Dynamics 2 Hw 1

1) $\begin{cases} \dot{x} = \sigma(y-x) = 0 \\ \dot{y} = rx - xz = 0 \\ \dot{z} = xy - bz = 0 \end{cases}$ $\quad \begin{cases} y = x \\ rx - xz - x = 0 \\ xy - bz = 0 \end{cases}$

$\begin{cases} y = x \\ x(r-z-1) = 0 \\ xy - bz = 0 \end{cases}$ $\quad x=0 \quad \text{or} \quad (r-z-1) = 0$

if $x=0$, Solution is $(0,0,0)$

else $x \neq 0$

$\begin{cases} y = x \\ r - z - 1 = 0 \\ z = \frac{xy}{b} = \frac{x^2}{b} \end{cases} = \begin{cases} y = x \\ r - \frac{x^2}{b} - 1 = 0 \\ z = \frac{x^2}{b} \end{cases} = \begin{cases} y = x \\ x = \pm\sqrt{(r-1)b} \\ z = \frac{x^2}{b} \end{cases}$

Solutions are

$$\left(\pm\sqrt{(r-1)b},\ \pm\sqrt{(r-1)b},\ r-1\right)$$

2) Symetry means $(x,y,z) = (-x,-y,z)$, so

$\sigma(-y+x) = -\sigma(y-x) = -\dot{x}$

$-rx + xz + y = -\dot{y}$

$xy - bz = \dot{z}$ $\quad$ z-axis doesn't change

3) $V(x,y,z) = rx^2 + \sigma y^2 + \sigma(z-2r)^2$

$\dot{V} = 2rx\dot{x} + 2\sigma y\dot{y} + 2\sigma(z-2r)\dot{z}$

Substitute

$\dot{x} = -\sigma x + \sigma y$

$\dot{y} = rx - y - xz$

$\dot{z} = -bz + xy$

$\dot{V} = 2rx(-\sigma x + \sigma y) + 2\sigma y(rx - y - xz) + 2\sigma(z-2r)(-bz+xy)$

$= 2\sigma(-rx^2 - y^2 - bz^2 + 2rbz$

$= -2\sigma(rx^2 + y^2 + b(z-r)^2 - br) = 0 \rightarrow$ $rx^2 + y^2 + b(z-r^2) = br^2$

Ellipsoid