Hall Effect and Magnetic Hysteresis

Equipment

- Electromagnet
- Anatek power supply (2)
- Fluke Multimeter (2)
- Philips PM2535 multimeter
- InAs Hall probe
- standard magnet
- laboratory stand
- fork clamp
- right angle clamp
- female BNC to male banana adaptor
- BNC coaxial cable
- set of connecting leads
- compass
- calipers

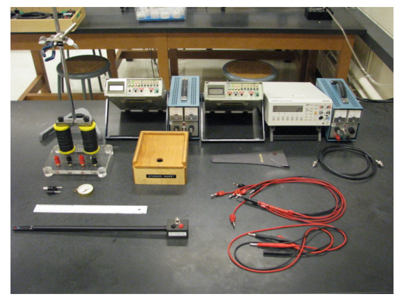

Figure 5.1: A photograph of the experimental setup.

Preparation

Review the fundamentals of the Hall effect. The derivation of the Hall voltage is provided in detail in the lab manual.

Purpose

To learn to calibrate a transducer, in this case a Hall probe, using known standards of magnetic field strength. To learn to identify hysteresis effects in physical systems. Lastly, be able to determine the density of charge carriers and drift velocity in a Hall probe.

Theory

Transducers are a general class of device that converts variations in a physical quantity into an electrical signal. One well studied example is a **magnetic field transducer**, called a **Hall probe**, which converts magnetic field intensity into potential differences. In this experiment the generation of Hall voltages through the Hall effect is examined. The Hall effect in an InAs semiconductor probe is explained in Section I. Hysteresis and the hysteresis curve of an iron core electromagnet, explained in section II, are then examined using a Hall probe. Lastly, the density of charge carriers and their drift velocity in a Hall probe are determined.

I - The Hall Effect

An investigation into the generation of potential differences across current carrying conductors immersed in a magnetic field, presently known as the **Hall Effect** was initiated by Edwin Hall, then a graduate student, in 1879. The effect is thought to have escaped the attention of other researchers mainly because the potential differences were so small and difficult to measure.

Charges moving with drift velocity , in the presence of both a magnetic field, $\vec{B}$, and an electric field, $\vec{E}$, experience a total force known as the Lorentz force, $\vec{F}$, given by

$$\vec{F} = \vec{F}_E + \vec{F}_M = q \cdot \vec{E} + q \cdot \vec{v} \times \vec{B} \quad (5.1)$$

where F_E and F_M are the electric and magnetic force vectors respectively and q is the elementary electronic charge. Since charge carriers in a conductor undergo random motion similar to the motion of molecules in a gas, the drift velocity, $\vec{v}$, is an average velocity of the charge carriers.

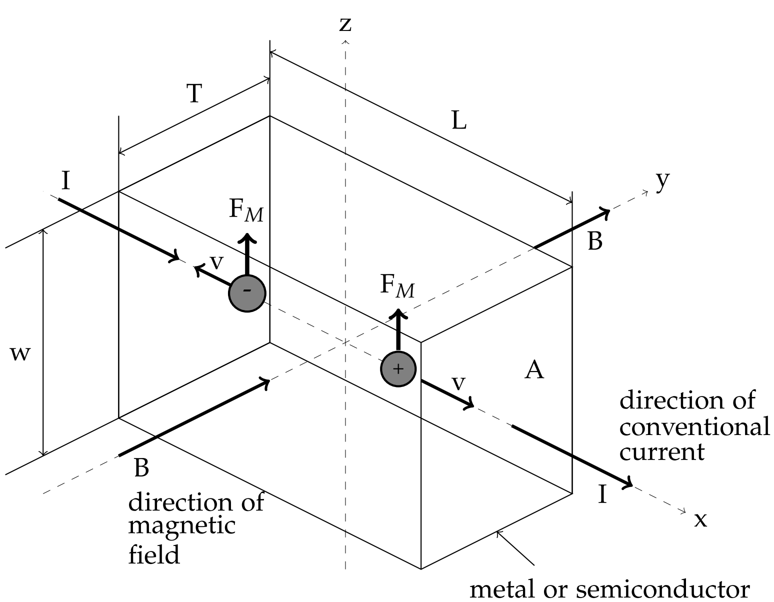

Figure 5.2: Current flow through a volume element of a metal or semiconductor.

As illustrated in Figure 5.2, current can be viewed either as a flow of positive charges in one direction or as a flow of negative charges in the opposite direction. If the polarity of the majority charge carriers is negative, the current will be due to the flow of electrons; otherwise it will be due to the flow of holes (positive charges). The direction of flow for positive charges is referred to as the direction of conventional current. Charges deflected by the action of the magnetic field, accumulate at the edges of the conductor and a transverse electric field, $\vec{E}$, appears across the conductor. The process of charge deflection and separation continues until the magnitudes of $\vec{F}_E$ and $\vec{F}_M$ have become equal and charges can no longer be deflected against the induced electric field. An equilibrium is established where the total Lorentz force, $\vec{F}$, is equal to zero and a small potential difference, called the **Hall voltage**, appears across the conductor. The polarity of the Hall voltage is determined by the polarity of the charge carriers alone because, as can be seen from Figure 5.2, the magnetic force on a charge carrier of either polarity is always in the same direction. The polarity of the Hall voltage is thus a function of the polarity of the majority charge carriers for that conductor. Another important factor is the relative abundance of carriers of either polarity. In a hypothetical situation, such as the one shown in Figure 5.2, where the ratio of the positive to negative carriers is one to one, the Hall voltage would vanish because of cancellation. Predominance of one type of current carrier, negative or positive is thus essential. The effect could not occur in semiconductors where both types of carriers appear in equal concentrations. Prior to Hall's work, conduction in metals was thought to have been caused entirely by a flow of negative charges (electrons). Experiments with various metals have shown, however, that this was not always true. It was discovered that current, in some metals, could be carried by positive charges (holes) not electrons. The phenomenon remained unexplained at the time and came to be known as the **Anomalous Hall Effect**. Semiconductors, unknown at the time of Hall's investigation, exhibit properties similar to those of metals. Here however, the current densities are lower and the Hall voltages are higher than in metals. Since higher voltages are easier to measure, semiconductor Hall probes are routinely used for the measurement of magnetic fields. The information required to find an expression for the Hall voltage is given in Equation 5.1. Since $\vec{F} = 0$ at equilibrium, it is true that

$$qvB = qE \tag{5.2}$$

so

$$vB = E \tag{5.3}$$

The **Hall voltage**, V_H, is then

$$V_H = Ew = vBw \tag{5.4}$$

since the potential difference, V_H, appearing across the conductor of width w, is given by the product of the electric field magnitude and the width of the conductor. Equation 5.4 can then be rearranged to read

$$B = KV_H \tag{5.5}$$

where K, a proportionality constant, is equal to $1/vw$. Equation 5.5 illustrates how the Hall voltage can be used in a transducer for the measurement of magnetic fields. If a semiconductor is placed in a magnetic field, then the Hall voltage developed across it can be measured directly with a volmeter. The magnetic field, B, is then found from Equation 5.5. Moreover, since Equation 5.5 is linear, once the proportionality constant K is known, any magnetic field can be determined. The procedure used to find K for a particular transducer is to subject the transducer to a known magnetic field and measure the Hall voltage. The proportionality constant for that transducer then is B/V_H. This procedure of **calibrating the transducer** must always be performed before a transducer can be used for making measurements.

Charge carrier density, n, defined as the number of charge carriers per unit volume, can be determined from the measurement of the current in a conductor, as current is related to the motion of charged particles. As a rule, charge densities tend to be higher in metals than in semiconductors.

For a volume element , like the one shown in Figure 5.3, it is true that

$$\Delta V = A\Delta x \tag{5.6}$$

So if the charge carrier density is constant,then ΔV contains $nA\Delta x$ charge carriers. A **charge element**, ΔQ, contained in the volume element, ΔV, is then given by

$$\Delta Q = nqA\Delta x \tag{5.7}$$

where q is the elementary electronic charge, 1.602×10^{-19} C.

For charges moving with drift velocity $\vec{v}$, Δx is the distance covered in a time interval Δt. Thus

$$\Delta x = v\Delta t \tag{5.8}$$

The charge element ΔQ can now be written as

$$\Delta Q = nqAv\Delta t \tag{5.9}$$

Dividing both sides of Equation 5.9 by Δt yields

$$\frac{\Delta Q}{\Delta t} = I = nqAv \tag{5.10}$$

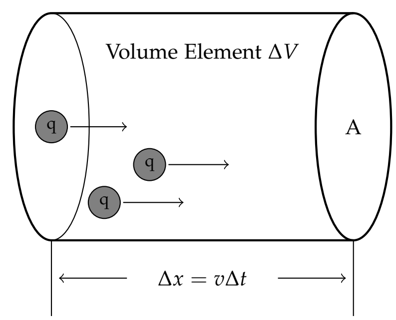

Figure 5.3: Charge flow through a volume element.

where I is the current flowing through the area A. Solving Equation 5.10 for drift velocity ν gives

$$\nu = \frac{I}{nqA} \tag{5.11}$$

Substituting an expression for the drift velocity ν from Equation 5.4 gives

$$V_H = \frac{IBw}{nqA} \tag{5.12}$$

As can be seen from Figure 5.2, A, the cross sectional area of the conductor, is equal to the product of its width, w, and thickness, T. Equation 5.12 can thus be simplified to

$$V_H = \frac{IB}{nqT} \tag{5.13}$$

The quantity $1/nq$ is known as the **Hall coefficient**, R_H. When the semiconductor is used as a probe to measure magnetic fields, it is excited with a constant current called the **control current**, I_c. So I is renamed to I_c and Equation 5.13 becomes

$$V_H = R_H \frac{I_c}{T} B \tag{5.14}$$

The sensor element of the Hall probe is made of indium-arsenide. To measure magnetic flux density, the probe is first energized by allowing a steady control current to flow through it. Figure 5.4 shows how the Hall sensor probe is connected for the measurements of magnetic fields. The probe is then immersed in a magnetic field such that its direction is perpendicular to the plane of the sensor element.

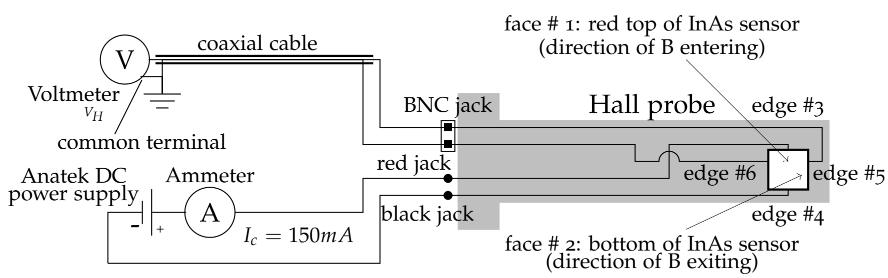

Figure 5.4: Schematic of the Hall probe sensor setup.

The Hall voltage that develops across the probe is small, of the order of a few millivolts, so it is good practice to shield it from sources of external noise, which is why a coaxial cable is used. The polarity of the Hall voltage depends on the type of charge carrier active in the semiconductor and can be determined from the polarity of the observed Hall voltage. The polarities of the electrical connections shown in Figure 5.4 are in agreement with the sign conventions introduced earlier in Figure 5.2.

The mounting of the semiconductor wafer inside the Hall probe makes it impractical to measure its dimensions directly. For the purposes of this experiment the following dimensions, given by the manufacturer, can be used in the calculations. The thickness of the semiconductor wafer, T, is equal to $(8 \pm 1) \times 10^{-5}$m. The width of the Hall plate, w, is equal to $(2.03 \pm 0.01) \times 10^{-3}$m. Its length, L, is equal to $(4.57 \pm 0.01) \times 10^{-3}$m.

Magnetic Hysteresis

All matter is made up of atoms that contain moving electrons. Associated with every atom, there is a tiny **magnetic moment** that is partly due to the orbital motion of the atomic electrons and partly due to their spin. In some materials the magnetic moments remain at random and the substance appears non-magnetic. In others, large numbers of magnetic moments are oriented in some sense or the presence of an external magnetic field causes them to orient in some sense. When this happens, the material becomes **magnetized** and begins to generate its own magnetic field.

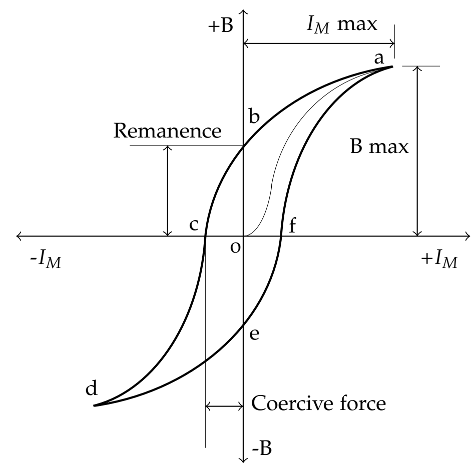

Figure 5.5: Hysteresis in a magnetized ferromagnetic material.

In ferromagnetic materials, strong interaction between atomic magnetic moments leads to an alignment of these moments in regions called **magnetic domains**, consisting of a large number of magnetic moments. The alignment within the magnetic domains exists even in the absence of an external magnetic field, but the domains are pointed randomly. When an external magnetic field is applied however, the domains tend to orient themselves parallel to the external field, producing a strong net magnetization effect. As the external field intensity is increased, more and more magnetic domains are aligned with the external field increasing the magnetization of the material. At some point virtually all magnetic domains are aligned and the sample reaches a state of **magnetic saturation**.

If a de-magnetizing field (magnetizing field of opposite polarity) is now applied to the sample, the de-magnetization of the sample lags behind the applied de-magnetizing field. This property, whereby the magnetic flux through the substance does not depend solely on the external magnetic field but also on the magnetic history, is referred to as the **magnetic hysteresis**. When such a substance is subjected to periodic changes in magnetization, the energy dissipated through the production of heat due to the realignment of the magnetic domains, is known as the magnetic hysteresis loss.

As indicated in Figure 5.5, initial magnetization of a ferromagnetic material corresponds to segment o-a on the curve. At point a, which corresponds to a maximum on the curve, the material reaches a state of positive magnetic saturation. At this point, all magnetic domains are aligned in the positive direction and increases in the intensity of the magnetizing current have no further effect on the degree of magnetization. Now, as the intensity of the magnetizing current decreases towards zero, the magnetic flux density does not decrease along path a-0. Instead, it decreases along path a-b. This is because some magnetic domains remain aligned and the material retains some degree of magnetization even in the absence of a magnetizing current. The property whereby ferromagnetic materials retain some of their flux density, is referred to as **remanence** or **residual magnetism**. To reduce the remanence to zero, a reverse magnetizing force must be applied. This is accomplished by reversing the magnetizing current (the reverse magnetizing current is sometimes called the de-magnetizing current) and increasing its intensity. With an increase in the reverse current, the flux density drops off to zero along path b-c. The de-magnetizing force required to accomplish this task is called the **coercive force**. As the de-magnetizing force increases beyond point c, the flux density begins to increase in the negative direction, completing segment c-d on the curve. At point d, which represents the minimum on the curve, negative magnetic saturation is reached and all magnetic domains are aligned in the opposite direction. At this point, further increases in the intensity of the de-magnetizing current have no effect on the state of magnetization. The return leg of the magnetization (re-magnetization) process, between points d-e-f-a, is just the reverse of the process described above. It is completed by first decreasing the de-magnetizing force to zero between points d-e, then increasing the magnetizing force in the positive direction between points e-a. The process is complete when the sample returns to its state of positive saturation at point a.

The magnetic field to be measured is generated in an **electromagnet**. Figure 5.6 is the the circuit diagram for the electromagnet, power supply, and ammeter. Direct current passing through the coils magnetizes the iron core of the electromagnet and the resultant magnetic flux density is measured in the air gap (see Figures 5.6 and 5.7). Since iron is ferromagnetic the characteristics of the magnetic field produced in it are determined by a magnetic hysteresis curve similar to the one shown in Figure 5.5.

Experimental Procedure

1. Fasten the Hall probe to the lab stand with the extension clamp. Connect the power supply to the control current inputs on the body of the Hall probe, as shown in Figure 5.4. Insert a dual banana plug across the voltage and common terminals of the multimeter. The

small rectangular tab on the body of the dual plug indicates ground or common polarity. Make the Hall voltage connection from the coaxial jack on the probe to the multimeter using a coaxial cable. Using a single connecting lead, connect the common terminal of the multimeter to the green grounding terminal of the power supply. This connection is necessary to ensure that the lower contact on the InAs probe is at ground potential while the control current potentials are floating. Do not ground either the positive or the negative terminal of the Anatek power supply as this would place one end of the Hall probe at the same ground potential as the negative end of the probe. As a result, the equipotential lines across the sensor would be skewed, producing a Hall voltage even in the absence of a magnetic field.

2. Have your lab instructor check your circuit before turning on any of the power supplies. Insert the Hall probe into the standard magnet (the magnitude of B_{cal} is stated on the magnet), center it in the gap, and at a constant control current of 150 mA, measure its Hall voltage output to find constant K from Equation 5.5. This calibrates the Hall probe so it can be used to measure other magnetic fields.

3. When K has been determined, set the air gap of the electromagnet to approximately 3.5 mm (see Figure 5.7). Place the Hall probe in the gap and take a series of Hall voltage versus electromagnet current readings (in steps of 0.1 A). Because of the residual magnetism in the core, take the readings for the full range of electromagnet currents in both directions. This means starting from $I = 0$ (Figure 5.5), increasing the current to $I = +I_M$, decreasing back to $I = 0$, reversing the electromagnet polarity and increasing to $I = -I_M$, decreasing back to $I = 0$, reversing the polarity on the electromagnet again, and increasing back to $I = +I_M$. To get stable Hall voltage readings, the electromagnet current should be adjusted gradually, in one direction only.

4. Using the calibrated Hall probe as your measuring instrument, set the magnetic intensity of the electromagnet to 0.2 T. It is necessary to use the same probe control current that was used for probe calibration. Keeping the magnetic field constant at 0.2 T, record a series of Hall voltage versus control current readings over a range of 0 to 150 mA.

5. Make a sketch of the Hall probe, magnetic field and control current orientations with respect to the coordinate axes. Use a compass to determine the direction of magnetic field if necessary. Using the information from Figures 5.2 and 5.4 and the polarity of the Hall voltage output, find which type of majority charge carrier is responsible

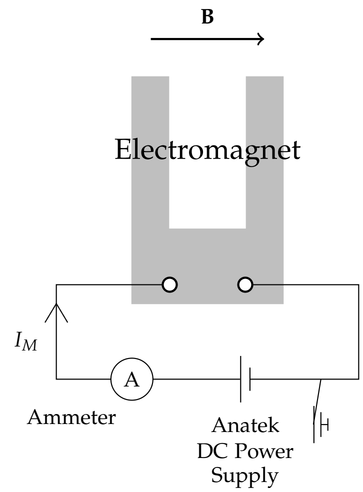

Figure 5.6: Circuit diagram for electromagnet power supply.

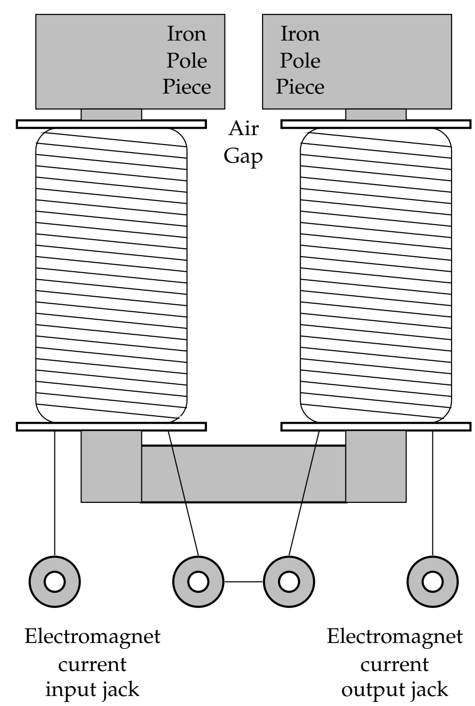

Figure 5.7: Diagram of the electromagnet.

for the conduction of current through the probe.

Error Analysis

A significant source of systematic error lies in the instability of the Hall voltage readings due to the heating of the Hall probe and the electromagnet. Other sources of error may be found in the non-uniformity of the magnetic fields produced in both the electromagnet and calibration magnet air gaps. Further errors may be introduced by non-uniform positioning of the probe in the calibration magnet gap or the tilting of the probe in the electromagnet gap while taking measurements. Such experimental inconsistencies would lead to a measurement of a component of the magnetic flux density instead of the entire flux density. Lastly, there is an error associated with the exact value of the magnetic flux density of the calibration magnet. The experimental uncertainties in I and B are determined from the uncertainties of the measuring instruments and can be readily estimated. The uncertainty in K can be determined from the uncertainties associated with the Hall voltage and the calibration magnetic flux density readings.

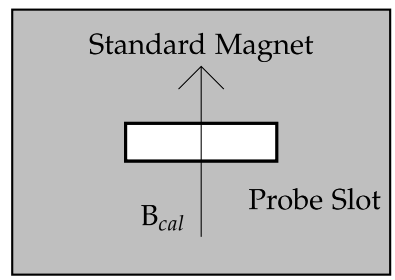

Figure 5.8: Standard magnet.

To be handed in to your lab instructor

Prelab

1. Provide clear and concise definitions for the following terms related to this experiment: transducer, Hall voltage, drift velocity, charge carrier density, hole, magnetic moment, hysteresis.

2. Provide several examples of physical processes that exhibit hysteresis.

3. A Hall probe of thickness 100 microns is being used at a current of 50 mA. The semiconductor in the probe is doped to a carrier density of 7.5×10^{20} m^{-3}. If the Hall voltage is measure to be 7 mV, what is the magnetic field strength?

Data Requirements

4. The magnitude of the Hall voltage for the standard magnet. From this, the numerical value of the proportionality constant, K, with error, can be determined using from Equation 5.5 (procedure step 2).

5. A data table of Hall voltage versus electromagnet current at a constant probe control current of 150 mA (procedure step 3).

6. A data table of Hall voltage versus control current (procedure step 4).
7. A sketch of your complete Hall probe experimental system as described in procedure step 5. What is the sign of the charge carriers in this Hall probe.

Calculations

8. Using the experimental value of K, calculate the numerical value of the drift velocity, v, with error.
9. Calculate the magnetic flux density for each electromagnet current at a constant probe control current of 150 mA from your results in data requirement 5.
10. Produce a graph of magnetic intensity versus electromagnetic current.
11. Produce a graph of the Hall voltage versus control current.
12. Calculate the numerical value of the Hall coefficient, R_H, with error.
13. Calculate the numerical value of the charge carrier density, n, with error, for Indium Arsenide.

Discussion

14. What does the graph of magnetic intensity versus electromagnetic current represent? Discuss the appearance of the plot you generate in Calculation part 11. Is this how you would expect an electromagnet to behave?
15. What is the remanence of the electromagnet?
16. Discuss the impact of magnetic hysteresis on the use of electromagnets.
17. What are the two main sources of heat responsible for warming the Hall probe?
18. What possible applications are there for the use of Hall probes and electromagnets.