# Exercise 3

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1. **Rayleigh-Bénard convection** Rayleigh studied the flow occurring in a layer of fluid of uniform depth $H$, when the temperature difference between the upper- and lower-surfaces is maintained at a constant value $\Delta T$. In the case where all motions are parallel to the $x-z$-plane, and no variations in the direction of the $y$-axis occur, the governing equations can be written as:

$$\begin{aligned}
\partial_t u + u\partial_x u + w\partial_z u &= -\frac{1}{\rho_0}\partial_x p + \nu\nabla^2 u && (1)\\
\partial_t w + u\partial_x w + w\partial_z w &= -\frac{1}{\rho_0}\partial_z p + \nu\nabla^2 w + g(1-\alpha(T-T_0)) && (2)\\
\partial_t T + u\partial_x T + w\partial_z T &= \kappa\nabla^2 T && (3)\\
\partial_x u + \partial_z w &= 0 && (4)
\end{aligned}$$

where $w$ and $u$ are the vertical and horizontal components of the velocity, respectively. Furthermore, $\nu = \eta/\rho_0$, $\kappa = \lambda/(\rho_0 C_v)$ the momentum diffusivity (kinematic viscosity) and thermal diffusivity, respectively.

The R codes are

```
rb_plot_functions.R;
rb_functions.R;
rayleigh-benard.R
```

In R, you shall make

```
source("rayleigh-benard.R")
```

and the model run will create a directory out and writes output:

```
#Output Parameter
N_out = 50;
out_dir = "./out/";        #Output directory
```

Tasks:

(a) Write down the dimensionless parameters and their relation to characteristic length and time scales charachterizing the flow!

(b) Vary the Rayleigh and the Prandtl number by $R_a = 20000, 40000, 60000$ and $Pr = 0.5, 1, 1.5, 5, 10$ and describe the dynamics (words, figures) ! For high values of $R_a$ the spatial resolution might be chosen higher (to the double). Here are the standart values:

```
lx      = 100;        #Number of horizontal cells
ly      = 52;         #Number of vertical cells
```

(c) Vary the initial perturbation and obtain the reversed circulation! Look at the line

```
#Set small trigger to break symmetry
T[lx/2+1, 1] = 1.1 * T_bot;
```

2. **Ocean-like Rayleigh-Bénard convection** A modifies version of the first problem is an ocean box with solid walls and free slip at the surface (no friction).

(a) Evaluate the effect of different external temperatures (hemispheric, double hemispheric). The R code is

`ocean_rb.R`

Here are two options:

```
# Pre-compute imposed temperature-profile on top (linear)
tempTop = array(0, c(lx));
for (x in 2:lx-1) {
  tempTop[x] = THot - (THot-TCold)*(x-2)/(lx-3);
 }
```

for a single hemisphere, and

```
# Pre-compute imposed temperature-profile on top (linear+sinus)
tempTop = array(0, c(lx));
bett= 0.2          # right boundary
alph= (0.1-bett)/lx  ;
gamma =1.-alph * lx/2 -bett;
for (x in 2:lx-1) {
        tempTop[x] = alph *x + bett + gamma * sin( 3.1416* x/lx);
}
```

for a double hemisphere version. Describe the dynamics with respect tothe temperature at the top layer tempTop !

(b) In lattice Boltzmann models, it is relatively easy to insert obstacles. The R code is

`ocean_rb_ridge.R`

Discuss the influnce of the ridge on the ocean circulation!

(c) Manage to change the Rayleigh-Bénard convection from 1. from a no-slip to free slip bounday conditions at the top. The upper plate is just removed and we have an air-water interface. What are the differences? Make a plot!