Exercise 2

May 12, 2014

1. **Several questions about the course:** (*0.5 points each*)

   (a) The Coriolis parameter f is defined as
   a) $f = \Omega \cos \varphi$
   b) $f = 2\Omega \cos \varphi$
   c) $f = 2\Omega \sin \varphi$
   d) $f = \beta y$

   (b) Please clarify: On the Northern Hemisphere, particles tend to go to the right or left relative to the direction of motion due to the Coriolis force?

   (c) Please write down the equation of state for the ocean and atmosphere!

   (d) What is the hydrostatic approximation in the momentum equations?

2. **Short programming questions.** *(1 point each)*

   Write down and explain the output for the following R-commands:

   (a) `0:10`

   (b) `a<-c(0,5,3,4); mean(a)`

   (c) `max(a)-min(a)`

   (d) `paste("The mean value of a is",mean(a),"for sure",sep="_")`

   (e) `a*2+c(1,1,1,0)`

   (f)
   ```
   my.fun<-function(n){return(n*n+1)}
   my.fun(10)-my.fun(1)
   ```

3. **Evaluate the double vector product:** (*2 points*)

$$\Omega \times (\Omega \times r)$$

   with vectors $\Omega = (0, 0, \omega)$ and $r = (x, y, z)$.

4. **Given $f(x, y, z, t)$. What is the definition of partial derivatives for this variable? What is the definition of nabla, Laplacean, divergence, total (substantial) derivative, total differential?** (*2 points*)

5. **Population Dynamics** (1 point each)
   Consider population dynamics with population $x > 0$ and reproduction (birth-death) r:
   $$\frac{dx}{dt} = x \cdot r(x) \tag{1}$$
   (a) Solve the differential equation for $r(x) = r_0 = const.$!
   What happens for $t \to \infty$ when $r_0 > 0$ or $r_0 < 0$?

   (b) Solve the differential equation for $r = r_0(1 - x)$! (limited growth)
   What happens for $t \to \infty$?

   (c) Consider the case $r = r_0(1 - x/K)$ with $K > 0$! Give a physical interpretation for K!
   Calculate the bifurcation with respect to parameter r; illustrate with a bifurcation diagram.

6. **Difference equations** (*1 point each*)
   Consider the discretised form of (1) using the Euler scheme
   $$\frac{dx}{dt} \approx \frac{x_{n+1} - x_n}{\Delta t}. \tag{2}$$
   (a) Write down the iteration x_{n+1} as a function of x_n .

   (b) What is the solution of x_{n+1} as a function of x_0?
   Consider the stability for the cases $r > 0$, $0 > \Delta t\, r > -1$, $-1 > \Delta t\, r > -2$, $-2 > \Delta t\, r$.
   Do you have a graphical interpretation of the oscillation/decay?

7. **Reynolds Number** (*3 points*)
   Let us consider a flow problem where the fluid behaves according to the incompressible Navier-Stokes equations:
   $$\begin{cases} \dfrac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\, \vec{u} = -\dfrac{1}{\rho_0} \nabla p + \nu \nabla^2 \vec{u} \\ \nabla \cdot \vec{u} = 0 \end{cases}$$
   where:
   - $\vec{u}$ = flow velocity;
   - t = time;

- ρ_0 = fluid density (constant);
- p = pressure;
- ν = kinematic viscosity (fluid property).

Using the reference density ρ_0 and further assuming that a reference lengthscale l_0 and a reference flow velocity U_0 have already been defined for the flow, transform these equations into a non-dimensional frame of reference. On how many parameters does the flow depend in the non-dimensional system? What is the role of the Reynolds number ($Re \equiv \frac{U_0 l_0}{\nu}$)?

Hint: Use U_0, ρ_0, l_0 *to construct reference values for all other involved quantities* (t, p) *and operators* $(\partial/\partial_t,\ \nabla)$*, insert them into the equations and reduce as much as possible the terms which have constant factors.*

Notes on submission of the exercises: *Working in study groups is encouraged, but each student is responsible for his/her own solution. Please email your own answers to the questions to Paul.Gierz@awi.de.*