Exercise 8: Last Exercise!!

August 11, 2014

1. **Ice Sheet Melting** In the present climate the volume of freshwater trapped in ice sheets over land is 33×10^6 km^3. If all this ice melted and ran into the ocean, estimate by how much the sea level would rise. What would happen to sea level if all the sea-ice melted?

2. **Solution** We know the volume of ice from the problem, the ocean area is 71%, and the earth radius is 6371 km. Therefore, we can calculate the amount of sea level rise:

$$\begin{aligned} h &= \frac{V_ice}{A_ocean} && (1)\\ &= \frac{33 \times 10^6}{362.145 \times 10^6} && (2)\\ &= 91.1 && (3) \end{aligned}$$

Since sea ice floats on the ocean surface, it displaces just as much volume of water as it would generate when melting. Therefore, sea level rise can only be caused by land ice melting, along with accompanying second order effects (eg thermal expansion of the ocean)

3. **Wind-driven ocean circulation** When the windstress is only zonal, the Sverdrup transport is:

$$\rho_0 \beta V = \text{curl } \tau = -\frac{\partial}{\partial y}\tau^x \tag{4}$$

and Ekman transports and Ekman pumping velocity are:

$$\rho_0 f V_E = -\tau^x \rho_0 w_E = \text{curl } \tau = -\frac{\partial}{\partial y}\tau^x. \tag{5}$$

Assume furthermore

$$\tau^x = -\tau_0 \cos(\pi y/B) \tag{6}$$

for an ocean basin $0 < x < L, 0 < y < B$.

(a) At what latitudes y are $|V|$ and $|V_E|$ maximum? Calculate their magnitudes. Take constant $f = 10^{-4}$ s^{-1} and $\beta = 1.8 \cdot 10^{-11}$ m^{-1}s^{-1} and $B =$ 5000 km, $\tau_0/\rho_0 = 10^{-4}$ m^{2}s^{-2}.

(b) Calculate the maximum of w_E for constant f (value see above). Is this measurable?

4. **Stochastic Climate Model** Imagine that the temperature of the ocean mixed layer of depth h is governed by

$$\frac{dT}{dt} = -\lambda T + \frac{Q_{net}}{\gamma_O}, \tag{7}$$

where coefficient γ_O is given by the heat capacity $c_p \rho h$, and λ is the typical damping rate of a temperature anomaly. The air-sea fluxes due to weather systems are represented by a white-noise process $Q_{net} = \hat{Q}_\omega e^{i\omega t}$ where $\hat{Q}_\omega$ is the amplitude of the random forcing at frequency ω. $\hat{Q}^*$ is the complex conjugate.

   (a) What is a white-noise process? Remember that

$$\int_R \exp(i\omega t)\delta(t-0)dt = 1 \tag{8}$$

   and use the Fourier transformation.

   (b) Solve the equation (4) above for the temperature response $T = \hat{T}_\omega e^{i\omega t}$ and hence show that:

$$\hat{T}_\omega = \frac{\hat{Q}_\omega}{\gamma_O\left(\lambda + i\omega\right)} \tag{9}$$

   (c) Show that it has a spectral density $\hat{T}_\omega \hat{T}_\omega^*$ is given by:

$$\hat{T}\hat{T}^* = \frac{\hat{Q}\hat{Q}^*}{\gamma_O^2\left(\lambda^2 + \omega^2\right)} \tag{10}$$

   and the spectrum

$$S(\omega) = <\hat{T}\hat{T}^*> = \frac{1}{\gamma_O^2\left(\lambda^2 + \omega^2\right)}. \tag{11}$$

   The brackets $< \cdots >$ denote the ensemble mean. Make a sketch of the spectrum using a log-log plot and show that fluctuations with a frequency greater than λ are damped.

5. **Angular Momentum and Hadley Cell** Consider a zonally symmetric circulation (i.e., one with no longitudinal variations) in the atmosphere. In the inviscid upper troposphere one expects such a flow to conserve absolute angular momentum, i.e.,

$$\frac{DA}{Dt} = 0 \tag{12}$$

where A is the absolute angular momentum per unit mass (parallel to the Earth's rotation axis)

$$A = r\left(u + \Omega r\right) = \Omega R^2 \cos^2\varphi + uR\cos\varphi \tag{13}$$

Ω is the Earth rotation rate, u the eastward wind component, $r = R\cos\varphi$ is the distance from the rotation axis, R the Earth's radius, and φ latitude.

(a) Show for inviscid zonally symmetric flow that the relation $\frac{DA}{Dt} = 0$ is consistent with the zonal component of the equation of motion

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{14}$$

in (x, y, z) coordinates, where $y = R\varphi$.

(b) Use angular momentum conservation to describe in words how the existence of the Hadley circulation explains the existence of both the subtropical jet in the upper troposphere and the near-surface trade winds.

(c) If the Hadley circulation is symmetric about the equator, and its edge is at 20° latitude, determine the strength of the subtropical jet. Use (10, 11).

(d) Is the Hadley cell geostrophically driven or not?

Notes on submission form of the exercises: *Students can work together in groups, but each student must submit his or her own solutions. The answers to the questions shall be send to paul.gierz@awi.de.*