Voltage Dividers and Voltage Sources

Equipment

- Anatek power supply
- Philips multimeter
- Sanwa 501 analog multimeter
- Set of 3 resistors
- Set of 2 resistors
- Eico 1171 decade resistor box
- 25 Ω Potentiometer
- Black box voltage divider
- Set of connecting leads (3)

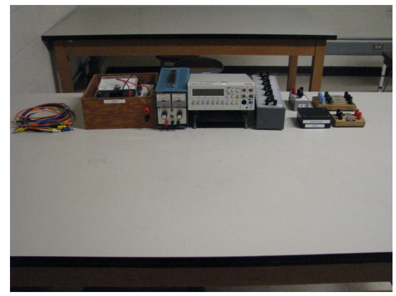

Figure 2.1: Equipment Setup

Preparation

Review the basic ideas of circuit analysis including Ohm's law and Kirchhoff's laws.

Goals of the Experiment

- To investigate voltage dividers and their uses.
- To understand the concepts of internal resistance and Thévenin equivalence.
- To get experience with potentiometers, power supplies, and voltmeters, and gain a better understanding of how they work.
- To observe the effects of circuit loading and how this affects measurements.

Theory

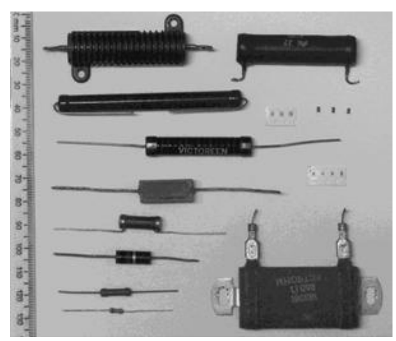

Figure 2.2: Examples of resistors of different shapes and sizes

The use of electric circuits today is a large part of everyday life. The radio, television, telephone and computer are all examples of devices that use electric circuits. In almost all circuits **resistors** are among the most common components. Figure 2.2 shows some of the many different kinds of resistors. The different shapes and sizes play a role in their behavior and application. Resistors are used to control the current and voltage in a circuit. They were extensively studied by physicists due to this property. Whether it is a complex circuit, such as the ones found in a computer, or one as simple as a battery connected to a light bulb, there are fundamental rules that all resistor circuits follow.

Georg Ohm (1789-1854) contributed largely to what is now known about resistors. He speculated how current might work and formulated the law governing resistors that now bears his name. Later, Gustav Kirchhoff (1824-1887) made further contributions to the understanding of electric circuits. He extended Ohm's work describing what are now call **Kirchhoff's Laws**, which explained how current and voltage in electric circuits are related. In 1883 Léon Thévenin (1857-1926), a French telegraph engineer, described what he thought to be a new theory of equivalent circuits. He showed that any resistor circuit could be simplified to make analysis easier. Coincidentally, the concept of equivalency in circuits had already been proposed almost 30 years earlier by the physicist Herman von Helmholtz (1821-1894). Thévenin was unaware of Helmholtz's work and both theories met with resistance during their time. It was due to Thévenin's engineering approach, and the growth of electrical engineering in the coming years, that his result is now called **Thévenin's Theorem**.

Many combinations of resistors exist in circuitry, but the **voltage divider** is one combination that is seen everywhere. The circuit in Figure 2.3 contains a voltage source of some kind, connected in series loop with two resistors. A voltmeter is in parallel with one of the resistors to measure the voltage across it. Each resistor in the loop drops a portion of the voltage. This resistor combination, which accomplishes the splitting of voltages, is called a voltage divider. It can be used to control the voltages coming out of, or going into, a particular system, or even be used as a tool for analysis of circuits.

In Figure 2.4 the connections of the voltage divider to the voltage source and voltmeter are explicitly shown. The voltage source is supplying a voltage with magnitude V_{in} into the voltage divider and the voltmeter reads V_{out} across R_2. This suggests the voltage divider can be generalized further by extracting it from the circuit. Figure 2.5 shows only the voltage divider. The input voltage, V_{in}, can be generalized to be any source of voltage such as a battery, power supply or another device. The output voltage, V_{out}, is what would be applied to any device or system connected to these two terminals.

For the voltage divider in Figure 2.5, V_{in} is split and V_{out} is some portion of V_{in}. This can be shown using Ohms and Kirchhoff's laws. From Ohm's law, the voltage drop across a resistor is the product of its resistance and the current through it. So the voltages across R_1 and

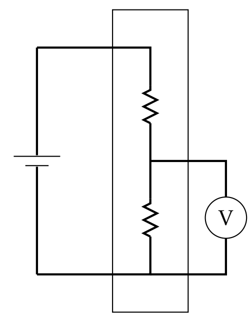

Figure 2.3: Basic voltage divider with power supply

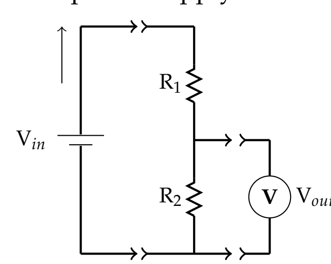

Figure 2.4: Connections for the voltage divider to voltage source and voltmeter

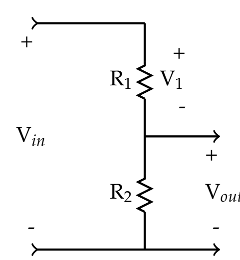

Figure 2.5: Isolated voltage divider

R_2 are V_1 and V_{out}, which are given by

$$V_1 = IR_1, \tag{2.1}$$

and

$$V_{out} = IR_2, \tag{2.2}$$

where I is the current flowing in the loop. From Kirchhoff's Voltage Law, it is expected that the applied voltage equals the voltage drops across all the resistors in series, so that

$$V_{in} = V_1 + V_{out}. \tag{2.3}$$

Using Equations 2.1, 2.2 and 2.3, it is seen that

$$V_{in} = I\left(R_1 + R_2\right). \tag{2.4}$$

Substituting for the current using Equation 2.2 gives the **voltage divider formula**

$$V_{out} = V_{in}\frac{R_2}{R_1 + R_2}. \tag{2.5}$$

Equation 2.5 implies that the ratio of $\frac{V_{out}}{V_{in}}$ is equal to the ratio of $\frac{R_2}{R_1+R_2}$. It also implies that V_{out} cannot be larger than V_{in} since the ratio of one resistor to two resistors can only be equal to or less than one. A special case is seen when R_1 is zero. The ratio becomes equal to one and V_{out} is equal to V_{in}.

The voltage divider in Figure 2.5 supplies a fixed voltage ratio. It is also possible for voltage dividers to supply a variable ratio. This is done with a device called a **potentiometer**. A potentiometer is a variable resistor whose resistance can be changed by turning a dial. Figure 2.6 shows a schematic of a potentiometer used as a variable voltage divider. The resistance of the entire potentiometer is R_x and R_P is the portion of this controlled by the dial. If the dial is adjusted so that R_p is equal to R_x, then V_{out} is equal to V_{in}. Likewise V_{out} becomes zero if the dial is adjusted so that R_P is zero.

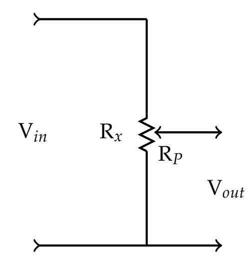

Figure 2.6: Variable voltage divider

Suppose two more resistors are added to the potentiometer to get the circuit shown in Figure 2.7. The derivation for Equation 2.5 involves a loop with two resistors, but in fact can work for any number of resistors. The total series resistance would be on the bottom of the ratio. The output voltage V_{out} could be taken across any or all of the resistors in the circuit and the value of these resistors would go on the top of the ratio. So, the output voltage will be

$$V_{out} = V_{in}\frac{R_P + R_2}{R_1 + R_2 + R_x}. \tag{2.6}$$

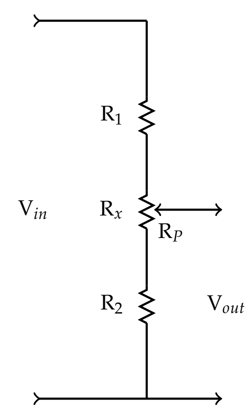

Figure 2.7: Voltage divider with three resistors

This setup produces a range of voltages for V_{out}, which can be controlled by the dial on the potentiometer. The output voltage has a minimum greater than zero and a maximum less than V_{in}, as determined by R_1 and R_2.The minimum output voltage, V_{min}, occurs when R_P is zero so

$$V_{min} = V_{in} \frac{R_2}{R_1 + R_2 + R_x}, \tag{2.7}$$

and the maximum output voltage, Vmax, arises when RP equals Rx which gives

$$V_{max} = V_{in} \frac{R_x + R_2}{R_1 + R_2 + R_x}. \tag{2.8}$$

Not only do voltage dividers exist explicitly as the circuits shown in Figures 2.3-2.7. They also exist implicitly whenever any two circuits are connected together. There is a division of voltage between the output of any circuit and the input of a second circuit. For example, imagine that a single resistor is connected to a power supply as shown in Figure 2.8. Here, the second circuit is composed of a single resistor R_L. Typically, R_L is called the **load**, or in this case the **load resistor**.

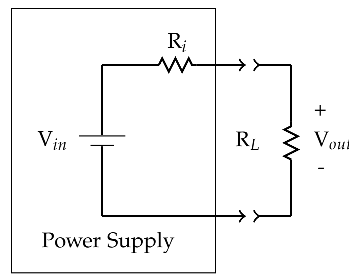

Figure 2.8: Power supply with internal resistor

Ideally, it would be expected that the entire voltage, V_{in}, would be developed across the load resistor. However, what happens in practice is that the voltage across the load, V_{out}, is always somewhat smaller than V_{in}. Moreover, the smaller the value of R_L, the larger the discrepancy between V_{out} and V_{in}. This situation can be neatly explained by the addition of an internal resistor, R_i. The voltage is now seen as being split between the load resistor R_L, and the internal resistance R_i. The basic voltage divider formula, Equation 2.5, can be used to find V_{out} as before to get

$$V_{out} = V_{in} \frac{R_L}{R_L + R_i}. \tag{2.9}$$

What is now observable is that the voltage across R_L is less than the voltage being supplied by the power supply. If R_i is much smaller than R_L, V_{out} is, to a reasonable approximation, equal to V_{in}. At the same time, if the value of R_i is close to the value of R_L, then V_{out} will only be a portion of V_{in}. If R_i were much larger than R_L this effect would be dramatically increased and almost no voltage would be across R_L. For perfect power supplies R_i is zero which means that V_{out} = V_{in} or any load resistor.

$$\frac{1}{V_{out}} = \frac{R_i}{V_{in}} \frac{1}{R_L} + \frac{1}{V_{in}}. \tag{2.10}$$

With Equation 2.10, R_i can be found by connecting a variable load and measuring V_{out} across different load resistances. It can be seen that plotting $\frac{1}{V_{out}}$ versus $\frac{1}{R_L}$ yields a straight line with a slope of $\frac{R_i}{V_{in}}$ from which R_i can be found.

So what is internal resistance exactly? Thévenin answered this by showing that from the point of view of any resistor in a circuit, the rest of the circuit is equivalent to a voltage source with a series internal resistance, just as in Figure 2.7. For example, the left side of Figure 2.9 shows a circuit containing a number of voltage sources and resistors. From the point of view of a single resistor such as R_5, the entire remaining part of the original circuit is equivalent to a single voltage source in series with a single resistor, as shown on the right side of Figure 2.9. This is called the Thévenin equivalent circuit. The voltage source, V_{th}, is called the Thévenin equivalent voltage and the resistor, R_{th}, is called the Thévenin equivalent resistance. Comparing this with Figure 2.8, it is seen that the Thévenin equivalent circuit is a power supply with an internal resistance driving a load consisting of the chosen component, in this case R_5.

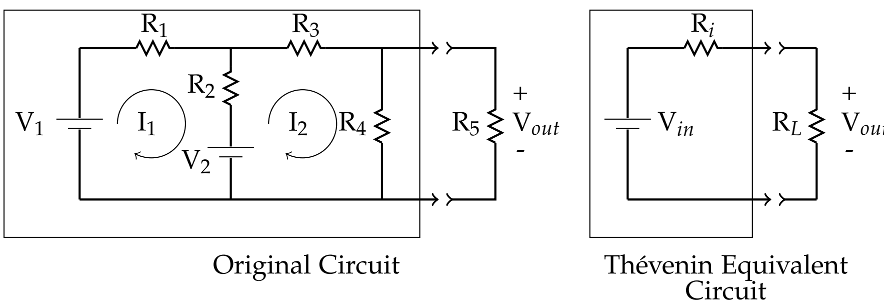

Figure 2.9: Thévenin equivalent circuit

There is a standard procedure for calculating V_{th} and R_{th}. The Thévenin equivalent voltage is equal to the voltage that would be found across the terminals without anything attached to it. Here, R_5 is the reference component so it is replaced with an open circuit. So the voltage across R_4 is the voltage being output by this circuit and is equal to V_{th}. In this example the Thévenin voltage is equal to the product of the current in the second loop, I_2, and the resistance R_4. If the value for all the voltage sources and resistors are known, the current across R_4 can be found by solving for the loop currents. This will yield the Thévenin voltage as being

$$V_{th} = I_2 R_4. \tag{2.11}$$

To find R_{th}, R_5 is again removed and all voltage sources in the circuit are replaced with their internal resistance. In this case the voltage sources are assumed to be ideal and are replaced with short circuits. The Thévenin resistance is then the equivalent resistance at the open terminals. For the circuit in Figure 2.9 this is

$$R_{th} = ((R_1 \parallel R_2) + R_3) \parallel R_4, \tag{2.12}$$

where $\parallel$ indicates that the resistors are to be added in parallel.

Another way to measure V_{th} and R_{th} is by knowing that the Thévenin equivalent circuit in Figure 2.9 is identical to the circuit depicted in Figure 2.8. Equation 2.10 can therefore be used to find V_{th} and R_{th} by varying R_5. Here, V_{out} is the voltage across R_5, and V_{in} and R_i are V_{th} and R_{th} respectively. Again, plotting $\frac{1}{V_{out}}$ versus $\frac{1}{R_5}$ should yield a straight line. This method produces a graph giving R_{th} and V_{th} from the slope and intercept.

So by attaching a load resistor to a circuit, in the case of Figure 2.9 this was R_5, the voltage across the resistor is dependent on the internal resistance as explained by the voltage divider rule. The effect that a load resistor has on circuits is called circuit loading. Imagine the basic voltage divider in Figure 2.5 where a voltmeter is used to measure V_{out}. Just like the power supply, voltmeters have some internal resistance. In the analysis of Equation 2.5, it was assumed that no current goes into the voltmeter, which is true only for an ideal voltmeter.

The effect that non ideal voltmeters have on voltage dividers is shown in Figure 2.10. This schematic is similar to Figure 2.4 with the exception that the voltmeter now has an internal resistance R_i. Ideally, the voltage across R_2 is the ratio of R_2 to the entire resistance, $R_1 + R_2$. Once a voltmeter is connected, the voltage is split between R_1 and the combined resistance of R_2 and R_i, in parallel. In Figure 2.10, V_r is the voltage read by the voltmeter and V_{out}, although not explicitly shown, is the actual voltage across R_2 without the voltmeter attached.This equivalent resistance is given by

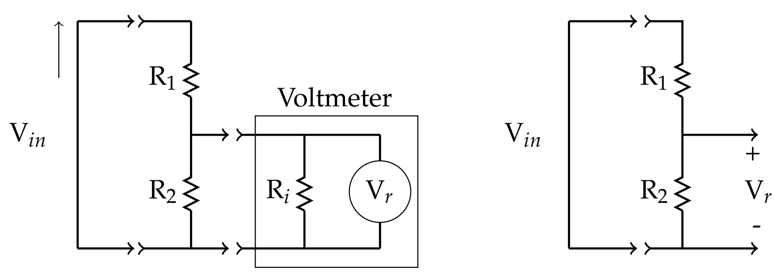

Figure 2.10: Internal Resistance of Voltmeter

$$R_{eq} = R_2 \parallel R_i = \frac{R_2 R_i}{R_2 + R_i}. \tag{2.13}$$

It is seen that if R_i is much larger than R_2, then R_{eq} is nearly equal to R_2, and Equation 2.5 still holds. This is what would be expected from an ideal voltmeter. If this is not the case then R_{eq} will be smaller than R_2. So a larger portion of voltage will develop across R_1 than before the voltmeter was connected. The voltage measured by the voltmeter will now be smaller than expected. This effect is called meter loading and is seen when R_2 in Equation 2.5 is replaced with R_{eq} to get

$$V_r = V_{in}\frac{R_{eq}}{R_{eq}+R_1} \neq V_{out}. \tag{2.14}$$

Equation 2.14 gives the output voltage, taking the effect of the voltmeter on the circuit into account. It can be used to calculate the internal resistance of the voltmeter. Furthermore, it is possible to rearrange Equation 2.14 to find the voltmeter internal resistance by

$$R_i = \frac{R_1 R_2}{R_2\left(\frac{V_{in}}{V_r}-1\right)-R_1}, \tag{2.15}$$

if R_1, R_2 and V_{in} are known, and V_r is the reading from the voltmeter.

It has been shown that voltage dividers occur in many circuits containing resistors and/or devices in series. In this experiment several different voltage dividers are examined. A simple voltage divider similar to the one seen in Figure 2.3 is constructed to examine the voltage divider formula. A potentiometer can then be inserted to check Equations 2.6-2.8. The power supply can then be probed to find its internal resistance and see the effects of having a power supply with a large internal resistance. Thévenin equivalence can be tested using the three methods previously described, calculation of theoretical values for V_{th} and R_{th}, taking direct measurements, and the use of a variable load resistor.

Two multimeters are used in this experiment, the Philips digital meter and the Sanwa analog meter. The Philips multimeter is close to ideal in many situations and is used for most of the measurements in this experiment. The Sanwa multimeter, which is farther from ideal, is used to see the effects of meter loading. The Philips multimeter also has the necessary precision required for observing the small internal resistance of the power supply.

In this experiment the Philips multimeter is used for taking voltage and resistance measurements. All connections are made to the red V Jack and the black COM jack. Switching between voltage and resistance readings is done via buttons along the bottom of the multimeter, under the display. The DC voltage function is selected by the V⎓ button and the resistance function is chosen with the 2W button. For some measurements it may be necessary to change the range of the meter. The three top right buttons on the instrument are the ranging controls. The range can be changed to automatic or manual with the AUT/MAN button. In manual mode the UP and DOWN buttons are used to change the range. Auto ranging is recommended unless otherwise specified in the procedure. When measuring resistance, be sure to disconnect any power supplies from the circuit. The internal resistance of the Philips voltmeter is 10 MΩ.

The Sanwa multimeter is an analog meter with a fairly low input resistance. This multimeter will be used to witness the effect of voltmeter loading. For this experiment the multimeter is in a box and all connections are done using the terminals on the outside of this box. The range can be set using the dial on the front of the device. Useful ranges for this experiment are 0.5 V, 2.5 V, 10 V and 50 V. The internal resistance of the Sanwa multimeter is directly proportional to the voltage range it is reading in and is 20 kΩ/V. For example the internal resistance at the 10 V setting should be 10 V x 20 kΩ/V, which is 200 kΩ.

Experimental Procedure

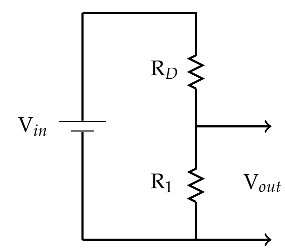

Figure 2.11: Step 1 Basic voltage divider

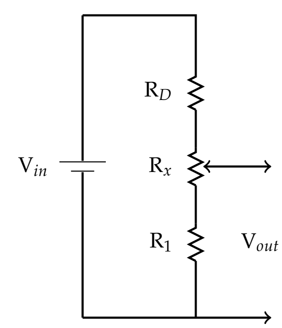

Figure 2.12: Voltage divider with three resistors

1. Construct the basic voltage divider depicted in Figure 2.11 with R_D = 100 Ω and R_1 = 20 Ω . With the power supply disconnected, measure the resistance of R_D and R_1 directly with the Philips multimeter. Turn on the power supply and connect the Philips multimeter across R_1 and take measurements of V_{out} while changing V_{in} from 1 V to 5 V. Take a minimum of 9 data points.

2. Measure the resistance of the potentiometer, R_x, using the Ohmmeter across the red and black terminals. Build the circuit in Figure 2.12 with R_D = 100 Ω and R_1 = 20 Ω. This circuit has a V_{min} and V_{max} which can be found experimentally. Set V_{in} to approximately 1 V and measure V_{in} with the Phillips multimeter. Disconnect the Phillips multimeter from the power supply and use it to take measurements of V_{out} for dial readings 1-10 on the potentiometer.

3. Connect R_D = 100 Ω to the positive output terminal of the power supply. This will show the effects of having a power supply with a high output resistance. Connect a resistance decade box in series to serve as a variable load. The setup should resemble Figure 2.8. Because the 100 Ω resistor is much larger than the actual internal resistance of the power supply, R_i is essentially equal to the value of R_D. Set the power supply to 1 V. Using the Philips multimeter, take at least 10 measurements of V_{out} while varying the load resistance from 10 Ω to 10 kΩ.

4. Remove R_D from the circuit in step 3. Now, R_i is the actual internal resistance of the power supply to be measured. Again set the power supply to 1 V and take measurements, with the Philips multimeter, of V_{out} for load resistance's ranging from 1 Ω to 10 Ω.

5. Choose one of the circuits in Figures 2.13-2.15 and build it. The Thévenin equivalent voltage and resistance can be found by direct measurements and by calculation. Set the power supply between 1 V and 5 V. Measure V_{th} directly with a voltmeter. Replace the power supply with a short and measure R_{th} directly with an ohmmeter. Remember to disconnect the power from a circuit before taking measurements of resistance with an ohmmeter.

6. Next, use an indirect approach to find the Thévenin equivalent voltage and resistance. Connect a resistance decade box across Vout for the circuit chosen in step 5. This is the load resistor R_L and creates a voltage divider between R_{th} and R_L. Take at least 10 measurements of V_{out} across the load resistor, as the resistance is varied from 10 Ω to 10 kΩ. Equation 2.10 can then be used to deduce the Thévenin equivalent voltage and resistance.

7. Similar to Figure 2.5, a black box containing a voltage divider is provided as seen in Figure 2.16. Using resistance measurements only, deduce the contents of the box.

8. Using the same black box as step 7, connect the positive end of the power supply to the red terminal of the black box, and the negative end to the black terminal. Set V_{in} to 3 V and take voltage readings across the red and white terminals. Call this voltage V_r. Measure V_r with the Philips multimeter at voltage ranges 3, 30 and 300.

9. Measure V_r with the Sanwa analog meter at voltage ranges 0.5, 2.5, 10 and 50.

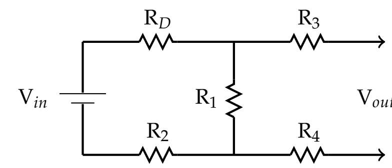

Figure 2.13: Step 5 Optional circuit diagram A

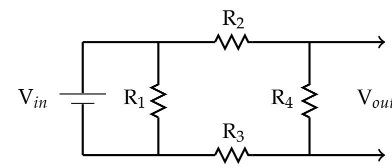

Figure 2.14: Step 5 Optional circuit diagram B

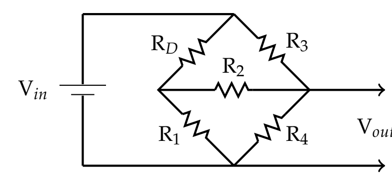

Figure 2.15: Step 5 Optional circuit diagram C

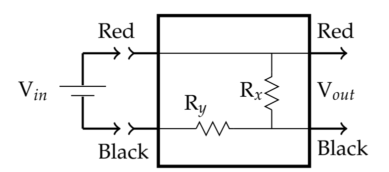

Figure 2.16: Voltage divider black box

Error Analysis

The wires used are assumed to be ideal, but they do have a small amount of resistance. For some steps such as finding the internal resistance of the power supply, the resistance in the wires may affect the value obtained. Another source of uncertainty arises from the self heating of the resistors when current passes through them. Also for a 5 digit multimeter like the Philips instrument, external noise can be seen in the variance of the lower digits. In this case, the error can be estimated as half the smallest non-varying digit. The uncertainties for voltage and resistance measurements for the multimeter are given in Tables 2.1 and 2.2, respectively. The Sanwa multimeter accuracy is given by half the resolution of the scale for that range.

To be handed in to the laboratory instructor

Range	% of reading	% of reading
300 mV	0.0025	0.0013
3 V	0.0020	0.0010
30 V	0.0025	0.0013
300 V	0.0025	0.0010

Table 2.1: The accuracy of the voltage measurements for the Philips multimeter.

Range	% of reading	% of reading
3 kΩ	0.01	0.0033
300 kΩ	0.01	0.0033
3 MΩ	0.02	0.0033

Table 2.2: The accuracy of the resistance measurements for the Philips multimeter.

Prelab

1. Design a fixed voltage divider for a V_{in} of 9 V and a V_{out} of 1 V with a total resistance (R1+R2) of 1 MΩ.

2. Design a variable voltage divider for a V_{in} of 12 V, with an output ranging from 1 V to 4 V.

3. Suppose a power supply outputs 10.0 V with no load and the output drops to 9.8 V with a 1 kΩ load. What is the internal resistance of the power supply.

4. Calculate the Thévenin equivalent circuit for one of the three circuits in Figures 2.13-2.15. Assume that R_D = 100 Ω, R_1 = 20.0 Ω, R_2 = 27.0 Ω, R_3 = 47.0 Ω, R_4 = 100 Ω and V_{in} = 3.0 V.

5. Using Figure 2.10, suppose R_1 = 110 kΩ, R_2 = 330 kΩ and V_{in} is 1 V. For this voltage divider calculate V_{out}. What will a voltmeter with 100 kΩ internal resistance measure for V_{out}.

Data Requirements

6. A table containing V_{out} and V_{in} for the basic voltage divider and values of R_D and R_1 as measured with the ohmmeter. Include all associated uncertainties.

7. A graph of V_{out} versus V_{in} for the basic voltage divider, including error bars.

8. The measured value of the potentiometer, R_x, as well as a table with V_{out} and the dial reading numbers from step 2 of the **Experimental Procedure**. Include all relevant uncertainties.

9. A graph of V_{out} versus dial reading and values of V_{min} and V_{max} for the variable divider.

10. A table with Vout, R_L, $\frac{1}{V_{out}}$, and $\frac{1}{R_L}$ from step 3 of the **Experimental Procedure**. Include uncertainties and the value of V_{in}.

11. A graph of $\frac{1}{V_{out}}$ versus $\frac{1}{R_L}$ including error bars. Present measured and calculated values of R_D.

12. A table with V_{out}, R_L, $\frac{1}{V_{out}}$ and $\frac{1}{R_L}$ from step 4 of the **Experimental Procedure**. Include uncertainties and the value of V_{in}.

13. A graph of $\frac{1}{V_{out}}$ versus $\frac{1}{R_L}$ including error bars. Show the derived value for the internal resistance of the power supply, R_i.

14. Direct measurements of R_{th} and V_{th} from step 5.

15. A table with V_{out}, R_L, $\frac{1}{V_{out}}$ and $\frac{1}{R_L}$ from step 6. Include uncertainties and the value of V_{in}.

16. A graph of $\frac{1}{V_{out}}$ versus $\frac{1}{R_L}$ and values of V_{th} and R_{th} obtained from the slope and intercept.

17. Measured values of R_x and R_y in the black box from step 7 of the **Experimental Procedure**.

18. Values for V_r from step 8 of the **Experimental Procedure** and the internal resistance of the Philips voltmeter at each range.

19. Values for V_r from step 9 of the **Experimental Procedure** and the internal resistance of the Sanwa voltmeter at each range.

Discussion

20. Based on the graph from the basic voltage divider, compare the ratio obtained from the graph to the calculated ratio.

21. Compare the behaviour of the variable voltage divider with the theoretically predicted performance.

22. For parts 3 and 4, compare the output voltage behaviour under varying loads for the high and low internal resistance power supplies. Why is a small internal resistance preferred for a power supply? What is the internal resistance of an ideal power supply?

23. For the circuit chosen in parts 5 and 6, compare the calculated, directly measured, and indirectly measured values of V_{th} and R_{th} of the Thévenin equivalent circuit.

24. Using the Measured values for R_x and R_y, compare the voltmeter readings taken with the Philips and Sanwa multimeter to the expected value. Which multimeter is better for taking voltage measurements and why? What is the internal resistance of an ideal voltmeter?

Voltage Dividers and Voltage Sources - Companion Guide

Equipment

- Anatek power supply
- Philips multimeter
- Sanwa 501 analog multimeter
- Set of 3 resistors
- Set of 2 resistors
- Eico 1171 decade resistor box
- 25 Ω Potentiometer
- Black box voltage divider
- Set of connecting leads (3)

Setup

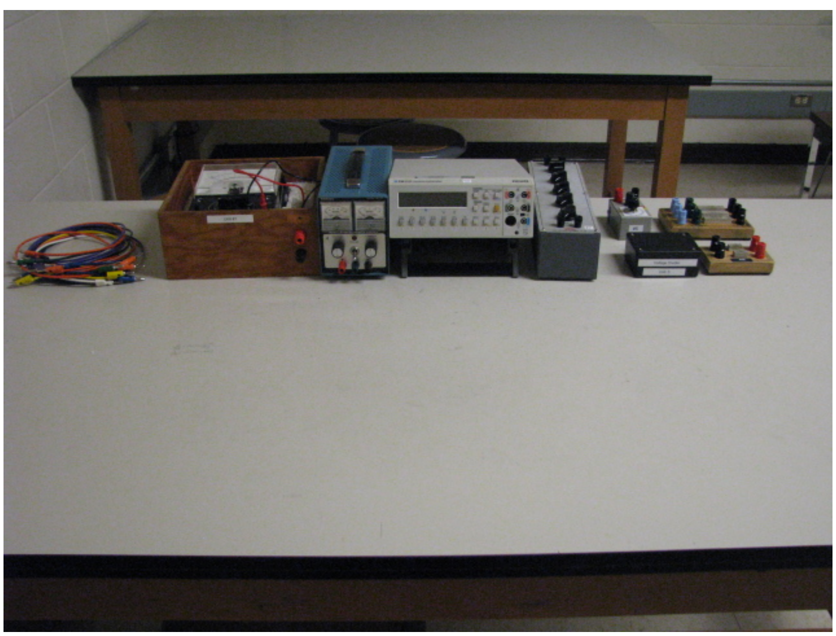

Figure 2.17: Equipment Setup

Setup bench as shown in Figure 2.17.

Maintenance

1. Periodically check the Eico 1171 decade resistor boxes to ensure that they have not been damaged.

2. Periodically check that the connecting leads are still working correctly

Critical Points of Failure

There are currently no known critical points of failure.

Notes to the Instructor

1. All connections should be made while power is off

2. The voltmeter on the Anatek have an error of 0.5 V. It is best to use the Phillips meter to measure the voltage of the Anatek whenever asked. To do this have the student attach the Phillips multimeter to the red and black terminals of the Anatek power supply, turn on the power and set voltage, record the voltage, and then turn off the power supply before disconnecting multimeter. Have the students then connect their circuit and turn the power supply back on without touching the voltage or current knobs. If voltage or current knobs are bumped when turning the Anatek back on have the student re-measure the voltage of the power supply.

3. Power supplies should not be on when measurements are not being taken. This goes especially for step 4 when measuring the internal resistance of the Anatek power supply. When the load resistance in low (ie in the ones and tens of Ω) the power through the resistors can get high, which can lead to a change in resistance, or even the melting of the resistor.

Prelab Questions

These are example answers and derivations to the prelab questions. These are not necessarily the only possible derivations or answers possible

1.
$$V_{out} = V_{in}\frac{R_2}{R_1 + R_2} \tag{2.16}$$

   $R_2 = 111.1\ k\Omega$, and $R_1 = 888.9\ k\Omega$

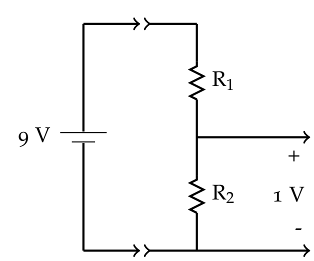

Figure 2.18: Circuit diagram for question 1

2.
$$V_{min} = V_{in}\frac{R_2}{R_1 + R_2 + R_x}, \tag{2.17}$$

$$V_{max} = V_{in}\frac{R_x + R_2}{R_1 + R_2 + R_x}. \tag{2.18}$$

Rearrange Equations 2.17 and 2.18 and plug in given values of V_{in}, V_{max}, and V_{min} to obtain

$$R_1 + R_2 + Rx = 12R_2, \tag{2.19}$$

from Equation 2.17, and

$$R_1 + R_2 + Rx = 3R_2 + 3R_x, \tag{2.20}$$

from Equation 2.18. Divide Equations 2.19 by 2.20 and simplify to obtain

$$R_2 = \frac{1}{3}R_x. \tag{2.21}$$

plug the result into either Equation 2.19 or 2.20 and simplify to obtain

$$R_1 = \frac{8}{3}R_x \tag{2.22}$$

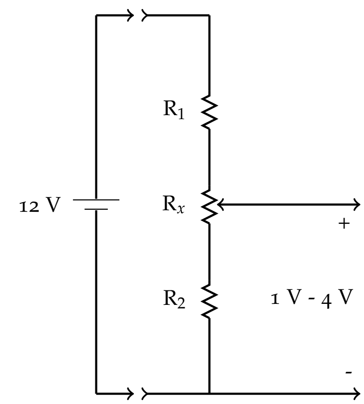

Figure 2.19: Circuit diagram for question 2

3.
$$V_{out} = V_{in}\frac{R_L}{R_L + R_i} \tag{2.23}$$

In this case V_{out}=9.8 V, V_{in}=10.0 V, and R_L=1 kΩ.

$$R_i = V_{in}\frac{R_L}{V_{out}} - R_L = 20\ \Omega \tag{2.24}$$

4. To find V_{th} determine the voltage across the output with no load. To find R_{th} replace the battery with short and calculate resistance across the output. $R_D = 100\ \Omega, R_1 = 20.0\ \Omega, R_2 = 27.0\ \Omega, R_3 = 47.0\ \Omega, R_4 = 100\ \Omega, V_{in} = 3.0\ V$. Note that $\|$ indicates that the resistors are to be added in parallel.

(a)
$$V_{th} = V_1 = \frac{V_{in}}{(R_D + R_1 + R_2)}R_1 = 0.41\ V \tag{2.25}$$

$$R_{th} = [(R_D + R_2) \parallel R_1] + R_3 + R_4 = 164\ \Omega \tag{2.26}$$

(b)

$$V_{th} = V_4 = \frac{V_{in}}{(R_2 + R_3 + R_4)} R_4 = 1.7\ V \tag{2.27}$$

$$R_{th} = (R_2 + R_3) \parallel R_4 = 42.5\ \Omega \tag{2.28}$$

(c) Try and find the voltage drop across V_4, but to do this first find the current through V_4.

$$V_4 - V_1 + V_2 = R_4(I_2 + I_3) - R_1(I_1 - I_3) + R_2 I_3 = 0 \tag{2.29}$$

$$V_{in} - V_3 - V_4 = V_{in} - R_3 I_2 - R_4(I_2 + I_3) = 0 \tag{2.30}$$

$$V_{in} - V_D - V_1 = V_{in} - R_D I_1 - R_1(I_1 - I_3) = 0 \tag{2.31}$$

Using Equations 2.29-2.31 solve for I_2 in term of resistance and V_{in}. Use the result to determine I_3 and then $I_2 + I_3$ each in terms of resistances and V_{in}.

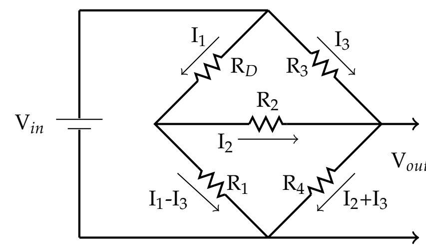

Figure 2.20: Setting up Kirchhoff's laws for finding V_{th} of optional diagram C in question 4c

$$V_{th} = V_4 = R_4(I_2 + I_3) = 1.4\ V \tag{2.32}$$

$$R_{th} = [(R_D \parallel R_1) + R_2] \parallel (R_3 \parallel R_4) = 18.5\ \Omega \tag{2.33}$$

5.

$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2} = 0.75\ V \tag{2.34}$$

$$V_r = V_{in} \frac{R_{eq}}{R_{eq} + R_1} = 0.41\ V,\ where\ R_{eq} = R_2 \parallel R_i \tag{2.35}$$

Data Requirements

6. Table of data collect in procedure step #1, along with sample calculations.

$$u(R_D) = (0.0001)(0.10039k\Omega) + (0.000033)(3k\Omega) \tag{2.36a}$$
$$u\left(R_D\right) = 2E^{-4}\ k\Omega \tag{2.36b}$$
$$R_D = 100.4\ \Omega \pm 0.1\ \Omega \tag{2.36c}$$

$$u(V_{out}) = (0.000020)(0.42906\ V) + (0.00010)(3\ V) \tag{2.37a}$$
$$u(V_{out}) = 3.09E^{-4}\ V \tag{2.37b}$$

V_{in} (V)	$u(V_{in})$ (V)	V_{out} (V)	$u(V_{out})$ (V)
1.0	0.5	0.224505	1E-5
1.5	0.5	0.32841	4E-5
2.0	0.5	0.42906	4E-5
2.5	0.5	0.50662	4E-5
3.0	0.5	0.60073	4E-5
3.5	0.5	0.68801	4E-5
4.0	0.5	0.76444	5E-5
4.5	0.5	0.84698	5E-5
5.0	0.5	0.93345	5E-5

$R_1 = 21.3\ \Omega \pm 0.1\ \Omega$
$R_D = 100.4\ \Omega \pm 0.1\ \Omega$

7. Graph of the response of a basic voltage divider from step #1.

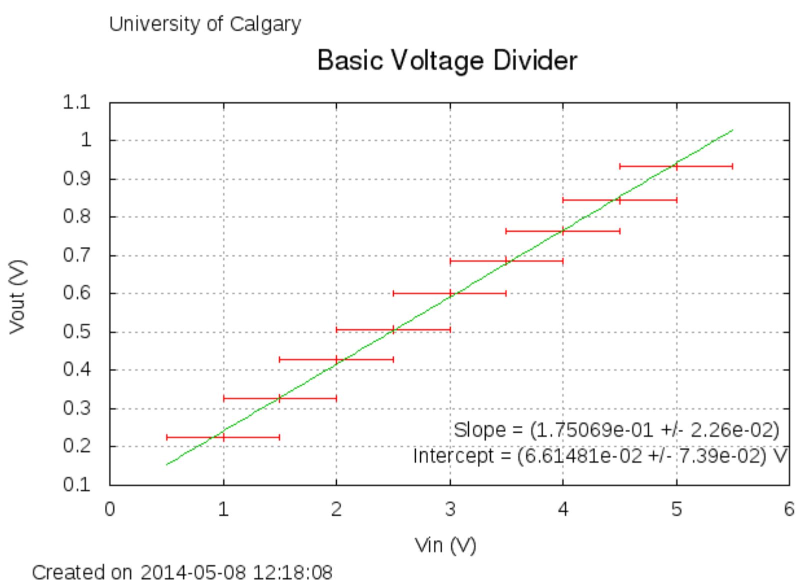

Figure 2.21: Response of a basic voltage divider

8. Table of data collected in procedure step #2.

Pot Reading	u(Pot Reading) (V)	Vout (V)	u(Vout) (V)
1	0	0.42730	4E-5
2	0	0.40349	4E-5
3	0	0.37975	4E-5
4	0	0.35317	4E-5
5	0	0.32618	4E-5
6	0	0.29983	4E-5
7	0	0.273036	1E-5
8	0	0.247345	1E-5
9	0	0.225489	1E-5
10	0	0.206331	9E-6

Table 2.3: Data for the variable voltage divider

$R_1 = 21.3\ \Omega \pm 0.1\ \Omega$
$R_D = 100.4\ \Omega \pm 0.1\ \Omega$
$R_x = 25.14\ \Omega \pm 0.1\ \Omega$
$V_{in} = 1.0\ V \pm 0.5\ V$

9. Graph of the response of a variable voltage divider from step #2.

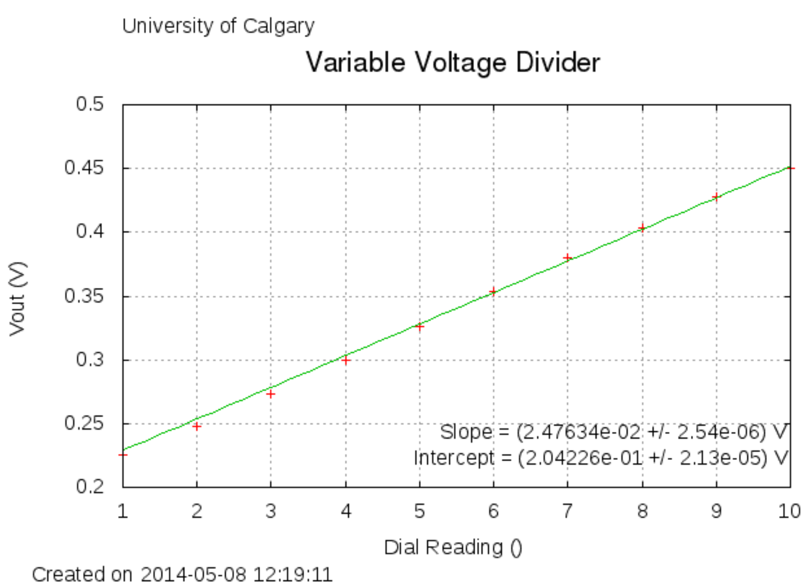

Figure 2.22: Response of a basic voltage divider

An experimental value of V_{min} can be found directly from the y-intercept of the graph. While V_{max} can be found from the slope and y-intercept of the graph.

$$V_{out} = m(DialReading) + b \tag{2.38a}$$

$$V_{min} = 0.20423 \pm 2E^{-5}\ V \tag{2.38b}$$
$$V_{max} = 2.47634E^{-2}(10) + 0.204226 = 0.45186\ V \tag{2.38c}$$
$$u(V_{max}) = \sqrt{(u(m)x)^2 + u(b)^2} \tag{2.38d}$$
$$u(V_{max}) = \sqrt{(2.54E^{-6}(10))^2 + (2.13E^{-5})^2} = 3E^{-5}\ V. \tag{2.38e}$$

10. Table of data collected from the poor powers supply with a high internal resistance in procedure step #3.

$$u\left(\frac{1}{V_{out}}\right) = \left|\frac{1}{V_{out}^2}\right| u\left(V_{out}\right) \tag{2.39a}$$
$$u\left(\frac{1}{V_{out}}\right) = \frac{1}{(0.43115)^2}(4E^{-5}) = 2E^{-4} \tag{2.39b}$$

R_L (Ω)	u(R_L) (Ω)	V_{out} (V)	u(V_{out}) (V)	$\frac{1}{R_L}$ ($\frac{1}{\Omega}$)	u($\frac{1}{R_L}$) ($\frac{1}{\Omega}$)	$\frac{1}{V_{out}}$ ($\frac{1}{V}$)	u($\frac{1}{V_{out}}$) ($\frac{1}{V}$)
10	0	0.097726	6E-6	0.1	0	10.2327	7E-4
20	0	0.179834	8E-6	0.05	0	5.5607	3E-4
50	0	0.35947	4E-5	0.02	0	2.7819	3E-4
100	0	0.53807	4E-5	0.01	0	1.8585	1E-4
200	0	0.71969	4E-5	0.005	0	1.38949	9E-5
500	0	0.89906	5E-5	0.002	0	1.11227	6E-5
1000	0	0.98074	5E-5	0.001	0	1.01964	5E-5
2000	0	1.02764	5E-5	0.0005	0	0.97310	5E-5
5000	0	1.05806	5E-5	0.0002	0	0.94513	5E-5
10000	0	1.06859	5E-5	0.0001	0	0.93581	4E-5

$R_i = 100.4\ \Omega \pm 0.1\ \Omega$

$V_{in} = 1.08259\ V \pm 5E^{-5}\ V$

Table 2.4: Data for the internal resistance of a poor power supply

11. Graph to determine the internal resistance of the poor power supply from step #3.

$$\frac{1}{V_{out}} = \frac{R_D}{V_{in}}\frac{1}{R_L} + \frac{1}{V_{in}} \tag{2.40a}$$
$$\frac{R_D}{V_{in}} = m \tag{2.40b}$$
$$R_D = mV_{in} = 92.8773(1.08259) = 100.55\ \Omega \tag{2.40c}$$
$$u(R_D) = \sqrt{[u(m)V_{in}]^2 + [u(V_{in})m]^2} \tag{2.40d}$$
$$u(R_D) = \sqrt{0.003^2 + 38^2} = 0.01\ \Omega \tag{2.40e}$$

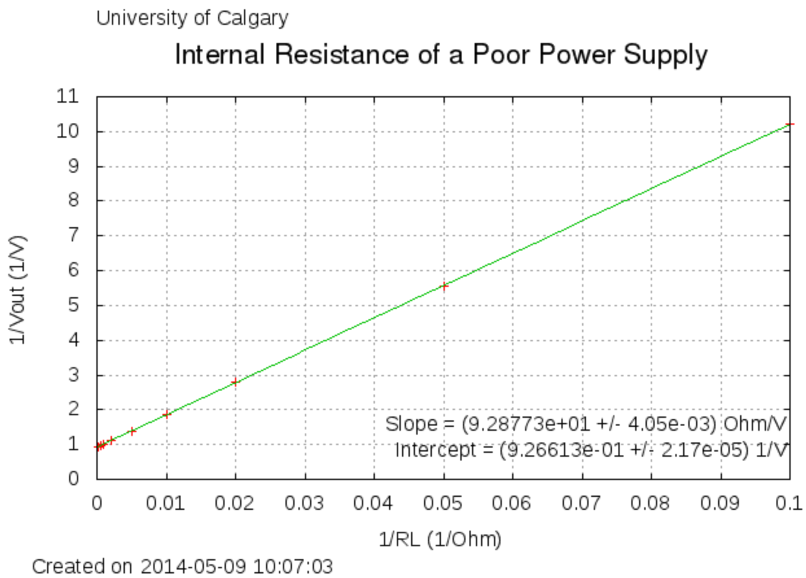

Figure 2.23: Internal resistance of a poor power supply

$R_D = 100.39\ \Omega \pm 0.1\ \Omega$, as measured directly by Phillips multimeter
$R_D = 100.55\Omega \pm 0.1\ \Omega$, as determined from the graph "Internal Resistance of a Poor Power Supply"

12. Table of data collected in procedure step #4.

R_L (Ω)	u(R_L) (Ω)	V_{out} (V)	u(V_{out}) (V)	$\frac{1}{R_L}$ ($\frac{1}{\Omega}$)	u($\frac{1}{R_L}$) ($\frac{1}{\Omega}$)	$\frac{1}{V_{out}}$ ($\frac{1}{V}$)	u($\frac{1}{V_{out}}$) ($\frac{1}{V}$)
1	0	1.04961	5E-5	1	0	0.95273	5E-5
2	0	1.06394	5E-5	0.5	0	0.93990	5E-5
3	0	1.06899	5E-5	0.3333	0	0.93546	4E-5
4	0	1.07166	5E-5	0.25	0	0.93313	4E-5
5	0	1.07316	5E-5	0.2	0	0.93183	4E-5
6	0	1.07424	5E-5	0.1667	0	0.93089	4E-5
7	0	1.07496	5E-5	0.1429	0	0.93027	4E-5
8	0	1.07559	5E-5	0.125	0	0.92972	4E-5
9	0	1.07597	5E-5	0.1111	0	0.92939	4E-5
10	0	1.07631	5E-5	0.1	0	0.92910	4E-5

$V_{in} = 1.08259\ V \pm 5e^{-5}\ \Omega$

Table 2.5: Data for the internal resistance of an Anatek power supply

13. Graph to determine the internal resistance of an Anatek power supply from step #4.

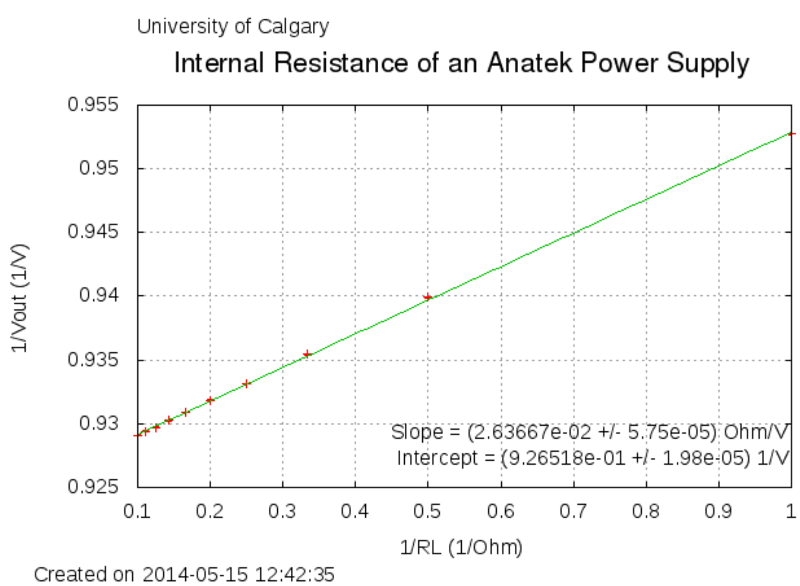

Figure 2.24: Internal resistance of an Anatek power supply

$R_i = 0.02854\ \Omega \pm 6E^{-5}\ \Omega$, as determined from the graph "Internal Resistance of a Poor Power Supply"

14. Direct measurement of R_{th} and V_{th} in step #5.

	V_{th} (V)	R_{th} (Ω)
Optional Circuit A	$V_{th} = 0.32829 \pm 4E^{-5}V$	$R_{th} = 166.2 \pm 0.1\Omega$
Optional Circuit B	$V_{th} = 1.35846 \pm 3E^{-5}V$	$R_{th} = 42.9 \pm 0.1\Omega$
Optional Circuit C	$V_{th} = 1.10700 \pm 3E^{-5}V$	$R_{th} = 18.7 \pm 0.1\Omega$

$V_{in} = 2.25425\ V \pm 3E^{-5}\ V$

Table 2.6: Direct measurement of the Thévenin equivalent resistances and voltages for optional circuits A, B, and C

15. Indirect measurement of R_{th} and V_{th} in step #6.

R_L (Ω)	u(R_L) (Ω)	V_{out} (V)	u(V_{out}) (V)	$\frac{1}{R_L}$ ($\frac{1}{\Omega}$)	u($\frac{1}{R_L}$) ($\frac{1}{\Omega}$)	$\frac{1}{V_{out}}$ ($\frac{1}{V}$)	u($\frac{1}{V_{out}}$) ($\frac{1}{V}$)
10	0	0.018897	4E-6	0.1	0	52.9185	1E-2
20	0	0.035905	5E-6	0.05	0	27.8513	4E-3
50	0	0.077231	6E-6	0.02	0	12.9482	1E-3
100	0	0.125039	7E-6	0.01	0	7.9975	4E-4
200	0	0.18238	8E-6	0.005	0	5.48306	3E-4
500	0	0.25021	1E-5	0.002	0	3.99664	2E-4
1000	0	0.285713	1E-5	0.001	0	3.50002	1E-4
2000	0	0.30762	4E-5	0.0005	0	3.25076	4E-4
5000	0	0.32249	4E-5	0.0002	0	3.10087	4E-4
10000	0	0.32780	4E-5	0.0001	0	3.05064	3E-4

$V_{in} = 2.35629\ V \pm 8E^{-5}\ V$

Table 2.7: Indirect measurement of the Thévenin equivalent resistances and voltages for optional circuits A

R_L (Ω)	$u(R_L)$ (Ω)	V_{out} (V)	$u(V_{out})$ (V)	$\frac{1}{R_L}$ ($\frac{1}{\Omega}$)	$u(\frac{1}{R_L})$ ($\frac{1}{\Omega}$)	$\frac{1}{V_{out}}$ ($\frac{1}{V}$)	$u(\frac{1}{V_{out}})$ ($\frac{1}{V}$)
10	0	0.256729	1E-5	0.1	0	3.8952	2E-4
20	0	0.43304	4E-5	0.05	0	2.3093	2E-4
50	0	0.73213	4E-5	0.02	0	1.3659	8E-5
100	0	0.95019	5E-5	0.01	0	1.0524	5E-5
200	0	1.11952	5E-5	0.005	0	0.89324	4E-5
500	0	1.25133	6E-5	0.002	0	0.79915	4E-5
1000	0	1.30259	6E-5	0.001	0	0.76770	3E-5
2000	0	1.32996	6E-5	0.0005	0	0.75190	3E-5
5000	0	1.34696	6E-5	0.0002	0	0.74241	3E-5
10000	0	1.35273	6E-5	0.0001	0	0.73925	3E-5

$V_{in} = 2.35629\ V \pm 8E^{-5}\ V$

Table 2.8: Indirect measurement of the Thévenin equivalent resistances and voltages for optional circuits B

16. Determine Thévenin voltage and resistance of optional circuits using the indirect method in step #6.

    The Thévenin voltage is the inverse of the intercept, and the Thévenin resistance the slope divided by the intercept.

$$\frac{1}{V_{out}} = \frac{R_{th}}{V_{th}}\frac{1}{R_L} + \frac{1}{V_{th}} \tag{2.41a}$$

$$V_{th} = \frac{1}{b} = \frac{1}{3.00128} = 0.33319\ V \tag{2.41b}$$

$$u(V_{th}) = \frac{u(b)}{b^2} = \frac{9.25E^{-5}}{(3.00128)^2} = 1E^{-5} \tag{2.41c}$$

$$R_{th} = \frac{m}{b} = \frac{498.169}{3.00128} = 165.99\ \Omega \tag{2.41d}$$

$$u(R_{th}) = \sqrt{\left(\frac{u(m)}{b}\right)^2 + \left(\frac{u(b)m}{b^2}\right)^2} \tag{2.41e}$$

$$= \sqrt{\left(\frac{2.85E^{-2}}{3.00218}\right)^2 + \left(\frac{9.25E^{-5}(4.98169E^2)}{(3.00128)^2}\right)^2} = 0.01\ \Omega \tag{2.41f}$$

where m is the slope of the graph, and b is the intercept.

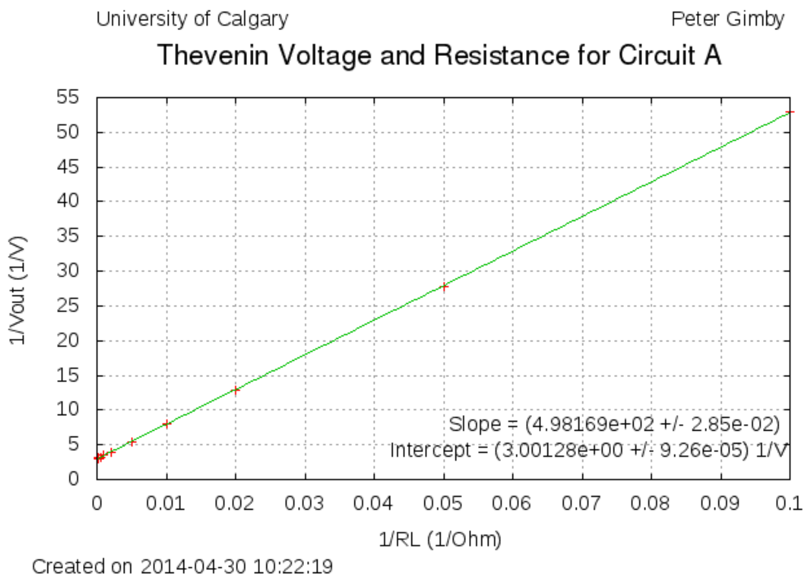

Figure 2.25: Thevein equivalents for circuit A

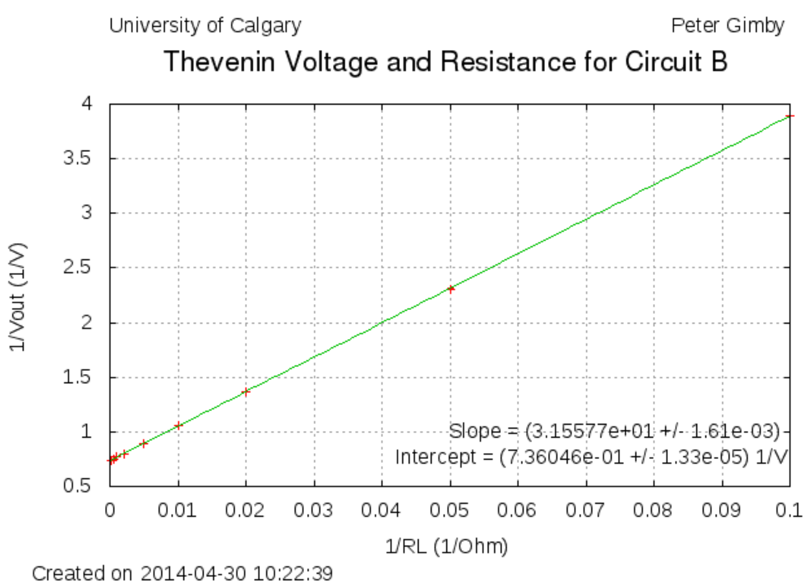

Figure 2.26: Thevein equivalents for circuit B

R_L (Ω)	u(R_L) (Ω)	V_{out} (V)	u(V_{out}) (V)	$\frac{1}{R_L}$ ($\frac{1}{\Omega}$)	u($\frac{1}{R_L}$) ($\frac{1}{\Omega}$)	$\frac{1}{V_{out}}$ ($\frac{1}{V}$)	u($\frac{1}{V_{out}}$) ($\frac{1}{V}$)
10	0	0.38485	4E-5	0.1	0	2.5984	3E-4
20	0	0.57225	4E-5	0.05	0	1.7475	1E-4
50	0	0.80546	5E-5	0.02	0	1.2415	7E-5
100	0	0.93158	5E-5	0.01	0	1.0734	6E-5
200	0	1.01206	5E-5	0.005	0	0.98808	5E-5
500	0	1.06651	5E-5	0.002	0	0.93764	5E-5
1000	0	1.08604	5E-5	0.001	0	0.92078	4E-5
2000	0	1.09613	5E-5	0.0005	0	0.91230	4E-5
5000	0	1.10226	5E-5	0.0002	0	0.90723	4E-5
10000	0	1.10434	5E-5	0.0001	0	0.90552	4E-5

$V_{in} = 2.35629\ V \pm 8E^{-5}\ V$

Table 2.9: Indirect measurement of the Thévenin equivalent resistances and voltages for optional circuits C

	V_{th} (V)	R_{th} (Ω)
Optional Circuit A	$V_{th} = 0.33319 \pm 1E^{-5}$	$R_{th} = 165.99 \pm 0.01$
Optional Circuit B	$V_{th} = 1.35861 \pm 2E^{-5}$	$R_{th} = 42.87 \pm 0.002$
Optional Circuit C	$V_{th} = 0.32829 \pm 2E^{-5}$	$R_{th} = 18.694 \pm 0.002$

Table 2.10: Indirect measurement of the Thévenin equivalent resistances and voltages for optional circuits A, B, and C

17. Directly measured values of R_x and R_y from step #7.

    R_x = 480.1 kΩ ± 0.2 kΩ
    R_y = 148.4 kΩ ± 0.2 kΩ

18. Table of data collected for the internal resistance of the Phillips multimeter in step #8.

    The voltage in was set to $V_{in} = 3.3759\ V\ \pm\ 1E^{-5}\ V$. The expected V_{out} can therefore be found using the voltage divider equation

$$V_{out} = V_{in}\frac{R_x}{R_x + Ry} = 2.579\ V \tag{2.42}$$

    The internal resistance of the meter can then be found by applying the meter reading to the equation

$$R_i - \frac{R_y R_x}{R_x\left(\frac{V_{in}}{V_r} - 1\right) - R_y} \tag{2.43}$$

V_r (V)	u(V_r) (V)	R_i (kΩ)	Range (V)	Theoretical R_i (kΩ)
2.54973	8E-5	9946.2	3	10000
2.5495	5E-4	9867.2	30	10000
2.548	3E-3	9381.0	300	10000

Table 2.11: Internal resistance of the Phillips multimeter

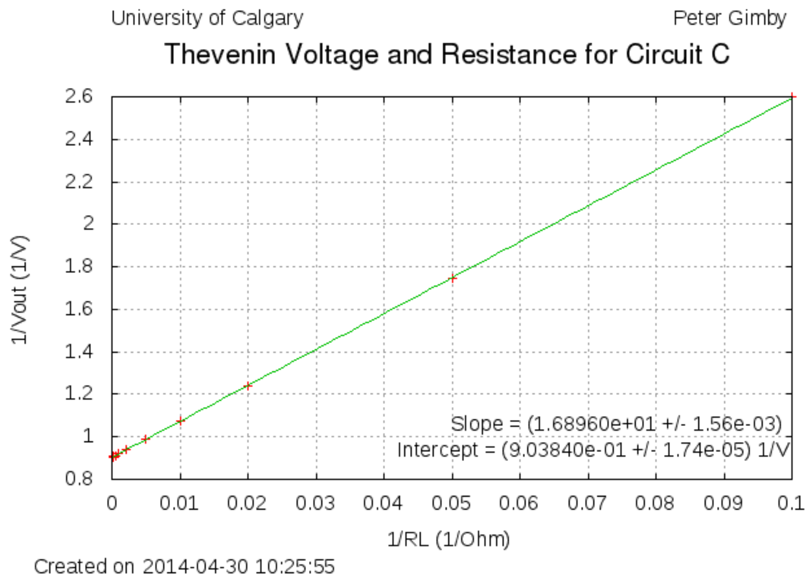

Figure 2.27: Thevein equivalents for circuit C

19. Table of data collected for the internal resistance of the Sanwa multimeter in step #9.

    $V_{in} = 3.3759\ V$
    $V_{out} = 2.579\ V$

V_r (V)	u(V_r) (V)	R_i (kΩ)	Range (V)	Theoretical R_i (kΩ)
0.205	0.005	9.79	0.5	10
0.78	0.005	49.2	2.5	50
1.7	0.1	219	10	200
2.3	0.5	935	50	1000

Table 2.12: Internal resistance of the Sanwa multimeter

Discussion

20. A plot of V_{in} verses V_{out} for the basic voltage divider would theoretically produce a linear graph with a slope and intercept of

$$m = \frac{R_1}{R_1 + R_D} = 0.18 \tag{2.44a}$$

$$u(m) = \sqrt{\left[\left(\frac{1}{R_D + R_1} + \frac{R_1}{(R_D + R_1)^2}\right) u(R_1)\right]^2 + \left[\left(\frac{R_1}{R_1 + R_D}\right) u(R_D)\right]^2} \tag{2.44b}$$

$$u(m) = 0.05 \tag{2.44c}$$

$$b = 0 \tag{2.44d}$$

$$u(b) = 0 \tag{2.44e}$$

The voltage divider ratio, as determined from the direct measurements of the resistors, is 0.18 ± 0.05. The voltage divider ration, as determined from the slope of the graph "Basic Voltage Divider" is 0.175 ± 0.023. These results are in agreement with each other. This would indicate that the voltage divider equation is a valid method of characterizing a voltage divider.

21. The values of V_{min} and V_{max}, as determined by using the measured values of V_{in}, R_1, R_D, and R_x. The error in V_{min} and V_{max} is overwhelmingly as a result of the error in V_{in}, therefore the values of the resistors we be considered as exact.

$$V_{min} = V_{in}\left(\frac{R_1}{R_1 + R_D + R_x}\right) \tag{2.45a}$$

$$= 1.0\left(\frac{21.3}{21.3 + 100.4 + 25.14}\right) = 0.16\ V \tag{2.45b}$$

$$u(V_{min}) = u(V_{in})V_{min} = (0.5)0.16 = 0.08\ V \tag{2.45c}$$

$$V_{max} = V_{in}\left(\frac{R_1 + R_x}{R_1 + R_D + R_x}\right) \tag{2.45d}$$

$$= 1.0\left(\frac{21.3 + 25.14}{21.3 + 100.4 + 25.14}\right) = 0.32\ V \tag{2.45e}$$

$$u(V_{max}) = u(V_{in})V_{max} = (0.5)(0.32) = 0.16\ V \tag{2.45f}$$

The values of V_{min} and V_{max}, as determined from the graph "Variable Voltage Divider".

$$V_{min} = 0.20424 \pm 2E^{-5}\ V \tag{2.46a}$$

$$V_{max} = 0.45186 \pm 3E^{-5}\ V \tag{2.46b}$$

$$\tag{2.46c}$$

Both methods of determining the minimum and maximum voltages from the variable voltage divider are in agreement, which would support the validity of the variable voltage divider equation.

22. The voltage supplied to a circuit by the poor power supply will be less than that supplied by a good power supply. There is a voltage drop caused by the internal resistance which is proportional, by Ohm's law, to the size of the internal resistor. Therefore the smaller the internal resistor the less the power supply will adversely effect the circuit. Ideally a power supply would have zero internal resistance.

23. Direct measurement of R_{th} and V_{th}.

An Ideal voltmeter will have an infinite internal resistance. The internal resistance of a voltmeter can be thought of as a resistor in parallel with the circuit. If the internal resistance of the voltmeter

Direct Method	V_{th} (V)	R_{th} (Ω)
Optional Circuit A	$V_{th} = 0.32829 \pm 4E^{-5} V$	$R_{th} = 166.2 \pm 0.k\Omega$
Optional Circuit B	$V_{th} = 1.35846 \pm 3E^{-5} V$	$R_{th} = 42.9 \pm 0.1\Omega$
Optional Circuit C	$V_{th} = 1.10700 \pm 3E^{-5} V$	$R_{th} = 18.7 \pm 0.1\Omega$
Indirect Method	**V_{th} (V)**	**R_{th} (Ω)**
Optional Circuit A	$V_{th} = 0.33319 \pm 1E^{-5}$	$R_{th} = 165.99 \pm 0.01$
Optional Circuit B	$V_{th} = 1.35861 \pm 2E^{-5}$	$R_{th} = 42.870 \pm 0.002$
Optional Circuit C	$V_{th} = 0.32829 \pm 2E^{-5}$	$R_{th} = 18.694 \pm 0.002$

$V_{in} = (2.25425 \pm 0.00003)\ V$

Table 2.13: Measurement of the Thévenin equivalent resistances and voltages for optional circuits A, B, and C

is not much much greater than the resistance of the circuit then the voltmeter will give an alternate route for the current. There is therefore a significant load put on the circuit by the voltmeter. This load will change the behaviour of the circuit.

In the case if the Sanwa meter the internal resistance is within a couple orders of magnitude of the resistance of the black box. This will cause a significant change in the output voltage of the black box voltage divider.

The effect that the Phillips voltmeter has on the circuit is much less, as can be seen by how the output voltages measured agree with the predicted values. This happens because the internal resistance of the Phillips multimeter is much much greater than the resistance of the black box voltage divider.

24.

$R_x = 480.1\ k\Omega \pm 0.2\ k\Omega$
$R_y = 148.4\ k\Omega \pm 0.2\ k\Omega$
$V_{in} = 3.3759\ V \pm 1E^{-5}\ V$
$V_{out} = 3.3759\ V \pm 1E^{-5}\ V$ (Expected)

Phillips Multimeter

Range (V)	R_i (MΩ)	V_{out} (V) (Measured)
3	10	2.54973
30	10	2.5495
300	10	2.548

Sanwa Multimeter

Range (V)	R_i (kΩ)	V_{out} (V) (Measured)
0.5	10	0.205
2.5	50	0.78
10	200	1.7
50	1000	2.3

Table 2.14: Actual output voltages from the black box voltage divider for the Phillips and Sanwa multimeters