X-Ray Diffraction - Companion Guide

Equipment

- Tel-X-ometer x-ray spectrometer
- LiF crystal diffraction grating
- Geiger counter detector
- 1mm and 3mm lead collimator slides
- Tel-Atomic Digicounter digital pulse counter
- X-ray tube current monitoring cable
- Fluke multimeter
- Acrylic plastic x-ray attenuator slide plate

Setup

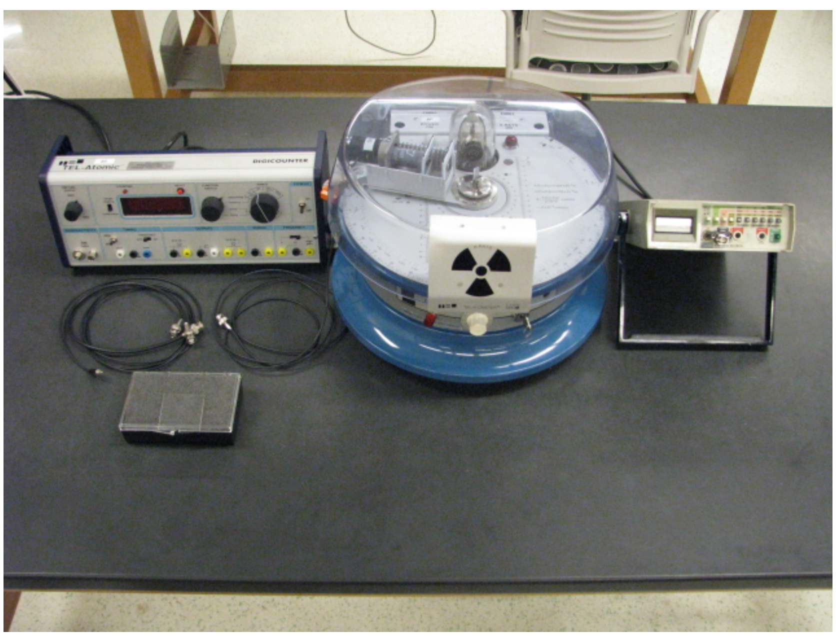

Figure 1: Equipment Setup

Set up bench as shown in Figure 1.

Maintenance

1.

2.

Critical Points of Failure

There are currently no known critical points of failure.

Notes to the Instructor

1. It is practical to take standard single-degree steps when measuring the bremsstrahlung continuum however it is recommended that students take smaller, half-degree steps around the characteristic peaks to improve the resolution of their spectra.

Prelab Questions

1. **The x-ray region of the electromagnetic spectrum extends between the frequencies of 10^{17} Hz and 10^{20} Hz. For x-rays of these two frequencies, calculate the corresponding wavelength in nanometers and the corresponding energy in keV.**

   We know that $c = \lambda\nu$ and that E=hν. Using these relationships we can find that 10^{17} Hz=3 nm=0.4136 keV and 10^{20} Hz=0.003 nm=413.6 keV.

2. **Find the energy, in keV, of an electron accelerated across a 30kV potential difference. What is the cut-off energy, in keV, of a 30kV x-ray tube?**

   1 keV is the energy gained by a single electron crossing a potential difference of 1 V. In this case, the electron will gain 30 keV of energy. If we have a 30 keV x-ray tube, we should see no photons possessing higher energy than 30 keV, thus 30 keV would be the cut-off energy.

3. **In Figure 4, explain whether it is the left side or the right side of the x-ray spectrum that is the high energy side.**

   Using the following relationship,

$$n\lambda = 2dsin\theta \tag{1}$$

We can see that λ is maximal when $\theta = 90°$ and minimal when $\theta = 0°$. This means that the left side of the plot, i.e., the smaller angles, correspond to shorter wavelengths and higher energies and the right side of the plot, i.e., the larger angles, correspond to longer wavelengths and lower energies.

4. **Look up the atomic masses of lithium and fluorine, and the density of LiF. Using these values calculate the atomic spacing of LiF.**

   A_{Li}=6.94 g mol^{-1} and A_F=19.0 g mol^{-1} so A_{LiF}=25.94 g mol^{-1}. The density of LiF, ρ, is 2.64 g cm^{-3}. We use the following relation to find the interatomic spacing of LiF,

$$d = \sqrt[3]{\frac{A_{LiF}}{2N_A\rho}} = \sqrt[3]{\frac{25.94}{2(6.022 \times 10^{23})(2.64)}} = 0.2013nm \qquad (2)$$

5. **Look up the K-shell characteristic x-ray energies for copper.**

   K-shell characteristic energies for copper are $K_\alpha = 8.038$ keV and $K_\beta = 8.905$ keV[1]. These are first order lines. With the LiF crystal used in this lab, students will also see second order lines.

[1] http://www.phywe-es.com/index.php/fuseaction/download/lrn_file/versuchsanleitungen/P2540101/e/P2540101E.pdf

Data Requirements

1. **A data table of the x-ray spectrum together with appropriate errors.**

   See Tables 1-3.

2. **A data table, with errors, of the x-ray spectrum with the plastic plate inserted into the beam.**

   See Tables 4-5.

3. **A graph of counts versus angle θ with error bars for both x-ray spectra.**

   See Figure 2.

4. **A graph of counts versus wavelength in nanometers with error bars for both x-ray spectra.**

   See Figure 3.

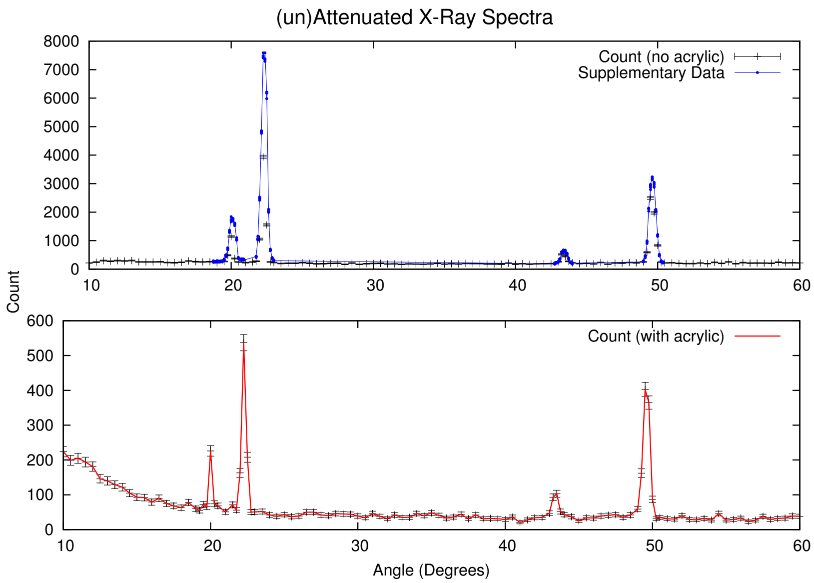

Figure 2: The top plot includes higher angular resolution data (in blue) to illustrate more clearly the location of the peaks. Additionally, multiple sets of counts were collected for each high resolution angle to verify that the variance in measurements was at least Poissonian. The bottom plot shows the data obtained for measurements of every degree with the acrylic slide in place. Uncertainties in the angle measurements are not included here to keep the plots uncluttered.

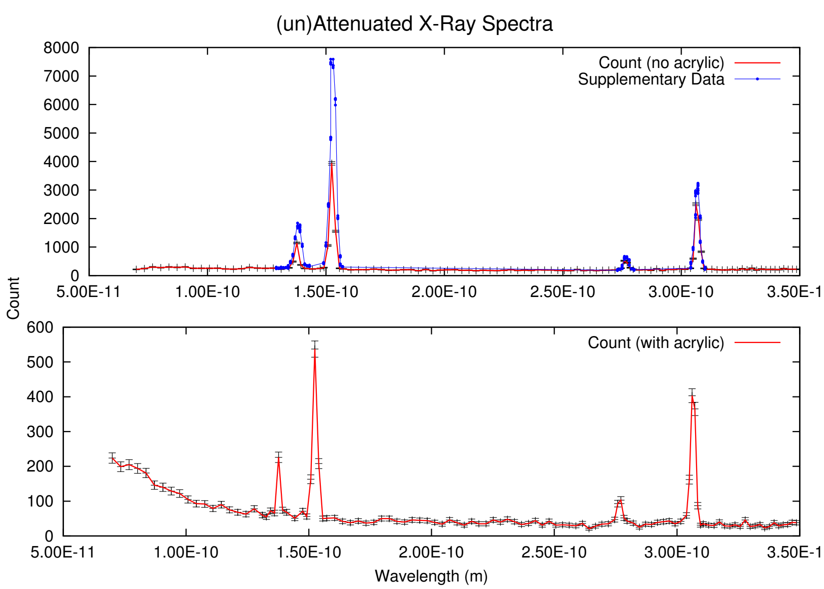

Figure 3: This plot shows the wavelength (m) of the characteristic peaks of copper with and without the acrylic slide. Supplementary data is in blue.

5. **A graph of counts versus energy in keV with error bars for both x-ray spectra.**

See Figure 4.

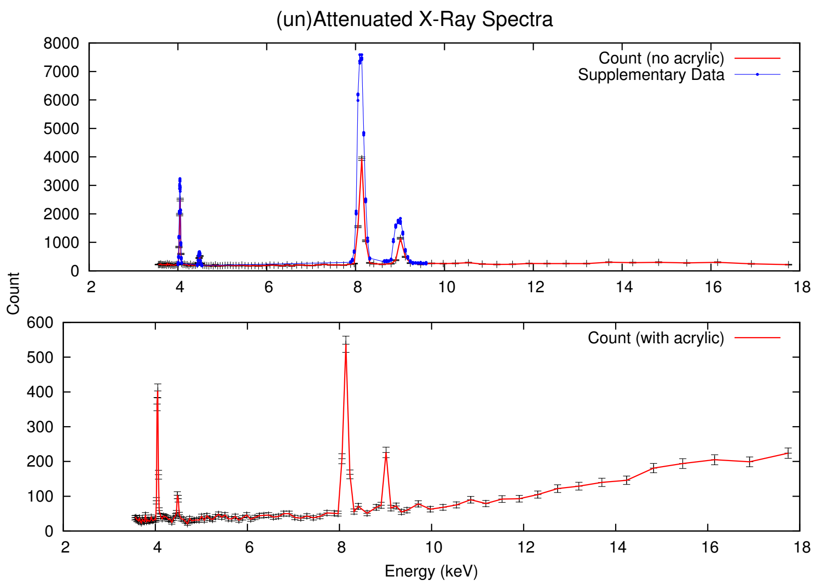

Figure 4: This plot shows the energies (keV) of the characteristic peaks of copper with and without the acrylic slide. Supplementary data is in blue. The cut-off energy cannot be seen due to the physical constraints of the equipment.

6. **A value, with error, for the estimated cut-off wavelength, λ_{min}, of the of the x-ray tube.**

Depending on the condition of the Bragg spectrometer that students use there may be varying degrees of success in obtaining a satisfactory data set for determining the cut-off energy. Ideally, students will obtain a spectrum such as that shown in Figure 5 from which they can discern the cut-off angle and thus the cut-off energy. It is likely that many won't however and so it is necessary to record appropriate magnitudes of uncertainty in extrapolations of the graph.

Using the data from Figure 5 we estimate the cut-off angle to be $6\pm1°$. This corresponds to a wavelength of 42.1 ± 6.99 picometers

7. **A value, with error, for Planck's constant as derived from the cut-off wavelength.**

The Planck constant can be calculated as follows,

$$h = \frac{eV}{c} \cdot 2d \cdot sin(\theta_{min}) = 4.208 \times 10^{-15} \tag{3}$$

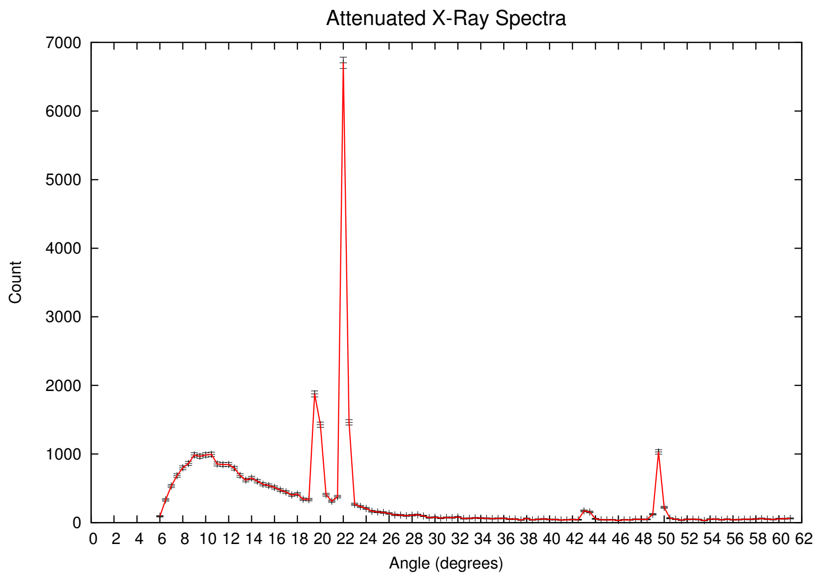

Figure 5: Ideal spectrum for experimentally determining the cut-off energy of the Bragg x-ray spectrometer. Here the cut-off angle is easily extrapolated from the plot.

Where eV is 30 keV corresponding to the spectrometer's energy setting and θ_{min} is the value extrapolated from the graph. The uncertainty is found from

$$u(h) = \frac{eV}{c} \cdot 2d \cdot cos(\theta_{min}) \cdot u(\theta_{min}) = 6.988 \times 10^{-16} \tag{4}$$

8. **Values, with error, for the observed K-shell characteristic x-rays of copper.**

The two values desired here are for K_α and K_β (see Figure blah). K_α is actually a doublet but it is beyond the ability of this spectrometer to resolve it so we take the average value of the doublet to be E=8.038 keV. The energy of K_β is taken to be E=8.905 keV.

Reading from our data (see Tables 1-3 we see that the highest counts are obtained at energies of 8.139$\pm$ 0.174 keV and 9.010$\pm$0.216 keV. This gives us percent errors of 1.26% for the K_α line and 1.18% for the K_β line. These values are well within experimental uncertainty. Accepted values for the characteristic energies of copper can be found in the Handbook of Chemistry and Physics, CRC Press, available online.

Discussion

9. **A comparison of the observed x-ray spectrum with the expected spectrum.**

In our spectrum we see evidence of the expected braking radiation, smoothly spread across all energies and ending at the cut-off energy of the spectrometer. We also see the first and second order characteristic spikes of copper at the expected energies. We do not, however, see the K_α doublet. This is likely beyond the resolution of the spectrometer because of its relatively large beam width.

10. **A comparison of the observed x-ray spectrum with the observed x-ray spectrum with the plastic inserted.**

We again see the characteristic energy spikes of copper in their expected positions. Regarding the braking radiation, the right-skewed shape is more prominent when we attenuate the lower energy x-rays with the acrylic slide. This makes it somewhat easier to extrapolate the curve towards the cut-off energy (see Figure 5).

11. **A comparison of the observed cut-off and derived Planck's constant with the expected values.**

Experimentally determined values for the cut-off energy (29.482 ± 4.896 keV) and the Planck constant ($4.208 \times 10^{-15} \pm 6.988 \times 10^{-16}$ eV s) agreed with literature values (30 keV and 4.136×10^{-15} eV s, respectively) to within uncertainty.

12. **A comparison of the K-shell characteristic x-rays of copper with the accepted values.**

Reading from our data (see Tables 1-3 we see that the highest counts are obtained at energies of 8.139 ± 0.174 keV and 9.010 ± 0.216 keV. This gives us percent errors of 1.26% for the K_α line and 1.18% for the K_β line. These values are well within experimental uncertainty. Accepted values for the characteristic energies of copper can be found in the CRC Handbook, available online.

13. **Discuss whether your results support or refute the hypothesis that x-rays are electromagnetic waves produced in the copper target of the x-ray tube by bremsstrahlung and K-shell interactions and diffracted by the LiF crystal.**

All the predictions made under the assumption of the correctness of electromagnetic and spectroscopic theory as presented in this lab are supported by our results to within experimental uncertainties.

14. **Using your results, explain why the third and fourth (if visible) peaks are more more likely to be n=2 versions of the first two peaks rather than separate characteristic x-rays.**

Because the K_β line is only $\sim$73 keV away from the ionization energy for copper we would expect that any higher energy lines would need to be very tightly packed and immediately following the K_β line and at lower angles, i.e., higher energy. This is not what we see from the second set of lines in our spectra. These lines are at higher angles and lower energy. Given that the K_α line is predicted to be a transition from n=2 to n=1 we would expect no characteristic lines to be found at lower energies than K_α. This leaves one possibility; the additional lines seen are the result of higher order diffraction.

15. **The method of tilting a grating at an extreme angle to effectively get a very small spacing between lines is called the grazing incidence technique. Figure 10 shows a tilted grating with distance AB between adjacent grooves. Calculate the angle θ required so that the effective grating spacing BC is 1/1000th the spacing of AB.**

The effective spacing, d_{eff}, is given by,

$$d_{eff} = dsin(\theta) \tag{5}$$

We want d_{eff} to be d/1000.

$$\frac{d}{1000} = dsin(\theta) \tag{6}$$

$$\theta = sin^{-1}\left(\frac{1}{1000}\right) = 0.057^\circ \tag{7}$$

So the minimum angle necessary to achieve a line spacing that is $1/1000^{th}$ of the original, AB, is 0.057°.

2θ	u(2θ)	θ	u(θ)	Count	u(Count)	λ (m)	u(λ) (m)	Energy (keV)	u(Energy) (keV)
20	1.0	10	0.5	217	14.7	6.99E-11	3.46E-12	17.7469	0.8783
21	1.0	10.5	0.5	247	15.7	7.34E-11	3.45E-12	16.9106	0.7962
22	1.0	11	0.5	300	17.3	7.68E-11	3.45E-12	16.1508	0.7251
23	1.0	11.5	0.5	279	16.7	8.03E-11	3.44E-12	15.4574	0.6630
24	1.0	12	0.5	300	17.3	8.37E-11	3.44E-12	14.8223	0.6085
25	1.0	12.5	0.5	288	17.0	8.71E-11	3.43E-12	14.2382	0.5605
26	1.0	13	0.5	303	17.4	9.06E-11	3.42E-12	13.6995	0.5178
27	1.0	13.5	0.5	256	16.0	9.40E-11	3.42E-12	13.2010	0.4798
28	1.0	14	0.5	253	15.9	9.74E-11	3.41E-12	12.7385	0.4459
29	1.0	14.5	0.5	255	16.0	1.01E-10	3.40E-12	12.3082	0.4153
30	1.0	15	0.5	257	16.0	1.04E-10	3.39E-12	11.9069	0.3878
31	1.0	15.5	0.5	232	15.2	1.08E-10	3.39E-12	11.5317	0.3629
32	1.0	16	0.5	227	15.1	1.11E-10	3.38E-12	11.1803	0.3403
33	1.0	16.5	0.5	240	15.5	1.14E-10	3.37E-12	10.8505	0.3197
34	1.0	17	0.5	286	16.9	1.18E-10	3.36E-12	10.5404	0.3009
35	1.0	17.5	0.5	261	16.2	1.21E-10	3.35E-12	10.2483	0.2836
36	1.0	18	0.5	256	16.0	1.24E-10	3.34E-12	9.9727	0.2678
37	1.0	18.5	0.5	263	16.2	1.28E-10	3.33E-12	9.7122	0.2533
38	1.0	19	0.5	225	15.0	1.31E-10	3.32E-12	9.4657	0.2399
39	1.0	19.5	0.5	269	16.4	1.34E-10	3.31E-12	9.2320	0.2275
39.5	1.0	19.75	0.5	491	22.2	1.36E-10	3.31E-12	9.1198	0.2217
40	1.0	20	0.5	1143	33.8	1.38E-10	3.30E-12	9.0103	0.2160
40.5	1.0	20.25	0.5	373	19.3	1.39E-10	3.30E-12	8.9037	0.2106
41	1.0	20.5	0.5	250	15.8	1.41E-10	3.29E-12	8.7997	0.2054
42	1.0	21	0.5	231	15.2	1.44E-10	3.28E-12	8.5993	0.1955
43	1.0	21.5	0.5	249	15.8	1.48E-10	3.27E-12	8.4085	0.1863
43.5	1.0	21.75	0.5	280	16.7	1.49E-10	3.26E-12	8.3164	0.1819
44	1.0	22	0.5	1056	32.5	1.51E-10	3.26E-12	8.2265	0.1777
44.5	1.0	22.25	0.5	3936	62.7	1.52E-10	3.25E-12	8.1387	0.1736
45	1.0	22.5	0.5	1552	39.4	1.54E-10	3.25E-12	8.0529	0.1697
45.5	1.0	22.75	0.5	253	15.9	1.56E-10	3.24E-12	7.9691	0.1658
46	1.0	23	0.5	235	15.3	1.57E-10	3.23E-12	7.8871	0.1621
47	1.0	23.5	0.5	200	14.1	1.61E-10	3.22E-12	7.7285	0.1551
48	1.0	24	0.5	209	14.5	1.64E-10	3.21E-12	7.5767	0.1485
49	1.0	24.5	0.5	207	14.4	1.67E-10	3.20E-12	7.4313	0.1423
50	1.0	25	0.5	228	15.1	1.70E-10	3.18E-12	7.2920	0.1365
51	1.0	25.5	0.5	210	14.5	1.73E-10	3.17E-12	7.1583	0.1310
52	1.0	26	0.5	189	13.7	1.76E-10	3.16E-12	7.0299	0.1258
53	1.0	26.5	0.5	191	13.8	1.80E-10	3.14E-12	6.9066	0.1209
54	1.0	27	0.5	204	14.3	1.83E-10	3.13E-12	6.7881	0.1163
55	1.0	27.5	0.5	207	14.4	1.86E-10	3.12E-12	6.6740	0.1119
56	1.0	28	0.5	170	13.0	1.89E-10	3.10E-12	6.5642	0.1077
57	1.0	28.5	0.5	214	14.6	1.92E-10	3.09E-12	6.4585	0.1038
58	1.0	29	0.5	170	13.0	1.95E-10	3.07E-12	6.3566	0.1001
59	1.0	29.5	0.5	193	13.9	1.98E-10	3.06E-12	6.2583	0.0965
60	1.0	30	0.5	199	14.1	2.01E-10	3.04E-12	6.1634	0.0932

Table 1: Data for unattenuated x-rays.

2θ	u(2θ)	θ	u(θ)	Count	u(Count)	λ (m)	u(λ) (m)	Energy (keV)	u(Energy) (keV)
61	1.0	30.5	0.5	204	14.3	2.04E-10	3.03E-12	6.0719	0.0900
62	1.0	31	0.5	182	13.5	2.07E-10	3.01E-12	5.9835	0.0869
63	1.0	31.5	0.5	186	13.6	2.10E-10	3.00E-12	5.8980	0.0840
64	1.0	32	0.5	170	13.0	2.13E-10	2.98E-12	5.8155	0.0812
65	1.0	32.5	0.5	184	13.6	2.16E-10	2.96E-12	5.7356	0.0786
66	1.0	33	0.5	178	13.3	2.19E-10	2.95E-12	5.6583	0.0760
67	1.0	33.5	0.5	173	13.2	2.22E-10	2.93E-12	5.5835	0.0736
68	1.0	34	0.5	183	13.5	2.25E-10	2.91E-12	5.5110	0.0713
69	1.0	34.5	0.5	205	14.3	2.28E-10	2.90E-12	5.4408	0.0691
70	1.0	35	0.5	178	13.3	2.31E-10	2.88E-12	5.3728	0.0670
71	1.0	35.5	0.5	190	13.8	2.34E-10	2.86E-12	5.3069	0.0649
72	1.0	36	0.5	180	13.4	2.37E-10	2.84E-12	5.2429	0.0630
73	1.0	36.5	0.5	198	14.1	2.39E-10	2.82E-12	5.1809	0.0611
74	1.0	37	0.5	195	14.0	2.42E-10	2.81E-12	5.1207	0.0593
75	1.0	37.5	0.5	194	13.9	2.45E-10	2.79E-12	5.0623	0.0576
76	1.0	38	0.5	181	13.5	2.48E-10	2.77E-12	5.0055	0.0559
77	1.0	38.5	0.5	179	13.4	2.51E-10	2.75E-12	4.9504	0.0543
78	1.0	39	0.5	175	13.2	2.53E-10	2.73E-12	4.8969	0.0528
79	1.0	39.5	0.5	212	14.6	2.56E-10	2.71E-12	4.8449	0.0513
80	1.0	40	0.5	175	13.2	2.59E-10	2.69E-12	4.7943	0.0499
81	1.0	40.5	0.5	193	13.9	2.61E-10	2.67E-12	4.7451	0.0485
82	1.0	41	0.5	173	13.2	2.64E-10	2.65E-12	4.6973	0.0472
83	1.0	41.5	0.5	187	13.7	2.67E-10	2.63E-12	4.6508	0.0459
84	1.0	42	0.5	193	13.9	2.69E-10	2.61E-12	4.6056	0.0446
85	1.0	42.5	0.5	195	14.0	2.72E-10	2.59E-12	4.5615	0.0434
86	1.0	43	0.5	258	16.1	2.75E-10	2.57E-12	4.5187	0.0423
86.5	1.0	43.25	0.5	518	22.8	2.76E-10	2.56E-12	4.4977	0.0417
87	1.0	43.5	0.5	449	21.2	2.77E-10	2.55E-12	4.4769	0.0412
87.5	1.0	43.75	0.5	273	16.5	2.78E-10	2.54E-12	4.4565	0.0406
88	1.0	44	0.5	221	14.9	2.80E-10	2.53E-12	4.4363	0.0401
89	1.0	44.5	0.5	194	13.9	2.82E-10	2.51E-12	4.3967	0.0390
90	1.0	45	0.5	204	14.3	2.85E-10	2.48E-12	4.3582	0.0380
91	1.0	45.5	0.5	185	13.6	2.87E-10	2.46E-12	4.3207	0.0371
92	1.0	46	0.5	235	15.3	2.90E-10	2.44E-12	4.2841	0.0361
93	1.0	46.5	0.5	176	13.3	2.92E-10	2.42E-12	4.2485	0.0352
94	1.0	47	0.5	215	14.7	2.94E-10	2.40E-12	4.2137	0.0343
95	1.0	47.5	0.5	211	14.5	2.97E-10	2.37E-12	4.1799	0.0334
96	1.0	48	0.5	233	15.3	2.99E-10	2.35E-12	4.1469	0.0326
97	1.0	48.5	0.5	222	14.9	3.02E-10	2.33E-12	4.1147	0.0318
98	1.0	49	0.5	255	16.0	3.04E-10	2.30E-12	4.0833	0.0310
98.5	1.0	49.25	0.5	592	24.3	3.05E-10	2.29E-12	4.0679	0.0306
99	1.0	49.5	0.5	2502	50.0	3.06E-10	2.28E-12	4.0527	0.0302
99.5	1.0	49.75	0.5	1975	44.4	3.07E-10	2.27E-12	4.0377	0.0298
100	1.0	50	0.5	837	28.9	3.08E-10	2.26E-12	4.0229	0.0295
100.5	1.0	50.25	0.5	252	15.9	3.10E-10	2.25E-12	4.0083	0.0291

Table 2: Data for unattenuated x-rays.

2θ	$u(2\theta)$	θ	$u(\theta)$	Count	u(Count)	λ (m)	$u(\lambda)$ (m)	Energy (keV)	u(Energy) (keV)
101	1.0	50.5	0.5	219	14.8	3.11E-10	2.23E-12	3.9938	0.0287
102	1.0	51	0.5	224	15.0	3.13E-10	2.21E-12	3.9654	0.0280
103	1.0	51.5	0.5	205	14.3	3.15E-10	2.19E-12	3.9378	0.0273
104	1.0	52	0.5	196	14.0	3.17E-10	2.16E-12	3.9108	0.0267
105	1.0	52.5	0.5	196	14.0	3.19E-10	2.14E-12	3.8844	0.0260
106	1.0	53	0.5	216	14.7	3.22E-10	2.11E-12	3.8587	0.0254
107	1.0	53.5	0.5	194	13.9	3.24E-10	2.09E-12	3.8337	0.0248
108	1.0	54	0.5	238	15.4	3.26E-10	2.07E-12	3.8092	0.0242
109	1.0	54.5	0.5	204	14.3	3.28E-10	2.04E-12	3.7854	0.0236
110	1.0	55	0.5	255	16.0	3.30E-10	2.02E-12	3.7621	0.0230
111	1.0	55.5	0.5	193	13.9	3.32E-10	1.99E-12	3.7394	0.0224
112	1.0	56	0.5	231	15.2	3.34E-10	1.96E-12	3.7172	0.0219
113	1.0	56.5	0.5	212	14.6	3.36E-10	1.94E-12	3.6956	0.0213
114	1.0	57	0.5	214	14.6	3.38E-10	1.91E-12	3.6745	0.0208
115	1.0	57.5	0.5	213	14.6	3.40E-10	1.89E-12	3.6540	0.0203
116	1.0	58	0.5	208	14.4	3.41E-10	1.86E-12	3.6339	0.0198
117	1.0	58.5	0.5	243	15.6	3.43E-10	1.84E-12	3.6143	0.0193
118	1.0	59	0.5	221	14.9	3.45E-10	1.81E-12	3.5952	0.0189
119	1.0	59.5	0.5	225	15.0	3.47E-10	1.78E-12	3.5766	0.0184
120	1.0	60	0.5	219	14.8	3.49E-10	1.76E-12	3.5585	0.0179

Table 3: Data for unattenuated x-rays.

2θ	u(2θ)	θ	u(θ)	Count	u(Count)	λ (m)	u(λ) (m)	Energy (keV)	u(Energy) (keV)
20	1.0	10	0.5	224	15.0	6.99E-11	3.46E-12	17.7469	0.8783
21	1.0	10.5	0.5	199	14.1	7.34E-11	3.45E-12	16.9106	0.7962
22	1.0	11	0.5	205	14.3	7.68E-11	3.45E-12	16.1508	0.7251
23	1.0	11.5	0.5	194	13.9	8.03E-11	3.44E-12	15.4574	0.6630
24	1.0	12	0.5	181	13.5	8.37E-11	3.44E-12	14.8223	0.6085
25	1.0	12.5	0.5	146	12.1	8.71E-11	3.43E-12	14.2382	0.5605
26	1.0	13	0.5	140	11.8	9.06E-11	3.42E-12	13.6995	0.5178
27	1.0	13.5	0.5	129	11.4	9.40E-11	3.42E-12	13.2010	0.4798
28	1.0	14	0.5	122	11.0	9.74E-11	3.41E-12	12.7385	0.4459
29	1.0	14.5	0.5	105	10.2	1.01E-10	3.40E-12	12.3082	0.4153
30	1.0	15	0.5	93	9.6	1.04E-10	3.39E-12	11.9069	0.3878
31	1.0	15.5	0.5	92	9.6	1.08E-10	3.39E-12	11.5317	0.3629
32	1.0	16	0.5	79	8.9	1.11E-10	3.38E-12	11.1803	0.3403
33	1.0	16.5	0.5	90	9.5	1.14E-10	3.37E-12	10.8505	0.3197
34	1.0	17	0.5	75	8.7	1.18E-10	3.36E-12	10.5404	0.3009
35	1.0	17.5	0.5	68	8.2	1.21E-10	3.35E-12	10.2483	0.2836
36	1.0	18	0.5	63	7.9	1.24E-10	3.34E-12	9.9727	0.2678
37	1.0	18.5	0.5	78	8.8	1.28E-10	3.33E-12	9.7122	0.2533
38	1.0	19	0.5	60	7.7	1.31E-10	3.32E-12	9.4657	0.2399
38.5	1.0	19.25	0.5	55	7.4	1.33E-10	3.32E-12	9.3473	0.2336
39	1.0	19.5	0.5	72	8.5	1.34E-10	3.31E-12	9.2320	0.2275
39.5	1.0	19.75	0.5	66	8.1	1.36E-10	3.31E-12	9.1198	0.2217
40	1.0	20	0.5	226	15.0	1.38E-10	3.30E-12	9.0103	0.2160
40.5	1.0	20.25	0.5	74	8.6	1.39E-10	3.30E-12	8.9037	0.2106
41	1.0	20.5	0.5	68	8.2	1.41E-10	3.29E-12	8.7997	0.2054
42	1.0	21	0.5	51	7.1	1.44E-10	3.28E-12	8.5993	0.1955
43	1.0	21.5	0.5	71	8.4	1.48E-10	3.27E-12	8.4085	0.1863
43.5	1.0	21.75	0.5	56	7.5	1.49E-10	3.26E-12	8.3164	0.1819
44	1.0	22	0.5	163	12.8	1.51E-10	3.26E-12	8.2265	0.1777
44.5	1.0	22.25	0.5	537	23.2	1.52E-10	3.25E-12	8.1387	0.1736
45	1.0	22.5	0.5	208	14.4	1.54E-10	3.25E-12	8.0529	0.1697
45.5	1.0	22.75	0.5	50	7.1	1.56E-10	3.24E-12	7.9691	0.1658
46	1.0	23	0.5	51	7.1	1.57E-10	3.23E-12	7.8871	0.1621
47	1.0	23.5	0.5	52	7.2	1.61E-10	3.22E-12	7.7285	0.1551
48	1.0	24	0.5	42	6.5	1.64E-10	3.21E-12	7.5767	0.1485
49	1.0	24.5	0.5	38	6.2	1.67E-10	3.20E-12	7.4313	0.1423
50	1.0	25	0.5	43	6.6	1.70E-10	3.18E-12	7.2920	0.1365
51	1.0	25.5	0.5	37	6.1	1.73E-10	3.17E-12	7.1583	0.1310
52	1.0	26	0.5	39	6.2	1.76E-10	3.16E-12	7.0299	0.1258
53	1.0	26.5	0.5	50	7.1	1.80E-10	3.14E-12	6.9066	0.1209
54	1.0	27	0.5	50	7.1	1.83E-10	3.13E-12	6.7881	0.1163
55	1.0	27.5	0.5	42	6.5	1.86E-10	3.12E-12	6.6740	0.1119
56	1.0	28	0.5	40	6.3	1.89E-10	3.10E-12	6.5642	0.1077
57	1.0	28.5	0.5	46	6.8	1.92E-10	3.09E-12	6.4585	0.1038
58	1.0	29	0.5	45	6.7	1.95E-10	3.07E-12	6.3566	0.1001
59	1.0	29.5	0.5	44	6.6	1.98E-10	3.06E-12	6.2583	0.0965
60	1.0	30	0.5	39	6.2	2.01E-10	3.04E-12	6.1634	0.0932

Table 4: Data for x-rays attenuated by an acrylic slide.

2θ	u(2θ)	θ	u(θ)	Count	u(Count)	λ (m)	u(λ) (m)	Energy (keV)	u(Energy) (keV)
61	1.0	30.5	0.5	34	5.8	2.04E-10	3.03E-12	6.0719	0.0900
62	1.0	31	0.5	46	6.8	2.07E-10	3.01E-12	5.9835	0.0869
63	1.0	31.5	0.5	38	6.2	2.10E-10	3.00E-12	5.8980	0.0840
64	1.0	32	0.5	31	5.6	2.13E-10	2.98E-12	5.8155	0.0812
65	1.0	32.5	0.5	42	6.5	2.16E-10	2.96E-12	5.7356	0.0786
66	1.0	33	0.5	35	5.9	2.19E-10	2.95E-12	5.6583	0.0760
67	1.0	33.5	0.5	35	5.9	2.22E-10	2.93E-12	5.5835	0.0736
68	1.0	34	0.5	46	6.8	2.25E-10	2.91E-12	5.5110	0.0713
69	1.0	34.5	0.5	40	6.3	2.28E-10	2.90E-12	5.4408	0.0691
70	1.0	35	0.5	48	6.9	2.31E-10	2.88E-12	5.3728	0.0670
71	1.0	35.5	0.5	41	6.4	2.34E-10	2.86E-12	5.3069	0.0649
72	1.0	36	0.5	33	5.7	2.37E-10	2.84E-12	5.2429	0.0630
73	1.0	36.5	0.5	36	6.0	2.39E-10	2.82E-12	5.1809	0.0611
74	1.0	37	0.5	44	6.6	2.42E-10	2.81E-12	5.1207	0.0593
75	1.0	37.5	0.5	31	5.6	2.45E-10	2.79E-12	5.0623	0.0576
76	1.0	38	0.5	42	6.5	2.48E-10	2.77E-12	5.0055	0.0559
77	1.0	38.5	0.5	31	5.6	2.51E-10	2.75E-12	4.9504	0.0543
78	1.0	39	0.5	32	5.7	2.53E-10	2.73E-12	4.8969	0.0528
79	1.0	39.5	0.5	31	5.6	2.56E-10	2.71E-12	4.8449	0.0513
80	1.0	40	0.5	29	5.4	2.59E-10	2.69E-12	4.7943	0.0499
81	1.0	40.5	0.5	36	6.0	2.61E-10	2.67E-12	4.7451	0.0485
82	1.0	41	0.5	21	4.6	2.64E-10	2.65E-12	4.6973	0.0472
83	1.0	41.5	0.5	28	5.3	2.67E-10	2.63E-12	4.6508	0.0459
84	1.0	42	0.5	34	5.8	2.69E-10	2.61E-12	4.6056	0.0446
85	1.0	42.5	0.5	35	5.9	2.72E-10	2.59E-12	4.5615	0.0434
86	1.0	43	0.5	47	6.9	2.75E-10	2.57E-12	4.5187	0.0423
86.5	1.0	43.25	0.5	93	9.6	2.76E-10	2.56E-12	4.4977	0.0417
87	1.0	43.5	0.5	103	10.1	2.77E-10	2.55E-12	4.4769	0.0412
87.5	1.0	43.75	0.5	52	7.2	2.78E-10	2.54E-12	4.4565	0.0406
88	1.0	44	0.5	43	6.6	2.80E-10	2.53E-12	4.4363	0.0401
89	1.0	44.5	0.5	37	6.1	2.82E-10	2.51E-12	4.3967	0.0390
90	1.0	45	0.5	25	5.0	2.85E-10	2.48E-12	4.3582	0.0380
91	1.0	45.5	0.5	34	5.8	2.87E-10	2.46E-12	4.3207	0.0371
92	1.0	46	0.5	33	5.7	2.90E-10	2.44E-12	4.2841	0.0361
93	1.0	46.5	0.5	39	6.2	2.92E-10	2.42E-12	4.2485	0.0352
94	1.0	47	0.5	41	6.4	2.94E-10	2.40E-12	4.2137	0.0343
95	1.0	47.5	0.5	43	6.6	2.97E-10	2.37E-12	4.1799	0.0334
96	1.0	48	0.5	35	5.9	2.99E-10	2.35E-12	4.1469	0.0326
97	1.0	48.5	0.5	42	6.5	3.02E-10	2.33E-12	4.1147	0.0318
98	1.0	49	0.5	59	7.7	3.04E-10	2.30E-12	4.0833	0.0310
98.5	1.0	49.25	0.5	162	12.7	3.05E-10	2.29E-12	4.0679	0.0306
99	1.0	49.5	0.5	403	20.1	3.06E-10	2.28E-12	4.0527	0.0302
99.5	1.0	49.75	0.5	365	19.1	3.07E-10	2.27E-12	4.0377	0.0298

Table 5: Data for x-rays attenuated by an acrylic slide.

2θ	$u(2\theta)$	θ	$u(\theta)$	Count	u(Count)	λ (m)	$u(\lambda)$ (m)	Energy (keV)	u(Energy) (keV)
100	1.0	50	0.5	87	9.3	3.08E-10	2.26E-12	4.0229	0.0295
100.5	1.0	50.25	0.5	31	5.6	3.10E-10	2.25E-12	4.0083	0.0291
101	1.0	50.5	0.5	35	5.9	3.11E-10	2.23E-12	3.9938	0.0287
102	1.0	51	0.5	31	5.6	3.13E-10	2.21E-12	3.9654	0.0280
103	1.0	51.5	0.5	30	5.5	3.15E-10	2.19E-12	3.9378	0.0273
104	1.0	52	0.5	40	6.3	3.17E-10	2.16E-12	3.9108	0.0267
105	1.0	52.5	0.5	29	5.4	3.19E-10	2.14E-12	3.8844	0.0260
106	1.0	53	0.5	28	5.3	3.22E-10	2.11E-12	3.8587	0.0254
107	1.0	53.5	0.5	32	5.7	3.24E-10	2.09E-12	3.8337	0.0248
108	1.0	54	0.5	27	5.2	3.26E-10	2.07E-12	3.8092	0.0242
109	1.0	54.5	0.5	46	6.8	3.28E-10	2.04E-12	3.7854	0.0236
110	1.0	55	0.5	27	5.2	3.30E-10	2.02E-12	3.7621	0.0230
111	1.0	55.5	0.5	29	5.4	3.32E-10	1.99E-12	3.7394	0.0224
112	1.0	56	0.5	33	5.7	3.34E-10	1.96E-12	3.7172	0.0219
113	1.0	56.5	0.5	23	4.8	3.36E-10	1.94E-12	3.6956	0.0213
114	1.0	57	0.5	27	5.2	3.38E-10	1.91E-12	3.6745	0.0208
115	1.0	57.5	0.5	38	6.2	3.40E-10	1.89E-12	3.6540	0.0203
116	1.0	58	0.5	29	5.4	3.41E-10	1.86E-12	3.6339	0.0198
117	1.0	58.5	0.5	32	5.7	3.43E-10	1.84E-12	3.6143	0.0193
118	1.0	59	0.5	33	5.7	3.45E-10	1.81E-12	3.5952	0.0189
119	1.0	59.5	0.5	39	6.2	3.47E-10	1.78E-12	3.5766	0.0184
120	1.0	60	0.5	38	6.2	3.49E-10	1.76E-12	3.5585	0.0179

Table 6: Data for x-rays attenuated by an acrylic slide.