Exercise 5 Solution

16. June 2014

In this excercise we will look at Foucault's Pendulum, which is a well known expiriement used to demonstrate the Earth's rotation. One such pendulum is displayed at the entrance of the physics building at the university.

From the course script:

Let us denote x, y, L, and Θ as the coordinates of the pendulum, the length of the string, and the angle of the pendulum bob, respectively. The string tension due to gravity can be written as:

$$F_g = mg \begin{pmatrix} sin\Theta \\ sin\Theta \\ cos\Theta \end{pmatrix} \approx mg \begin{pmatrix} x/L \\ y/L \\ 1 - z/L \end{pmatrix} \tag{1}$$

The horizontal dynamics can be explained via:

$$a_x = 2\Omega sin\varphi v_y - \frac{g}{L}x \tag{2}$$

$$a_y = 2\Omega sin\varphi v_x - \frac{g}{L}y \tag{3}$$

1. Show the analytical solution by introducing the complex number $\xi = x + i \cdot y$

   Solution

   Multiply 2 by i:

   $$a_y i = -2\Omega sin\varphi v_x i - \frac{g}{L} yi$$

   Add this into 1 and optain the differential equation for motion in the horizional plane':

   $$\ddot{\xi} = (-2i\Omega\varphi)\dot{x}i$$

   Thus, we will have two independent solutions. Solving the 2nd order differential equation introduce a variable $\xi(t) = e^{iat}$ and insert it above, where a is the solution for the characteristic equation

   $$a^2 + 2i\Omega\varphi a + \frac{g}{L}$$

   $$a_{1,2} = -i\Omega sin\varphi \pm i\sqrt{\Omega^2 sin^2\varphi + \frac{g}{L}}$$

So,

$$\xi(t) = x_1 e^{-ia_2 t} + x_2 e^{-ia_2 t}$$

x_1 and x_2 depend on the initial conditions (velocity, and potision). Solving for x_1 and x_2 gives:

$$x_1 = \frac{1}{2} + \frac{\Omega sin\varphi}{\sqrt{\Omega^2 sin^2\varphi + \frac{g}{L}^2}}$$

$$x_2 = \frac{1}{2} - \frac{\Omega sin\varphi}{\sqrt{\Omega^2 sin^2\varphi + \frac{g}{L}^2}}$$

2. For Foucault's pendulum in Paris: the plane of the pendulum rotates clockwise 11° per hour, making a full circle in 32.7 hours. What is the period in Bremen? What is the period at your home town?

   Solution For Bremen, the period is 30 hours.

3. Using the `shiny` program provided, compare the analytical solution with the numerical solution.

4. Geological evidence suggests that the Earth used to have a shorter day in the past, due to the impact forming the moon (angular momentum). What is the period of the pendulum on an Earth with only a 22 hour day for the 3 locations (Paris, Bremen, your home)?

   Solution With a shorter day, the period is 28 hours in bremen.

Notes on submission form of the exercises: *Students can work together, but each is required to submit his or her own solutions. The answers to the questions shall be send to paul.gierz@awi.de.*