

RLC Resonant Circuits

Equipment

- Series and parallel RLC circuit board
- Oscilloscope
- B & K 3011 function generator
- set of connecting leads (2)

Preparation

Review the theory of RLC resonant circuits. Study the principles of AC signals and their application to resistors, inductors, and capacitors.

Purpose

- To learn to construct series and parallel RLC circuits.
- To be able to calculate through theory and measurements, the amplitude resonance and the phase resonance of them.
- To learn to measure phase resonance via x-y oscilloscope measurements.

Theory

The current delivered to homes and factories is an oscillating function of time. This is called **alternating current**, or **AC**. All the appliances connected to ordinary outlets in homes therefore involve circuits with oscillating currents. Furthermore, electronic devices, such as radio transmitters and receivers, involve a variety of circuits with oscillating currents of high frequency. Many of these circuits have natural frequencies of oscillation. Such circuits exhibit the phenomenon of **resonance** when the natural frequency matches the frequency of a signal applied to the circuit. For instance, the tuning of a radio relies on an oscillating circuit whose frequency of oscillation is adjusted by means of a variable capacitor so that it matches the frequency of the radio signal.

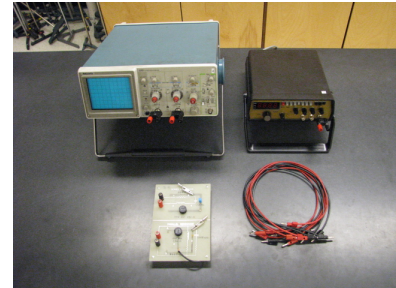


Figure 6.1: A photograph of the experimental setup.

A common circuit known as the **RLC circuit** exhibits the phenomenon of resonance. It is named RLC because it includes a resistor, capacitor, and inductor. There are two different kinds of RLC circuit. A **series RLC circuit** is explained in section I and the **parallel RLC circuit** is explained in section II.

I - Series RLC Circuits

The series RLC circuit used in this experiment is shown in Figure 6.2. The values of resistance, R_r , capacitance, C , and inductance, L , are written on the components themselves. The total resistance of the circuit, R , is the sum of the resistances of the resistor, R_r , and the inductor, R_L .

Assume that the internal resistance of the function generator can be ignored and choose the input signal to be $v(t) = E \sin(\omega t)$, where E is the maximum emf. The emf acts as a driving force that moves the charge in the circuit. An application of **Kirchoff's voltage law** gives

$$\sum V = 0 = V_L + V_C + V_R - E \sin(\omega t) \quad (6.1)$$

where

$$V_C = \frac{1}{C} \int i \cdot dt \quad (6.2)$$

$$V_L = L \frac{di}{dt} \quad (6.3)$$

and

$$V_R = iR \quad (6.4)$$

are the AC voltages across each component.

Substituting Equations 6.2, 6.3, and 6.4 into Equation 6.1 and rearranging gives the expression

$$E \sin(\omega t) = \frac{1}{C} \int i \cdot dt + L \frac{di}{dt} + iR \quad (6.5)$$

Differentiating both sides with respect to time, t , to remove the integral, yields the differential equation

$$E \omega \cos(\omega t) = \frac{i}{C} + L \frac{d^2 i}{dt^2} + R \frac{di}{dt} \quad (6.6)$$

The steady state solution of this equation is

$$i(t) = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}} \cdot \sin(\omega t - \phi) \quad (6.7)$$

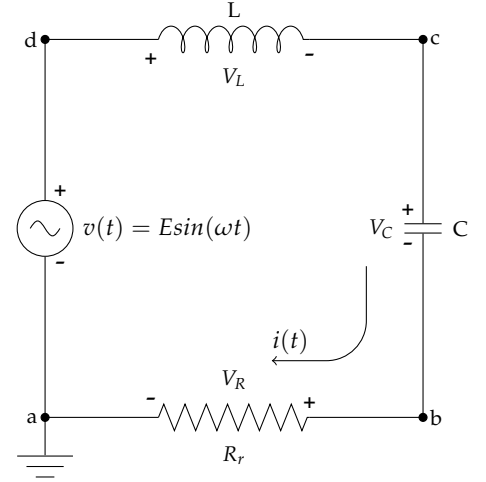


Figure 6.2: Series RLC circuit.

where

$$\phi = \tan^{-1} \left[\frac{1}{R} \left(\omega L - \frac{1}{\omega C} \right) \right] \quad (6.8)$$

The **angular frequency**, ω , is $2\pi f$, where f is the frequency in cycles per second or Hertz (Hz). Equations 6.7 and 6.8 imply that the current flowing in a series RLC circuit is a sine wave with an amplitude that changes with frequency and a phase difference from the signal generator that also changes with frequency. So the behaviour of the circuit strongly depends on the frequency of the applied voltage.

For Equation 6.7, the current, $i(t)$, becomes an extremum when $\omega^2 = 1/LC$. The condition when the amplitude of $i(t)$ is an extremum is known as **amplitude resonance**. For series RLC circuits, the extremum turns out to be a **maximum**. Thus, at amplitude resonance the current flowing in the circuit has maximum amplitude. At all other frequencies, the current flowing in the circuit is smaller. This property makes the series RLC circuit useful in filters where a response at one frequency is preferred over all other frequencies. This is what happens in radio receivers when one station is selected. The frequency corresponding to the carrier of that station is chosen to be the amplitude resonant frequency.

When $\omega^2 = 1/LC$, Equation 6.8 implies that $\phi = 0$. The condition when $\phi = 0$ is called **phase resonance**. At phase resonance, the signals at points **d** and **b** in Figure 6.2 are in phase. So there is no phase difference between the applied voltage and the current. In series RLC circuits, amplitude and phase resonance both happen at the **same frequency**. In other types of circuits this is not necessarily true.

The oscilloscope screen in Figure 6.3 shows two signals that are out of phase. A phase difference between two waveforms is usually reported as an angle. Since horizontal distances on an oscilloscope are in terms of time, a conversion from time to angle has to be made. This is done by noting that the time for a full cycle is 360° , so the actual phase difference as an angle is found by determining it as a fraction of one cycle and multiplying by 360° .

To measure a phase difference between two sine waves necessarily implies that they have the same frequency. The phase between two sine waves of different frequency is not constant. Also, notice that the two sinusoidal waveforms depicted in Figure 2 have the same amplitude. This is deliberate. An accurate phase measurement can only be made on an oscilloscope when the amplitudes of the signals are the same. Since in general, it is unlikely that any two signals have the same amplitude, a special control is available on the oscilloscope to shift amplitudes without requiring a change to the input signal. This control is called the **vertical calibration** adjustment. To make two signals have

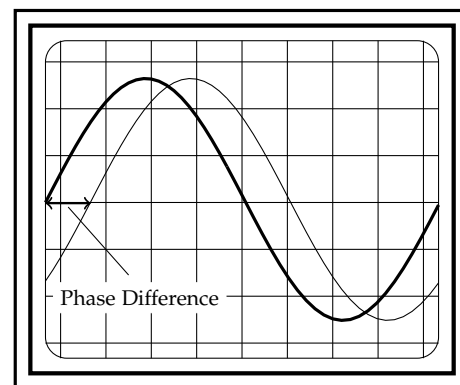


Figure 6.3: An illustration of phase difference.

the same displayed amplitude simply modify the vertical calibration of one channel. After the phase measurement is completed, remember to return the vertical calibration setting back to normal, otherwise further vertical measurements will be incorrect.

When $i(t)$ becomes an extremum, the input frequency matches the **natural frequency** of the circuit. This is manifested as a voltage extremum on the oscilloscope screen. Thus, when searching for frequencies at which $i(t)$ becomes an extremum, look for frequencies corresponding to a voltage extremum on the oscilloscope screen. The reason for voltage measurements rather than current measurements is due to the fact that the oscilloscope only displays voltages. Moreover, the signal at point **b** is the voltage across the resistor, which is directly proportional to the current by Ohm's law.

II - Parallel RLC Circuits

The parallel RLC circuit used in this experiment is depicted in Figure 6.4. The values of resistance, R_r , capacitance, C , and inductance, L , are written on the components themselves. Again, the total resistance, R , of the circuit is the sum of the resistances of the resistor, R_r , and the inductor, R_L .

Again, ignore the internal resistance of the function generator and choose the input signal to have the form $v(t) = E \sin(\omega t)$, where E is the maximum emf. This emf acts as a driving force that moves the charge in the circuit. However, in parallel circuits, the behavior of the circuit is given by two simultaneous equations. This is due to the fact that the positions of the inductor and the capacitor are in parallel to each other, thus, the input current splits. Let $i_1(t)$ be the current in the inductive branch and $i_2(t)$ be the current in the capacitive branch. Then **Kirchoff's current law** yields

$$i_{tot}(t) = i_1(t) + i_2(t) \quad (6.9)$$

Furthermore, **Kirchoff's voltage law** gives the equations

$$L \frac{di_1}{dt} + Ri_1 = E \sin(\omega t) \quad (6.10)$$

and

$$\frac{i_2}{C} = E \omega \cos(\omega t) \quad (6.11)$$

These equations have the solution

$$i_{tot}(t) = E \cdot \sqrt{\frac{(1 - \omega^2 LC)^2 + (\omega RC)^2}{R^2 + (\omega L)^2}} \cdot \sin(\omega t + \phi) \quad (6.12)$$

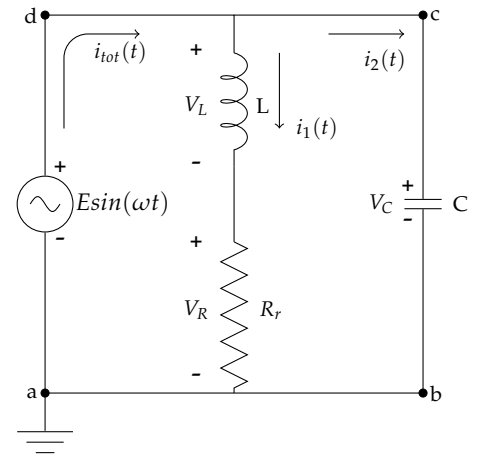


Figure 6.4: Parallel RLC circuit.

where

$$\phi = \tan^{-1} \left[\omega RC + \frac{\omega^3 L^2 C}{R} - \frac{\omega L}{R} \right] \quad (6.13)$$

In the case of the parallel RLC circuit, the amplitude and the phase resonances do occur at **different frequencies**. For the amplitude resonance frequency, ω_A , Equation 6.12 implies that

$$\omega_A^2 = \frac{1}{LC} \sqrt{1 + \frac{2R^2 C}{L}} - \frac{R^2}{L^2} \quad (6.14)$$

whereas, for the phase resonance frequency, ω_P , Equation 6.13 yields

$$\omega_P^2 = \frac{1}{LC} - \frac{R^2}{L^2} \quad (6.15)$$

This experiment examines the resonance phenomena of RLC circuits. The circuits to be examined are shown in Figures 6.2 and 6.4. The input frequency to the circuit is varied to determine at what frequencies resonances occur. Then, the experimental values obtained can be compared with the theoretical values to assess the validity of the resonance theory.

Experimental Procedure

1. Inspect the function generator and the dual channel oscilloscope. Make sure that the function generator is set to a sine wave output. Also, set the scope to show both channels on the screen simultaneously as this will be needed to verify the phase resonance of the circuits.
2. Before proceeding further, calculate the theoretical amplitude and phase resonant frequencies for both the series and parallel RLC circuits using the component values displayed on the circuit boards. This will help the experimental search for resonance frequencies.
3. For the **series RLC circuit**, setup the experiment as in Figure 6.5. Connect both the function generator and channel 1 of the oscilloscope between **d** and **a**, the input pins. Thus the trace on channel 1 will represent the input signal from the function generator. Connect channel 2 of the oscilloscope between **b** and **a**. The channel 2 trace will then be directly proportional to the current going through the circuit.

On the circuit board there is a red and a black connection terminal. When connecting the function generator and the dual channel oscilloscope onto the circuit board, the common ground on the circuit

board is the **RED** terminal, while the common ground on the instruments is the black terminal. The reason for this is that the resistor voltage needs to be measured so the oscilloscope ground has to be connected to point **a**.

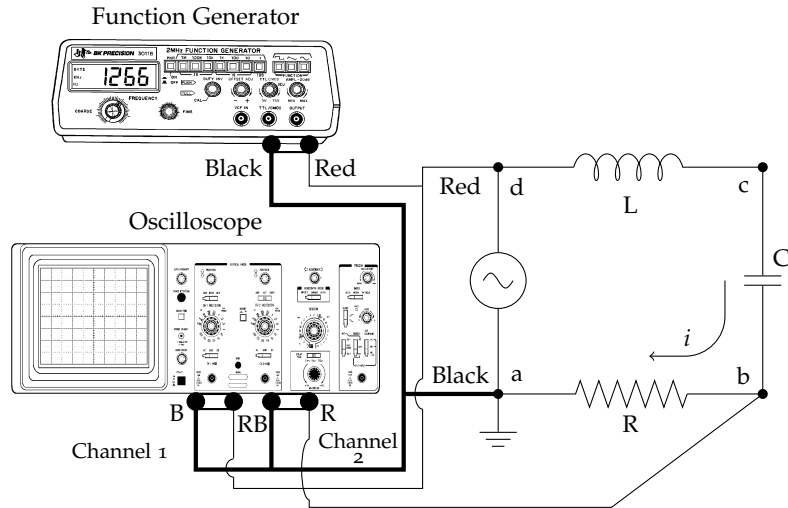


Figure 6.5: Series RLC circuit setup.

4. Find the amplitude/phase resonance by scanning through frequencies near the theoretical value. Stop varying the frequency when the channel 2 trace has its amplitude extremum. Record the experimental resonant frequency from the function generator. Verify that the frequency for amplitude resonance is the same as the frequency for phase resonance ($\phi = 0$).
5. For the resonant frequency and at least 20 other frequencies, some below and some above the resonant frequency, record the peak-to-peak voltage value from the oscilloscope.
6. For the **parallel RLC circuit**, setup the experiment as in Figure 6.6. Connect both the function generator and channel 1 of the oscilloscope between the red and black input pins (between point **a** and ground). Thus the trace on channel 1 represents the input signal from the function generator. The hidden resistor under the circuit board is a necessity for the output signal to be different from the input signal and to measure the current.
 - a) Connect channel 2 of the oscilloscope between **b** and ground. Remember that black lead goes into the black terminal. Also, short out the resistor positioned below the inductor using the shorting wire provided. This is necessary as amplitude resonance is better seen without the resistor since there is less voltage sharing between the inductor and the resistor and the hidden resistor has less effect.

- b) Find the amplitude resonance by scanning through frequencies near the theoretical value. Record the experimental resonant frequency from the function generator.
- c) For the resonant frequencies and at least 20 other frequencies, some below and some above the resonant frequencies, record the peak-to-peak voltage value from the oscilloscope.
- d) Remove the shorting wire from the resistor and find the phase resonance. The reason for removing the shorting clip is that the frequency difference between amplitude and phase resonance is magnified for a larger resistance in series with the inductor. There are two common methods for measuring a phase difference. One method was outlined previously. Another technique is to switch the oscilloscope timebase to x-y mode which plots channel 1 versus channel 2 on the screen. When the signals are out of phase a loop appears on the screen. When the two signals are in phase the loop becomes completely closed into a line of positive slope. When the two signals are 180° out of phase the loop again closes but this time into a line of negative slope.

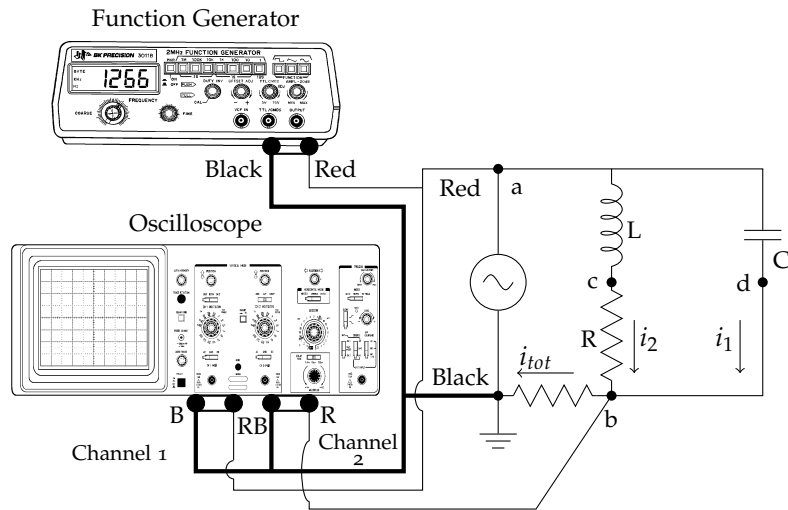


Figure 6.6: Parallel RLC circuit setup

Error Analysis

The internal resistance of the function generator is ignored in this experiment. Consider all values written on the circuit boards as exact. The error in an oscilloscope measurement is half the resolution of the measurement scale. Remember to record which voltage scale and time scale the oscilloscope is set on when recording data.

There are two errors associated with the function generator frequency. A **drift error** caused by temperature variations inside the function generator and a **quantization error** which is associated with any digital instrument. The drift error can be estimated by observing variations in the output frequency. While you are making measurements, take note of the maximum and minimum frequency values displayed on the function generator. The difference between the maximum and the minimum value divided by two gives the associated drift error. If the function generator has been on for some time then no drift in frequency values may be observable. In this case, the only error associated with the frequency will be the quantization error.

The error resulting from the conversion of a frequency to a digital display is called the quantization error. Manufacturers usually give the value of this error as plus or minus one number in the smallest digit of the display. The manual accompanying the B & K Precision Function Generator lists this value as ± 1 count.

Take the error in frequency to be the sum of the drift and quantization errors. If, for example, the frequency fluctuates from 767 to 771 Hz in the course of resonance measurements, then the frequency is recorded as 769 Hz and the associated drift error will be ± 2 Hz. To take into account the quantization error, add ± 1 of the last digit shown on the function generator. Thus, if the last digit shown on the function generator is a ones digit, then the quantization error is ± 1 Hz. Therefore, the total error would be ± 3 Hz.

To be handed in to your lab instructor

Prelab

1. Explain what resonance is and give four examples from physical systems.
2. For the series RLC circuit, derive $\omega^2 = 1/LC$ from Equations 6.7 and 6.8 with the condition that $\phi = 0$. Confirm that phase and amplitude resonance occur at the same frequency.
3. For the parallel RLC circuit, derive Equation 6.15 from Equation 6.13.
4. Justify the use of Equations 6.6, 6.9, 6.10, and 6.11 for the particular circuits, series and parallel.
5. Show that when $R=0$ in the parallel RLC circuit, both the amplitude resonant frequency and the phase resonant frequency are equal and have the same expression as the resonant frequency of the series RLC circuit.

Data Requirements

6. Calculation of the theoretical amplitude and phase resonance frequencies for both the series and parallel RLC circuits (procedure step 2).
7. Experimental amplitude and phase resonance frequencies, with uncertainties, for both the series and parallel RLC circuits (procedure steps 4, 7, and 9).
8. Table of peak-to-peak voltage measurements from the oscilloscope for at least 20 nonresonant frequencies for the series and parallel RLC circuits (procedure steps 5 and 8).
9. For the series RLC circuit, provide a table with numerical values for ω , f , and $V(p-p)$, and their uncertainties. Remember that function generators give frequency readings not angular frequencies.
10. For the series RLC circuit, plot $V(p-p)$ versus ω .
11. On the same plot, graph the corresponding theoretical curve of $\frac{2E \cdot R_r}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$ versus ω .
12. For the parallel RLC circuit, provide a table with numerical values for ω , f and $V(p-p)$, and their uncertainties.
13. For the parallel RLC circuit, plot $V(p-p)$ versus ω .

Discussion

14. Compare the theoretical resonant frequencies for both series and parallel RLC circuits with the measured values. Discuss possible sources of error.
15. Comment on the $V(p-p)$ versus ω plot of the series circuit. Indicate the theoretical and the experimental curves. Also, indicate the corresponding frequency extremum point(s) on your graph. Comment on how well your theoretical curves match the experimental points.
16. Do your results confirm or refute the theory of resonance? At resonance, is i at its extremum? Justify your conclusion with reference to your results. Consider uncertainties and errors involved in this experiment if you feel that they played significant role in obtaining your conclusion.

