed Optimization: Solving a central optimization problem using multiple processes in parallel. In particular, the Dual Ascent method described in [BPC+11] is used in this work.

   Distributed Dual Ascent:

   - For a central optimization problem (obtained using a Potential Function in this work), determine the dual problem to the central, primal problem.
   - Evaluate if the dual problem can be separated into sub-problems, each with independent optimization variables.
   - Perform an ascent of the dual problem by iterating the following:
     (a) Solve the separate dual function of the sub-problems.
     (b) Broadcast these solutions and perform the gradient step of the dual problem.

   If the problem is convex with strong duality, the dual ascent will converge to the optimal (in this case a Nash Equilibrium).

1.3 Outline

The above tools are used in the steps outlined below.

- 2.1: Introduce the system model for the general game.
- 2.1.4: Analyze the general setup to see why further refinements are useful in arriving at a distributed solution.
- 2.2: Introduce refinements to the system model.
- 2.2.4: Develop a distributed solution using the refined system model.
- 3: Discuss numerical results of the proposed solution.

Chapter 2

System Model

2.1 General System Model

2.1.1 Players: Femtocell Base Stations

Each player of the game, a femtocell base station (FBS) $\mathrm{f} \in \{1...F\}$, is characterized by the following.

- T_{f} antennas with which to transmit to K_{f} femtocell users (it is assumed that $T_{\mathrm{f}} \geq K_{\mathrm{f}}$).

- The beamforming matrix $\mathbf{U}_{\mathrm{f}} \in \mathbb{C}_{T_{\mathrm{f}} \times K_{\mathrm{f}}}$ such that the transmitted signal is $\mathbf{s}_{\mathrm{f}} = \mathbf{U}_{\mathrm{f}}\mathbf{x}_{\mathrm{f}}$. The vector of symbols for users of FBS f, $\mathbf{x}_{\mathrm{f}}$, is normalized such that $E[\|x_{\mathrm{f,i}}\|_2^2] = 1$ and $E[\mathbf{x}_{\mathrm{f}}\mathbf{x}_{\mathrm{f}}^H] = \mathbf{I}_{K_{\mathrm{f}} \times K_{\mathrm{f}}}$.

- The power constraint $trace(\mathbf{U}_{\mathrm{f}}^H\mathbf{U}_{\mathrm{f}}) \leq P_{\mathrm{f}}^{Total}$.

- The cost function $U_{\mathrm{f}}() = \sum_{\mathrm{i=1}}^{\mathrm{K_f}} U_{\mathrm{f,i}}(\gamma_{\mathrm{f,i}})$ in which $U_{\mathrm{f,i}}()$ is non-decreasing (no convexity assumption is made at this point).

- Perfect knowledge of the downlink channel matrix $\mathbf{H}_{\mathrm{f}} \in \mathbb{C}_{K_{\mathrm{f}} \times T_{\mathrm{f}}}$ to the K_{f} FBS users.

- FBSs are assumed to be spaced far apart in distance such that FBS f causes no interference to the users of any other FBS.

2.1.2 Macrocell Users

Macrocell users $m \in \{1...M\}$ introduce constraints into the game. These users are characterized by the following.

- Received interference constraint $\sum_{\mathrm{f=1}}^{F} \tilde{\mathbf{h}}_{\mathrm{m,f}}^{T} \mathbf{U}_{\mathrm{f}} \mathbf{U}_{\mathrm{f}}^{\mathrm{H}} \tilde{\mathbf{h}}_{\mathrm{m,f}}^{*} \leq I_{\mathrm{m}}^{Threshold}$. In which $\tilde{\mathbf{h}}_{\mathrm{m,f}}$ is the channel from FBS f to macro user m
- FBS f is assumed to know the downlink channel matrix $\tilde{\mathbf{H}}_{\mathrm{f}} \in \mathbb{C}_{M \times T_{\mathrm{f}}}$ to all M Macrocell users.

2.1.3 Femtocell Users

- User i of FBS f has signal to interference plus noise ratio (SINR)

$$\gamma_{\mathrm{f,i}} = \frac{\|\mathbf{h}_{\mathrm{f,i}}^{\mathbf{H}} \mathbf{u}_{\mathrm{f,i}}\|^2}{\sigma_{\mathrm{noise}}^2 + \underbrace{\sum_{\tilde{\mathrm{f}}=1, \tilde{\mathrm{f}} \neq f}^{\mathrm{F}} \sum_{\mathrm{u}=1}^{K_{\tilde{\mathrm{f}}}} \|\mathbf{h}_{\mathrm{f,u}}^{\mathbf{H}} \mathbf{u}_{\tilde{\mathrm{f}},\mathrm{i}}\|^2}_{\mathrm{inter-cell}} + \underbrace{\sum_{\tilde{\mathrm{k}} \neq \mathrm{i}}^{\mathrm{K_f}} \|\mathbf{h}_{\mathrm{f},\tilde{\mathrm{k}}}^{\mathbf{H}} \mathbf{u}_{\mathrm{f},\tilde{\mathrm{k}}}\|^2}_{\mathrm{intra-cell}}}, \ \mathrm{i} \in \{1...\mathrm{K_f}\}$$

with noise power $\sigma_{\mathrm{noise}}^2$.

By the FBS spacing assumption above, this reduces to

$$\gamma_{\mathrm{f,i}} = \frac{\|\mathbf{h}_{\mathrm{f,i}}^{\mathbf{H}} \mathbf{u}_{\mathrm{f,i}}\|^2}{\sigma_{\mathrm{noise}}^2 + \sum_{\tilde{\mathrm{k}} \neq \mathrm{i}}^{\mathrm{K_f}} \|\mathbf{h}_{\mathrm{f},\tilde{\mathrm{k}}}^{\mathbf{H}} \mathbf{u}_{\mathrm{f},\tilde{\mathrm{k}}}\|^2}, \ \mathrm{i} \in \{1...\mathrm{K_f}\}$$

2.1.4 General Optimization Problem

Each FBS f attempts to maximize utility function $U_{\mathrm{f}}()$ while satisfying the interference constraints imposed by the macrocell users and a transmission power constraint. This results in the following optimization problem at each FBS.

$$\underset{\mathbf{U}_\mathrm{f}}{\text{minimize:}} \quad -\sum_{\mathrm{i}=1}^{\mathrm{K}_\mathrm{f}} U_{\mathrm{f,i}}(\gamma_{\mathrm{f,i}}) \tag{2.1a}$$

$$\text{subject to:} \quad \sum_{f=1}^{F} \tilde{\mathbf{h}}_{\mathrm{m,f}}^{T} \mathbf{U}_\mathrm{f} \mathbf{U}_\mathrm{f}^\mathrm{H} \tilde{\mathbf{h}}_{\mathrm{m,f}}^{*} \leq I_\mathrm{m}^{Threshold} \qquad m \in \{1...M\} \tag{2.1b}$$

$$trace(\mathbf{U}_\mathrm{f}^{H}\mathbf{U}_\mathrm{f}) \leq P_\mathrm{f}^{Total} \tag{2.1c}$$

$$\langle \mathbf{h}_{\mathrm{f,j}} \mathbf{u}_{\mathrm{f,i}} \rangle = 0 \qquad \forall\, \mathrm{j} \in \{1...K_\mathrm{f}\} \backslash \mathrm{i},\ \forall\, \mathrm{i} \in \{1...K_\mathrm{f}\} \tag{2.1d}$$

Constraint (2.1d) restricts $\mathbf{U}_\mathrm{f}$ such that

$$\gamma_{\mathrm{f,i}} = \frac{\|\mathbf{h}_{\mathrm{f,i}}^{\mathbf{H}} \mathbf{u}_{\mathrm{f,i}}\|^2}{\sigma_{noise}^2} = \frac{\mathbf{u}_{\mathrm{f,i}}^{\mathbf{H}} \mathbf{h}_{\mathrm{f,i}} \mathbf{h}_{\mathrm{f,i}}^{\mathbf{H}} \mathbf{u}_{\mathrm{f,i}}}{\sigma_{noise}^2}. \tag{2.2}$$

As a result, $U_{\mathrm{f,i}}(\gamma_{\mathrm{f,i}})$ is concave with respect to $\gamma_{\mathrm{f,i}}$.

2.1.5 Concave N-Person game analysis of general setup

The convexity of the resulting problem is analyzed to investigate if this system admits the Concave N-Person game framework described above.

1. First, check that the constraints form a convex, closed and bounded set.

   - Constaint (2.1b) contains M quadratic constraints on $\mathbf{U_f}$, each of which can be rewritten as

     $$\sum_{f=1}^{F} trace(\mathbf{U_f^H} \tilde{\mathbf{h}}_{\mathbf{m},\mathbf{f}} \tilde{\mathbf{h}}_{\mathbf{m},\mathbf{f}}^{\mathbf{H}} \mathbf{U_f}) \leq I_m^{Threshold}.$$

     This can be further decomposed into

     $$\sum_{f=1}^{F} \sum_{i=1}^{f_i} \mathbf{u}_{\mathrm{f,i}}^{\mathbf{H}} \tilde{\mathbf{h}}_{\mathrm{m,f}} \tilde{\mathbf{h}}_{\mathrm{m,f}}^{H} \mathbf{u}_{\mathrm{f,i}} \leq I_m^{Threshold}$$

     in which the matrix $\tilde{\mathbf{h}}_{\mathrm{m,f}} \tilde{\mathbf{h}}_{\mathrm{m,f}}^{H}$ is positive semi-definite creating in a convex set [BV04, p. 8,9].

- Constraint (2.1c) can be similarly decomposed into the sum

$$\sum_{i=1}^{K_f} \mathbf{u}_{\mathrm{f,i}}^{\mathrm{H}} \mathbf{I} \mathbf{u}_{\mathrm{f,i}} \leq P_f^{Total}$$

  in which $\mathbf{I}$ is positive definite, creating a strictly convex set.
- Constaint (2.1d) is an affine constraint.

$$\langle \mathbf{h}_{\mathrm{f,j}} \mathbf{u}_{\mathrm{f,i}} \rangle = 0$$

2. Second, check if the utility function is concave.

- With

$$\gamma_{\mathrm{f,i}} = \frac{\mathbf{u}_{\mathrm{f,i}}^{\mathbf{H}} \mathbf{h}_{\mathrm{f,i}} \mathbf{h}_{\mathrm{f,i}}^{\mathbf{H}} \mathbf{u}_{\mathrm{f,i}}}{\sigma_{noise}^2},$$

  $\mathbf{h}_{\mathrm{f,i}} \mathbf{h}_{\mathrm{f,i}}^{H}$ is positive semi-definite so $\gamma_{\mathrm{f,i}}$ is convex in $\mathbf{u}_{\mathrm{f,i}}$. The resulting composition $U_{\mathrm{f,i}}(\gamma_{\mathrm{f,i}})$ is concave only if $U_{\mathrm{f,i}}()$ is concave and non-increasing; violating the non-decreasing definition of $U_{\mathrm{f,i}}()$. If $U_{\mathrm{f,i}}()$ is a general, non-decreasing function, the result is quasiconvex [BV04, p. 102]. If $U_{\mathrm{f,i}}()$ is non-decreasing and convex, the resulting function is convex which can be solved using Concave Programming to find a *maximum*.

2.1.6 Potential Game for General Problem

Under the current system model, the game is not a Concave N-Person game. Nevertheless, because of the independence of the utility functions due to the spacing assumption, the game still admits a potential function. Investigation of distributed Concave Programming techniques for solving this problem is left for future work.

2.2 Concave System Model

The general setup of this game seen in the previous section prohibits the use of established tools to reach a NE. The following refinement of the system model permits a unique NE to be found via a single convex problem which may be distributed across base stations and users in the network.

Rather than jointly optimizing the power allocation over the set of all zero-forcing beamformers, we now pre-select normalized, zero-forcing beamformers independently, *before* the allocation of power. As seen later, one result of this ordering is that additional degrees of freedom at the FBS cannot be exploited with knowledge of the final power allocation.

2.2.1 Players: Femtocell Base Stations

Femtocell Base Stations are adapted from the general setup to the refined setup by the following.

- FBSs with multiple antennas ($T_f \geq 1$) can beamform their transmission using the precoding matrix $\mathbf{U}_\mathrm{f} \in \mathbb{C}_{T_\mathrm{f} \times K_\mathrm{f}}$. The columns of $\mathbf{U}_\mathrm{f}$ are now *normalized* such that $\|\mathbf{u}_\mathrm{fi}\|^2 = 1 \; \forall i \in \{1...K_\mathrm{f}\}$.

- $\mathbf{U}_f$ is selected as a pseudo-inverse to $\mathbf{H}_\mathrm{f}$. Such that

$$\mathbf{H}_\mathrm{f}\mathbf{U}_\mathrm{f} = \mathbf{I}.$$

- FBS f allocates transmission power using the diagonal, power allocation matrix diag($\mathbf{p}_\mathrm{f}$) with $p_\mathrm{fi} \geq 0, \forall i \in \{1...K_\mathrm{f}\}$ such that the transmitted signal is $\mathbf{s}_\mathrm{f} = \mathbf{U}_\mathrm{f}\mathrm{diag}(\mathbf{p}_\mathrm{f})^{\frac{1}{2}}\mathbf{x}_\mathrm{f}$.

- FBS f enforces power constraint:

$$trace(E[\mathbf{s}_\mathrm{f}\mathbf{s}_\mathrm{f}^H]) = \sum_{\mathrm{i}=1}^{\mathrm{K_f}} p_\mathrm{fi} \leq P_\mathrm{f}^{Total}$$

- Utility function $U_\mathrm{f}() = \sum_{\mathrm{i}=1}^{\mathrm{K_f}} U_{\mathrm{f,i}}(\gamma_{\mathrm{f,i}})$ in which all $U_{\mathrm{f,i}}()$ are non-decreasing and *strictly* concave.

- The SINR of the Femto Cell users is now

$$\gamma_{\mathrm{f,i}} = \frac{\mathbf{u}_{\mathrm{f,i}}^{\mathbf{H}}\mathbf{h}_{\mathrm{f,i}}\mathbf{h}_{\mathrm{f,i}}^{\mathbf{H}}\mathbf{u}_{\mathrm{f,i}}}{\sigma_{noise}^2} = \frac{\sum_{\mathrm{i=i}}^{\mathrm{K_f}} p_{\mathrm{f,i}}\|h_{\mathrm{f,i}}u_{\mathrm{f,i}}\|^2}{\sigma_{noise}^2} \quad (2.3)$$

2.2.2 Macrocell Users

No change from general setup.

2.2.3 Femtocell Users

No change from general setup.

2.2.4 Optimization Problem of player f

The optimization problem at FBS f is now:

$$\underset{\mathbf{p}_\mathrm{f}}{\text{minimize:}} \quad -\sum_{\mathrm{i}=1}^{\mathrm{K_f}} U_{\mathrm{f,i}}(\gamma_{\mathrm{f,i}}) \tag{2.4a}$$

$$\text{subject to:} \quad \sum_{\mathrm{f}=1}^{F} \tilde{\mathbf{h}}_{\mathrm{m,f}}^{T}\mathbf{s}_\mathrm{f}\mathbf{s}_\mathrm{f}^{\mathrm{H}}\tilde{\mathbf{h}}_{\mathrm{m,f}}^{*} = \sum_{\mathrm{f}=1}^{\mathrm{F}}\sum_{\mathrm{i}=1}^{\mathrm{K_f}} p_{\mathrm{f,i}}\|\tilde{\mathbf{h}}_{\mathrm{m,f}}^{T}\mathbf{u}_{\mathrm{f,i}}\|_2^2 \leq I_{\mathrm{m}}^{Threshold} \quad m \in \{1...\mathrm{M}\} \tag{2.4b}$$

$$\sum_{\mathrm{i}=1}^{\mathrm{K_f}} p_{\mathrm{f,i}} \leq P_{\mathrm{f}}^{\mathrm{Total}} \tag{2.4c}$$

$$p_{\mathrm{f,i}} \geq 0 \quad \mathrm{i} \in \{1...\mathrm{K_f}\} \tag{2.4d}$$

Note that because $\mathbf{U}_\mathrm{f}$ is pre-selected as a pseudo-inverse to $\mathbf{H}_\mathrm{f}$, the zero-forcing constraint (2.1d) has been removed.

2.2.5 Concave N-Person game analysis of concave setup

The convexity of the resulting problem is analyzed to investigate if this system admits the Concave N-Person game framework.

1. First, check that the constraints form a convex, closed and bounded set.

   - Constaint (2.4b) contains M affine constraints on $diag(\mathbf{p}_\mathrm{f})$.

   $$\sum_{\mathrm{f}=1}^{F} \tilde{\mathbf{h}}_{\mathrm{m,f}}^{T}\mathbf{s}_\mathrm{f}\mathbf{s}_\mathrm{f}^{\mathrm{H}}\tilde{\mathbf{h}}_{\mathrm{m,f}}^{*} = \sum_{\mathrm{f}=1}^{\mathrm{F}}\sum_{\mathrm{i}=1}^{\mathrm{K_f}} p_{\mathrm{fi}}\|\tilde{h}_{\mathrm{mf}}^{T}u_{\mathrm{f,i}}\| \leq I_{\mathrm{m}}^{Threshold}$$

   - Constraint (2.4c) is affine in $diag(\mathbf{p}_\mathrm{f})$.
   - Constraint (2.4d) is affine in $diag(\mathbf{p}_\mathrm{f})$.

2. The utility function was defined to be concave in $p_{\mathrm{f,i}}$.

This problem satisfies the conditions for the Concave N-Persion game and therefore a pure strategy Nash Equilibrium exists due to [Ros64, Thm1].

Assuming continuously differentiable $U_{\mathrm{f,i}}(\gamma_{\mathrm{f,i}})$ and $\gamma_{\mathrm{f,i}}(p_{\mathrm{f,i}})$, the diagonal elements of $[G(b,r)+G^T(b,r)]$ correspond to

$$\underbrace{U''_{\mathrm{f,i}}(\gamma_{\mathrm{f,i}}(p_{\mathrm{f,i}}))}_{<0}\underbrace{\gamma'_{\mathrm{f,i}}(p_{\mathrm{f,i}})\gamma'_{\mathrm{f,i}}(p_{\mathrm{f,i}})}_{>0}+\underbrace{U'_{\mathrm{f,i}}(\gamma_{\mathrm{f,i}}(p_{\mathrm{f,i}}))}_{>0}\underbrace{\gamma''_{\mathrm{f,i}}(p_{\mathrm{f,i}})}_{0}; \tag{2.5}$$

satisfying the negative definite condition on $[G(b,r)+G^T(b,r)]$ for a unique Nash Equilibrium [Ros64, Thm4].

(TODO: Discuss this)

2.2.6 Potential Game for Concave Problem

The potential function

$$\Psi(\gamma)=\sum_{f=1}^{F}U_{\mathrm{f}}(\gamma_{\mathrm{f}})$$

satisfying (1.2) is proposed to give the central optimization problem

$$\underset{\gamma}{\text{minimize:}} \quad \Psi(\gamma) \tag{2.6a}$$

$$\text{subject to:} \quad \sum_{f=1}^{F}\tilde{\mathbf{h}}_{\mathrm{m,f}}^{T}\mathbf{s}_{\mathrm{f}}\mathbf{s}_{\mathrm{f}}^{\mathrm{H}}\tilde{\mathbf{h}}_{\mathrm{m,f}}^{*}\leq I_{\mathrm{m}}^{Threshold} \qquad m\in\{1...\mathrm{M}\} \tag{2.6b}$$

$$trace(\mathbf{s}_{\mathrm{f}}\mathbf{s}_{\mathrm{f}}^{H})\leq P_{\mathrm{f}}^{\mathrm{Total}} \qquad \forall f\in\{1...\mathrm{F}\} \tag{2.6c}$$

$$p_{\mathrm{f,i}}\geq 0 \qquad \forall i\in\{1...K_{\mathrm{f}}\}\ \forall f\in\{1...F\} \tag{2.6d}$$

with $\gamma=[\gamma_1,\gamma_2...\gamma_{\mathrm{F}}]$.

2.2.7 Distributed Solution to the Game

A distributed solution to (2.6) is now proposed using the steps from section 1.2.

Central Problem Resulting from Potential Game

The Lagrangian to the potential function (the primal problem) is

$$L(\mathbf{p}, \lambda, \chi, \nu) = \sum_{\mathrm{f}=1}^{F} U_{\mathrm{f}}(\gamma_{\mathrm{f}}) + \sum_{\mathrm{m}=1}^{M} \lambda_{\mathrm{m}}(\sum_{f=1}^{F} \tilde{\mathbf{h}}_{\mathrm{m,f}}^{T} \mathbf{s}_{\mathrm{f}} \mathbf{s}_{\mathrm{f}}^{\mathrm{H}} \tilde{\mathbf{h}}_{\mathrm{m,f}}^{*} - I_{\mathrm{m}}^{Threshold}) + \sum_{\mathrm{f}=1}^{\mathrm{F}} \chi_{\mathrm{f}}(trace(\mathbf{s}_{\mathrm{f}} \mathbf{s}_{\mathrm{f}}^{H}) - P_{\mathrm{f}}^{Total}) + \sum_{\mathrm{f}=1}^{\mathrm{F}} \sum_{i=1}^{K_{\mathrm{f}}} \nu_{\mathrm{f,i}}(-p_{\mathrm{f,i}}). \quad (2.7)$$

Using $\mathbf{p} = [\mathrm{p}_1, \mathrm{p}_2 ... \mathrm{p}_{\mathrm{F}}]$, corresponding dual function is

$$g(\lambda, \chi, \nu) = \underset{\mathbf{p}}{\operatorname{argmin}}\, L(\mathbf{p}, \lambda)$$

with dual problem

$$\underset{\lambda,\chi,\nu}{\operatorname{argmax}}\, \underset{\mathbf{p}}{\operatorname{argmin}}\, L(\mathbf{p}, \lambda) = \underset{\lambda,\chi,\nu}{\operatorname{argmax}}\, g(\lambda, \chi, \nu).$$

This dual function can be decomposed into F component functions

$$g_f(\lambda, \chi, \nu) = \underset{p_{\mathrm{f}}}{\operatorname{argmin}}\{\, U_{\mathrm{f}}(\gamma_{\mathrm{f}}) + \sum_{\mathrm{m}=1}^{M} \lambda_{\mathrm{m}}(\tilde{\mathbf{h}}_{\mathrm{m,f}}^{T} \mathbf{s}_{\mathrm{f}} \mathbf{s}_{\mathrm{f}}^{\mathrm{H}} \tilde{\mathbf{h}}_{\mathrm{m,f}}^{*} - \frac{I_{\mathrm{m}}^{Threshold}}{F}) + \chi_{\mathrm{f}}(trace(\mathbf{s}_{\mathrm{f}} \mathbf{s}_{\mathrm{f}}^{H}) - P_{\mathrm{f}}^{Total}) + \sum_{i=1}^{K_f} \nu_{\mathrm{f,i}}(-p_{\mathrm{f,i}})\} \quad (2.8)$$

The problem can now be solved using a distributed version of dual ascent to reach the unique NE of the potential game.

Distributed Algorithm

The non-decreasing, strictly concave utility function $U_{\mathrm{f,i}}(\gamma_{\mathrm{f,i}}) = log(1 + \gamma_{\mathrm{f,i}})$ is used in the following algorithms and simulation results.

1. Individual players (FBSs) can solve $g_f(\lambda, \chi, \nu)$ independently and analytically setting $\frac{\partial g_f(\lambda,\chi,\nu)}{\partial p_{\mathrm{f,i}}} = 0$ and solving for p_{fi} using (2.3).

$$p_{\mathrm{fi}} = (\sum_{\mathrm{m}=1}^{\mathrm{M}} \lambda_{\mathrm{m}} \|\tilde{\mathbf{h}}_{m,f}^{T} \mathbf{u}_{\mathrm{f,i}}\|_2^2 + \chi_{\mathrm{f}} \|\mathbf{u}_{\mathrm{f,i}}\|_2^2 - \nu_{\mathrm{f,i}})^{-1} - \sigma_{\mathrm{noise}}^2. \quad (2.9)$$

2. With $g(\lambda,\chi,\nu)=\sum_{f=1}^{F} g_f(\lambda,\chi,\nu)$ the dual variables are updated independently in the direction of the positive gradient of the dual function using

$$\lambda_{\mathrm{m}}^{\mathrm{k+1}} = (\lambda_{\mathrm{m}}^{\mathrm{k}} + \alpha^{\mathrm{k}} * (\underbrace{\sum_{\mathrm{f=1}}^{\mathrm{F}}\sum_{\mathrm{i=1}}^{\mathrm{K_f}} p_{\mathrm{fi}} \|\tilde{\mathbf{h}}_{\mathrm{mf}}^{T}\mathbf{u}_{\mathrm{fi}}\|_2^2}_{Interference} - I_{\mathrm{m}}))^{+} \tag{2.10}$$

at the macrocell users,

$$\chi_{\mathrm{f}}^{\mathrm{k+1}} = (\chi_{\mathrm{f}}^{\mathrm{k}} + \alpha^{\mathrm{k}} * (\underbrace{\sum_{\mathrm{i=1}}^{\mathrm{K_f}} p_{\mathrm{fi}}}_{UsedPower} - P_{\mathrm{f}}^{Total}))^{+} \tag{2.11}$$

at individual FBSs, and

$$\nu_{\mathrm{fi}}^{\mathrm{k+1}} = (\nu_{\mathrm{fi}}^{\mathrm{k}} + \alpha^{\mathrm{k}} * (-p_{\mathrm{fi}}))^{+} \tag{2.12}$$

also at individual FBSs. As each of dual variables correspond an inequality constraint, the implementation of this algorithm directly enforces non-negativity using $(arg)^{+}$. The step size α^{k} is predefined and constant (TODO show proof of convergence for this).

These local updates impose additional information passing in the network. In particular, macrocell users must monitor their incoming interference then broadcast the updated dual variable $\lambda_{\mathrm{m}}^{\mathrm{k+1}}$ with which FBSs can update the power allocation. In contrast, however, a central solution would require the same information plus addition information from FBSs to be passed to a central location; a much higher overhead on the system than the proposed distributed solution.

(Note also that the dual ascent algorithm uses the dual and thus cannot guarantee a feasible solution to the primal problem originally posed. (Discuss constraint tolerance used in simulation?))

2.2.8 Beamformer Optimization for Convex Setup

When additional degrees of freedom for the choice of $\mathbf{U}_{\mathrm{f}}$ are available after satisfying the zero-forcing constraint (i.e. $T_{\mathrm{f}} > K_{\mathrm{f}}$), a preliminary optimization can be performed. In the following, variations of an objective function are considered. Note that because the beamformer $\mathbf{U}_\mathrm{f}$ is chosen prior to the power allocation, the final values of $p_{\mathrm{f,i}}$ are not yet known for this optimization.

The first objective function considered corresponds to the Moore-Penrose pseudoinverse of $\mathbf{H}_\mathrm{f}$ which minimizes the received signal noise enhancement.

$$\begin{aligned} \underset{\mathbf{U}_\mathrm{f}}{\text{minimize:}} \quad & tr(\sigma_n \mathbf{U}_\mathrm{f} {\mathbf{U}_\mathrm{f}}^H) \\ \text{subject to:} \quad & \mathbf{H}_\mathrm{f}\mathbf{U}_\mathrm{f} = \mathbf{I} \end{aligned} \tag{2.13}$$

This problem is convex and therefore can be found easily at individual FBSs.

Next, the objective function is optimized to minimize the correlation of the chosen beam-former $\mathbf{U}_\mathrm{f}$ with the known macrocell user channels $\tilde{\mathbf{H}}_\mathrm{f}$.

$$\begin{aligned} \underset{\mathbf{U}_\mathrm{f}}{\text{minimize:}} \quad & \textstyle\sum_{\mathrm{f}=1}^{F} \|\tilde{\mathbf{H}}_\mathrm{f}\mathbf{U}_\mathrm{f}\|^2 \\ \text{subject to:} \quad & \mathbf{H}_\mathrm{f}\mathbf{U}_\mathrm{f} = \mathbf{I} \end{aligned} \tag{2.14}$$

This is problem also convex in $\mathbf{U}_\mathrm{f}$ and can be found at individual FBSs.

Lastly, we consider the case in which FBSs minimize correlation with macrocell users while also maximizing correlation with the channels of its own users. The relative importance of these two terms is weighted by α.

$$\begin{aligned} \underset{\mathbf{U}_\mathrm{f}}{\text{minimize:}} \quad & \alpha \textstyle\sum_{\mathrm{f}=1}^{F} \|\tilde{\mathbf{H}}_\mathrm{f}\mathbf{U}_\mathrm{f}\|_2^2 - (1-\alpha)\textstyle\sum_{f=1}^{F} \|\mathbf{H}_\mathrm{f}\mathbf{U}_\mathrm{f}\|^2, \ 0 \leq \alpha \leq 1 \\ \text{subject to:} \quad & \mathbf{H}_\mathrm{f}\mathbf{U}_\mathrm{f} = \mathbf{I} \end{aligned} \tag{2.15}$$

This problem is not convex in $\mathbf{U}_\mathrm{f}$, however, the first term of the utility function is convex and the second, subtracted term is also convex. Combined with the convex zero-forcing constraint, this type of problem, known as a Difference of Convex Functions problem (DC), has been extensively studied due to its prevalence in many applications can be solved (at least up to local optima) via DC programming methods. Note that as this optimization is performed locally at a FBS, there is no restriction to distributed methods.

Chapter 3

Simulation and Results

3.0.1 Simulation System Description

The Heterogeneous Network is simulated by randomly placing F FBSs and M macrocell users within a predefined area. Each FBS is assigned K_{f} unique Femto Users. All channels between FBSs and users (both Femtro and Macro) are simulated using rayleigh fading and the attenuation coefficient $d^{-\beta}$ with $\beta = 2$ and d as the distance between user and base-station. FBS f initializes power to user i as $p_{\mathrm{fi}} = \frac{P_{\mathrm{f}}^{\mathrm{Total}}}{K_{\mathrm{f}}}$ such that the corresponding power constraint is not violated.

3.0.2 Simulation Results

Using the algorithm from 2.2.7, convergence to a Nash Equilibrium is observed over different system scenarios.

Simulation Results for Concave Setup

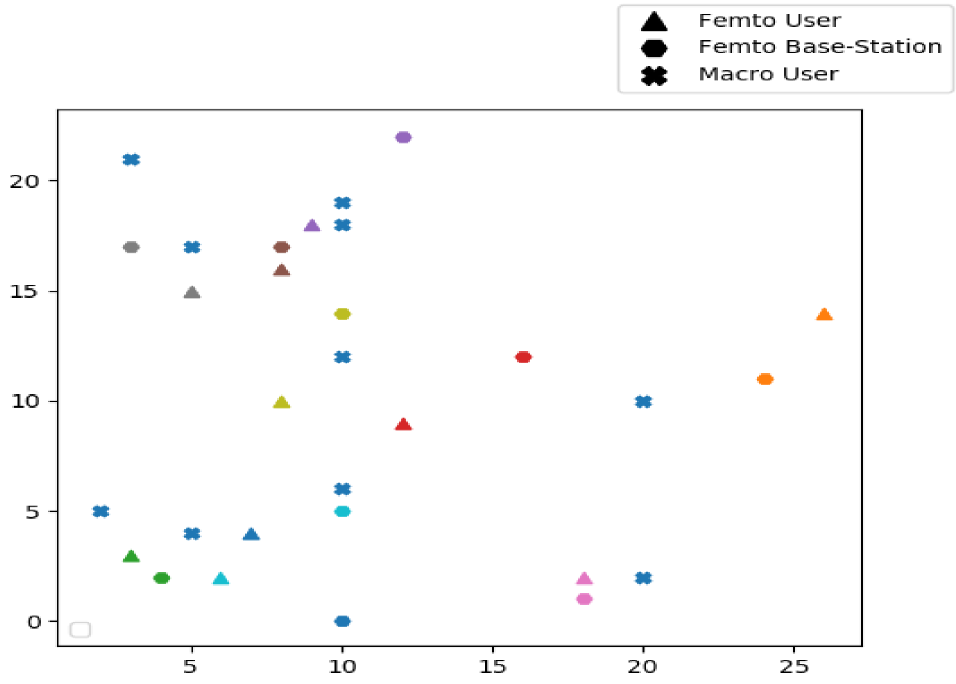

Figure 3.1: Simluation Environment: Single Antenna and User.

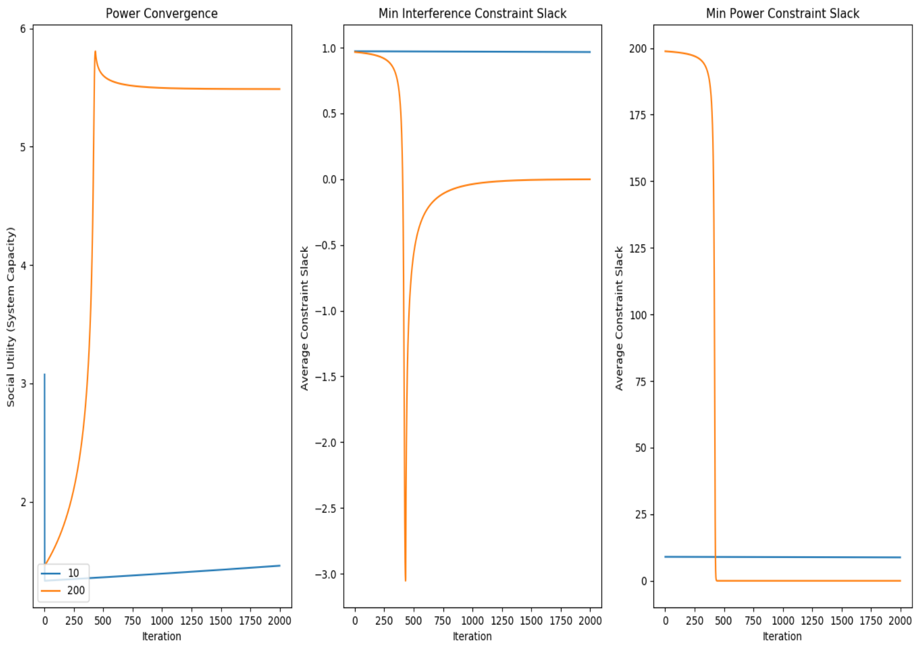

Figure 3.2: Single Antenna Game Simulation with different base station power constraints

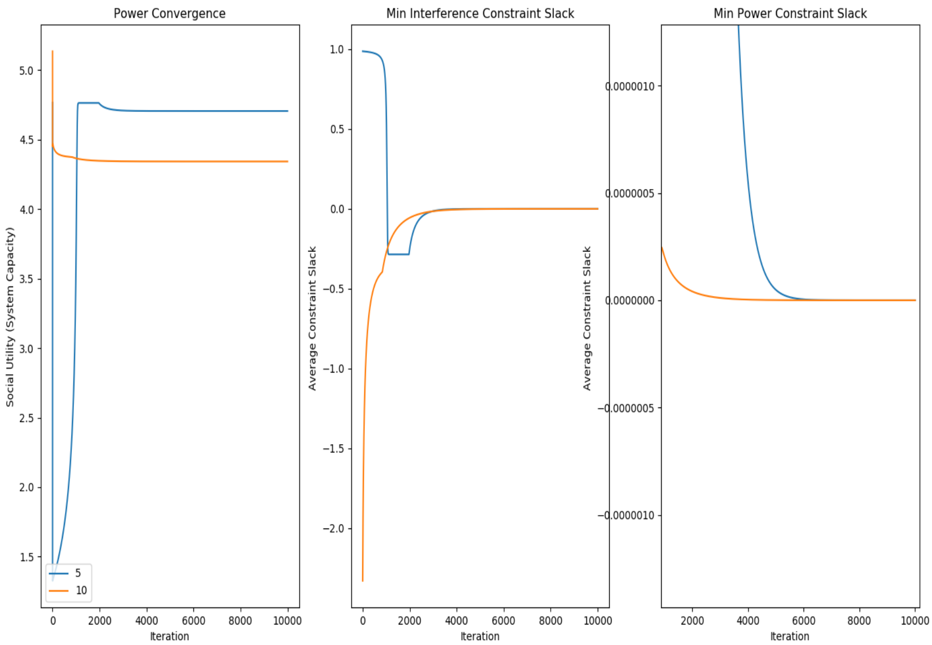

Figure 3.3: Single Antenna Game Simulation with different numbers of macrocell users

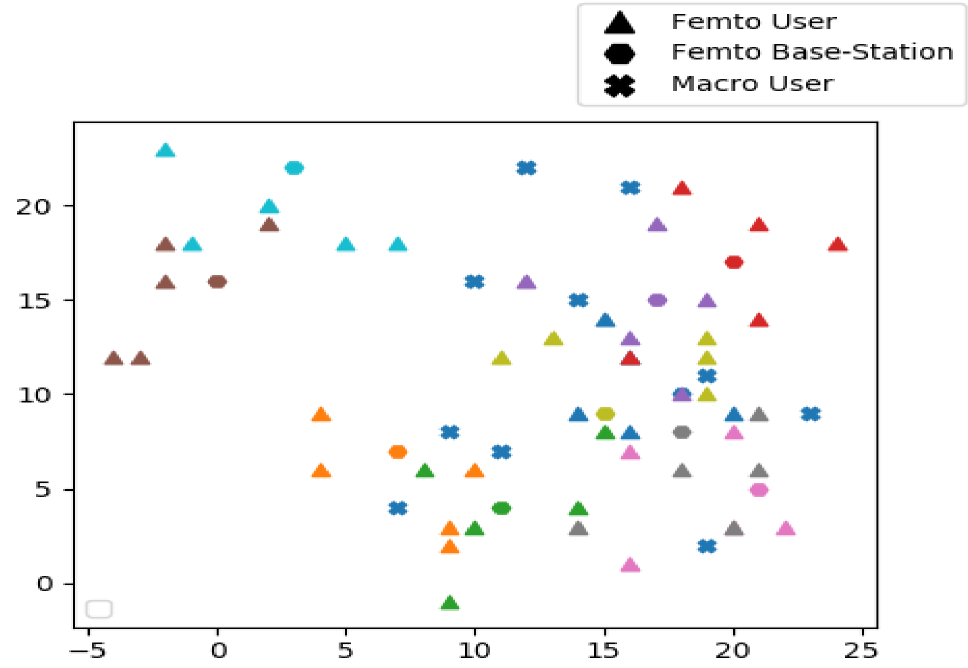

Figure 3.4: Simluation Setup Multiple Antenna and User.

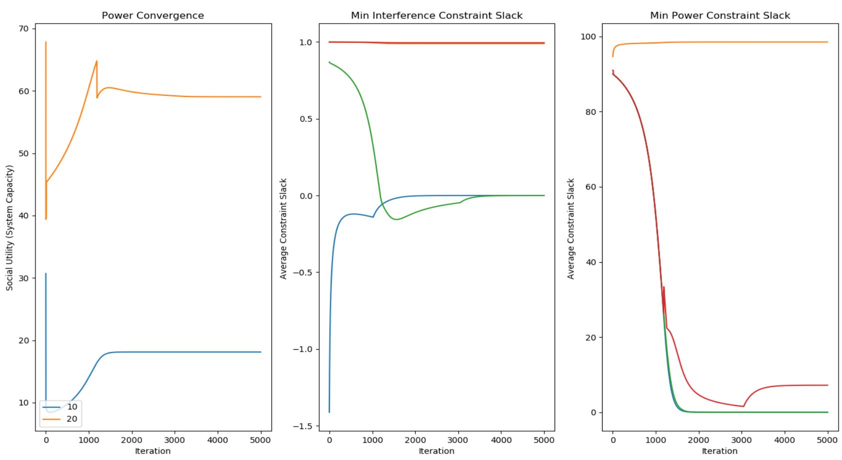

Figure 3.5: Convergence for different numbers of antennas at FBSs. Number of femto cell users and macrocell users constant. Each BS has 10 users.

Chapter 4

Conclusion

In this work, the power allocation problem for heterogeneous networks with MIMO capable basestations has been analyzed. The most general setup of this problem poses difficulties for distributed solutions due to the non-concavity of the resulting utility functions. It is shown that under additional beam-forming constraints on the MIMO capable base stations, a unique Nash Equilibrium can be achieved with a distributed solution.

Future work should extend the system presented here to further generalizations including interference between the femtocell basestations, and the potential for imperfect channel estimation.

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