Homework 8, Wind Driven Ocean Circulation

A) $\rho_0 \beta V = -\frac{\partial}{\partial y}\tau = -\frac{\partial}{\partial y}\left(-\tau\cos(\pi y/B)\right)$

$= -\frac{\pi\tau_0}{B}\left(\sin y/B\right)$

$V = \underbrace{\frac{-\pi\tau_0}{B\beta\rho_0}}_{C}\sin\left(\frac{\pi y}{B}\right)$

Take Derivative + set to 0 for Maximum

$\frac{\partial V}{\partial y} = \underbrace{\frac{-\pi^2\tau_0}{B^2\rho_0\beta}}_{A}\cdot\cos\left(\frac{\pi y}{B}\right) \overset{?}{=} 0$

- $\cos(x)$ is 0 for $\pi/2$ and multiples thereof
- $A = -\pi^2\cdot\frac{1}{1{,}8\times10^{-11}}\cdot\frac{1}{5000^2}\cdot 10^{-4} \cong -2{,}19$
- $C = \frac{-\pi\tau_0}{B\beta\rho_0} = -3490$

Using this, we can solve for $y$.

$y=0$ or $y=B$

Plugging in gives

$|V_{max}| = \frac{\pi\tau_0}{B\beta\rho_0} \cong \boxed{3.5\ m/s}$

Same for $|V_{E\,max}|$

$V_E = \frac{\tau}{f\rho_0}\cos\left(\frac{\pi y}{B}\right)$

$\frac{\partial V_E}{\partial y} = -\frac{\pi\tau_0}{fB\rho_0}\cdot\sin\left(\frac{\pi y}{B}\right) \overset{?}{=} 0$ ; $y = B/2$

Plugging in:

$|V_{E\,max}| = \frac{\tau}{f\rho_0}\cos\left(\cancel{\frac{\pi\frac{B}{2}}{B}}\right)^{=1} = \frac{\tau}{f\rho_0}$

B) $$w_E = -\frac{\partial}{\partial y}\left(\frac{\tau_0^x}{f\rho_0}\right)$$

$$w_E = -\frac{\partial}{\partial y}\left(\frac{-\tau_0 \cos \pi y / B}{f\rho_0}\right)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)\cdot x'$$

$$= \frac{\tau_0}{\rho_0}\cancel{\frac{1}{f}}\frac{\partial}{\partial y}\left(\cos\frac{\pi y}{B}\right)$$

$$= \frac{\tau_0}{\rho_0}\cancel{\frac{1}{f}}\cdot\frac{\pi}{B}\cdot -\sin\left(\frac{\pi y}{B}\right)$$

$$= \frac{\tau_0}{\rho_0}\cdot\frac{\pi}{fB}\cdot -\sin\left(\frac{\pi y}{B}\right)$$

take derivative, set to 0

$$\frac{d w_E}{dy} = -\frac{\tau_0}{\rho_0}\frac{\pi}{fB}\cdot\frac{\pi}{B}\cdot\cos\left(\frac{\pi y}{B}\right) = 0$$

$$\cos(x) = 0 \quad \text{if} \quad x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$$

$$y = \frac{B}{2}$$

Solve for $|w_E|_{max}$

$$-\frac{\tau_0}{\rho_0}\cdot\frac{\pi}{fB}\cdot\sin\left(\cancel{\frac{\pi\frac{B}{2}}{B}}\right)$$

$$= -10^{-4}\cdot\frac{\pi}{10^{-4}\cdot 5000\,\text{km}} = \frac{\pi}{5000\,\text{km}}$$

Homework 8, Stochastic Climate Model

a) What is a white noise process?

→ White noise is a random signal with constant power spectral density. The fourier transformation of white noise shows a <u>constant distribution</u> over all frequencies.

b) Given

$$\frac{dT}{dt} = -\lambda T + \frac{Q_{net}}{\gamma_0}$$

$$= -\lambda T + \frac{\hat{Q} e^{i\omega t}}{\gamma_0}$$

and

$$T = \hat{T} e^{i\omega t}, \quad \frac{dT}{dt} = i\omega \hat{T}_\omega e^{i\omega t}$$

Rearanging gives

$$i\omega \hat{T} e^{i\omega t} = -\lambda \hat{T} e^{i\omega t} + \frac{Q_\omega}{\gamma} e^{i\omega t}$$

Move across + factor out gives

$$\hat{T}_\omega (\lambda + i\omega) = \frac{\hat{Q}_\omega}{\gamma}$$

or, the desired sol'n:

$$\boxed{\hat{T}_\omega = \frac{\hat{Q}_\omega}{\gamma(\lambda + i\omega)}}$$

Homework 8: Stochastic Climate Model

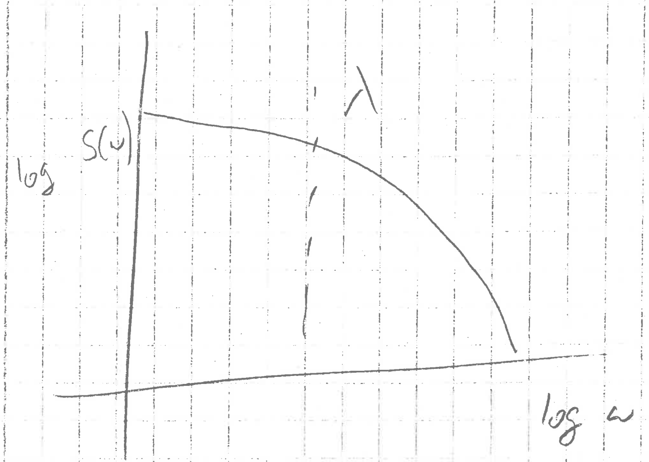
c) $\hat{T}_w = \dfrac{Q_w}{\gamma(\lambda + iw)}$

$$\hat{T}\hat{T}^* = \frac{Q_w}{\gamma(\lambda+iw)} \cdot \frac{\hat{Q}_w^*}{\gamma(\lambda+iw)} = \frac{\hat{Q}_w \hat{Q}_w^*}{(\gamma(\lambda+iw))^2}$$

The Spectrum

$$S(w) = \langle \hat{T}\hat{T}_* \rangle = \iint_{e\,e} \underbrace{\varepsilon(\hat{T}(t_1)\,\hat{T}^*(t_2))}_{\text{Becomes } 1,\ \text{Since white noise is Constant}} e^{-iw(t_1-t_2)}\,dt_1\,dt_2$$

$$= \frac{1}{\gamma^2(\lambda^2 + w^2)}$$



4 Homework 8: Angular Momentum and the Hadley Cell

We are given

$$\frac{DA}{Dt} = 0, \quad A = r(u+\Omega r) = \Omega R^2 \cos^2\phi - uR\cos\phi$$

This means

$$\frac{DA}{dt} = 0 = \frac{\partial A}{\partial t} + \underbrace{\cancel{u\frac{\partial A}{\partial x}}}_{0,\ \text{since zonally same}} + v\frac{\partial A}{\partial y} \qquad \boxed{\begin{array}{l} y = R\phi \\ \frac{dy}{d\phi} = R \end{array}}$$

$$\frac{DA}{dt} = \frac{\partial A}{\partial t} + \frac{v}{R}\frac{\partial A}{\partial \phi}$$

Plug in A

$$= -2\Omega R^2 \cos\phi \sin\phi \cdot \frac{\partial \phi}{\partial t} + \frac{\partial u}{\partial t} R\cos\phi - uR\sin\phi \frac{\partial\phi}{\partial t} +$$

$$\frac{v}{R}\left(-2\Omega R^2 \cos\phi \sin\phi + R\cos\phi \cdot \frac{\partial u}{\partial \phi} - uR\sin\phi\right)$$

$$= -2\Omega R^2 \cos\phi \sin\phi \frac{\partial\phi}{\partial t} - uR\sin\phi \frac{\partial \phi}{\partial t} - 2\Omega v R\cos\phi \sin\phi - uv\sin\phi$$

$$+ R\cos\phi \frac{Du}{dt}$$

$$= \left(-2\Omega R^2 \cos\phi \sin\phi - uR\sin\phi\right) \frac{Du}{Dt} + R\cos\phi \frac{Du}{Dt}$$

$$= \Big(\underbrace{-2\Omega\sin\phi}_{\text{this is } f!} R\cos\phi - u\sin\phi\Big) \cdot v + R\cos\phi \frac{Du}{Dt}$$

$$= \left(-f R\cos\phi - u\sin\phi\right)v + R\cos\phi \frac{Du}{Dt} = 0$$

OR

$$\frac{Du}{Dt} - fv = \underbrace{\frac{uv\sin\phi}{R\cos\phi}}$$

We can approximate this as 0 since u, v are much smaller than R and $\sin\phi$ is much less than 1

$$\frac{Du}{Dt} - fv = 0$$

B. The conservation of angular momentum generates converging flow at both the tip of the cell (jet stream) and at the bottom (surface wind)

c) $A = \Omega R^2 \cos^2 \phi + u R \cos \phi$

$A_{equator} = \Omega R^2$

Conservation of impulse says $\frac{DA}{Dt} = 0$, $A = \text{constant}$ $\phi = 20°$

$A_{20°} = \Omega R^2 \cos^2(20°) + u R \cos(20) = \Omega R^2 = A_{eq}$

Solve for $u$

$$u = \frac{\Omega R^2 - \Omega R^2 \cos^2 20}{R \cos 20} = \frac{\Omega R^2 (1 - \cos^2(20))}{R \cos 20}$$

$$= \Omega R \frac{\sin^2 20}{\cos^2 20} = \boxed{63.4 \text{ m/s}}$$

d) The Hadley Cell cannot be described as a geostrophic balance.