Exercise 7

August 11, 2014

The aim of this exercise is to understand the propegation of shallow water waves and examine deep ocean circulation.

There is a R program to calculate both 1D and 2D waves.

1. For both 1D and 2D waves:
    - Run the program: Which type of waves do you see?
    - **Solution** Both the 1D and 2D waves shown are *gravity waves*
    - Change the constants of water depth H, gravity g, describe your observations!
    - **Solution** Increasing g causes the waves to propegate faster. Increasing H has a similar effect, although the peak shapes are altered; the waves do not propegate as well in deeper water and are damped more quickly.
    - Can you roughly estimate the phase speed of the waves?
    - **Solution** The phase speed of a wave is given mathematically by $c = \sqrt{gH}$
2. Consider a geostrophic flow (u, v)

$$-fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \tag{1}$$

with pressure $p(x, y, z, t)$.
Use the hydrostatic approximation

$$\frac{\partial p}{\partial z} = -g\rho \tag{2}$$

and equation (1) in order to derive the meridional overturning stream function $\Phi(y, z)$ as a fuction of density ρ at the basin boundaries! Φ is defined via

$$\Phi(y, z) = \int_0^z \frac{\partial \Phi}{\partial \tilde{z}}\, d\tilde{z} \tag{3}$$

$$\frac{\partial \Phi}{\partial \tilde{z}} = \int_{x_e}^{x_w} v(x, y, \tilde{z})\, dx \quad \text{(zonally integrated transport)}, \tag{4}$$

where x_e and x_w are the eastward and westward boundaries in the ocean basin (think e.g. of the Atlantic Ocean). Units of Φ are $m^3 s^{-1}$. At the surface $\Phi(y, 0) = 0$.

3. **Solution** Here, we will find that the overturning strength *only depends on the density differences between East and West.* We know that at $z = 0, \Phi(y, 0) = 0$, since at the surface there is no overturning. We can start by finding $\Phi(\rho)$, the overturning as a function of density.

$$\frac{\partial \Phi}{\partial z} = \int_{x_e}^{x_w} v(x, y, z) dx \tag{5}$$
$$= \int_{x_e}^{x_w} \frac{1}{\rho_0 g} \frac{\partial p}{\partial x} dx \tag{6}$$
$$= \frac{1}{\rho_0 g} p(x_w) - p(x_e) \tag{7}$$

We can now substitute this into the equation for $\Phi(y, z)$.

$$\Phi(y, z) = \int_0^z \frac{\partial \Phi}{\partial z} dz \tag{8}$$
$$= \int_0^z \frac{1}{\rho_0 g} p(x_w) - p(x_e) dz \tag{9}$$
$$= \frac{1}{-2\rho_0 f g \rho} * [p_w(z)^2 - p_e(z)^2 - p_w(0)^2 + p_e(0)^2] \tag{10}$$

We know that pressure at the surface is simply atmospheric standard pressure, and we can neglect this as small fluctuations of p_0 will not impact the overturning strength. We therefore arrive at:

$$\Phi = -\frac{1}{2\rho_0 \rho f g} [p_w(z)^2 - p_e(z)^2] \tag{11}$$

You can easily convert pressure to density, since all other factors remain the same (ie gravity, depth, temperature, and so on). Therefore, the strength of the overturning depends only on the pressure (and thereby density) differences between the eastern and western edge of the basin.

4. It is observed that water sinks in to the deep ocean in polar regions of the atlantic basin at a rate of 15 Sv.

   - How long would it take to 'fill up' the Atlantic basin? (Assume $10^{14} m^2$ and $5km$ depth)
   - **Solution** $\frac{5\times10^{17}}{15\times10^{6}} = 0.33 \times 10^{11} s = 1046.6 yrs$
   - Supposing that the local sinking is balanced by large-scale upwelling, estimate the strength of this upwelling. Express your answer in $m\ y^{-1}$.
   - **Solution** $\frac{15\times10^{6}}{10^{14}} = 4.7 \frac{m}{y}$

Notes on submission form of the exercises: *Two students work together in one group. Each group has to submit only one solution. The answers to the questions shall be send to paul.gierz@awi.de.*