Exercise 3

NEW

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1. **Rayleigh-Bénard convection** Rayleigh studied the flow occurring in a layer of fluid of uniform depth H, when the temperature difference between the upper- and lower-surfaces is maintained at a constant value ΔT. In the case where all motions are parallel to the $x-z$-plane, and no variations in the direction of the y-axis occur, the governing equations can be written as:

$$\partial_t u + u\partial_x u + w\partial_z u = -\frac{1}{\rho_0}\partial_x p + \nu\nabla^2 u \quad (1)$$

$$\partial_t w + u\partial_x w + w\partial_z w = -\frac{1}{\rho_0}\partial_z p + \nu\nabla^2 w + g(1-\alpha(T-T_0)) \quad (2)$$

$$\partial_t T + u\partial_x T + w\partial_z T = \kappa\nabla^2 T \quad (3)$$

$$\partial_x u + \partial_z w = 0 \quad (4)$$

where w and u are the vertical and horizontal components of the velocity, respectively. Furthermore, $\nu = \eta/\rho_0$, $\kappa = \lambda/(\rho_0 C_v)$ the momentum diffusivity (kinematic viscosity) and thermal diffusivity, respectively.

You have been provided with the `R-shiny` interface for the `rayleigh-benard.R` codes. More description can be found in the file README_Shiny_ex3.pdf.

Tasks:

(a) Write down the dimensionless parameters and their relation to characteristic length and time scales charachterizing the flow!

(b) Describe what the dimensionless parameters mean and where they appear in the equations!

(c) Vary the Rayleigh and the Prandtl number by $R_a = 20000, 40000, 60000$ and $Pr = 0.5, 1, 1.5, 5, 10$ and describe the dynamics (words, figures)! For high values of R_a the spatial resolution might be chosen higher (to the double). Here are the standard values:

```
lx    = 100;      #Number of horizontal cells
ly    = 52;       #Number of vertical cells
```

What does a high R_a mean? A low R_a? High Pr? Low Pr?