Exercise 3: Solution

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1. **Rayleigh-Bénard convection** Rayleigh studied the flow occurring in a layer of fluid of uniform depth H, when the temperature difference between the upper- and lower-surfaces is maintained at a constant value ΔT. In the case where all motions are parallel to the $x-z$-plane, and no variations in the direction of the y-axis occur, the governing equations can be written as:

$$\partial_t u + u\partial_x u + w\partial_z u = -\frac{1}{\rho_0}\partial_x p + \nu\nabla^2 u \quad (1)$$

$$\partial_t w + u\partial_x w + w\partial_z w = -\frac{1}{\rho_0}\partial_z p + \nu\nabla^2 w + g(1-\alpha(T-T_0)) \quad (2)$$

$$\partial_t T + u\partial_x T + w\partial_z T = \kappa\nabla^2 T \quad (3)$$

$$\partial_x u + \partial_z w = 0 \quad (4)$$

where w and u are the vertical and horizontal components of the velocity, respectively. Furthermore, $\nu = \eta/\rho_0$, $\kappa = \lambda/(\rho_0 C_v)$ the momentum diffusivity (kinematic viscosity) and thermal diffusivity, respectively.

The R codes are

```
rb_plot_functions.R;
rb_functions.R;
rayleigh-benard.R
```

In R, you shall make

```
source("rayleigh-benard.R")
```

and the model run will create a directory out and writes output:

```
#Output Parameter
N_out = 50;
out_dir = "./out/";        #Output directory
```

Tasks:

(a) Write down the dimensionless parameters and their relation to characteristic length and time scales charachterizing the flow!

Solution The Dimensionless parameters are:

Prandl Number $\frac{c_p \mu}{k}$ Rayleigh Number $R_a = \frac{g \cdot \beta \cdot \Delta T \cdot L^3}{\nu}$

(b) Vary the Rayleigh and the Prandtl number by $R_a = 20000, 40000, 60000$ and $Pr = 0.5, 1, 1.5, 5, 10$ and describe the dynamics (words, figures) !

Figures for various combinations of Ra and Pr are presented in the course script.