# *RLC Resonant Circuits - Companion Guide*

## *Equipment*

- Series and parallel RLC circuit board
- B & K 3011 function generator
- set of connecting leads (2)
- Oscilloscope

## *Setup*

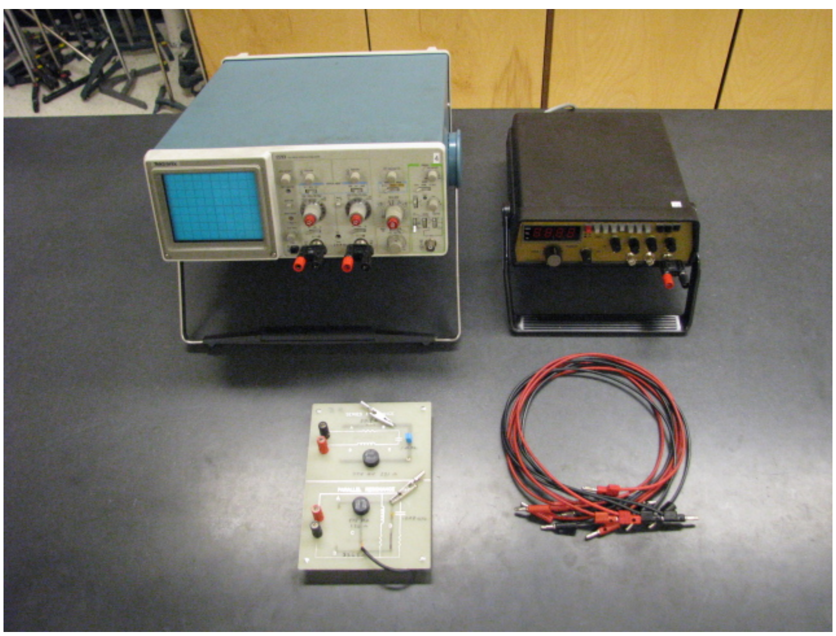

Figure 1: Equipment Setup

Set up bench as shown in Figure **??**.

## *Maintenance*

1.

2.

## *Critical Points of Failure*

There are currently no known critical points of failure.

## *Notes to the Instructor*

1. The function generator outputs a dip on each node for the parallel circuit at amplitude resonant frequency. This caused difficulty in measuring the experimental amplitude resonant frequency since the output frequency could be adjusted within a 100 Hz range while still exhibiting resonant attributes on the oscilloscope.

2.

3.

## *Prelab Questions*

1. Resonance is the tendency of a system to oscillate with a greater amplitude at some frequencies than at others (Wikipedia - Resonance). Four examples of physical systems that exhibit resonant qualities: musical instruments (acoustic resonance), suspension systems (mechanical resonance), electrical circuits (electrical resonance), and optical cavities (optical resonance).

2. Derivation of $\omega^2 = 1/LC$: First set $\phi = 0$ in Equation 8 (the condition for phase resonance),

$$0 = tan^{-1}\left[\frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right)\right]$$

Rearrange to get,

$$\omega L - \frac{1}{\omega C} = 0$$

Yielding,

$$\omega^2 = \frac{1}{LC}$$

Then we can see that Equation 7 is at an extremum (condition for amplitude resonance) when $\omega^2 = \frac{1}{LC}$ giving

$$E \sin \omega t = i(t)R$$

which is the maximal current possible for the circuit. Therefore, amplitude resonance and phase resonance occur at the same frequency $\omega = 1/\sqrt{LC}$.

3. Parallel phase frequency derivation: First set $\phi = 0$ in Equation 13 (the condition for phase resonance) and factor $\omega$ out to get,

$$\omega \left[ \frac{\omega^2 L^2 C}{R} + RC - \frac{L}{R} \right] = 0$$

Rearrange to get Equation 15,

$$\omega_P^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

4. Justification for Equation 6 (series circuit): Substituting voltage values into Equation 1 we have,

$$L\frac{di}{dt} + \frac{Q}{C} + iR - E\sin\omega t = 0$$

Taking the derivative of this gives,

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} - E\omega\cos\omega t = 0$$

which is Equation 6 from the lab manual. Therefore, this equation is valid to use for a series circuit.

Justification for Equations 9, 10, and 11 (parallel circuit): Equation 9 comes from the current splitting at the node (Kirchhoff's current law) that can be seen in Figure 4. We can then treat each current, $i_1$ and $i_2$, as part of a separate circuit. Equation 10 corresponds to $i_1$ where the voltage source, inductor, and resistor are seen as one circuit. Then Kirchhoff's voltage law is applied to give Equation 10. Equation 11 corresponds to $i_2$ in the same way where just the capacitor and voltage sources are seen as the second circuit. Kirchhoff's voltage law is applied to the circuit and then the derivative is taken with respect to time to give Equation 11.

5. Setting R=0 in Equation 14 gives,

$$\omega_A^2 = \frac{1}{LC}$$

Similarly for Equation 15,

$$\omega_P^2 = \frac{1}{LC}$$

which implies that

$$\omega_A = \omega_P = \omega = \frac{1}{\sqrt{LC}}$$

So, the resonant frequencies in the parallel circuit are equal when there is no resistance and are also equal to the resonant frequency of the series circuit.

## *Data Requirements*

6. The series circuit values were recorded to be: R=50.8 Ω, C=$1.00\times10^{-6}$ F, L=0.474 H. When substituted into,

   $$f = \frac{1}{2\pi\sqrt{LC}}$$

   the theoretical resonant frequency was found to be f=231.17 Hz.

   The parallel circuit values were recorded to be: R=3263 Ω, C=$9.00\times10^{-9}$ F, L=0.475 H. When substituted into,

   $$f_A = \frac{1}{LC}\sqrt{1+\frac{2R^2C}{L}} - \frac{R^2}{L^2}$$

   and

   $$f_P = \frac{1}{2\pi}\sqrt{\frac{1}{LC}\sqrt{1+\frac{2R^2C}{L}} - \frac{R^2}{L^2}}$$

   the theoretical amplitude resonant frequency was found to be $f_A$=2404.49 Hz and the theoretical phase resonant frequency was found to be $f_P$=2135.10 Hz.

7. The experimental resonant frequency for the series circuit was found to be f=230± 2 Hz. For the parallel circuit, the experimental amplitude resonant frequency was found to be $f_A$=2300± 20 Hz and the experimental phase resonant frequency was found to be $f_P$=2000± 20 Hz.

8. See Data Requirement 9 (asks for the same table)

9. Table: Series Voltage vs. Frequency and Angular Frequency

| f (Hz) | Δf (Hz) | $\omega$ (rad/s) | $\Delta\omega$ (rad/s) | V (V) | ΔV (V) |
|---|---|---|---|---|---|
| 158 | 2 | 993 | 10 | 0.34 | 0.05 |
| 169 | 2 | 1060 | 10 | 0.39 | 0.05 |
| 180 | 1 | 1130 | 6 | 0.44 | 0.05 |
| 192 | 2 | 1210 | 10 | 0.5 | 0.05 |
| 200 | 2 | 1260 | 10 | 0.54 | 0.05 |
| 211 | 1 | 1330 | 6 | 0.59 | 0.05 |
| 220 | 2 | 1380 | 10 | 0.62 | 0.05 |
| 225 | 2 | 1410 | 10 | 0.62 | 0.05 |
| 230 | 2 | 1450 | 10 | 0.62 | 0.05 |
| 235 | 2 | 1480 | 10 | 0.62 | 0.05 |
| 240 | 1 | 1510 | 6 | 0.61 | 0.05 |
| 245 | 2 | 1540 | 10 | 0.6 | 0.05 |
| 247 | 1 | 1550 | 6 | 0.59 | 0.05 |
| 250 | 2 | 1570 | 10 | 0.58 | 0.05 |
| 258 | 2 | 1620 | 10 | 0.56 | 0.05 |
| 269 | 1 | 1690 | 6 | 0.52 | 0.05 |
| 284 | 1 | 1780 | 6 | 0.46 | 0.05 |
| 295 | 2 | 1850 | 10 | 0.43 | 0.05 |
| 309 | 1 | 1940 | 6 | 0.39 | 0.05 |
| 322 | 2 | 2020 | 10 | 0.36 | 0.05 |

Table 1: Series Resonant Circuit Data

10. Graph: Series voltage vs. angular frequency

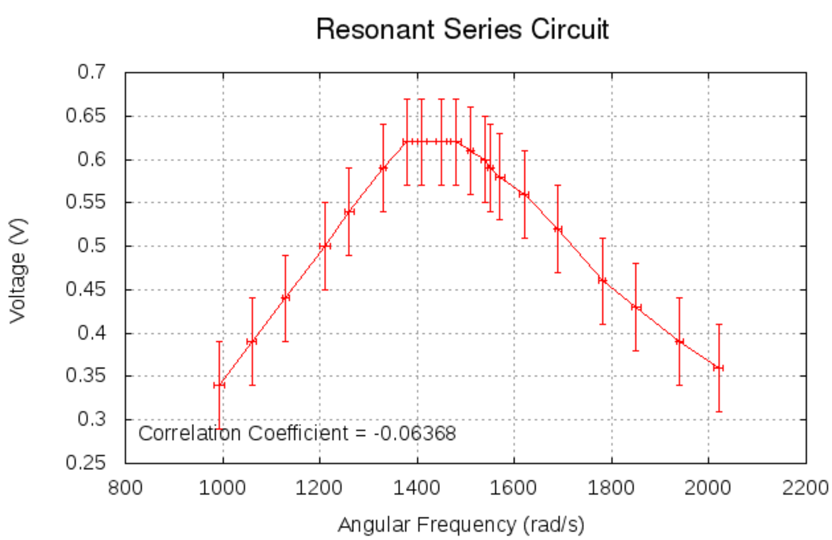


Figure 2: Resonant Series Circuit

11. Graph: Series voltage vs. angular frequency: Experimental vs. Theoretical

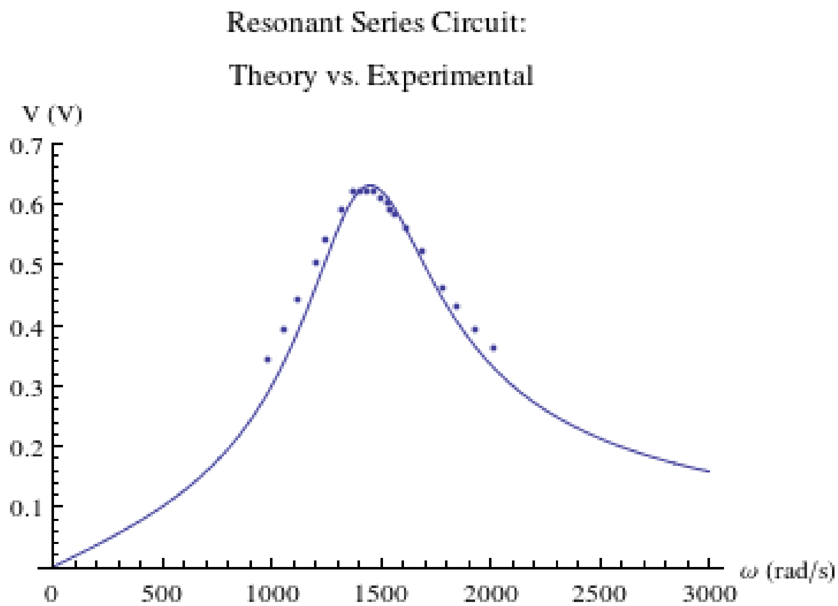


Figure 3: Resonant Series Circuit: Theoretical vs. Experimental

12. Table: Parallel voltage vs. frequency vs. angular frequency

| f (kHz) | Δf (kHz) | $\omega$ (krad/s) | $\Delta\omega$ (krad/s) | V (V) | ΔV (V) |
|---|---|---|---|---|---|
| 1.50 | 0.02 | 9.42 | 0.13 | 8.80 | 1.00 |
| 1.60 | 0.01 | 10.1 | 0.06 | 8.00 | 0.50 |
| 1.70 | 0.02 | 10.7 | 0.13 | 7.20 | 0.50 |
| 1.80 | 0.02 | 11.3 | 0.13 | 6.30 | 0.50 |
| 1.90 | 0.02 | 11.9 | 0.13 | 5.30 | 0.50 |
| 2.00 | 0.02 | 12.6 | 0.13 | 4.30 | 0.50 |
| 2.10 | 0.02 | 13.2 | 0.13 | 2.90 | 0.25 |
| 2.20 | 0.02 | 13.8 | 0.13 | 1.75 | 0.25 |
| 2.25 | 0.02 | 14.1 | 0.13 | 1.18 | 0.10 |
| 2.30 | 0.02 | 14.5 | 0.13 | 0.94 | 0.10 |
| 2.35 | 0.02 | 14.8 | 0.13 | 0.92 | 0.10 |
| 2.40 | 0.02 | 15.1 | 0.13 | 1.40 | 0.10 |
| 2.50 | 0.02 | 15.7 | 0.13 | 2.50 | 0.25 |
| 2.60 | 0.02 | 16.3 | 0.13 | 3.50 | 0.25 |
| 2.72 | 0.02 | 17.1 | 0.13 | 4.50 | 0.50 |
| 2.80 | 0.02 | 17.6 | 0.13 | 5.20 | 0.50 |
| 2.90 | 0.02 | 18.2 | 0.13 | 5.90 | 0.50 |
| 3.00 | 0.02 | 18.8 | 0.13 | 6.50 | 0.50 |
| 3.10 | 0.02 | 19.5 | 0.13 | 7.10 | 0.50 |
| 3.20 | 0.02 | 20.1 | 0.13 | 7.50 | 0.50 |

Table 2: Parallel Resonant Circuit Data

13. Graph: Parallel voltage vs. angular frequency

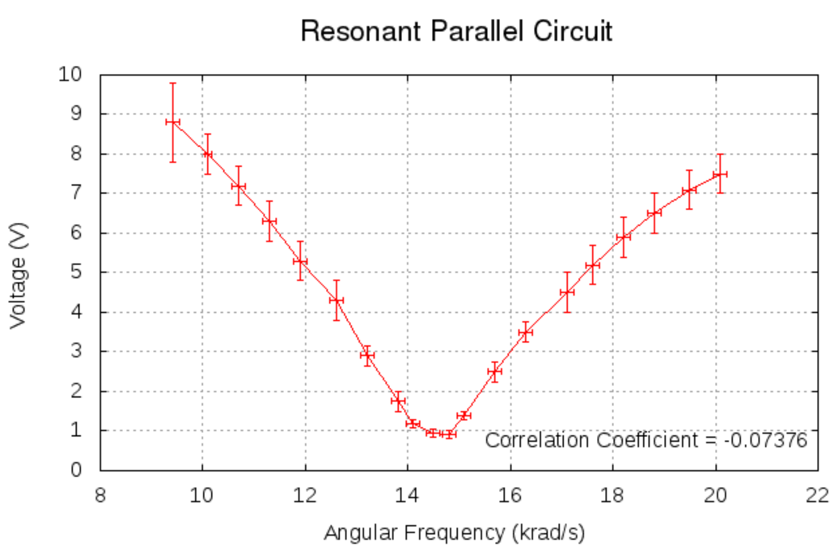


Figure 4: Resonant Parallel Circuit

## *Discussion*

14. The theoretical series resonant frequency was found to be f=231.17 Hz and the experimental value was found to be f=230± 2 Hz. These values agree within error.

    For the parallel circuit, the theoretical amplitude resonant frequency was found to be $f_A$=2404.49 Hz and the experimental amplitude resonant frequency was found to be $f_A$=2350± 70 Hz where the drift on the function generator occurred from 2300-2400 Hz and was accounted in the error by adding an extra 50 Hz to the original 20 Hz error inherent in the device. The theoretical phase resonant frequency was found to be $f_P$=2135.10 Hz and the experimental phase resonant frequency was found to be $f_P$=2120± 20 Hz. Both the phase and amplitude resonant frequencies agreed within error.

    Possible sources of error include: degraded devices on the circuit boards, degraded wiring, and the difficulties associated with measuring the theoretical amplitude frequency discussed in the notes below.

    *Notes: The function generator output exhibited a dip on each node, which made it difficult to record a proper measurement for the resonant frequencies on the parallel circuit. This error was taken into account for the amplitude resonant frequency measurement as discussed above.*

15. In Figure **??**, the experimental data was plotted along with the theoretical curve. The theoretical extremum point, resonance, occurred at about 6.25 V which corresponded to the experimental resonance at $0.62 \pm 0.05V$ within the error range.

16. The results of this experiment confirm the theory of resonance, as all theoretical values matched experimental within the error range (discussed in Discussion 14).

    At resonance, the current was found to be at an extremum for both the series and parallel circuits as predicted by Equations 7 and 12 from the lab manual within the experimental error range. This can be seen when values for angular frequency are substituted into Equations 7 and 12.