Peter Gifford, Ren Wall, Kyle Brekke, Madison Hanson Group: 7 due: 20 September 2019

CSCI 432 Problem 2-1

Collaborators:

Give a linear-time algorithm that takes two sorted arrays of real numbers as input, and returns a merged list of sorted numbers. You should give your answer in pseudocode. Your answer should contain:

- A prose explanation of the algorithm.
- Psuedocode. (Be sure to review the two resources on pseudocode that were posted as readings for Week 2! I also suggest the algorithm / algorithmx package in LaTex.)
- The decrementing function for any loop or recursion.
- Justification of why the runtime is linear.

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CSCI 432 Problem 2-2

Collaborators:

EPI 15.4 (Generate the Power Set) gives code to compute the power set of a set (without duplicates). Present this problem and solution in your own words using pseudocode.

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CSCI 432 Problem 2-3

Collaborators:

In EPI 15.1 (The Towers of Hanoi Problem), prove that the algorithm as presented terminates. In particular, you should give the decrementing function for the recursion.

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CSCI 432 Problem 2-4

Collaborators:

For the stock market problem discussed in class on September 6th (and in CLRS 4.1), walk through the algorithm for the following input:

$$\texttt{price} = \{3, 6, 8, 2, 1, 10, 5, 7\}.$$

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CSCI 432 Problem 2-5

Collaborators:

Prove using induction that the closed form of:

$$T(n) = \begin{cases} 1 & n = 1 \\ T(n-1) + n & n > 1 \end{cases}$$

is $O(n^2)$.

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CSCI 432 Problem 2-6

Collaborators:

What is the closed form of the following recurrence relations? Use Master's theorem to justify your answers:

1. $T(n) = 16T(n/4) + \Theta(n)$
2. $T(n) = 2T(n/2) + n \log n$
3. $T(n) = 6T(n/3) + n^2 \log n$
4. $T(n) = 4T(n/2) + n^2$
5. $T(n) = 9T(n/3) + n$

Note: we assume that $T(1) = \Theta(1)$ whenever it is not explicitly given.

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CSCI 432 Problem 2-7

Collaborators:

The skyline problem: You are waiting for the ferry across the river to get into a big city, and notice n buildings in front of you. You take a photo, and notice that each building has the silhouette of a rectangle. Suppose you represent each building as a triple (x_1, x_2, y), where the building can be seen from x_1 to x_2 horizontally and has a height of y. Let `rect(b)` be the set of points inside this rectangle (including the boundary). Let `building` be the set of n triples. Design an algorithm that takes `buildings` as input, and returns the skyline, where the skyline is a sequence of (x, y) coordinates defining $\cup_{b \in \texttt{buildings}} \texttt{rect}(b)$.

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CSCI 432 Problem 2-8

Collaborators:

The `rand()` function in the standard C library returns a uniformly random number in `[0,RANDMAX-1]`. Does `rand()` mod n generate a number uniformly distributed in $[0, n-1]$?

Note I: This is the second variant in EPI 5.12.

Note II: When asked questions of this form, you are expected to justify your answer.

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CSCI 432 Problem 2-9

Collaborators:

Algorithms where we use randomization to find a deterministic answer are known as Las Vegas algorithms. Monte Carlo algorithms also use randomization, but might not always give the right answer; however, they either have a high probability of being correct or close to correct.

(a) Give a Monte Carol algorithm to estimate π.

(b) Let n be the number of random numbers used by your algorithm. Explain why as $n \to \infty$, the expectation of the output for your algorithm is π.

(c) Implement this algorithm and plot a line graph of the values returned for at least 10 values of n.

Note: We can use the function `randReal`(a, b) that returns a random real number between a and b inclusive.