Exercise 8: Last Exercise!!

July 7, 2014

1. **Ice Sheet Melting** In the present climate the volume of freshwater trapped in ice sheets over land is 33×10^6 km^3. If all this ice melted and ran into the ocean, estimate by how much the sea level would rise. What would happen to sea level if all the sea-ice melted?

2. **Wind-driven ocean circulation** When the windstress is only zonal, the Sverdrup transport is:

$$\rho_0 \beta V = \text{curl } \tau = -\frac{\partial}{\partial y}\tau^x \tag{1}$$

and Ekman transports and Ekman pumping velocity are:

$$\rho_0 f V_E = -\tau^x \rho_0 w_E = \text{curl } \tau = -\frac{\partial}{\partial y}\tau^x. \tag{2}$$

Assume furthermore

$$\tau^x = -\tau_0 \cos(\pi y/B) \tag{3}$$

for an ocean basin $0 < x < L, 0 < y < B$.

(a) At what latitudes y are $|V|$ and $|V_E|$ maximum? Calculate their magnitudes. Take constant $f = 10^{-4}$ s^{-1} and $\beta = 1.8 \cdot 10^{-11}$ m^{-1}s^{-1} and $B =$ 5000 km, $\tau_0/\rho_0 = 10^{-4}$ m^{2}s^{-2}.

(b) Calculate the maximum of w_E for constant f (value see above). Is this measurable?

3. **Stochastic Climate Model** Imagine that the temperature of the ocean mixed layer of depth h is governed by

$$\frac{dT}{dt} = -\lambda T + \frac{Q_{net}}{\gamma_O}, \tag{4}$$

where coefficient γ_O is given by the heat capacity $c_p \rho h$, and λ is the typical damping rate of a temperature anomaly. The air-sea fluxes due to weather systems are represented by a white-noise process $Q_{net} = \hat{Q}_\omega e^{i\omega t}$ where $\hat{Q}_\omega$ is the amplitude of the random forcing at frequency ω. $\hat{Q}^*$ is the complex conjugate.

(a) What is a white-noise process? Remember that

$$\int_R \exp(i\omega t)\delta(t-0)dt = 1 \tag{5}$$

and use the Fourier transformation.

(b) Solve the equation (4) above for the temperature response $T = \hat{T}_\omega e^{i\omega t}$ and hence show that:

$$\hat{T}_\omega = \frac{\hat{Q}_\omega}{\gamma_O\left(\lambda + i\omega\right)} \tag{6}$$

(c) Show that it has a spectral density $\hat{T}_\omega \hat{T}_\omega^*$ is given by:

$$\hat{T}\hat{T}^* = \frac{\hat{Q}\hat{Q}^*}{\gamma_O^2\left(\lambda^2 + \omega^2\right)} \tag{7}$$

and the spectrum

$$S(\omega) =< \hat{T}\hat{T}^* >= \frac{1}{\gamma_O^2\left(\lambda^2 + \omega^2\right)}. \tag{8}$$

The brackets $< \ldots >$ denote the ensemble mean. Make a sketch of the spectrum using a log-log plot and show that fluctuations with a frequency greater than λ are damped.

4. **Angular Momentum and Hadley Cell** Consider a zonally symmetric circulation (i.e., one with no longitudinal variations) in the atmosphere. In the inviscid upper troposphere one expects such a flow to conserve absolute angular momentum, i.e.,

$$\frac{DA}{Dt} = 0 \tag{9}$$

where A is the absolute angular momentum per unit mass (parallel to the Earth's rotation axis)

$$A = r\left(u + \Omega r\right) = \Omega R^2 \cos^2\varphi + uR\cos\varphi \tag{10}$$

Ω is the Earth rotation rate, u the eastward wind component, $r = R\cos\varphi$ is the distance from the rotation axis, R the Earth's radius, and φ latitude.

(a) Show for inviscid zonally symmetric flow that the relation $\frac{DA}{Dt} = 0$ is consistent with the zonal component of the equation of motion

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{11}$$

in (x, y, z) coordinates, where $y = R\varphi$.

(b) Use angular momentum conservation to describe in words how the existence of the Hadley circulation explains the existence of both the subtropical jet in the upper troposphere and the near-surface trade winds.

(c) If the Hadley circulation is symmetric about the equator, and its edge is at 20° latitude, determine the strength of the subtropical jet. Use (10, 11).

(d) Is the Hadley cell geostrophically driven or not?

<u>Notes on submission form of the exercises:</u> *Students can work together in groups, but each student must submit his or her own solutions. The answers to the questions shall be send to paul.gierz@awi.de.*